UNIVERSITY OF TORONTO
3 1761 01179808 9

Digitized by the Internet Archive
in 2007 with funding from
Microsoft Corporation

http://www.archive.org/details/collectedmathema03hilluoft

20

THE COLLECTED
MATHEMATICAL WORKS
OF
GEORGE WILLIAM HILL

VOLUME THREE

Ἄστρων κάτοιδα νυκτέρων ὁμήγυριν.—Æschylus.

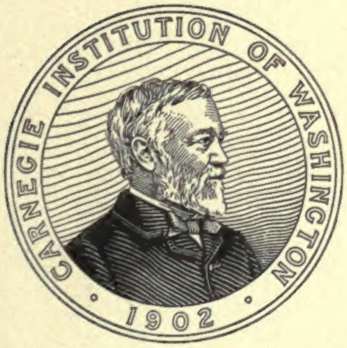
THE COLLECTED

MATHEMATICAL WORKS

OF

GEORGE WILLIAM HILL

VOLUME THREE



70922
26/7/06

Published by the Carnegie Institution of Washington
January, 1906

CARNEGIE INSTITUTION OF WASHINGTON

PUBLICATION NO. 9 (VOLUME THREE)

QB
3
H55
v.3

MEMOIR No. 50

A NEW THEORY OF JUPITER AND SATURN

(ASTRONOMICAL PAPERS OF THE AMERICAN EPHEMERIS, VOL. IV.)

CONTENTS.

INTRODUCTION.

Page.

History of previous attempts at the solution of this problem . . . 11
Guiding principles of the present investigation . . . 14

CHAPTER I. MUTUAL ACTION OF JUPITER AND SATURN. DEVELOPMENT OF THE RECIPROCAL OF THE DISTANCE BETWEEN THE PLANETS AND ITS ODD POWERS IN PERIODIC SERIES WHEN ELLIPTIC VALUES ARE SUBSTITUTED FOR THE CO-ORDINATES.

Elements of Jupiter and Saturn adopted . . . 19
Correction of the semi-axes for the constant terms of the radii vectores . . . 20
Relative position of the planes of the orbits . . . 21
Degree of accuracy required in the developments . . . 21
Value of certain constants employed in the developments . . . 22
Values of certain quantities for sixteen points of the circumference with respect to the mean anomaly of Saturn . 23
Separation of the distance into factors . . . 24
Values of the Laplace coefficients $b_s^{(i)}$ for the sixteen points of the circumference . . . 26
Formulæ and values belonging to the development of the second factor of the distance . . . 32
Values of certain coefficients for the first power of the reciprocal of the distance . . . 38
The similar coefficients for the third power . . . 40
The similar coefficients for the fifth power . . . 41
The similar coefficients for the seventh power . . . 43
Application of mechanical quadratures to these coefficients . . . 44
Derivation of coefficients beyond the limits by induction . . . 45
Expressions of the odd powers of the reciprocal of the distance in terms of the mean anomaly of Saturn and the eccentric of Jupiter . . . 46
Values of the Besselian functions corresponding to multiples of the eccentricities of Jupiter and Saturn . . . 51
Expressions of the odd powers of the reciprocal of the distance in terms of the mean anomalies of both planets . 53

CHAPTER II. PERTURBATIONS OF JUPITER AND SATURN ARISING FROM THEIR MUTUAL ACTION AND OF THE FIRST ORDER WITH RESPECT TO DISTURBING FORCES.

Values of four factors required in obtaining the forces . . . 59
Products of the cube of the reciprocal of the distance by these factors . . . 60
Developments of the reactions of Jupiter and Saturn on the Sun . . . 63
Expressions for the three functions denoting the action of Saturn on Jupiter . . . 66
The same relative to the action of Jupiter on Saturn . . . 69
Expressions for the multipliers A, B, and C for Jupiter and A′, B′, and C′ for Saturn . . . 72
Values of the constants η and θ . . . 74
Developments of the functions to be integrated for Jupiter . . . 74
Developments of the similar functions for Saturn . . . 83
Values of the integrating factors for Jupiter . . . 92
Values of the similar factors for Saturn . . . 93
Illustration of the method of forming $\overline{W}$ and $\overline{W'}$ . . . 94
Values assumed for the arbitrary constants . . . 94
Expressions for the differentials of δz and ν . . . 96
Expressions for the differentials of $\delta z'$ and ν' . . . 98
Formulæ for the secular portions of the six co-ordinates of Jupiter and Saturn . . . 101
Expressions for the perturbations of the co-ordinates of Jupiter . . . 103
Expressions for the perturbations of the co-ordinates of Saturn . . . 106

CHAPTER III. PERTURBATIONS OF SATURN BY URANUS OF THE FIRST ORDER WITH RESPECT TO THE DISTURBING FORCE.

Adopted elements of Uranus 109
Values of certain constants employed in the developments 109
Values of certain quantities for twelve points of the circumference with respect to the mean anomaly of Uranus . . 110
Values of the Laplace coefficients $b_s^{(i)}$ 112
Values of the A for the first power of the reciprocal of the distance 114
The similar coefficients for the third power 116
The similar coefficients for the fifth power 117
Application of mechanical quadratures to these coefficients 118
Expressions of the odd powers of the reciprocal of the distance in terms of the mean anomaly of Uranus and the eccentric of Saturn 119
The same in terms of the mean anomalies of both planets 122
Values of Besselian functions corresponding to the eccentricity of Uranus 124
Products of the cube of the reciprocal of the distance by three factors 125
Expressions for the reaction of Uranus on the Sun 127
Expressions for the three functions denoting the action of Uranus on Saturn 128
Expression for T 130
Expression for the differential of R 134
Values of the integrating factors 135
Expression for the differentials of δz and ν 136
Expressions for the perturbations of the co-ordinates of Saturn 138

CHAPTER IV. PERTURBATIONS OF JUPITER BY URANUS OF THE FIRST ORDER WITH RESPECT TO DISTURBING FORCES.

Values of certain constants employed in the developments 140
Values of certain quantities for twelve points of the circumference in reference to the mean anomaly of Uranus . . 141
Values of the a 142
Values of the A 144
Expressions for the odd powers of the reciprocal of the distance in terms of the mean anomaly of Uranus and eccentric of Jupiter 147
The same in terms of the mean anomalies of both planets 148
Values of Besselian functions corresponding to the eccentricity of Uranus 150
Products of the cube of the reciprocal of the distance by four factors 151
Expression for the reaction of Uranus on the Sun 153
Expressions for the three functions denoting the action of Uranus on Jupiter 154
Expression for T and the differential of R 156
Values of the integrating factors 158
Expressions for the differentials of δz and ν 158
Expressions for the perturbations of the co-ordinates of Jupiter 160

CHAPTER V. PERTURBATIONS OF SATURN BY NEPTUNE.

Elements of Neptune and resulting constants 161
Values of certain quantities for twelve points of the circumference with reference to the mean anomaly of Neptune . 162
Values of the a 163
Values of the A for two powers of the reciprocal of the distance 165
Expressions of the odd powers of the distance in terms of the mean anomaly of Neptune and the eccentric of Saturn . 168
The same in terms of the mean anomalies of both planets 169
Products of the cube of the reciprocal of the distance by two factors 171
Expressions for the reaction of Neptune on the Sun 172
Expressions for the three functions which denote the action of Neptune on Saturn 173
Expressions for T and the differential of R 175
Values of the integrating factors 177
Expressions for the differentials of δz and ν 178
Expressions for the perturbations of the co-ordinates of Saturn 179

CHAPTER VI. PERTURBATIONS OF JUPITER BY NEPTUNE.

Values of constants employed in the developments 180
Values of quantities for eight points of the circumference in reference to the mean anomaly of Neptune 181
Values of the a and of the A 181
Application of mechanical quadratures 183
Expressions for the odd powers of the reciprocal of the distance in terms of the mean anomaly of Neptune and eccentric of Jupiter 183

CHAPTER VI. PERTURBATIONS OF JUPITER BY NEPTUNE—Continued.

The same in terms of the mean anomalies of both planets 184
Values of Besselian functions corresponding to the eccentricity of Neptune 185
Products of the cube of the reciprocal of the distance by two factors 186
Expression of the reaction of Neptune on the Sun 187
Expressions of the three functions denoting the action of Neptune on Jupiter 187
Expressions for T and the differential of R 189
Values of the integrating factors 190
Expressions for the differentials of δz and ν 190
Expressions for the perturbations of the co-ordinates of Jupiter 191

CHAPTER VII. PERTURBATIONS OF JUPITER AND SATURN BY THE FOUR INTERIOR PLANETS.

Adopted values of the elements 192
Action of Mercury on Jupiter 192
Action of Venus on Jupiter 193
Action of the Earth on Jupiter 194
Action of Mars on Jupiter 195
Action of Mercury on Saturn 196
Action of Venus on Saturn 196
Action of the Earth on Saturn 197
Action of Mars on Saturn 198

CHAPTER VIII. PERTURBATIONS OF THE SECOND ORDER WITH RESPECT TO DISTURBING FORCES IN THE LONGITUDES AND RADII VECTORES ARISING FROM THE MUTUAL ACTION OF JUPITER AND SATURN. DERIVATION OF THE FACTORS OF δT AND δT′.

The eight terms of δT and δT′ 199
Formulæ for the eight factors 199
Products of the fifth power of the reciprocal of the distance by four factors 202
Expressions for the reactionary terms 204
Expressions for four derivatives of the perturbative function for Jupiter 205
The similar quantities for Saturn 208
The two multipliers M and N, and their values for Jupiter and Saturn 211
Expressions for the factors V, X, B, and G 212
Expression for $\overline{\mathrm{T}}$ 220
Expressions for the factors C, D, E, and H 221
Expressions for the factors V′, X′, B′, and G′ 228
Expression for $\overline{\mathrm{T}'}$ 236
Expressions for the factors C′, D′, E′, and H′ 237
Expressions for two of the secondary factors of δT 244
Expressions for two of the secondary factors of δT′ 246

CHAPTER IX. CALCULATION OF THE TERMS OF δT AND δT′ WHOSE ARGUMENTS ARE γ AND γ'.

Reasons for including portions of the third-order terms in those of the second order 249
Constituent parts of the coefficients of the terms here discussed 250
Explanation of the slowness of convergence met with here 251
Division of the terms in reference to their mass-multipliers 252
The secular terms in sum for Jupiter and Saturn 253

CHAPTER X. CALCULATION OF THE PORTION OF δT NOT FACTORED BY nt.

Value of limits for neglecting terms 255
Expressions for the eight separate terms of δT 255

CHAPTER XI. CALCULATION OF THE PORTION OF δT FACTORED BY nt.

Degree of precision adopted 262
Separate expressions for the eight terms of δT 262

CHAPTER XII. SECOND-ORDER PERTURBATIONS OF THE MEAN ANOMALY AND RADIUS-VECTOR OF JUPITER ARISING FROM THE MUTUAL ACTION OF THIS PLANET AND SATURN.

Sum of the eight portions of δT 269
Special treatment of long-period terms in integration 274
Expressions for $\overline{\delta \mathrm{W}_0}$ and its γ derivative 276
Expressions for two supplementary quantities needed to complete the differential of $n\delta^2 z$ 281

CHAPTER XII. SECOND-ORDER PERTURBATIONS, ETC.—Continued.

Expression for the supplementary quantity of the differential of δv 284
The great inequality as modified by second-order terms 288
The modification of the constant term of v by second-order terms 289
Expressions for $n\delta^2 z$ and v 290

CHAPTER XIII. CALCULATION OF THE PORTION OF $\delta T'$ NOT FACTORED BY $n't$.

Limits for neglecting terms 295
Separate expressions for the eight terms of $\delta T'$ 295

CHAPTER XIV. CALCULATION OF THE PORTION OF $\delta T'$ FACTORED BY $n't$.

Degree of precision employed 304
Separate expressions for the eight terms of $\delta T'$ 304

CHAPTER XV. SECOND-ORDER PERTURBATIONS OF THE MEAN ANOMALY AND RADIUS-VECTOR OF SATURN, ARISING FROM THE MUTUAL ACTION OF THIS PLANET AND JUPITER.

Sum of the eight portions of $\delta T'$ 313
Special treatment of long-period terms in integration 319
Expressions for $\overline{\delta W_0'}$ and its γ' derivative 320
Expressions for the two supplementary quantities completing the differential of $n'\delta^2 z'$ 325
Expression for the supplementary quantities completing the differential of $\delta v'$ 329
The modification of the great inequality by second-order terms 334
The modification of the constant term of v' by second-order terms 334
The modification of the long-period inequality in v' 334
Expressions for $n'\delta^2 z'$ and v' 335

CHAPTER XVI. PERTURBATIONS OF THE THIRD ORDER WITH RESPECT TO DISTURBING FORCES IN THE LONGITUDES AND RADII VECTORES ARISING FROM THE MUTUAL ACTION OF JUPITER AND SATURN. DETERMINATION OF THE FACTORS OF $\delta^3 T$ AND $\delta^3 T'$.

The terms retained of $\delta^3 T$ and $\delta^3 T'$ 340
Formulæ for their factors 340
Products of the fifth and seventh powers of the reciprocal of the distance by certain factors 343
Expressions for the third derivatives of the perturbative functions with reference to the radii vectores 345
Expressions for Y and Y′ 347
Developments of three factors of the terms of $\delta^3 T$ 351
The similar factors of $\delta^3 T'$ 356
Portions of the secondary factors of $\delta^3 T$ and $\delta^3 T'$ independent of the multipliers nt and $n't$ 361
Derivation of the terms of $\delta^3 T$ and $\delta^3 T'$ having the arguments γ and γ' 365
Portions of the secondary factors of $\delta^3 T$ and $\delta^3 T'$ multiplied by nt or $n't$ 367
Portions of the secondary factors of $\delta^3 T$ and $\delta^3 T'$ multiplied by n^2t^2 or n'^2t^2 370

CHAPTER XVII. CALCULATION OF THE SEVERAL PORTIONS OF $\delta^3 T$.

Expressions for the fourteen parts independent of nt 373
Expressions for the fourteen parts multiplied by nt 381
Expressions for the fourteen parts multiplied by n^2t^2 386

CHAPTER XVIII. THIRD-ORDER PERTURBATIONS OF THE MEAN ANOMALY AND RADIUS-VECTOR OF JUPITER ARISING FROM THE MUTUAL ACTION OF THIS PLANET AND SATURN.

Sum of the fourteen portions of $\delta^3 T$ 389
Special treatment in integration of the long-period terms 393
The expression for $\overline{\delta^3 W_0}$ 394
Expression for the γ derivative of the preceding 396
Supplementary quantities completing the value of the differential of $n\delta^3 z$ 397
Expression for the differential of $\delta^3 z$ 401
Modification of the great inequality by third-order terms 403
Expression for $n\delta^3 z$ 404
Expression for $\delta^3 v$ 406

CHAPTER XIX. CALCULATION OF THE SEVERAL PORTIONS OF $\delta^3 T'$.

Expressions for the fourteen parts independent of $n't$ 408
Expressions for the fourteen parts multiplied by $n't$ 417
Expressions for the fourteen parts multiplied by n'^2t^2 426

CHAPTER XX. THIRD-ORDER PERTURBATIONS OF THE MEAN ANOMALY AND RADIUS-VECTOR OF SATURN ARISING FROM THE MUTUAL ACTION OF THIS PLANET AND JUPITER.

Sum of the fourteen portions of $\delta^3 T'$ 431
Special treatment in integration of long-period terms 436
Expression for $\overline{\delta^3 W_0'}$ 437
Expression for the γ' derivative of the preceding 439
Supplementary quantities completing the value of the differential of $n'\delta^3 z'$ 441
Expression for the differential of $\delta^3 z'$ 447
Modification of the great inequality by third-order terms 449
Expression for $n'\delta^3 z'$ 450
Supplementary quantities completing the value of the differential of $\delta^3 v'$ 452
Special treatment of the long-period term in $\delta^3 v'$ 454
Expression for $\delta^3 v'$ 455

CHAPTER XXI. PERTURBATIONS OF SATURN OF THE SECOND ORDER FROM THE ACTION OF URANUS AND FACTORED BY $n't$.

The terms of $\delta T'$ 457
Their secondary factors 457
Formulæ for their primary factors 458
Expression for the second derivative of the perturbative function with respect to the radius 459
Expression for V′ 459
Expression for X′ 461
Expressions for the derivatives of T′ with respect to the radii 463
Expression for C′ 467
Expression for $\delta T'$ 468
Expressions for $\overline{\delta W_0'}$ and its γ' derivative 470
Supplementary quantities completing the values of the differentials of $\delta^2 z'$ and $\delta v'$ 471
Expressions for the differentials of $\delta^2 z'$ and $\delta v'$ 472
Expressions for $n'\delta^2 z'$ and $\delta v'$ 474
Long-period term in $n'\delta^2 z'$ not multiplied by $n't$ 475

CHAPTER XXII. PERTURBATIONS OF JUPITER PROPORTIONAL TO THE PRODUCT OF THE MASSES OF SATURN AND URANUS.

The several terms of δT 476
Derivation of certain of their primary factors 476
Combinations in the products which give sensible results 477
Mode of taking account of third-order terms 478
Expression for δT 479
Expressions for $n\delta z$ and ν 479

CHAPTER XXIII. PERTURBATIONS OF SATURN PROPORTIONAL TO THE PRODUCT OF THE MASSES OF JUPITER AND URANUS.

The several terms of $\delta T'$ 480
Expressions for certain of their factors 480
Quantities required in considering certain third-order terms 481
Separate terms of, and complete value of $\delta T'$ 482
Expressions for $\overline{\delta W_0'}$ and its γ' derivative 484
Supplementary quantities completing the value of the differential of $\delta^2 z'$ 484
Expressions for $n'\delta^2 z'$ and $\delta v'$ 485
Additional long-period terms in $n'\delta^2 z'$ of minor importance 486

CHAPTER XXIV. PERTURBATIONS OF THE LATITUDE OF JUPITER OF THE SECOND ORDER WITH RESPECT TO DISTURBING FORCES.

Terms of δU and formulæ for their primary factors 487
Expressions for the primary factors 487
Separate portions of δU having the argument γ 492
Expression for δU 492
Integral of δU 493
Supplementary quantity completing the value of δu 494
Expression for δu 495
Secular terms of the third order in the latitude 496

CHAPTER XXV. PERTURBATIONS OF THE LATITUDE OF SATURN OF THE SECOND ORDER WITH RESPECT TO DISTURBING FORCES.

Terms of $\delta U'$ and formulæ for their primary factors . . . 498
Expressions for the primary factors . . . 500
Separate portions of $\delta U'$ having the argument γ' . . . 503
Expression for $\delta U'$ . . . 503
Integral of $\delta U'$ . . . 504
Supplementary quantity completing the value of $\delta u'$ . . . 505
Expression for $\delta u'$ . . . 506
Secular terms of the third order in the latitude . . . 508

CHAPTER XXVI. FORMULÆ FOR THE MOTION OF THE PLANE OF THE ECLIPTIC AND FOR PRECESSION.

Values of certain planetary elements for 1600, 1850, and 2100 . . . 509
Separate actions of the planets in producing the motion of the ecliptic . . . 510
Formulæ of this motion . . . 512
Differential equations of precession . . . 512
Their integrals obtained by mechanical quadratures . . . 517

CHAPTER XXVII. REFERENCE OF THE LONGITUDES AND LATITUDES OF JUPITER AND SATURN TO THE MEAN EQUINOX AND ECLIPTIC OF DATE.

Reduction to the primitive plane of the orbit . . . 518
Formulæ in the case of Jupiter and Saturn . . . 519
Formulæ for referring longitude and latitude to the ecliptic of date . . . 520
Modification of the fundamental argument z so as to take into account the inequalities of the reduction and precession . . . 523
Application of the formulæ to Jupiter . . . 524
Parts of the sine of Jupiters's latitude resulting severally from the motion of the plane of its orbit and from that of the ecliptic . . . 526
Expression for $\varDelta\beta$ . . . 526
Reduction of the longitude to the ecliptic and mean equinox of date . . . 527
Expression for the correction to be applied to the fundamental argument . . . 528
The corresponding corrections of the logarithm of the radius-vector and of the sine of the latitude . . . 529
Application of the formulæ to Saturn . . . 530
Parts of the sine of Saturn's latitude resulting severally from the motion of the plane of its orbit and from that of the ecliptic . . . 531
Expression for $\varDelta\beta'$ . . . 531
Reduction of the longitude to the ecliptic and mean equinox of date . . . 533
Expression for the correction to be applied to the fundamental argument . . . 534
Corresponding corrections of the logarithm of the radius-vector and of the latitude . . . 535

CHAPTER XXVIII. PRELIMINARY COMPARISON OF THE PRECEDING THEORY WITH OBSERVATION, AND DERIVATION OF APPROXIMATE CORRECTIONS FOR THE ELEMENTS EMPLOYED IN THE CALCULATION OF THE PERTURBATIONS.

Normals in heliocentric longitude for Saturn . . . 537
Details of the positions calculated for comparison . . . 538
Equations of conditions in three different suppositions . . . 539
Resulting corrections of the elements . . . 540
Residuals . . . 540
Normals in heliocentric longitude for Jupiter . . . 541
Details of the positions calculated for comparison . . . 541
Equations of condition in three different suppositions . . . 543
Resulting corrections of the elements . . . 543
Residuals . . . 543

CHAPTER XXIX. RECTIFICATION OF THE FORMULÆ FOR THE PERTURBATIONS ON ACCOUNT OF THE CORRECTIONS OF THE ELEMENTS JUST DETERMINED.

Formulæ employed for this purpose . . . 545
Values of the secondary factors . . . 550
Application to Jupiter . . . 551
Application to Saturn . . . 552

CHAPTER XXX. ADDITION OF THE SEVERAL PORTIONS OF THE EXPRESSIONS FOR THE CO-ORDINATES OF JUPITER AND SATURN AND REDUCTION OF THEM TO THEIR FINAL FORM.

Values of planetary masses finally adopted 554
Corrections to certain terms in the co-ordinates for changes in these values 554
Corrections to pass from ν and ν' to log $(1+\nu)$ and log $(1+\nu')$ 555
Corrected elements of the two planets 558
Logarithms of their semi-axes 558
Values of the constituents of the arguments 559
Inequalities of the fundamental argument of Jupiter 559
Inequalities of the logarithm of the radius-vector of Jupiter 563
Periodic inequalities of the latitude of Jupiter 566
Formulæ for the principal terms of the co-ordinates of Jupiter 566
Inequalities of the fundamental argument of Saturn 567
Inequalities of the logarithm of the radius-vector of Saturn 571
Periodic inequalities of the latitude of Saturn 574
Formulæ for the principal terms of the co-ordinates of Saturn 575
ADDENDA 577

INTRODUCTION.

Jupiter and Saturn must have presented to the earliest observers of the celestial motions less difficulties than the interior planets. The first things noted, undoubtedly, were, that the first made a circuit of the heavens in about twelve and the second in about thirty years. Then the retrograde motion, at the time of opposition, and its extent would be perceived. The slowness and steadiness of the motion would naturally suggest the hypothesis of circular motion, but it was certainly reserved for a later and more philosophic age to explain the later-observed phenomenon by means of an epicycle.

The earliest tables of the motions of Jupiter and Saturn, as well as those of the other large planets which have come down to us, are those contained in the *Syntaxis* of Claudius Ptolemy. The annual parallax is there taken into account by one epicycle and the proper eccentricity of the orbit by a second. This, in the main, is the character of all the tables of the planets until the publication of Kepler's *Tabulæ Rudolphinæ* in 1627, where, for the first time, the equation of the center is derived from an elliptic formula, and we pass from heliocentric to geocentric positions in the modern way. From Kepler onwards the fact of the deviation of Jupiter and Saturn from a purely elliptic theory was recognized. Many attempts were made to better the theory; but it was found that no observations, embracing a long period of time, could be satisfied by elliptic elements varying proportionally to the time. Halley seems to have been the most successful in his tables; he adopted terms in the mean longitudes varying as the square of the time.

It was not until 1748 that any computation of the perturbations of Jupiter and Saturn, in accordance with the theory of gravitation, was undertaken. This was by Euler. He appears to have limited himself to the terms which have the mean elongation of the planets from each other as their argument. Later the terms factored by the simple power of the eccentricities were added by himself, Lalande, Lagrange, Bailly, and Lambert. But these terms not bringing about a reconciliation between observation and theory, Lagrange and Laplace were led to make their notable researches on the possibility of secular equations in the mean motions of the planets. At length the whole difficulty with Jupiter and Saturn was removed by Laplace's discovery of the great inequalities in 1786.

Delambre almost immediately constructed tables for these planets which far exceeded in accuracy any previously possessed. They are those which appear in the third edition of Lalande's *Astronomie.* This great success seems to have stirred up

LAPLACE and his colaborers to pushing the approximations still further. On the publication of the third volume of the *Mécanique Céleste*, terms of the fifth order with respect to the eccentricities and mutual inclination, as well as some of two dimensions with respect to disturbing forces, had been added to the coefficients of the great inequalities. That these advances might be utilized BOUVARD constructed tables of the planets founded on observed oppositions from 1747 to 1803. The formulæ used are very nearly those given in the *Mécanique Céleste*, Tom. III. These tables were published by the Bureau des Longitudes in 1808. It was discovered, however, that the terms of the fifth order, mentioned above, had been taken with the wrong sign. This led BOUVARD to prepare a new edition of his tables, which appeared in 1821, and in which this error was rectified, and the observations employed in the discussion extended to 1814.* Although BOUVARD himself speaks in admiration of the small residuals shown by the comparison of his theory with the observations, yet a glance shows their tendency to a systematic character, and this, too, with observations rather rudely reduced.

PLANA undertook, shortly after, to compute the portions of the great inequalities which arise from considering the square of the disturbing force.† The results he obtained failed to satisfy an equation of condition which LAPLACE had employed in his investigation. After some discussion LAPLACE abandoned his equation and substituted for it another, which PLANA's results were as far from satisfying as before. PONTÉCOULANT then, taking up the subject, discovered that LAPLACE's results had been taken with the wrong sign, and that PLANA had made errors of some importance in his investigation. When these oversights had been corrected the different results were brought into tolerable agreement.

However, the failure of BOUVARD's tables to better represent observations, and his getting for the mass of Jupiter a value so much smaller than was shortly after obtained from the action of this planet on the asteroids and on its own satellites, can not be explained by this error of sign. It is somewhat singular that no one has yet pointed out the real cause, which, it seems, must be either some error in the coefficients of his formulæ or some error in putting his equations in tables.

Neither LAPLACE's, PLANA's, nor PONTÉCOULANT's determination of these second-order terms can be regarded as anything else than a very rude and inadequate approximation.

HANSEN had, a short time previous, imagined a new method of treating perturbations. In the *Mécanique Céleste* LAPLACE had determined all long-period inequalities as if they were to be applied to the mean longitude, and had so directed they should, while the short-period ones were derived as if they were to be added to the true longitude. There is, therefore, a want of congruity, and even of rigor, in this way of proceeding. For LAPLACE has nowhere shown how these two modes of application can be employed in unison. It is plain there would be as many methods of perturbations as there were opinions as to the dividing line separating long from short-period

---

*Comparisons extending to 1819 are given in the preface to the tables; but it appears those of the last five years were added after the discussion was completed. See, on this point, LAPLACE, *Théorie Analytique des Probabilités*, Supplement I, p. 21.

†Memoirs Roy. Ast. Soc., Vol. II.

inequalities. These imperfections no doubt attracted the attention of HANSEN, whose thought must have been: Since it is advantageous to apply the long-period terms to the mean longitude, and indifferent whether the short-period ones are applied to the mean or true, why not apply all to the mean, and, moreover, compute the radius-vector and latitude with this equated quantity? Then the additional quantities necessary to complete the values of the latter co-ordinates would be, for the first, a function of three variations of the elements, and for the second, a function of two only. This, undoubtedly, was the origin of HANSEN'S new method.

He determined to apply it to Jupiter and Saturn, and his memoir, crowned by the Berlin Academy, must be regarded as the earliest example of an adequate treatment of perturbations of the second order with respect to disturbing forces. In all previous investigations it is impossible to form a conception of the probable magnitude of the terms passed over on account of the habit of the investigator of selecting here and there a term to be computed. But in HANSEN the continuity in the computed terms enables one to form a fair judgment as to the importance of those neglected. However, Saturn alone is treated with a fair degree of completeness. The expressions for Jupiter are limited to the terms arising from the first power of the disturbing force. Had this theory of Saturn been completed by the addition of the terms due to the action of Uranus and the whole compared with the observations more carefully reduced, as they then could have been by the aid of BESSEL'S *Tabulæ Regiomontanæ*, very excellent tables would have been obtained. But HANSEN seems to have been carried away with the ambition of applying his peculiar method of treatment to the lunar theory.

A long period of over forty years now elapsed without anything being contributed to the theories of Jupiter and Saturn, for the expressions of the perturbations given in PONTÉCOULANT'S *Théorie Analytique du Système du Monde*, beyond the correction of the error of sign in the second-order terms of the great inequalities, do not seem to be in anything more perfect than those found in the *Mécanique Céleste.*

In 1868 Mr. HUGH BREEN published a memoir containing equations of condition for the whole series of observations of the two planets made at Greenwich from 1750 to 1865, and, dividing them into four groups, obtained solutions for each. But as the equations contained no terms for the corrections of the acting masses, and no investigation was made of the errors of the formulæ actually employed in the construction of BOUVARD'S tables, the corrections obtained, when applied to BOUVARD'S elements, do not give anything like a fair approximation to the actual elements.

HANSEN, in 1875, published a memoir on Jupiter. But here, deserting his earlier notions on the lack of convergence in algebraical developments, he confines himself to calculating the easier terms of the co-ordinates. Hence this memoir can not be regarded as advancing much our knowledge of the subject.

In the years 1874 to 1876 appeared LEVERRIER'S investigation, concluding with the tables which are at present employed for all the European ephemerides. The method followed is that of attributing the perturbations to the six elements of the Keplerian ellipse; and, contrary to the mode followed in his earlier planetary theories, these are also the quantities tabulated. LEVERRIER'S labor is very much increased by his undertaking to exhibit, in the first instance, all his developments in a form where all the elements, save the mean distances, appear as indeterminates. This he does on the plea that, however far in the future the observations may be prolonged, they ought to be represented by one and the same theory. This notion must be approved by all, but it must be pointed out that it is not completely attained by substituting, in a set of formulæ belonging to an old epoch, the values the varying elements have arrived at, at the new epoch. It is true the two sets of formulæ may be in perfect consonance, but they can not be considered as one and the same. At best, this is but a distant imitation of the method of obtaining integrals by mechanical quadratures. If we wish to have theories good for all time we shall find ourselves driven to making the coefficients of our periodic terms perfectly constant, and to admitting about three elementary constituents into the arguments for every planet that acts. LEVERRIER's hope that his work would serve as a foundation for future investigations is not warranted by past experience. For if at any time it is found not adequate to present wants the suspicion is sure to arise that this is due, in part at least, to not carefully enough performed calculations. Besides, we can not expect that work of this difficult nature will ever be undertaken except by well-trained experts, who will feel that they ought to be permitted to choose methods satisfactory to themselves. Thus every investigator of the planetary theories sets out *ab origine.*

In consequence of this adoption of indeterminate elements in the formulæ LEVERRIER's values of the coefficients are less precise than if the latter had been treated as wholes. In the case of Saturn, however, he made an additional development by mechanical quadratures. There is not a very close agreement between the results of this and the algebraically derived formulæ. The difference, for instance, in the case of the coefficient of the great inequality exceeds 40″. With the exception of the terms constituting the great inequality and a term denoting a secular acceleration of the mean longitude LEVERRIER employed, in the construction of his tables, formulæ resulting from this process of mechanical quadratures. The way in which the observations of Saturn were represented by this theory was not satisfactory, the residuals being larger at times than could be accepted as errors of the observers; and the comparisons made with observation since the publication of the tables have shown residuals somewhat larger. With Jupiter LEVERRIER was more successful, but his discussion led him to assign to Saturn a mass which astronomers at present regard as too small. However, I have not been able to discover any oversight in LEVERRIER's theories which would account for these discrepancies. A few trifling errors, having plainly no effect on the representation of observation, were all that were found.

The desirableness of a new investigation of the subject has been generally admitted, and fault has been found with the amount of labor required to deduce positions of the planets from LEVERRIER's tables. But I had not these inducements to take up the subject when I began work, for these tables were then unpublished. The long interval which occurred between the publication of LEVERRIER's theory of Mars and the appearance of anything from him on Jupiter and Saturn was the occasion of leading me to consider the undertaking. On making known to the Superintendent of the American Ephemeris my desire to take up the problem I was relieved from all other routine work, and supplied with the assistance necessary to duplicate all my computations which required this safeguard against error. It was desired to abandon the use of the antiquated tables of **Bouvard**, and it appeared uncertain when **Leverrier** would publish his.

The plan, therefore, was to form theories of Jupiter and Saturn which would be practically serviceable for a space of three hundred years on each side of a central epoch taken near the center of gravity of all the times of observation; theories whose errors in this interval would simply result not from neglected terms in the developments, but from the unavoidable imperfections in the values of the arbitrary constants and masses adopted from the indications of observation.

Such were the considerations which influenced the adoption of the course to be followed. As there was no desire to lose time by forming a special method of treatment for the problem in hand it was decided to employ the method of **Hansen**, with such slight modifications as the exigencies of the case might suggest. On account of the presence of the great inequalities this method seemed to me to give expressions best suited to tabulation. The latest form of this method appears in **Hansen**'s memoir entitled *Auseinandersetzung, etc.* The employment of the eccentric anomaly of the planet whose co-ordinates are sought as the independent variable undoubtedly augments the convergence of the series; but the adoption of this mode of proceeding would bring about the use of two independent variables, one for the co-ordinates of Jupiter, another for those of Saturn. As the developments have to be pushed to terms of three dimensions with respect to disturbing forces the heaviest part of the labor consists in forming products of periodic series, one of which belongs to Jupiter, the other to Saturn; and as integration can not be performed unless these products are transformed so as to involve but one variable we should have an endless series of transformations to make. It therefore seems a necessity to have a single independent variable for the whole work. In consequence the final form adopted for all the periodic series is in terms of the mean anomalies, so that the time is always the independent variable. Fortunately very slight and readily perceived changes only are necessary in the formulæ of the *Auseinandersetzung* to render them applicable to the modified mode of proceeding.

Hansen's method makes two transformations of the series representing the odd powers of the distance between the acting planets in which Besselian functions are employed as multipliers; and he has thus no less than three different forms for these series, the first being that in which the eccentric anomalies of both planets appear, the second that in which one of the eccentric anomalies is replaced by a mean anomaly, and the third in which the expression is so transformed that it may be integrated by treating the first eccentric anomaly as the independent variable. One of these transformations and forms for the series can be avoided. By making the division of the circumference with reference to the mean anomaly instead of the eccentric, and computing all the auxilliary quantities to be employed so that they correspond to the points of this division, we obtain at once, and without any need of a transformation, the series in **Hansen**'s second form. The additional labor required to make the auxilliary quantities correspond to given values of the mean anomaly instead of the eccentric is very trifling. Consequently I have adopted this way of proceeding.

On arriving at the treatment of terms of the second and third orders with respect to disturbing forces, another, and as it seems to me quite advantageous, modification was made in HANSEN'S method, by which the more important terms of the third order were included in the computation of those of the second order; and again, on making the computation of the third-order terms the more important terms of the fourth order were included. Past experience has shown that in the second-order terms, and presumably in all higher orders, the secular terms of lower orders cause larger modifications than the periodic terms. Thus the great inequalities, in their modification by the consideration of second-order terms, are affected about six times more by the secular than the periodic terms. Again, it is known that the second-order terms contribute very important portions to the secular terms. Hence, if on commencing the calculation of the second-order terms one could attribute to the secular terms, not their first-order values, but these augmented by an approximate estimate of the corrections which in the obvious course of proceeding would be found for them, it is evident that the third-order terms, to be computed afterwards, would be much reduced in magnitude. And in computing the third-order terms a like method of treatment could be used with like benefit, and thus the fourth-order terms be rendered more easily negligible. This way of proceeding, of course, requires that the computation of every succeeding order of terms should be modified; but this is not difficult.

Thus the method I have followed is this: On arriving at the treatment of second-order terms ascertain first the two terms of δT, which are rigorously proportional to $\sin \gamma$ and $\cos \gamma$, and apply the corresponding secular terms to the secular terms of the first order, and with these terms so corrected proceed to the general determination of δT; and in third-order terms proceed in like manner. The terms of δT having the argument γ constitute a very fair approximation to the second-order portion of the secular terms, as the supplementary quantities added after the integration of T contribute in comparison quite insignificant corrections.

For the sake of brevity in the tables I have ventured, at the end, to make an alteration in the signification of the fundamental argument. In the *Auseinandersetzung* HANSEN defines this so that, on entering a table, constructed from a purely elliptic theory, we obtain the exact angle described by the radius-vector between any given times. To flatten this out into a plane he is obliged to compute the expression of a fourth co-ordinate, Γ. HANSEN'S only apparent reason for adopting such a signification for his z is that the differential equation determining it is thereby slightly simplified. But astronomers care nothing for the description of angle by the radius; all they wish to know is whereabouts is the planet. In the *Tables du Soleil* HANSEN has modified his z in such a way that it gives directly, by the employment of a purely elliptic table, the apparent tropical longitude of the Sun. But by making his z thus include the effect of precession he was led into a difficulty. By stopping with terms multiplied by the square of the time his tables, at remote epochs, failed to give sufficiently approximate longitudes of the Sun. Hence he published a supplement containing the terms involving the next two higher powers of the time. Should we adopt a similar mode of treating Jupiter and Saturn our difficulties would be greater, as the eccentricities of these planets are three or four times greater than that of the

Earth. However all embarassment is escaped if we agree to apply the principal term of precession, viz, that which is proportional to the time, outside of the table in which the argument is z.

Therefore I have equated z in such a way that it takes account of all the inequalities of the reduction to the ecliptic, secular and periodic, and also of all that part of precession which involves the square and higher powers of the time. This modification of z in no respect complicates its expression, which has the same form as before, only with numerical coefficients changed by amounts generally very small. In this way we escape the necessity of giving separate tables for the reduction to the ecliptic and for its periodic and secular perturbations. This method of treatment resembles that of PLANA and DELAUNAY in the lunar theory, for they pay no attention to the orbit longitude of the Moon, but proceed immediately to the ecliptic longitude.

As the formulæ which have been given for the motion of the plane of the ecliptic and for precession appear to me to be lacking in the precision sufficient for these late times, on account of their being limited to terms involving the time and its square, I have devoted a chapter to this subject, in which the expressions are prolonged so as to include the cubes of the time. These expressions are not, however, the definitive ones to be employed in the construction of the tables. The determination of the motions of the equator and ecliptic belongs to the theory of the four inner planets, and when this theory is completed the necessary modifications in the final formulæ of the present work can readily be made.

I have everywhere employed the notation of HANSEN without explanation, as it must now be very familiar to all who work in the planetary theories. Also, no demonstrations of the formulæ used are given whenever it is possible to make a reference to places where they may be found, generally the memoirs of HANSEN. But in all important cases the formulæ actually used are set down in the interest of the detection of any errors; and it seems to me that it would be possible for one to repeat all the computations of this investigation without hindrance from obscurity in the explanations.

The arrangement of the work will be easily seen from a glance at the table of contents. The terms arising from the first power of the disturbing force are considered by themselves. This occupies Chapters I to VII. The second-order terms in the fundamental arguments and the radii vectores, due to the interaction of Jupiter and Saturn, follow. This investigation is comprised in Chapters VIII to XV. The similar terms of the third order are next treated, and fill up Chapters XVI to XX. The perturbations of the two planets which are of the second order and have the mass of Uranus as factor follow. They are contained in Chapters XXI to XXIII. The second-order terms of the latitudes are then derived in Chapters XXIV and XXV. The motions of the ecliptic and precession are treated in Chapter XXVI. The longitudes and latitudes are referred to the mean equinox and ecliptic in Chapter XXVII. Corrections for the provisionally adopted elements are, in the next place, found by a comparison of the previously obtained theory with observation. This fills Chapter XXVIII. The formulæ for the perturbations are rectified on account of these corrections in Chapter XXIX. In fine, the summed expressions for the co-ordinates of Jupiter and Saturn are given in Chapter XXX.

In the long period of seven and a half years in which the computations of this investigation were carried on some modifications of the ideas entertained as to the proper values to be attributed to the planetary masses could scarcely fail to occur, and one will notice some incongruity in this respect; but the final results, given in Chapter XXX, are reduced to perfect uniformity in this respect. The values to which they correspond are those stated at the beginning of Chapter XXVI.

A careful comparison of the expressions obtained for the co-ordinates by LEVERRIER with those obtained in this investigation would undoubtedly have much interest, and might lead to the detection of the causes of disagreement. But the very great labor involved in carrying it out must be my excuse for not undertaking it. In the case of Saturn, too, whether LEVERRIER's elaboration by the algebraical process or that by mechanical quadratures should be taken for comparison would be an embarrassing question. The latter is very incomplete, yet LEVERRIER very nearly, as far as he could, used it for his tables. In the circumstances I must limit myself to pointing out, in a rude way, the more striking discrepancies. I have made a reduction of LEVERRIER's algebraically-determined coefficient of the great inequality of Saturn to HANSEN's form of the perturbations and to BESSEL's value of the mass of Jupiter, and it comes out 2944″.80. The value assigned in Chapter XXX is 2907″.85. LEVERRIER's is therefore the larger by 36″.95. Had we taken for reduction the value given by LEVERRIER's other process the difference would have been smaller. The motions assigned to the eccentricity and perihelion of Saturn in LEVERRIER's tables are considerably greater than those given by my theory. In the case of Jupiter LEVERRIER's values of the coefficients of the large terms are quite as large as mine, although they profess to correspond to the value $\frac{1}{3529.6}$ of the mass of Saturn, while mine have been computed with the mass $\frac{1}{3501.6}$. This, perhaps, explains why LEVERRIER's discussion led him to the too small mass of Saturn.

In performing the very large mass of computations demanded by this investigation, by direction of the Superintendent, I have been assisted by various gentlemen connected with this office; all, however, to small amounts, with the exception of Mr. W. F. McK. RITTER, who, with the greatest efficiency, has made a duplicate of about two-thirds of the computations. Without help such as he has rendered it would have been impossible for me to have brought this undertaking to a conclusion. But all the original computations have been performed by myself.

A NEW THEORY OF JUPITER AND SATURN.

CHAPTER I.

MUTUAL ACTION OF JUPITER AND SATURN.—DEVELOPMENT OF THE RECIPROCAL OF THE DISTANCE BETWEEN THE PLANETS AND ITS ODD POWERS IN PERIODIC SERIES, WHEN ELLIPTIC VALUES ARE SUBSTITUTED FOR THE CO-ORDINATES.

The first step in determining the absolute perturbations of the elliptic motions arising from the mutual action of two planets is to obtain the expansion of the reciprocal of their distance and its odd powers in periodic series. The first and third powers alone are necessary in treating the perturbations due to the first power of the disturbing force; the fifth, in addition, is required when the second and the seventh when the third power of this force is taken into consideration. In this chapter we shall be engaged in developing these four powers of the reciprocal of the distance between Jupiter and Saturn, it being understood that elliptic values are attributed to the co-ordinates. The method pursued demands that these developments in their final form should appear as functions of the mean anomalies of the two planets, or in other words as functions of the time.

The elements adopted for the two planets are:

Elements of Jupiter and Saturn.

Epoch, 1850, Jan. 0.0^d, Greenwich M. T.

$L = 159^\circ\ 56'\ 26''.60$
$\pi = 11^\circ\ 56'\ 9''.33$
$\theta = 98^\circ\ 56'\ 19''.79$
$i = 1^\circ\ 18'\ 42.''10$
$e = 0.04824277$
$n = 109256''.55563$
$m = \frac{1}{1047.879}$

$L' = 14^\circ\ 49'\ 34.''04$
$\pi' = 90^\circ\ 6'\ 46''.22$
$\theta' = 112^\circ\ 20'\ 49.''05$
$i' = 2^\circ\ 29'\ 40''.19$
$e' = 0.05605688$
$n' = 43996''.07844$
$m' = \frac{1}{3501.6}$

It is scarcely necessary to go into details as to the derivation of these elements. They are supposed to be mean elements derived in such a way that the perturbations of the fundamental argument and the latitude have no terms whose period is the same as that of the mean anomaly, and the former no constant term or term proportional to the time. The elements of Saturn have been obtained from a previous investigation, and are supposed to be quite approximate. Those of Jupiter have been got by applying to Bouvard's values the corrections found by Mr. Breen.* On account of the errors of Bouvard's formulæ for the perturbations these values will probably be found to need quite large corrections. But they are sufficiently accurate to serve our purpose, as it is proposed at the end of this investigation to ascertain rudely their errors and correct the expressions for the perturbations by the differential method where the squares and products of these errors may be neglected.

In computing the mutual perturbations of two planets we need consider only the ratio of the mean distances. This is given by the equation

$$\alpha = \frac{a}{a'} = \left[\frac{1+m}{1+m'}\right]^{\frac{1}{3}} \left(\frac{n'}{n}\right)^{\frac{2}{3}}$$

whence, on substitution of the numerical values,

$$\log \alpha = 9.7367410563$$

But a considerable portion of the perturbations of the second order will be included in the computation of the terms of the first order if we apply severally to log a and log a' the constant parts of the perturbations of log r and log r'. For the action of all the planets excepting Jupiter and Saturn these parts can be, with sufficient accuracy, obtained from the following formulæ:

For the action of an outer on an inner planet

$$\delta \log a = -\frac{1}{6} \mathrm{M} m' \alpha^2 \frac{db_{\frac{1}{2}}^{(0)}}{d\alpha}$$

for the action of an inner on an outer planet

$$\delta \log a' = \frac{1}{6} \mathrm{M} m' \left(b_{\frac{1}{2}}^{(0)} + \alpha \frac{db_{\frac{1}{2}}^{(0)}}{d\alpha} \right)$$

where M is the modulus of common logarithms. For the action of Jupiter and Saturn the values are obtained from Hansen.†

		$\delta \log a$	$\delta \log a'$
Action of Mercury, mass	$\frac{1}{5000000}$	+.00000003	+.00000003
Action of Venus, mass	$\frac{1}{425000}$	+.00000035	+.00000034
Action of the Earth, mass	$\frac{1}{322800}$	+.00000046	+.00000045
Action of Mars, mass	$\frac{1}{3095000}$	+.00000005	+.00000005
Action of Jupiter,			+.00018522
Action of Saturn,		−.00000502	
Action of Uranus, mass	$\frac{1}{21000}$	−.00000007	−.00000058
Action of Neptune, mass	$\frac{1}{19700}$	−.00000002	−.00000013
Sums,		−.00000423	+.00018538

---

* Correction of Bouvard's Elements of Jupiter and Saturn. Appendix I to Greenwich Observations, 1868.

† Untersuchung über die gegenseitigen Störungen des Jupiter und Saturn. Berlin, 1831.

To avoid confusion we will designate a and a', after their correction by these quantities, as a and a', but α will be employed as the equivalent of $\frac{\mathrm{a}}{\mathrm{a}'}$. When we come to the treatment of the perturbations of the second order it will be necessary to remember that this modification has been made.

The quantities which define the relative position of the planes of the orbits can be obtained from the equations

$$\begin{array}{ll}
\cos \mathrm{J} & = \cos i \cos i' + \sin i \sin i' \cos(\theta' - \theta) \\
\sin \mathrm{J} \cos \Phi & = \sin i \cos i' - \cos i \sin i' \cos(\theta' - \theta) \\
\sin \mathrm{J} \sin \Phi & = \sin i' \sin(\theta' - \theta) \\
\sin \mathrm{J} \cos \Psi & = \cos i \sin i' - \sin i \cos i' \cos(\theta' - \theta) \\
\sin \mathrm{J} \sin \Psi & = \sin i \sin(\theta' - \theta) \\
\Pi = \pi - \theta - \Phi & \Pi' = \pi' - \theta' - \Psi
\end{array}$$

Here J denotes the inclination of the planes of the orbits, and Π and Π', severally, the angular distances of the perihelia of Jupiter and Saturn from the ascending node of Saturn's orbit on that of Jupiter.

The coefficients of the terms of the developments of the reciprocal of the distance between the planets $\frac{\mathrm{a}'}{\Delta}$ and its odd powers $\left(\frac{\mathrm{a}'}{\Delta}\right)^3$, $\left(\frac{\mathrm{a}'}{\Delta}\right)^5$, etc., as periodic functions of the two anomalies, are then functions of the following six elements:

$$\begin{array}{ll}
\log \alpha = 9.73655145 & \mathrm{J} = 1^\circ\ 15'\ 21''.31 \\
e = 0.04824277 & \Pi = 245^\circ\ 34'\ 48''.81 \\
e' = 0.05605688 & \Pi' = 323^\circ\ 45'\ 1''.87
\end{array}$$

We must now consider the degree of accuracy with which these coefficients must be computed in order that a proposed degree of accuracy may be obtained in the coefficients of the perturbations. Let it be demanded that the latter shall be got correct to $0''.001$ in the longitude and latitude. The coefficients of the terms in $\frac{\mathrm{a}'}{\Delta}$, whose argument is $5g' - 2g$, will be multiplied by the factor

$$15 \frac{m}{1 + m'} \frac{n'^2}{(5n' - 2n - 82'')^2} \times 206264''.8, \ (\log = 6.4738)$$

in order to obtain the coefficients of the perturbation of the mean anomaly of Saturn. It is thus seen that $0''.001$ in the latter is equivalent to 0.0000000034 in the former. Consequently ten-place logarithms will have to be used until we get the coefficients of the terms in $\frac{\mathrm{a}'}{\Delta}$, which have the argument $5g' - 2g$. Recourse has been had to VEGA'S *Thesaurus Logarithmorum* for the logarithms.

In the case of $\left(\frac{\mathrm{a}'}{\Delta}\right)^3$ a far less degree of accuracy will suffice; nevertheless, for those terms on which depend the secular perturbations a degree of accuracy has been adopted which is equivalent to the employment of eight-place logarithms. For $\left(\frac{\mathrm{a}'}{\Delta}\right)^5$ and $\left(\frac{\mathrm{a}'}{\Delta}\right)^7$, respectively, seven and five-place logarithms suffice.

In developing these quantities after the manner of HANSEN,* it is preferable to take Saturn as the disturbed planet, for the reason that in this way the quantity he has denoted by γ_2 is less than one-fifteenth of what it is when Jupiter is so taken.

We have the well-known equations

$$\begin{aligned} \mathrm{H} &= \cos(f+\Pi)\cos(f'+\Pi') + \cos \mathrm{J} \sin(f+\Pi)\sin(f'+\Pi') \\ \triangle^2 &= r'^2 - 2rr'\,\mathrm{H} + r^2 \end{aligned}$$

If the constants k, K, k_1, K_1 are obtained from the equations

$$\begin{aligned} k\cos(\Pi'-\mathrm{K}) &= \cos\Pi & \qquad k_1\cos(\Pi'-\mathrm{K}_1) &= \cos\mathrm{J}\cos\Pi \\ k\sin(\Pi'-\mathrm{K}) &= \cos\mathrm{J}\sin\Pi & \qquad k_1\sin(\Pi'-\mathrm{K}_1) &= \sin\Pi \end{aligned}$$

we have

$$\mathrm{H} = k\cos(f'+\mathrm{K})\cos f + k_1\sin(f'+\mathrm{K}_1)\sin f$$

The numerical substitutions give

$$\begin{aligned} \log k &= 9.9999134935 & \qquad \mathrm{K} &= 78^\circ\ 10'\ 31''.71660 \\ \log k_1 &= 9.9999821696 & \qquad \mathrm{K}_1 &= 78^\circ\ 9'\ 54''.40637 \end{aligned}$$

The square of the distance between the planets is reduced to the form

$$\left(\frac{\triangle}{\mathrm{a}'}\right)^2 = \gamma_0 - f\cos(\varepsilon-\mathrm{F}) + \gamma_2\cos^2\varepsilon$$

by computing p, P, v, V, w, W, w_1, W_1 from the equations (φ and φ' denoting the angles of the eccentricities)

$$\begin{aligned} p\sin\mathrm{P} &= 2\alpha\left(\alpha\frac{e}{e'} - k\cos\mathrm{K}\right) & \qquad p\cos\mathrm{P} &= 2\,\alpha k_1\cos\varphi\sin\mathrm{K}_1 \\ v\sin\mathrm{V} &= 2\alpha k\cos\varphi'\sin\mathrm{K} & \qquad v\cos\mathrm{V} &= 2\,\alpha k_1\cos\varphi\cos\varphi'\cos\mathrm{K}_1 \\ w\sin\mathrm{W} &= p - 2\alpha^2\frac{e}{e'}\sin\mathrm{P} & \qquad w\cos\mathrm{W} &= v\cos(\mathrm{V}-\mathrm{P}) \\ w_1\sin\mathrm{W}_1 &= v\sin(\mathrm{V}-\mathrm{P}) & \qquad w_1\cos\mathrm{W}_1 &= 2\alpha^2\frac{e}{e'}\cos\mathrm{P} \\ \mathrm{R} &= 1+\alpha^2-2\alpha^2e^2 & \qquad \gamma_2 &= \alpha^2e^2 \end{aligned}$$

After which

$$\begin{aligned} f\sin(\mathrm{F}-\mathrm{P}) &= w\sin(\varepsilon'+\mathrm{W}) - e'p \\ f\cos(\mathrm{F}-\mathrm{P}) &= w_1\cos(\varepsilon'+\mathrm{W}_1) \\ \gamma_0 &= \mathrm{R} - 2e'\cos\varepsilon' + e'^2\cos^2\varepsilon' + ef\cos\mathrm{F} \end{aligned}$$

The numerical substitutions give

$$\begin{aligned} \log p &= 0.0430483461 & \qquad \mathrm{P} &= 15^\circ\ 7'\ 49''.25766 \\ \log v &= 0.0368088942 & \qquad \mathrm{V} &= 78^\circ\ 10'\ 37''.64429 \\ \log w &= 0.0369667932 & \qquad \mathrm{W} &= 63^\circ\ 3'\ 26''.51160 \\ \log w_1 &= 0.0369024323 & \qquad \mathrm{W}_1 &= 63^\circ\ 1'\ 21''.06684 \\ e'p &= 0.0618980929 & \qquad \log\gamma_2 &= 6.83996737 \end{aligned}$$

$$\begin{aligned} \gamma_0 = 1.2958534630 - [9.0496589179]\cos\varepsilon' + [7.4972578444]\cos^2\varepsilon' \\ + [8.6834322362]\,f\cos\mathrm{F} \end{aligned}$$

---

*Auseinandersetzung einer zweckmässigen Methode zur Berechnung der absoluten Störungen der kleinen Planeten. Abhandlung I, s. 138.

In the last equation the brackets denote that the common logarithms are given instead of the numbers corresponding.

It will suffice to divide the circumference with respect to the mean anomaly of Saturn (for the reasons stated in the Introduction we adopt the mean anomaly instead of the eccentric, which HANSEN uses) into sixteen parts. In this way the errors committed in the coefficients belonging to the great inequality are of the 16 — 3 = thirteenth order with respect to the eccentricities and inclination of the orbits. The errors relative to double the argument of this inequality are of the 16 — 6 = tenth order. This is certainly a sufficient degree of exactitude.

We compute by trial from the equation

$$g' = \varepsilon' - e' \sin \varepsilon'$$

where log (e' in seconds of arc) = 4.0630540554, the values of ε' corresponding to the seven following values of g':

g'	ε'		
22°.5	23°	47′	45″.26135
45	47	21	46 .08482
67.5	70	31	41 .24336
90	93	12	24 .45612
112.5	115	24	4 .75552
135	137	10	58 .60666
157.5	158	40	6 .06463

The values for the remaining points of division of the circumference are either known or readily deducible from these. By substituting them in the preceding equations which give the values of γ_0, f, and F, we get the following table:

g'	γ_0	log f	F — g'		
°			°	′	″
0	1.1984385810	0.0146441389	76	36	30.05448
22.5	1.1858045730	0.0116067484	79	15	27.87825
45	1.1916240568	0.0131809243	81	42	56.61757
67.5	1.2147457246	0.0188912301	83	32	2.35338
90	1.2509960366	0.0273706837	84	26	6.62580
112.5	1.2942528243	0.0369329790	84	20	48.21352
135	1.3377252007	0.0460462355	83	21	48.24421
157.5	1.3750575191	0.0535462990	81	41	8.79693
180	1.4011119502	0.0586511882	79	34	7.28496
202.5	1.4124496841	0.0608956692	77	17	19.95693
225	1.4075886817	0.0600677450	75	7	38.21369
247.5	1.3871073227	0.0561863167	73	21	27.02105
270	1.3536269388	0.0495348409	72	13	53.72792
292.5	1.3116584293	0.0407480405	71	57	17.20268
315	1.2672436572	0.0309155839	72	38	35.73266
337.5	1.2272790242	0.0216041043	74	16	5.05534

These quantities can be subjected to the following important test: The sum of the values which correspond to 0°, 45°, 90° . . . 315° ought to be nearly equal to the sum of the values which correspond to 22°.5, 67°.5, 112°.5 . . . 337°.5. It is known that the difference is of the eighth order with respect to the eccentricities and mutual inclination. Calling these sums S and S′ the result here is:

	γ_0	$\log f$	$F - g'$		
			°	′	″
S	10.4083551030	0.3004113404	625	41	36.50129
S′	10.4083551013	0.3004113872	625	41	36.47808

This test is applicable to many of the following computations, and the result of its application will always be given.

Following **Hansen**'s procedure, we separate the square of the distance into two factors, such that

$$\left(\frac{\triangle}{a'}\right)^2 = \left[C - q\cos(\varepsilon - Q)\right]\left[1 - \frac{\gamma_2}{q}\cos(\varepsilon + Q)\right]$$

which gives, for determining C, q, and Q, the equations

$$C = \gamma_0 + \gamma_2 \sin^2 Q$$

$$f \sin F = \left(q - \frac{\gamma_2 C}{q}\right) \sin Q$$

$$f \cos F = \left(q + \frac{\gamma_2 C}{q}\right) \cos Q$$

By eliminating C and q we get

$$\sin(Q - F) = \frac{\gamma_2}{f^2}\frac{\gamma_0 + \gamma_2 \sin^2 Q}{\sin(Q + F)} \sin^2 2Q$$

from which, by trial, Q may readily be obtained. In like manner q may be got from the equation

$$q = f\cos(Q - F) - \frac{\gamma_2 C}{q}\cos 2Q$$

By development from these may be obtained series, proceeding according to ascending powers of γ_2, which give the values of $Q - F$ and $\log\frac{q}{f}$. They have been derived by **Hansen**.[*] In our case we have preferred the method by trial. The numerical results obtained are

---

[*] Auseinandersetzung, Abhandlung I, S. 146, Gl. (120).

g'	C	$\log q$	$Q-g'$		
°			°	′	″
0	1.1990933606	0.0149445349	76	37	41.97691
22.5	1.1864677121	0.0119163222	79	14	23.98190
45	1.1920690737	0.0132773656	81	40	23.28806
67.5	1.2149083561	0.0187137572	83	29	47.52405
90	1.2510025615	0.0270452279	84	25	36.18120
112.5	1.2943110889	0.0366599027	84	22	14.80626
135	1.3379921669	0.0459719049	83	24	18.63371
157.5	1.3755681686	0.0536999596	81	43	23.61467
180	1.4017811164	0.0589513080	79	35	1.54460
202.5	1.4131215236	0.0611975819	77	16	29.02016
225	1.4081065680	0.0602270953	75	5	26.06779
247.5	1.3873834732	0.0561213072	73	18	57.30456
270	1.3536915381	0.0492712795	72	12	24.19588
292.5	1.3116626153	0.0404249841	71	57	41.28342
315	1.2673929280	0.0307276532	72	40	44.82897
337.5	1.2277063745	0.0216826028	74	18	39.15769
S	10.4111293132	0.3004163693	625	41	36.71712
S′	10.4111293123	0.3004164177	625	41	36.69271

It seems desirable to make one more transformation in the form of Δ^2; we put

$$\frac{q}{C} = \sin\chi \qquad a = \tan\frac{1}{2}\chi$$

$$\frac{\gamma_2}{q} = \sin\chi_1 \qquad b = \tan\frac{1}{2}\chi_1$$

$$N = \frac{\sec\frac{1}{2}\chi\sec\frac{1}{2}\chi_1}{\sqrt{C}}$$

In this way

$$\frac{a'}{\Delta} = N\left[1-2a\cos(\varepsilon-Q)+a^2\right]^{-\frac{1}{2}}\left[1-2b\cos(\varepsilon+Q)+b^2\right]^{-\frac{1}{2}}$$

The computations being performed, we have

g'	log N	log a	log b
°			
0	0.0223304134	9.7585753170	6.5239929
22.5	0.0254063724	9.7616990221	6.5270211
45	0.0240423963	9.7603321138	6.5256601
67.5	0.0185553673	9.7547944485	6.5202237
90	0.0101630309	9.7463412483	6.5118922
112.5	0.0005153668	9.7366605966	6.5022775
135	9.9912124592	9.7273667855	6.4929655
157.5	9.9835260741	9.7197220711	6.4852375
180	9.9783269754	9.7145752236	6.4799861
202.5	9.9761202182	9.7124079836	6.4777398
225	9.9771006514	9.7133983631	6.4787103
247.5	9.9811817483	9.7174547678	6.4828161
270	9.9879825162	9.7242062749	6.4896661
292.5	9.9967796223	9.7329541897	6.4985124
315	0.0064581878	9.7426139882	6.5082098
337.5	0.0155318861	9.7517163322	6.5172548
S	9.9976166306	7.8874093144	72.0110830
S′	9.9976166555	7.8874094116	72.0110829

The next step is the calculation of the values of LAPLACE's coefficients $b_s^{(i)}$, corresponding to each of the sixteen values of a just given. We proceed as follows:

Deriving θ from the equation

$$\sin\theta = \mathrm{a}$$

we get

$$\mathrm{b} = \cos\theta \qquad \mathrm{a}^0 = \tan^2\frac{1}{2}\theta \qquad \mathrm{b}^0 = \frac{\sqrt{\mathrm{b}}}{\cos^2\frac{1}{2}\theta}$$

$$p = \frac{\left(\frac{1}{2}\mathrm{a}^0\right)^2}{\mathrm{b}^0} \qquad \mathrm{P} = \mathrm{M}p^2 \qquad \log \mathrm{a}^{00} = \log p - \mathrm{P} \qquad \log \mathrm{b}^{00} = -\frac{1}{2}\mathrm{P}$$

where M is the modulus of common logarithms. Then

$$\mathrm{K} = \sqrt{\frac{\mathrm{b}^0\,\mathrm{b}^{00}}{\mathrm{b}}} \qquad \mathrm{H} = \frac{\mathrm{a}^0}{2}\left[1 + \frac{\mathrm{a}^{00}}{2\sqrt[4]{\mathrm{b}^{00}}}\right]$$

and

$$b_{\frac{1}{2}}^{(0)} = 2\mathrm{K} \qquad b_{\frac{1}{2}}^{(1)} = \mathrm{aK}\,(1+\mathrm{H}) \qquad b_{\frac{1}{2}}^{(2)} = \frac{2}{3}\mathrm{K}\left[\mathrm{a}^2(1+\mathrm{H}) + \mathrm{H}\right]$$

$$b_{\frac{3}{2}}^{(0)} = \frac{2\mathrm{K}}{\mathrm{b}^4}\left[1 - \mathrm{a}^2\mathrm{H}\right] \qquad b_{\frac{3}{2}}^{(2)} = b_{\frac{3}{2}}^{(0)} - 2\mathrm{K}\,(1+\mathrm{H}) \qquad b_{\frac{3}{2}}^{(1)} = \frac{\sin\chi}{4}\left(3b_{\frac{3}{2}}^{(0)} + b_{\frac{3}{2}}^{(2)}\right)^*$$

On account of the importance of $b_{\frac{1}{2}}^{(2)}$ and $b_{\frac{1}{2}}^{(4)}$ and as a check their values have also been computed from the following series given by LEVERRIER† (the coefficients are replaced by their logarithms):

---

* For the proof of these formulæ consult LEGENDRE, Traité des Fonctions Elliptiques, Tome II, pp. 6 and 548.

† Annales de l'Observatoire de Paris, Mémoires, Tome II, Additions, p. 2.

$$\begin{array}{rll} b_{\frac{1}{2}}^{(2)} = & 9.8750612634 & \alpha^{2} \\ + & 9.4948500216 & \alpha^{4} \\ + & 9.3119193380 & \alpha^{6} \\ + & 9.1869806013 & \alpha^{8} \\ + & 9.0912000935 & \alpha^{10} \\ + & 9.0132579195 & \alpha^{12} \\ + & 8.947440635 & \alpha^{14} \\ + & 8.89043237 & \alpha^{16} \\ + & 8.84012725 & \alpha^{18} \\ + & 8.79510028 & \alpha^{20} \\ + & 8.7543405 & \alpha^{22} \\ + & 8.7171038 & \alpha^{24} \\ + & 8.682826 & \alpha^{26} \\ + & 8.651069 & \alpha^{28} \\ + & 8.62149 & \alpha^{30} \end{array} \qquad \begin{array}{rll} b_{\frac{1}{2}}^{(4)} = & 9.7378880703 & \alpha^{4} \\ + & 9.3911005840 & \alpha^{6} \\ + & 9.2283732866 & \alpha^{8} \\ + & 9.117007357 & \alpha^{10} \\ + & 9.030986686 & \alpha^{12} \\ + & 8.960405612 & \alpha^{14} \\ + & 8.90034066 & \alpha^{16} \\ + & 8.84795259 & \alpha^{18} \\ + & 8.80144046 & \alpha^{20} \\ + & 8.7595835 & \alpha^{22} \\ + & 8.7215129 & \alpha^{24} \\ + & 8.6865862 & \alpha^{26} \\ + & 8.654315 & \alpha^{28} \\ + & 8.624316 & \alpha^{30} \\ + & 8.59629 & \alpha^{32} \end{array}$$

The remainders of these series can be summed as geometrical progressions. In this way have been obtained the following values, corresponding to intervals of 0.005 in log α. From these, by interpolation, it is easy to get the values corresponding to the sixteen values of log **a**:

log α	log $b_{\frac{1}{2}}^{(2)}$	log $b_{\frac{1}{2}}^{(4)}$
9.710	9.3493888435	8.636820535
9.715	9.3608488808	8.658411500
9.720	9.3723541687	8.680052169
9.725	9.3839064789	8.701744508
9.730	9.3955076774	8.723490591
9.735	9.4071597303	8.745292605
9.740	9.4188647109	8.767152856
9.745	9.4306248085	8.789073783
9.750	9.4424423364	8.811057961
9.755	9.4543197411	8.833108118
9.760	9.4662596138	8.855227147
9.765	9.4782647015	8.877418108

The remaining coefficients $b_{\frac{1}{2}}^{(i)}$ can be got from the formula

$$b_{\frac{1}{2}}^{(i)} = \frac{4\,(i-1)}{2i-1}\,\frac{b_{\frac{1}{2}}^{(i-1)}}{\sin\chi} - \frac{2i-3}{2i-1}\,b_{\frac{1}{2}}^{(i-2)}$$

taking care, however, to use one more decimal in calculating than we wish to retain. But for large values of i it will be more convenient to compute the continued fraction

$$\frac{b_{\frac{1}{2}}^{(i)}}{b_{\frac{1}{2}}^{(i-1)}} = \cfrac{\frac{2i-1}{2i}\,\alpha}{1 - \cfrac{\frac{\alpha^{2}}{4i\,(i+1)}}{1 - \cfrac{(2i+1)^{2}}{4\,(i+1)(i+2)}\,\alpha^{2}}} \\ \cdots\cdots\cdots$$

The remaining coefficients $b_{\frac{1}{2}}^{(i)}$ are got from the equation

$$b_{\frac{1}{2}}^{(i)} = b_{\frac{1}{2}}^{(i-2)} - \frac{2\,(i-1)}{\alpha}\, b_{\frac{1}{2}}^{(i-1)}$$

When i is large one can readily deduce the ratio between the quantities for two successive values of i by induction from the differences of the preceding logarithms.

The values of the $b_{\frac{3}{2}}^{(i)}$ are obtained from the formulæ

$$b_{\frac{3}{2}}^{(0)} + b_{\frac{3}{2}}^{(1)} = \frac{b_{\frac{1}{2}}^{(0)} + \frac{1}{3} b_{\frac{1}{2}}^{(1)}}{(1-\alpha)^2} \qquad b_{\frac{3}{2}}^{(0)} - b_{\frac{3}{2}}^{(1)} = \frac{b_{\frac{1}{2}}^{(0)} - \frac{1}{3} b_{\frac{1}{2}}^{(1)}}{(1+\alpha)^2} \qquad b_{\frac{3}{2}}^{(i)} = b_{\frac{3}{2}}^{(i-2)} - \frac{2\,(i-1)}{3\alpha}\, b_{\frac{1}{2}}^{(i-1)}$$

The formulæ used for obtaining the $b_{\frac{5}{2}}^{(i)}$ are

$$b_{\frac{5}{2}}^{(0)} + b_{\frac{5}{2}}^{(1)} = \frac{b_{\frac{3}{2}}^{(0)} + \frac{3}{5} b_{\frac{3}{2}}^{(1)}}{(1-\alpha)^2} \qquad b_{\frac{5}{2}}^{(0)} - b_{\frac{5}{2}}^{(1)} = \frac{b_{\frac{3}{2}}^{(0)} - \frac{3}{5} b_{\frac{3}{2}}^{(1)}}{(1+\alpha)^2} \qquad b_{\frac{5}{2}}^{(i)} = b_{\frac{5}{2}}^{(i-2)} - \frac{2\,(i-1)}{5\alpha}\, b_{\frac{3}{2}}^{(i-1)}$$

The following table contains all of these quantities it has been deemed necessary to compute. For convenience the points of division of the circumference relative to g' are designated as (0), (1), ... (15):

	$\log \frac{1}{2} b_{\frac{1}{2}}^{(0)}$	$\log b_{\frac{1}{2}}^{(1)}$	$\log b_{\frac{1}{2}}^{(2)}$	$\log b_{\frac{1}{2}}^{(3)}$	$\log b_{\frac{1}{2}}^{(4)}$	$\log b_{\frac{1}{2}}^{(5)}$	$\log b_{\frac{1}{2}}^{(6)}$
(0)	0.0422349759	9.8219019233	9.4628509859	9.1460177531	8.848917435	8.56330651	8.2852309
(1)	0.0429700812	9.8261268967	9.4703315379	9.1566921194	8.862759506	8.58030264	8.3053729
(2)	0.0426465422	9.8242752976	9.4670549712	9.1520177694	8.856698867	8.57286172	8.2965554
(3)	0.0413651511	9.8168178994	9.4538302440	9.1331333789	8.832200292	8.54277272	8.2608899
(4)	0.0394960133	9.8055641686	9.4337891208	9.1044619178	8.794964713	8.49700758	8.2066157
(5)	0.0374762256	9.7928569388	9.4110412050	9.0718430303	8.752546239	8.44482671	8.1446944
(6)	0.0356497314	9.7808259144	9.3893917767	9.0407286674	8.712031341	8.39494475	8.0854651
(7)	0.0342243503	9.7710449263	9.3717134206	9.0152727021	8.678847918	8.35405988	8.0368941
(8)	0.0333016727	9.7645151480	9.3598735555	8.9982002666	8.656575328	8.32660393	8.0042646
(9)	0.0329217162	9.7617784029	9.3549024197	8.9910266600	8.647212588	8.31505899	7.9905416
(10)	0.0330947278	9.7630281054	9.3571730733	8.9943037308	8.651490004	8.32033359	7.9968115
(11)	0.0338143165	9.7681630622	9.3664916805	9.0077455337	8.669029733	8.34195815	8.0225132
(12)	0.0350522873	9.7767700094	9.3820693919	9.0301900000	8.698297424	8.37802650	8.0653689
(13)	0.0367350622	9.7880398438	9.4023858750	9.0594117420	8.736365143	8.42490941	8.1210489
(14)	0.0387035953	9.8006495388	9.4250058437	9.0918765767	8.778605336	8.47688877	8.1827461
(15)	0.0406726683	9.8127022707	9.4465125737	9.1226718514	8.818619460	8.52608541	8.2411037
S	0.3001795459	8.3375301055	5.2772087190	2.5577966818	89.997580448	87.52997335	85.1230582
S′	0.3001795714	8.3375302408	5.2772089564	2.5577970178	89.997580879	87.52997391	85.1230587

	$\log b_{\frac{1}{2}}^{(7)}$	$\log b_{\frac{1}{2}}^{(8)}$	$\log b_{\frac{1}{2}}^{(9)}$	$\log b_{\frac{1}{2}}^{(10)}$	$\log b_{\frac{1}{2}}^{(11)}$	$\log b_{\frac{1}{2}}^{(12)}$	$\log \frac{1}{2} b_{\frac{3}{2}}^{(0)}$
(0)	8.012484	7.74371	7.4780	7.2147	6.9535	6.6939	0.3815770471
(1)	8.035767	7.77013	7.5076	7.2474	6.9893	6.7329	0.3882685965
(2)	8.025575	7.75856	7.4946	7.2331	6.9736	6.7158	0.3853231659
(3)	7.984342	7.71177	7.4423	7.1752	6.9102	6.6468	0.3736623431
(4)	7.921572	7.64051	7.3625	7.0870	6.8135	6.5416	0.3566660681
(5)	7.849925	7.55915	7.2715	6.9862	6.7030	6.4215	0.3383167742
(6)	7.781360	7.48126	7.1843	6.8897	6.5972	6.3064	0.3217379468
(7)	7.725113	7.41734	7.1127	6.8105	6.5103	6.2118	0.3088090996
(8)	7.687316	7.37438	7.0645	6.7572	6.4518	6.1482	0.3004441223
(9)	7.671418	7.35631	7.0443	6.7348	6.4273	6.1215	0.2970003660
(10)	7.678682	7.36457	7.0536	6.7450	6.4385	6.1337	0.2985684015
(11)	7.708455	7.39841	7.0915	6.7870	6.4845	6.1837	0.3050913479
(12)	7.758090	7.45482	7.1546	6.8569	6.5612	6.2672	0.3163179042
(13)	7.822556	7.52806	7.2366	6.9477	6.6608	6.3755	0.3315877222
(14)	7.893957	7.60916	7.3274	7.0482	6.7709	6.4953	0.3494650909
(15)	7.961462	7.68580	7.4132	7.1431	6.8749	6.6085	0.3673637317
S	82.759036	80.42696	78.1195	75.8318	73.5602	71.3021	2.7100997468
S′	82.759039	80.42697	78.1197	75.8319	73.5603	71.3022	2.7100999812

	$\log b_{\frac{3}{2}}^{(1)}$	$\log b_{\frac{3}{2}}^{(2)}$	$\log b_{\frac{3}{2}}^{(3)}$	$\log b_{\frac{3}{2}}^{(4)}$	$\log b_{\frac{3}{2}}^{(5)}$	$\log b_{\frac{3}{2}}^{(6)}$	$\log b_{\frac{3}{2}}^{(7)}$
(0)	0.5631118888	0.3981406377	0.21280182	0.0157694	9.8111164	9.601092	9.387073
(1)	0.5721300721	0.4099407726	0.22753282	0.0334998	9.8318830	9.624918	9.413973
(2)	0.5681679947	0.4047621302	0.22107214	0.0257271	9.8227817	9.614478	9.402188
(3)	0.5523652504	0.3840182539	0.19512648	9.9944604	9.7861294	9.572400	9.354660
(4)	0.5289763121	0.3530493494	0.15619400	9.9473895	9.7308256	9.508806	9.282738
(5)	0.5032062005	0.3185462420	0.11253865	9.8943923	9.6683838	9.436858	9.201243
(6)	0.4794072587	0.2863139951	0.07148990	9.8443547	9.6092648	9.368601	9.123813
(7)	0.4604721309	0.2604079767	0.03831122	9.8037684	9.5611982	9.313011	9.060671
(8)	0.4480304051	0.2432566373	0.01625366	9.7767170	9.5291062	9.275850	9.018424
(9)	0.4428622608	0.2361015454	0.00703035	9.7653894	9.5156546	9.260264	9.000693
(10)	0.4452188662	0.2393664182	0.01124053	9.7705612	9.5217972	9.267382	9.008791
(11)	0.4549614963	0.2528244432	0.02856678	9.7918252	9.5470335	9.296615	9.042030
(12)	0.4715112882	0.2755397030	0.05771104	9.8275149	9.5893332	9.345560	9.097650
(13)	0.4936087427	0.3055911562	0.09607087	9.8743422	9.6447152	9.409545	9.170277
(14)	0.5189305295	0.3396476025	0.13927240	9.9268738	9.7066745	9.480997	9.251252
(15)	0.5437484961	0.3726464779	0.18085806	9.9772307	9.7659032	9.549156	9.328384
S	4.0233545433	2.5400764734	0.88603549	9.1349076	7.3208996	5.462766	3.571929
S′	4.0233546498	2.5400768679	0.88603525	9.1349084	7.3209009	5.462767	3.571931

	$\log b_{\frac{3}{2}}^{(8)}$	$\log b_{\frac{3}{2}}^{(9)}$	$\log b_{\frac{3}{2}}^{(10)}$	$\log b_{\frac{3}{2}}^{(11)}$	$\log b_{\frac{3}{2}}^{(12)}$	$\log \frac{1}{2} b_{\frac{5}{2}}^{(0)}$	$\log b_{\frac{5}{2}}^{(1)}$
(0)	9.16997	8.95040	8.7288	8.5056	8.2810	0.9374315	1.2004172
(1)	9.19995	8.98348	8.7650	8.5449	8.3234	0.9525706	1.2165967
(2)	9.18682	8.96898	8.7491	8.5276	8.3047	0.9459100	1.2094831
(3)	9.13382	8.91051	8.6852	8.4582	8.2298	0.9194893	1.1811967
(4)	9.05354	8.82186	8.5882	8.3528	8.1160	0.8808271	1.1395928
(5)	8.96247	8.72119	8.4779	8.2329	7.9865	0.8388731	1.0941309
(6)	8.87584	8.62534	8.3728	8.1185	7.8629	0.8007651	1.0525155
(7)	8.80513	8.54706	8.2869	8.0251	7.7619	0.7709072	1.0196702
(8)	8.75779	8.49461	8.2294	7.9624	7.6940	0.7515206	0.9982205
(9)	8.73791	8.47257	8.2052	7.9360	7.6655	0.7435236	0.9893423
(10)	8.74699	8.48263	8.2162	7.9481	7.6785	0.7471661	0.9933884
(11)	8.78425	8.52391	8.2615	7.9974	7.7319	0.7622975	1.0101569
(12)	8.84655	8.59293	8.3373	8.0799	7.8211	0.7882634	1.0387900
(13)	8.92784	8.68290	8.4359	8.1873	7.9372	0.8234295	1.0773050
(14)	9.01837	8.78299	8.5456	8.3065	8.0660	0.8643897	1.1218225
(15)	9.10450	8.87814	8.6498	8.4197	8.1883	0.9051830	1.1658323
S	1.65587	89.71974	87.7674	85.8014	83.8242	6.7162735	8.7542300
S′	1.65587	89.71975	87.7674	85.8015	83.8245	6.7162738	8.7542310

	$\log b_{\frac{5}{2}}^{(2)}$	$\log b_{\frac{5}{2}}^{(3)}$	$\log b_{\frac{5}{2}}^{(4)}$	$\log b_{\frac{5}{2}}^{(5)}$	$\log b_{\frac{5}{2}}^{(6)}$	$\log b_{\frac{5}{2}}^{(7)}$	$\log b_{\frac{5}{2}}^{(8)}$
(0)	1.1161428	1.0021564	0.8677099	0.7183764	0.5577605	0.38831	0.21173
(1)	1.1342473	1.0226184	0.8907638	0.7441686	0.5863792	0.41981	0.24618
(2)	1.1262929	1.0136345	0.8806463	0.7328547	0.5738282	0.40598	0.23108
(3)	1.0945691	0.9777147	9.8401145	0.6874605	0.5234092	0.35043	0.17031
(4)	1.0476238	0.9242889	0.7795902	0.6194680	0.4477105	0.26688	0.07875
(5)	0.9959088	0.8650458	0.7121354	0.5433964	0.3627626	0.17288	9.97558
(6)	0.9481583	0.8099664	0.6490945	0.4720231	0.2828182	0.08420	9.87807
(7)	0.9101740	0.7658824	0.5984099	0.4144423	0.2181537	0.01233	9.79890
(8)	0.8852179	0.7367855	0.5648423	0.3762126	0.1751457	9.96446	9.74613
(9)	0.8748524	0.7246685	0.5508369	0.3602400	0.1571561	9.94441	9.72401
(10)	0.8795790	0.7301962	0.5572280	0.3675309	0.1653688	9.95357	9.73410
(11)	0.8991204	0.7530082	0.5835689	0.3975494	0.1991628	9.99120	9.77561
(12)	0.9323184	0.7916123	0.6280176	0.4480994	0.2559730	0.05439	9.84523
(13)	0.9766515	0.8428772	0.6868009	0.5147454	0.3306976	0.13735	9.93651
(14)	1.0274626	0.9012426	0.7533918	0.5899603	0.4147915	0.23048	0.03884
(15)	1.0772728	0.9580689	0.8178919	0.6625245	0.4956722	0.31983	0.13681
S	7.9627957	6.9098828	5.6805206	4.3245254	2.8733964	1.34827	9.76393
S′	7.9627963	6.9098841	5.6805222	4.3245271	2.8733934	1.34824	9.76391

	$\log b_{\frac{5}{2}}^{(9)}$	$\log b_{\frac{5}{2}}^{(10)}$	$\log b_{\frac{5}{2}}^{(11)}$	$\log b_{\frac{5}{2}}^{(12)}$	$\log \frac{1}{2} b_{\frac{7}{2}}^{(0)}$	$\log b_{\frac{7}{2}}^{(1)}$	$\log b_{\frac{7}{2}}^{(2)}$
(0)	0.0293	9.8420	9.6506	9.4556	1.57609	1.85858	1.80816
(1)	0.0667	9.8824	9.6940	9.5021	1.59982	1.88283	1.83370
(2)	0.0503	9.8647	9.6750	9.4817	1.58938	1.87216	1.82247
(3)	9.9843	9.7934	9.5983	9.3996	1.54794	1.82978	1.77778
(4)	9.8847	9.6856	9.4824	9.2755	1.48718	1.76753	1.71191
(5)	9.7723	9.5639	9.3512	9.1349	1.42105	1.69961	1.63971
(6)	9.6658	9.4484	9.2268	9.0014	1.36082	1.63758	1.57342
(7)	9.5793	9.3545	9.1254	8.8926	1.31349	1.58869	1.52093
(8)	9.5215	9.2918	9.0576	8.8198	1.28270	1.55683	1.48659
(9)	9.4973	9.2654	9.0292	8.7892	1.26999	1.54365	1.47236
(10)	9.5084	9.2775	9.0422	8.8032	1.27578	1.54965	1.47884
(11)	9.5538	9.3268	9.0955	8.8604	1.29982	1.57455	1.50570
(12)	9.6300	9.4095	9.1847	8.9563	1.34102	1.61714	1.55150
(13)	9.7297	9.5177	9.3014	9.0815	1.39666	1.67452	1.61293
(14)	9.8412	9.6385	9.4317	9.2212	1.46129	1.74097	1.68372
(15)	9.9478	9.7540	9.5559	9.3543	1.52547	1.80678	1.75348
S	8.1312	6.4580	4.7510	3.0146	11.37426	13.60044	13.11661
S′	8.1312	6.4581	4.7509	3.0146	11.37424	13.60041	13.11659

	$\log b_{\frac{7}{2}}^{(3)}$	$\log b_{\frac{7}{2}}^{(4)}$	$\log b_{\frac{7}{2}}^{(5)}$	$\log b_{\frac{7}{2}}^{(6)}$	$\log b_{\frac{7}{2}}^{(7)}$	$\log b_{\frac{7}{2}}^{(8)}$	$\log b_{\frac{7}{2}}^{(9)}$
(0)	1.73225	1.63615	1.52389	1.39853	1.2624	1.1171	0.9644
(1)	1.75959	1.66565	1.55573	1.43295	1.2993	1.1572	1.0067
(2)	1.74758	1.65267	1.54175	1.41781	1.2832	1.1395	0.9885
(3)	1.69964	1.60097	1.48566	1.35740	1.2178	1.0694	0.9120
(4)	1.62881	1.52421	1.40258	1.26723	1.1207	0.9646	0.8011
(5)	1.55073	1.43925	1.31004	1.16667	1.0116	0.8472	0.6742
(6)	1.47868	1.36047	1.22392	1.07276	0.9098	0.7370	0.5559
(7)	1.42134	1.29751	1.15483	0.99721	0.8274	0.6478	0.4595
(8)	1.38369	1.25603	1.10920	0.94720	0.7730	0.5885	0.3959
(9)	1.36806	1.23878	1.09021	0.92635	0.7502	0.5639	0.3715
(10)	1.37518	1.24664	1.09886	0.93585	0.7606	0.5746	0.3808
(11)	1.40466	1.27914	1.13464	0.97507	0.8031	0.6216	0.4306
(12)	1.45477	1.33424	1.19518	1.04135	0.8756	0.6999	0.5180
(13)	1.52168	1.40753	1.27541	1.12896	0.9707	0.8032	0.6268
(14)	1.59838	1.49114	1.36661	1.22819	1.0784	0.9192	0.7520
(15)	1.67357	1.57273	1.45524	1.32431	1.1822	1.0311	0.8719
S	12.39934	11.50155	10.46199	9.30892	8.0637	6.7404	5.3566
S′	12.39927	11.50156	10.46176	9.30892	8.0623	6.7414	5.3532

Putting

$$[1 - 2\mathbf{b}\cos(\varepsilon + Q) + \mathbf{b}^2]^{-\frac{n}{2}} = \frac{1}{2}B_{\frac{n}{2}}^{(0)} + B_{\frac{n}{2}}^{(1)}\cos(\varepsilon + Q) + B_{\frac{n}{2}}^{(2)}\cos 2(\varepsilon + Q) + \ldots$$

it will suffice to compute the $B_{\frac{n}{2}}^{(i)}$ by the following formulæ:

$$\log\left(\frac{1}{2}B_{\frac{1}{2}}^{(0)}\right) = \frac{1}{4}M\mathbf{b}^2 \qquad \log\left(\frac{1}{2}B_{\frac{3}{2}}^{(0)}\right) = \frac{9}{4}M\mathbf{b}^2 \qquad \log\left(\frac{1}{2}B_{\frac{5}{2}}^{(0)}\right) = 0 \qquad \log\left(\frac{1}{2}B_{\frac{7}{2}}^{(0)}\right) = 0$$

$$B_{\frac{1}{2}}^{(1)} = \mathbf{b} \qquad B_{\frac{3}{2}}^{(1)} = 3\mathbf{b} \qquad B_{\frac{5}{2}}^{(1)} = 5\mathbf{b} \qquad B_{\frac{7}{2}}^{(1)} = 7\mathbf{b}$$

$$B_{\frac{1}{2}}^{(2)} = \frac{3}{8}\mathbf{b}^2 \qquad B_{\frac{3}{2}}^{(2)} = \frac{15}{4}\mathbf{b}^2$$

Then, if we put θ for $Q - \varepsilon$, and

$$c_{\frac{n}{2}}^{(i)} = \frac{1}{2}N^n B_{\frac{n}{2}}^{(i)}\cos 2iQ \qquad s_{\frac{n}{2}}^{(i)} = \frac{1}{2}N^n B_{\frac{n}{2}}^{(i)}\sin 2iQ$$

and, for convenience, omit everywhere the subscript $\frac{n}{2}$ from the quantities b and c, according as 1 or 3 or 5 or 7 is substituted for n, we have the development of $\frac{\mathbf{a}'}{\triangle}$ or $\left(\frac{\mathbf{a}'}{\triangle}\right)^3$ or $\left(\frac{\mathbf{a}'}{\triangle}\right)^5$ or $\left(\frac{\mathbf{a}'}{\triangle}\right)^7$ given by the following series, the law of which is easily recognized:

$$\begin{aligned}
&\tfrac{1}{2}b^{(0)}c^{(0)} + b^{(1)}c^{(1)} + b^{(2)}c^{(2)} \\
&+ [b^{(1)}c^{(0)} + (b^{(0)} + b^{(2)})\,c^{(1)} + (b^{(1)} + b^{(3)})\,c^{(2)}]\cos\theta \\
&+ [\phantom{b^{(1)}c^{(0)} +}\ (b^{(0)} - b^{(2)})\,s^{(1)} + (b^{(1)} - b^{(3)})\,s^{(2)}]\sin\theta \\
&+ [b^{(2)}c^{(0)} + (b^{(1)} + b^{(3)})\,c^{(1)} + (b^{(0)} + b^{(4)})\,c^{(2)}]\cos 2\theta \\
&+ [\phantom{b^{(2)}c^{(0)} +}\ (b^{(1)} - b^{(3)})\,s^{(1)} + (b^{(0)} - b^{(4)})\,s^{(2)}]\sin 2\theta \\
&+ [b^{(3)}c^{(0)} + (b^{(2)} + b^{(4)})\,c^{(1)} + (b^{(1)} + b^{(5)})\,c^{(2)}]\cos 3\theta \\
&+ [\phantom{b^{(3)}c^{(0)} +}\ (b^{(2)} - b^{(4)})\,s^{(1)} + (b^{(1)} - b^{(5)})\,s^{(2)}]\sin 3\theta \\
&+ \ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}$$

If we put

$$\begin{aligned}
k_i\cos K_i &= b^{(i)}c^{(0)} + (b^{(i-1)} + b^{(i+1)})c^{(1)} + (b^{(i-2)} + b^{(i+2)})c^{(2)} \\
k_i\sin K_i &= \phantom{b^{(i)}c^{(0)} +}\ (b^{(i-1)} - b^{(i+1)})s^{(1)} + (b^{(i-2)} - b^{(i+2)})s^{(2)}
\end{aligned}$$

k_i will not differ much from $b^{(i)}$, and K_i will be a small positive or negative angle. The terms having $c^{(1)}$ and $c^{(2)}$ as factors are much smaller than $b^{(i)}c^{(0)}$, and the union of the terms can be made with addition and subtraction logarithms: when these do not give sufficiently accurate results, as in the cases where $\log k_i$ must have more than seven decimals, the following approximate formulæ can be used with advantage: x being very small, we have nearly

$$\log(1 + x) = \frac{Mx}{\sqrt{1 + x}} \qquad \log(1 - x) = -\frac{Mx}{\sqrt{1 - x}}$$

Our series now takes the form

$$\Sigma_i k_i \cos (i\theta - \mathrm{K}_i)$$

or, restoring for θ its value $\mathrm{Q} - \varepsilon$,

$$\Sigma_i \{ k_i \cos [i(\mathrm{Q} - g') - \mathrm{K}_i] \cos i(g' - \varepsilon) + k_i \sin [i(\mathrm{Q} - g') - \mathrm{K}_i] \sin i(g' - \varepsilon) \}$$

Let us put

$$\mathrm{A}_i^{(c)} = \frac{1}{8} k_i \cos [i(\mathrm{Q} - g') - \mathrm{K}_i] \qquad \mathrm{A}_i^{(s)} = -\frac{1}{8} k_i \sin [i(\mathrm{Q} - g') - \mathrm{K}_i]$$

This division by 8 is made in order to save the constant division by these integers which occurs in the quadratures to follow. For this purpose log 8 has been subtracted from log N^n.

The form of our series is now

$$\Sigma_i [\mathrm{A}_i^{(c)} \cos i(g' - \varepsilon) + \mathrm{A}_i^{(s)} \sin i(g' - \varepsilon)]$$

The values of

$$\delta \log k_i = \log k_i - \log \left(b^{(i)} c^{(0)}\right)$$

and K_i follow. In the case of the former the numbers given are units of the last decimal place employed; the small figure at the top of the column indicates the order.

The values for the development of $\frac{a'}{\Delta}$ are:

	$\delta \log k_0$	$\delta \log k_1$	$\delta \log k_2$	$\delta \log k_3$	$\delta \log k_4$	$\delta \log k_5$	$\delta \log k_6$	$\delta \log k_7$	$\delta \log k_8$
	10	10	10	9	9	8	7	[illegible]	6
(0)	−390183	−2436623	−1793306	−167087	−161998	−15924	−1576	−156	−16
(1)	406799	2504487	1845594	172035	166830	16401	1623	161	16
(2)	−126309	− 781866	− 577338	− 53765	− 52127	− 5124	− 506	− 50	− 5
(3)	+227251	+1443713	+1060737	+ 98785	+ 95753	+ 9411	+ 931	+ 92	+ 9
(4)	404078	2666780	1955126	181809	176117	17302	1711	170	17
(5)	326817	2254312	1645886	152880	148005	14535	1437	143	14
(6)	+ 85703	+ 617318	+ 447659	+ 41573	+ 40231	+ 3950	+ 391	+ 39	+ 4
(7)	−172497	−1285014	− 932793	− 86452	− 83607	− 8206	− 811	− 80	− 8
(8)	330048	2518043	1821824	168818	163226	16017	1583	157	16
(9)	329268	2536963	1834082	169912	164263	16117	1593	157	16
(10)	−174598	−1338582	− 969398	− 89782	− 86795	− 8516	− 841	− 83	− 8
(11)	+ 72156	+ 543770	+ 392696	+ 36431	+ 35235	+ 3458	+ 342	+ 34	+ 3
(12)	300855	2196182	+1595893	148012	143183	14054	1389	138	14
(13)	381342	2674751	1950814	181099	175282	17211	1701	169	17
(14)	+229910	+1543874	+1128914	+104963	+101659	+ 9986	+ 988	+ 98	+10
(15)	− 99593	− 641048	− 471966	− 43894	− 42530	− 4179	− 413	− 41	− 4

	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8
	″	″	″	″	″	″	″	″
(0)	+ 44.7526	+27.93535	+24.3214	+22.6863	+21.745	+21.15	+20.7	+20
(1)	− 39.4886	−24.62136	−21.4235	−19.9755	−19.142	−18.61	−18.2	−18
(2)	95.0603	59.37668	51.6595	48.1724	46.166	44.88	44.0	44
(3)	84.6114	53.04022	46.1733	43.0825	41.305	40.12	39.3	39
(4)	− 19.4464	−12.23565	−10.6651	− 9.9605	− 9.556	− 9.28	− 9.1	− 9
(5)	+ 56.3853	+35.57332	+31.0614	+29.0403	+27.882	+27.09	+26.5	+26
(6)	99.6608	62.96929	55.0840	51.5518	49.528	48.19	47.2	47
(7)	90.5860	57.27495	50.1812	47.0014	45.180	44.02	43.2	43
(8)	+ 36.7892	+23.27107	+20.4096	+19.1262	+18.392	+17.93	+17.6	+17
(9)	− 34.6673	−21.94106	−19.2491	−18.0423	−17.352	−16.92	−16.6	−16
(10)	89.7918	56.86664	49.8700	46.7373	44.945	43.80	43.0	43
(11)	101.0218	64.00032	56.0709	52.5251	50.497	49.18	48.2	48
(12)	− 59.6880	−37.79195	−33.0644	−30.9519	−29.743	−28.92	−28.3	−28
(13)	+ 15.7935	+ 9.97826	+ 8.7172	+ 8.1530	+ 7.830	+ 7.61	+ 7.5	+ 7
(14)	83.0739	52.28706	45.6163	42.6220	40.903	39.75	39.0	39
(15)	+ 97.3145	+60.96963	+53.1300	+49.5953	+47.563	+46.24	+45.3	+45

The values for the development of $\left(\frac{a'}{\Delta}\right)^3$ are:

	$\delta \log k_0$	$\delta \log k_1$	$\delta \log k_2$	$\delta \log k_3$	$\delta \log k_4$	$\delta \log k_5$	$\delta \log k_6$	$\delta \log k_7$	$\delta \log k_8$
	8	8	8	8	7	7	6	6	5
(0)	−29535	−38906	−41121	−42146	−4275	−4314	−435	−437	−44
(1)	30710	40185	42429	43472	4408	4450	448	450	45
(2)	− 9551	−12534	−13247	−13577	−1377	−1389	−140	−141	−14
(3)	+17248	+22904	+24234	+24851	+2520	+2545	+256	+257	+26
(4)	30886	41784	44347	45527	4621	4666	470	472	47
(5)	25169	34807	37061	38085	3868	3908	394	396	40
(6)	+ 6642	+ 9388	+10016	+10304	+1047	+1058	+107	+108	+11
(7)	−13455	−19372	−20737	−21361	−2172	−2196	−221	−222	−22
(8)	25831	37661	40367	41603	4233	4278	431	433	43
(9)	25807	37829	40575	41828	4254	4301	433	435	44
(10)	−13677	−19999	−21451	−22113	−2249	−2275	−229	−230	−23
(11)	+ 5631	+ 8151	+ 8722	+ 8984	+ 914	+ 923	+ 93	+ 93	+ 9
(12)	23382	33296	35586	36634	3724	3764	379	381	38
(13)	29450	41079	43786	45027	4575	4621	465	467	47
(14)	+17623	+24043	+25541	+26226	+2664	+2690	+271	+272	+27
(15)	− 7586	−10143	−10751	−11031	−1120	−1130	−114	−115	−12

	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8
	″	″	″	″	″	″	″	″
(0)	+29.449	+37.646	+ 41.712	+ 44.17	+ 45.82	+ 47.1	+ 48.0	+ 48
(1)	−25.781	−32.995	− 36.580	− 38.75	− 40.22	− 41.3	− 42.1	− 43
(2)	62.288	79.715	88.358	93.58	97.10	99.7	101.6	103
(3)	56.210	71.845	79.563	84.22	87.34	89.7	91.4	93
(4)	−13.178	−16.802	− 18.579	− 19.65	− 20.36	− 20.9	− 21.3	− 21
(5)	+39.025	+49.603	+ 54.760	+ 57.84	+ 59.90	+ 61.5	+ 62.7	+ 63
(6)	70.295	89.066	98.166	103.59	107.20	110.0	112.0	113
(7)	64.837	81.928	90.186	95.06	98.35	100.9	102.7	104
(8)	+26.582	+33.530	+ 36.880	+ 38.87	+ 40.19	+ 41.2	+ 42.0	+ 42
(9)	−25.147	−31.701	− 34.858	− 36.73	− 37.98	− 39.0	− 39.8	− 40
(10)	65.022	82.022	90.208	95.04	98.28	100.8	102.7	104
(11)	72.622	91.755	100.984	106.46	110.12	112.9	114.9	116
(12)	−42.367	−53.654	− 59.114	− 62.36	− 64.52	− 66.2	− 67.4	− 68
(13)	+11.016	+13.988	+ 15.433	+ 16.30	+ 16.87	+ 17.3	+ 17.6	+ 18
(14)	56.760	72.260	79.850	84.42	87.48	89.7	91.4	92
(15)	+65.114	+83.088	+ 91.954	+ 97.32	+100.88	+103.5	+105.4	+107

The values for the development of $\left(\frac{a'}{\Delta}\right)^5$ are:

	$\delta \log k_0$	$\delta \log k_1$	$\delta \log k_2$	$\delta \log k_3$	$\delta \log k_4$	$\delta \log k_5$	$\delta \log k_6$
	7	7	7	7	7	7	7
(0)	−5939	−6209	−6444	−6593	−6717	−6812	−6887
(1)	6158	6425	6648	6811	6936	7032	7108
(2)	−1919	−2003	−2074	−2126	−2164	−2194	−2219
(3)	+3479	+3644	+3776	+3880	+3953	+4010	+4057
(4)	6280	6615	6882	7076	7221	7331	7421
(5)	5164	5477	5718	5895	6023	6123	6200
(6)	+1374	+1467	+1538	+1588	+1626	+1654	+1677
(7)	−2805	−3014	−3171	−3282	−3363	−3424	−3463
(8)	5409	5837	6154	6379	6540	6662	6758
(9)	5415	5855	6177	6407	6573	6695	6792
(10)	−2868	−3098	−3268	−3388	−3474	−3540	−3591
(11)	+1176	+1264	+1332	+1379	+1413	+1439	+1460
(12)	4852	5194	5453	5637	5772	5875	5954
(13)	6064	6449	6744	6957	7116	7235	7326
(14)	+3599	+3796	+3955	+4073	+4157	+4223	+4275
(15)	−1536	−1613	−1674	−1720	−1754	−1780	−1800

	K_1	K_2	K_3	K_4	K_5	K_6
	″	″	″	″	″	″
(0)	+20.74	+34.52	+ 43.90	+ 50.70	+ 55.79	+ 59.76
(1)	−18.07	−30.11	− 38.36	− 44.33	− 48.83	− 52.35
(2)	43.78	72.85	92.80	107.27	117.97	126.44
(3)	39.98	66.38	84.10	96.98	106.63	114.13
(4)	− 9.51	−15.65	− 19.81	− 22.80	− 25.03	− 26.75
(5)	+28.58	+46.79	+ 59.03	+ 67.72	+ 74.20	+ 79.18
(6)	52.14	85.00	106.88	122.34	133.78	142.60
(7)	48.58	78.92	98.97	113.05	123.51	131.49
(8)	+20.04	+32.49	+ 40.66	+ 46.41	+ 50.64	+ 53.90
(9)	−19.02	−30.79	− 38.53	− 43.94	− 47.93	− 50.99
(10)	49.13	79.56	99.56	113.61	123.94	131.82
(11)	54.64	88.58	111.02	126.80	138.42	147.30
(12)	−31.61	−51.38	− 64.53	− 73.82	− 80.70	− 85.96
(13)	+ 8.12	+13.25	+ 16.70	+ 19.15	+ 20.96	+ 22.36
(14)	41.18	67.66	85.56	98.30	117.82	115.21
(15)	+46.45	+76.88	+ 97.57	+112.40	+123.54	+132.12

The values for the development of $\left(\frac{a'}{\Delta}\right)^7$ are:

	$\delta \log k_0$	$\delta \log k_1$	$\delta \log k_2$	$\delta \log k_3$	$\delta \log k_4$	$\delta \log k_5$	$\delta \log k_6$
	5	5	5	5	5	5	5
(0)	−89	−77	−80	−83	−84	−85	−86
(1)	−93	−80	−84	−87	−88	−89	−90
(2)	−29	−25	−26	−26	−26	−26	−27
(3)	+52	+46	+47	+47	+48	+48	+49
(4)	+94	+82	+86	+87	+90	+91	+92
(5)	+78	+69	+71	+72	+74	+75	+76
(6)	+20	+18	+19	+20	+20	+21	+21
(7)	−42	−37	−40	−41	−41	−42	−42
(8)	−81	−73	−77	−80	−81	−83	−84
(9)	−81	−74	−77	−80	−81	−83	−84
(10)	−43	−38	−41	−42	−43	−44	−45
(11)	+18	+16	+16	+16	+17	+17	+17
(12)	+73	+65	+68	+70	+72	+73	+74
(13)	+91	+80	+84	+86	+89	+90	+92
(14)	+54	+47	+50	+51	+52	+53	+53
(15)	−22	−20	−21	−21	−21	−21	−21

	K_1	K_2	K_3	K_4	K_5	K_6
	″	″	″	″	″	″
(0)	+16	+ 41	+ 44	+ 58	+ 67	+ 72
(1)	−16	− 35	− 38	− 50	− 58	− 62
(2)	−39	− 87	− 93	−122	−140	−151
(3)	−38	− 77	− 84	−111	−128	−138
(4)	− 7	− 18	− 20	− 26	− 30	− 32
(5)	+25	+ 55	+ 59	+ 78	+ 90	+ 97
(6)	+41	+102	+107	+139	+160	+171
(7)	+40	+ 92	+ 99	+130	+150	+161
(8)	+16	+ 39	+ 41	+ 53	+ 61	+ 65
(9)	−15	− 36	− 39	− 51	− 59	− 64
(10)	−39	− 95	−100	−130	−148	−158
(11)	−44	−104	−111	−145	−167	−180
(12)	−25	− 61	− 65	− 84	− 97	−103
(13)	+ 6	+ 15	+ 17	+ 23	+ 26	+ 28
(14)	+33	+ 80	+ 86	+112	+131	+138
(15)	+37	+ 90	+ 97	+128	+148	+159

Then in the expansion of $\frac{a'}{\triangle}$ we have:

	A_0	$A_1^{(c)}$	$A_1^{(s)}$	$A_2^{(c)}$	$A_2^{(s)}$	$A_3^{(c)}$
	15	10	11	11	11	11
(0)	1450228368	+ 202026552	— 849063514	— 3409958446	— 1718843096	—118646864
(1)	1463004962	165534743	871976671	3640004664	1434802629	101473601
(2)	1457425396	127235650	872119792	3710027586	1108633113	79125332
(3)	1435004518	96594253	850441063	3615257970	834129533	59175347
(4)	1401552842	79393961	814422998	3410294766	671741875	46834975
(5)	1364376712	76442920	773386511	3163570196	630325000	42931597
(6)	1329783024	85298720	734687422	2923596837	686107798	45577632
(7)	1302096579	102538880	702712207	2714169129	807553772	52363664
(8)	1283824520	125058969	679625103	2544173764	968420038	61418720
(9)	1276200852	150383334	666455050	2417898623	1150402242	71594188
(10)	1279640398	176449375	663868971	2341401552	1341099104	82325461
(11)	1293937512	201197979	672502830	2325293606	1530361315	93402992
(12)	1318180590	222136767	692842956	2384491313	1705621583	104636227
(13)	1350401285	236036572	724598535	2533891037	1846379573	115315760
(14)	1387058565	239080805	765531034	2777564222	1920143086	123519532
(15)	1422671462	+ 227952301	— 810089284	— 3091426682	— 1886658091	—125827915
S	1.0907693703	+1256680799	—6072161790	—23501508486	—10120609693	—662084743
S′	1.0907693882	+1256680982	—6072162151	—23501511907	—10120612155	—662085064

	$A_3^{(s)}$	$A_4^{(c)}$	$A_4^{(s)}$	$A_5^{(c)}$	$A_5^{(s)}$	$A_6^{(c)}$
	11	11	11	10	10	9
(0)	+140789048	+ 552656433	+ 746681339	+ 44256400	— 18909459	— 430248
(1)	160676350	705980548	659132055	40670344	29771961	1149843
(2)	169946161	793843408	521340176	32814529	36931974	1682626
(3)	167128543	796919804	388716382	24469122	38400170	1848808
(4)	155942105	738727075	302828944	18789498	35541528	1719023
(5)	141337426	654118754	271278616	16451022	30740457	1451923
(6)	126598317	565439767	280608214	16560520	25512780	1149523
(7)	113095826	481059746	313840044	17979990	20396865	847724
(8)	101197466	402679958	358400034	19894049	15494742	554261
(9)	91076536	331071140	407325962	21886552	10854325	273151
(10)	83150239	268703826	458447644	23891442	6610129	— 11412
(11)	78272599	220785424	513193884	26133786	3020832	+ 220900
(12)	77792910	196036282	574685970	29039408	— 528111	407502
(13)	83419434	207275019	643902015	33019360	+ 112284	513211
(14)	96589768	269480117	712983914	37983836	— 2246167	465530
(15)	+116999681	+ 390360914	+ 758591663	+ 42620115	— 8703060	+ 163229
S	+952006014	+3787566866	+3955976235	+223229682	−141774890	—4674061
S′	+952006395	+3787571349	+3955980619	+223230291	—141775386	—4674109

	$A_6^{(s)}$	$A_7^{(c)}$	$A_7^{(s)}$	$A_8^{(c)}$	$A_8^{(s)}$	$A_9^{(c)}$
	9	8	8	8	8	7
(0)	− 2500211	−135116	− 8518	−21291	+ 69732	+ 3413
(1)	2416648	139155	+ 36468	+ 5342	77852	4235
(2)	2001634	119178	73675	30086	69535	3985
(3)	1497651	89792	88198	41353	52961	3080
(4)	1135085	67264	83033	39845	39269	2264
(5)	971348	56259	68462	32055	32100	1808
(6)	950371	53425	51274	22448	29543	1611
(7)	998643	54156	33907	12681	28787	1503
(8)	1064833	55310	16996	+ 3267	27958	1377
(9)	1124512	55492	+ 899	− 5493	26296	1192
(10)	1176841	54807	− 14025	13417	23944	961
(11)	1241277	54617	27559	20607	21746	735
(12)	1353841	57396	39514	27439	21178	590
(13)	1557889	66507	48804	34008	24428	649
(14)	1876019	84979	51581	38690	34138	1101
(15)	− 2250948	−111506	− 40225	−36511	+ 51140	+ 2101
S	−12058835	−627475	+111340	− 5191	+315297	+15302
S′	−12058916	−627484	+111346	− 5188	+315310	+15303

	$A_9^{(s)}$	$A_{10}^{(c)}$	$A_{10}^{(s)}$	$A_{11}^{(c)}$	$A_{11}^{(s)}$	$A_{12}^{(c)}$	$A_{12}^{(s)}$
	7	7	7	7	7	7	7
(0)	+1999	+1491	−1559	− 642	− 992	− 613	+ 217
(1)	+ 507	+ 708	2233	1138	614	452	556
(2)	−1073	− 265	2244	1243	− 34	− 118	677
(3)	1887	825	1770	1006	+ 336	+ 120	566
(4)	1889	880	1292	731	400	175	410
(5)	1482	673	1009	557	297	126	305
(6)	954	387	868	462	+ 144	+ 47	244
(7)	− 417	− 98	772	390	− 7	− 32	193
(8)	+ 90	+ 166	659	306	140	96	137
(9)	544	389	511	204	242	139	72
(10)	935	565	339	− 90	313	161	+ 3
(11)	1282	714	167	+ [illegible]	365	171	− 63
(12)	1632	874	− 32	120	426	188	124
(13)	2039	1100	+ 12	180	539	237	175
(14)	2462	1408	− 167	+ 137	736	351	186
(15)	+2615	+1656	− 707	− 125	− 963	− 523	− 76
S	+3202	+2972	−7160	−3217	−2097	−1305	+1378
S′	+3201	+2971	−7157	−3218	−2097	−1308	+1378

In the expansion of $\left(\frac{a'}{\Delta}\right)^3$ we have:

	A_0	$A_1^{(c)}$	$A_1^{(s)}$	$A_2^{(c)}$	$A_2^{(s)}$	$A_3^{(c)}$	$A_3^{(s)}$
	8	8	8	8	8	8	8
(0)	35089976	+12331006	− 51840998	− 32542992	−16405728	−15328172	+ 18185642
(1)	36399108	10368258	54596399	35586989	14025867	13418173	21250198
(2)	35831498	7888818	54012776	35901058	10724124	10356352	22253821
(3)	33605553	5730867	50394167	33541581	7735653	7433543	21005313
(4)	30505218	4403645	45158364	29691228	5847739	5531546	18420444
(5)	27354241	3930701	39801819	25639731	5110397	4730376	15566695
(6)	24680381	4087409	35249120	22146607	5200308	4701661	13050975
(7)	22707163	4644416	31856614	19464988	5794001	5118886	11050226
(8)	21482105	5458113	29670020	17593578	6697861	5791861	9541327
(9)	20990030	6463806	28639546	16468922	7834727	6650155	8461126
(10)	21215032	7640861	28733896	16061400	9196986	7699370	7779549
(11)	22161697	8974548	29982329	16415743	10800639	8985035	7532876
(12)	23845661	10412474	32467052	17662465	12631871	10552267	7847264
(13)	26250114	11809186	36255364	19988913	14565988	12370479	8948193
(14)	29236542	12872141	41234772	23514502	16259050	14199125	11099630
(15)	32418555	+13172728	− 46840846	− 28007090	−17096490	−15453849	+ 14364166
S	2. 21886413	+65094467	−3. 18366998	−1. 95113830	−82963667	−74160354	+1. 08178652
S′	2. 21886461	+65094510	−3. 18367084	−1. 95113957	−82963762	−74160496	+1. 08178793

	$A_4^{(c)}$	$A_4^{(s)}$	$A_5^{(c)}$	$A_5^{(s)}$	$A_6^{(c)}$	$A_6^{(s)}$	$A_7^{(c)}$	$A_7^{(s)}$
	8	8	8	8	7	7	7	7
(0)	+ 8987554	+12145522	+ 8673540	− 3704752	− 98548	− 573100	− 354546	− 22400
(1)	11751919	10970062	8153752	5970082	269656	566582	373435	+ 97911
(2)	13086548	8590198	6514222	7335251	390879	464736	316759	195941
(3)	12617811	6151518	4667605	7328629	412819	334245	229471	225476
(4)	11003715	4510084	3375280	6385368	361504	238674	162012	200018
(5)	9093835	3772919	2761546	5158312	285325	190954	126732	154166
(6)	7364525	3657075	2606588	4013196	211855	175258	112934	108319
(7)	5942540	3878846	2685155	3044511	148300	174800	108717	68008
(8)	4802403	4275141	2868451	2233644	93667	180000	107227	32935
(9)	3891580	4787033	3109732	1542615	45528	187345	106040	+ 1729
(10)	3181133	5424561	3417701	946538	− 1968	197398	105446	− 26951
(11)	2688125	6243783	3843124	445356	+ 38016	214001	107992	54448
(12)	2500012	7325205	4471921	− 82081	73496	244336	118769	81732
(13)	2807451	8722558	5400372	+ 18601	98335	298456	146025	107167
(14)	3905202	10338624	6644708	− 391428	95436	384195	199357	121075
(15)	+ 6037771	+11739930	+ 7951480	− 1621558	+ 35769	− 491360	− 278717	−100637
S	+54831092	+56266410	+38572411	−25092258	−989489	−2457697	−1477050	+285055
S′	+54831032	+56266649	+38572766	−25092462	−989508	−2457743	−1477129	+285038

	$A_8^{(c)}$	$A_8^{(s)}$	$A_9^{(c)}$	$A_9^{(s)}$	$A_{10}^{(c)}$	$A_{10}^{(s)}$	$A_{11}^{(c)}$	$A_{11}^{(s)}$	$A_{12}^{(c)}$	$A_{12}^{(s)}$
	6	6	6	6	6	6	6	6	6	6
(0)	− 6296	+20609	+11218	+6569	+5392	− 5640	− 2537	−3918	−2623	+ 930
(1)	+ 1617	23513	14225	+1703	+2619	8256	4593	2478	1981	2437
(2)	9014	20816	13263	−3575	− 971	8222	4969	− 137	− 512	2931
(3)	11912	15247	9866	6045	2909	6243	3874	+1295	+ 501	2361
(4)	10815	10658	6838	5707	2928	4298	2654	1452	688	1613
(5)	8136	8151	5112	4187	2095	3140	1894	1010	465	1125
(6)	5346	7041	4274	2529	1131	2535	1475	+ 463	+ 162	843
(7)	2868	6516	3787	−1049	− 271	2141	1181	− 22	− 105	636
(8)	+ 714	6114	3351	+ 220	+ 446	1768	897	410	305	435
(9)	− 1183	5669	2860	1304	1028	1352	584	696	436	225
(10)	2910	5197	2325	2257	1505	902	− 261	910	509	+ 10
(11)	4593	4851	1827	3181	1957	457	+ 67	1090	556	− 204
(12)	6401	4942	1534	4241	2502	− 90	375	1332	636	421
(13)	8418	6046	1788	5619	3339	+ 36	596	1787	852	629
(14)	10233	9024	3238	7244	4561	− 543	+ 485	2604	1346	712
(15)	−10287	+14400	+ 6580	+8193	+5714	− 2437	− 471	−3627	−2124	− 309
S	+ 49	+84401	+46041	+8720	+9376	−23998	−11933	−7396	−5081	+5629
S′	+ 52	+84393	+46045	+8719	+9382	−23990	−11934	−7395	−5088	+5642

In the expansion of $\left(\frac{a'}{\Delta}\right)^5$ we have:

	A_0	$A_1^{(c)}$	$A_1^{(s)}$	$A_2^{(c)}$	$A_2^{(s)}$	$A_3^{(c)}$	$A_3^{(s)}$
	7	6	6	6	6	6	6
(0)	1. 3976550	+ 592456	−2. 491221	−1. 883176	− 949320	−1. 045379	+1. 240231
(1)	1. 4993293	513843	2. 705203	2. 119180	835265	940458	1. 489421
(2)	1. 4549244	386015	2. 641289	2. 111756	630886	717393	1. 541628
(3)	1. 2868392	265827	2. 335899	1. 876627	432856	490916	1. 387302
(4)	1. 0694995	188399	1. 931629	1. 540895	303491	340026	1. 132334
(5)	8687136	153604	1. 556182	1. 223261	243792	268332	882959
(6)	7142908	146793	1. 266890	976410	229253	247320	686426
(7)	6097718	155897	1. 069910	805313	239699	253352	546854
(8)	5489399	175228	952703	697844	265665	275293	453490
(9)	5254040	203732	902561	641865	305357	310798	395449
(10)	5361580	242957	913370	631077	361373	362639	366449
(11)	5824171	295627	987320	666748	438698	436873	366302
(12)	6692240	363961	1. 134661	758545	542510	541254	402529
(13)	8032381	446366	1. 370453	923946	673277	680769	492428
(14)	9861677	531488	1. 703027	1. 180903	816494	845868	661188
(15)	1. 2011551	+ 592412	− 2. 107287	−1. 523724	− 930071	− 993712	+ 923593
S	7. 3768593	+2. 627297	−13. 034790	−9. 780606	−4. 098992	−4. 375172	+6. 484275
S′	7. 3768682	+2. 627308	−13. 034815	−9. 780664	−4. 099015	−4. 375210	+6. 484308

	$A_4^{(c)}$	$A_4^{(s)}$	$A_5^{(c)}$	$A_5^{(s)}$	$A_6^{(c)}$	$A_6^{(s)}$	$A_7^{(c)}$	$A_7^{(s)}$
	6	6	6	6	6	6	4	4
(0)	+ 707914	+ 956717	+ 776027	− 331422	− 98757	− 574528	− 3938	− 249
(1)	950482	887199	748488	548082	277250	582456	4252	+1115
(2)	1.046573	686886	591413	666088	397615	472620	3568	2208
(3)	963619	469716	405153	636264	401857	325291	2476	2434
(4)	784005	321333	273852	518114	329325	217416	1638	2022
(5)	599415	248724	207718	387933	241255	161471	1191	1448
(6)	451178	224097	182558	280994	166988	138186	990	949
(7)	343180	224045	177502	201208	110443	130216	901	564
(8)	266662	237420	182468	142072	67180	129120	856	263
(9)	212618	261523	194670	96580	32157	132289	834	+ 14
(10)	175144	298598	215540	59723	− 1421	140411	835	− 213
(11)	152653	354476	249809	28917	+ 27831	156817	881	444
(12)	149515	438010	305799	− 5637	56588	188189	1018	700
(13)	179707	558365	394707	+ 1368	80858	245389	1335	979
(14)	269867	714591	523400	− 30781	84488	339942	1958	1190
(15)	+ 449238	+ 873662	+ 673097	− 137189	+ 34028	− 466563	− 2935	−1060
S	+3.850858	+3.877652	+3.051057	−2.034831	−920210	−2.200412	−1.4801	+3090
S′	+3.850912	+3.877710	+3.051044	−2.034805	−920245	−2.200492	−1.4805	+3092

	$A_8^{(c)}$	$A_8^{(s)}$	$A_9^{(c)}$	$A_9^{(s)}$	$A_{10}^{(c)}$	$A_{10}^{(s)}$	$A_{11}^{(c)}$	$A_{11}^{(s)}$	$A_{12}^{(c)}$	$A_{12}^{(s)}$
	4	4	4	4	4	4	4	4	4	4
(0)	− 768	+2513	+1490	+ 873	+ 775	− 811	− 392	− 606	−434	+ 154
(1)	+ 202	2940	1936	+ 231	+ 385	1216	727	392	335	412
(2)	1115	2574	1786	− 482	− 142	1198	779	− 21	− 86	492
(3)	1412	1807	1274	781	407	872	582	+ 195	+ 81	380
(4)	1202	1184	829	692	384	564	375	205	104	244
(5)	841	843	577	472	256	384	249	133	66	159
(6)	516	680	451	266	129	290	182	+ 57	+ 21	111
(7)	262	595	378	− 105	− 29	232	138	− 3	− 13	80
(8)	+ 63	538	323	+ 21	+ 47	185	101	46	37	53
(9)	− 103	492	271	124	106	139	65	78	52	27
(10)	254	454	222	215	156	93	− 29	102	61	+ 1
(11)	413	436	180	312	208	49	+ 8	125	69	− 25
(12)	604	467	158	437	280	− 10	45	161	82	54
(13)	847	608	196	617	397	+ 3	76	229	117	86
(14)	1106	975	381	854	582	− 69	+ 67	358	198	105
(15)	−1190	+1666	+ 829	+1034	+ 780	− 332	− 69	− 533	−334	− 49
S	+ 164	+9385	+5640	+ 960	+ 1185	−3220	−1746	−1032	−773	+ 896
S′	+ 164	+9387	+5641	+ 960	+ 1184	−3221	−1746	−1032	−773	+ 898

In the expansion of $\left(\frac{a'}{\triangle}\right)^7$ we have:

	A_0	$A_1^{(c)}$	$A_1^{(s)}$	$A_2^{(c)}$	$A_2^{(s)}$	$A_3^{(c)}$	$A_3^{(s)}$	$A_4^{(c)}$	$A_4^{(s)}$
(0)	6.7363	+ 2.9876	—12.5626	—10.2662	— 5.1752	— 6.2209	+ 7.3804	+ 4.6006	+ 6.2189
(1)	7.4755	2.6776	14.0955	11.9195	4.6978	5.7671	9.1336	6.3606	5.9367
(2)	7.1499	1.9830	13.5684	11.7182	3.5007	4.3425	9.3319	6.9158	4.5389
(3)	5.9601	1.2895	11.3305	9.8582	2.2733	2.8192	7.9670	6.0529	2.9500
(4)	4.5307	8385	8.5968	7.4569	1.4687	1.8052	6.0115	4.5650	1.8710
(5)	3.3293	6208	6.2910	5.4011	1.0766	1.3047	4.2932	3.2061	1.3305
(6)	2.4913	5423	4.6801	3.9569	9290	1.1077	3.0745	2.2294	1.1073
(7)	1.9708	5354	3.6757	3.0456	9067	1.0621	2.2925	1.5907	1.0386
(8)	1.6869	5737	3.1195	2.5213	9599	1.1046	1.8196	1.1849	1.0549
(9)	1.5810	6539	2.8964	2.2753	1.0823	1.2245	1.5581	9283	1.1417
(10)	1.6291	7869	2.9582	2.2566	1.2922	1.4407	1.4558	7708	1.3140
(11)	1.8415	9945	3.3208	2.4714	1.6259	1.7963	1.5062	6946	1.6126
(12)	2.2622	1.3047	4.0674	2.9871	2.1364	2.3586	1.7541	7195	2.1078
(13)	2.9644	1.7402	5.3430	3.9414	2.8722	3.2026	2.3166	9310	2.8929
(14)	4.0174	2.2785	7.3010	5.5143	3.8127	4.3397	3.3922	1.5203	4.0256
(15)	5.3812	+ 2.7831	— 9.9015	— 7.7644	— 4.7401	— 5.5435	+ 5.1524	+ 2.7439	+ 5.3358
S	30.5038	+11.2952	—56.8540	—46.6775	—19.2748	—22.7199	+34.2200	+22.5063	+22.2384
S′	30.5038	+11.2950	—56.8544	—46.6769	—19.2749	—22.7200	+34.2196	+22.5081	+22.2388

	$A_5^{(c)}$	$A_5^{(s)}$	$A_6^{(c)}$	$A_6^{(s)}$	$A_7^{(c)}$	$A_7^{(s)}$	$A_8^{(c)}$	$A_8^{(s)}$	$A_9^{(c)}$	$A_9^{(s)}$
										3
(0)	+ 5.4939	— 2.3463	— 7583	— 4.4113	— 3.271	— 206	— 684	+2.244	+1.424	+834
(1)	5.4500	3.9912	2.1885	4.5973	3.627	+ 950	+ 185	2.697	1.898	+227
(2)	4.2543	4.7914	3.1009	3.6857	3.007	1.858	1.008	2.331	1.732	—466
(3)	2.7729	4.3556	2.9873	2.4176	1.986	1.950	1.217	1.559	1.174	719
(4)	1.7424	3.2964	2.2785	1.5042	1.224	1.510	966	953	715	597
(5)	1.2171	2.2725	1.5399	1.0310	822	1.000	626	627	460	377
(6)	9903	1.5243	9888	8182	636	611	358	471	336	199
(7)	9050	1.0256	6153	7257	546	342	172	390	266	— 74
(8)	8920	6950	3596	6911	500	154	+ 40	339	219	+ 14
(9)	9362	4646	1693	6964	478	+ 8	— 64	305	182	83
(10)	1.0446	2895	— 75	7447	482	— 124	159	283	149	145
(11)	1.2505	1452	+ 1522	8584	524	264	266	280	124	216
(12)	1.6167	— 298	3267	1.0866	637	439	409	315	115	319
(13)	2.2419	+ 78	5008	1.5198	895	656	613	440	152	479
(14)	3.2248	— 1897	5665	2.2794	1.420	862	863	731	320	715
(15)	+ 4.4857	— 9142	+ 2464	— 3.3774	— 2.298	— 828	—1.001	+1.404	+ 749	+931
S	+19.2596	—13.1624	—6.6004	—15.2212	—11.177	+2.502	+ 257	+7.667	+5.010	+765
S′	+19.2593	—13.1611	—6.6009	—15.2236	—11.176	+2.502	+ 256	+7.702	+5.005	+766

The next step is to apply the process of mechanical quadratures to the preceding values. Let us denote any one of the coefficients $A_i^{(c)}$, $A_i^{(s)}$ by Y; and the sixteen special values of Y by $Y_0, Y_1, Y_2 \;.\;.\;.\;.\; Y_{15}$. If Y is developed in a periodic series as a function of g', we have

$$\begin{aligned} Y = c_0 + c_1 \cos g' + c_2 \cos 2g' + c_3 \cos 3g' + \;.\;.\;. \\ + s_1 \sin g' + s_2 \sin 2g' + s_3 \sin 3g' + \;.\;.\;. \end{aligned}$$

and the following formulæ determine c_0, c_1, s_1, etc., from the special values of Y.* Let

$$\begin{array}{ll} (0.8) = Y_0 + Y_8 & \left(\frac{0}{8}\right) = Y_0 - Y_8 \\ (1.9) = Y_1 + Y_9 & \left(\frac{1}{9}\right) = Y_1 - Y_9 \\ (2.10) = Y_2 + Y_{10} & \left(\frac{2}{10}\right) = Y_2 - Y_{10} \\ .\;.\;.\;.\;.\;.\;. & .\;.\;.\;.\;.\;.\;. \\ (7.15) = Y_7 + Y_{15} & \left(\frac{7}{15}\right) = Y_7 - Y_{15} \\ \hline (0.4) = (0.8) + (4.12) & (0.2) = (0.4) + (2.6) \\ (1.5) = (1.9) + (5.13) & (1.3) = (1.5) + (3.7) \\ (2.6) = (2.10) + (6.14) & \\ (3.7) = (3.11) + (7.15) & \end{array}$$

Then,

$$\begin{aligned}
8\,(c_0 + c_8) &= (0.2) \\
8\,(c_0 - c_8) &= (1.3) \\
4\,(c_2 + c_6) &= (0.8) - (4.12) \\
4\,(c_2 - c_6) &= \{[(1.9) - (5.13)] - [(3.11) - (7.15)]\} \cos 45^\circ \\
8c_4 &= (0.4) - (2.6) \\
4\,(s_2 + s_6) &= \{[(1.9) - (5.13)] + [(3.11) - (7.15)]\} \cos 45^\circ \\
4\,(s_2 - s_6) &= (2.10) - (6.14) \\
8s_4 &= (1.5) - (3.7) \\
4\,(c_1 + c_7) &= \left(\frac{0}{8}\right) + \left\{\left(\frac{2}{10}\right) - \left(\frac{6}{14}\right)\right\} \cos 45^\circ \\
4\,(c_1 - c_7) &= \left\{\left(\frac{1}{9}\right) - \left(\frac{7}{15}\right)\right\} \cos 22^\circ.5 + \left\{\left(\frac{3}{11}\right) - \left(\frac{5}{13}\right)\right\} \cos 67^\circ.5 \\
4\,(c_3 + c_5) &= \left(\frac{0}{8}\right) - \left\{\left(\frac{2}{10}\right) - \left(\frac{6}{14}\right)\right\} \cos 45^\circ \\
4\,(c_3 - c_5) &= \left\{\left(\frac{1}{9}\right) - \left(\frac{7}{15}\right)\right\} \sin 22^\circ.5 - \left\{\left(\frac{3}{11}\right) - \left(\frac{5}{13}\right)\right\} \sin 67^\circ.5 \\
4\,(s_1 + s_7) &= \left\{\left(\frac{1}{9}\right) + \left(\frac{7}{15}\right)\right\} \sin 22^\circ.5 + \left\{\left(\frac{3}{11}\right) + \left(\frac{5}{13}\right)\right\} \sin 67^\circ.5 \\
4\,(s_1 - s_7) &= \left\{\left(\frac{2}{10}\right) + \left(\frac{6}{14}\right)\right\} \cos 45^\circ + \left(\frac{4}{12}\right) \\
4\,(s_3 + s_5) &= \left\{\left(\frac{1}{9}\right) + \left(\frac{7}{15}\right)\right\} \cos 22^\circ.5 - \left\{\left(\frac{3}{11}\right) + \left(\frac{5}{13}\right)\right\} \cos 67^\circ.5 \\
4\,(s_3 - s_5) &= \left\{\left(\frac{2}{10}\right) + \left(\frac{6}{14}\right)\right\} \cos 45^\circ - \left(\frac{4}{12}\right)
\end{aligned}$$

---

*Auseinandersetzung, Abh. I, ss. 160, 161.

As the values of the Y have already been divided by 8 we have no need to make this division in obtaining the values of c_i and s_i.

If we suppose that

$$\begin{aligned} A_i^{(c)} &= C^c_{i,0} + C^c_{i,1} \cos g' + C^c_{i,2} \cos 2g' + \ldots \\ &\quad + C^s_{i,1} \sin g' + C^s_{i,2} \sin 2g' + \ldots \\ A_i^{(s)} &= S^c_{i,0} + S^c_{i,1} \cos g' + S^c_{i,2} \cos 2g' + \ldots \\ &\quad + S^s_{i,1} \sin g' + S^s_{i,2} \sin 2g' + \ldots \end{aligned}$$

the terms in the developed function, which belong to each value of ν, are*

$$\frac{1}{2}\left(C^c_{i,\nu} \pm S^s_{i,\nu}\right) \cos\left[(i \mp \nu)g' - i\varepsilon\right] \mp \frac{1}{2}\left(C^s_{i,\nu} \mp S^c_{i,\nu}\right) \sin\left[(i \mp \nu)g' - i\varepsilon\right]$$

except for $\nu = 0$, when we have

$$C^c_{i,0} \cos (ig' - i\varepsilon) + S^{(c)}_{i,0} \sin (ig' - i\varepsilon)$$

It may be observed that in all the following expansions the constant terms are not doubled as is generally done.

It is supposed that it will be sufficiently accurate to stop with the terms whose argument involves the twelfth multiple of the motion of either planet; but as the coefficients below this limit will be modified through corrections arising from terms involving higher multiplies, it has been deemed advisable to prolong the series considerably beyond this limit. Thus, some coefficients are needed which are not given by the division of the circumference into sixteen parts. However, it is possible to derive these with sufficient accuracy for our purpose by induction from the rest. For, when the terms involving the argument $(i + \nu)g' - i\varepsilon$ are put in the form

$$k \cos [(i + \nu)g' - i\varepsilon + \beta]$$

it is found that when i is tolerably large and ν not too large with reference to i, log k and β, for the same value of ν, will difference, and thus may be considered as continuous functions of i. These differences may be conjecturally prolonged, and values of k and β, for larger values of i, be obtained.

---

*Auseinandersetzung, Abh. I, s. 165, gl. (136).

I give here three specimens of this method of induction, taken from the development of $\frac{a'}{\Delta}$. All the quantities below the line have been inferred from the corresponding quantities above it:

Arg.	log k			β				
g' ε				°	′	°	′	
8 — 9	6.5752			160	16			
		— 2412				+ 79	5	
9 — 10	6.3340		— 52	239	21			
		2464				79	8	
10 — 11	6.0876		40	318	29			
		2504				79	4	
11 — 12	5.8372		30	37	33			
		2534				79	6	
12 — 13	5.5838		20	116	39			
		2554				79	6	
13 — 14	5.3284		— 13	195	45			
		— 2567				+ 79	6	
14 — 15	5.0717			274	51			
9 — 9	7.1941			348	11			
		— 3048				+ 79	16	
10 — 10	6.8893			67	27			+ 11
		3049				79	27	
11 — 11	6.5844			146	54			12
		3060				79	39	
12 — 12	6.2784			226	33			11
		3057				79	50	
13 — 13	5.9727			306	23			11
		3057				80	1	
14 — 14	5.6670			26	24			+ 11
		— 3057				+ 80	12	
15 — 15	5.3613			106	36			
10 — 9	7.1213			327	39			
		— 2516				+ 78	43	
11 — 10	6.8697		— 33	46	22			
		2549				78	49	
12 — 11	6.6148		17	125	11			
		2566				78	49	
13 — 12	6.3582		10	204	0			
		2576				78	47	
14 — 13	6.1006		5	282	47			
		2581				78	47	
15 — 14	5.8425		— 4	1	34			
		— 2585				+ 78	47	
16 — 15	5.5840			80	21			

All the coefficients in the following expressions, which belong to arguments having a multiple of ε higher than the twelfth, have been derived in this way.

These series are the developments of the first four odd powers of the reciprocal of the distance between the planets in terms of the mean anomaly of Saturn and the eccentric anomaly of Jupiter, having the general form, which may be denoted thus:

$$\Sigma_{i',i}[\mathrm{C}_{i',i}\cos(i'g'+i\varepsilon)+\mathrm{S}_{i',i}\sin(i'g'+i\varepsilon)]$$

Arg.	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
0 0	1.090769379		2.21886437		7.376863	
0 — 1	—0.009611762	—0.005222181	—0.13073646	—0.32233309	—0.919969	—2.722383
0 — 2	—0.000029218	+0.000403992	—0.01248822	+0.01678057	—0.228893	+0.216600
0 — 3	+0.000014879	—0.000003042	+0.00122596	+0.00012614	+0.026086	+0.008834
0 — 4	—0.000000353	—0.000000516	—0.00002614	—0.00006522	—0.000377	—0.002112
0 — 5	—0.000000011	+0.000000024	—0.00000220	+0.00000309	—0.000110	+0.000069

Arg.	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
1 + 4	−0.000000055	+0.000000028	−0.00000641	+0.00000630	−0.000173	+ 0.000218
1 + 3	+0.000001213	+0.000000886	+0.00013146	+0.00005714	+0.003737	+ 0.000801
1 + 2	+0.000010986	−0.000037004	−0.00025189	−0.00244367	−0.012724	− 0.045109
1 + 1	−0.001021715	+0.000073316	−0.03290290	+0.02458293	−0.369508	+ 0.328868
1 0	+0.066523370	+0.033717067	+0.54330098	+0.27546502	+3.381297	+ 1.715381
1 − 1	+0.125668089	−0.607216197	+0.65094488	−3.18367041	+2.627302	−13.034802
1 − 2	+0.003405534	+0.006381882	−0.16421930	+0.06482696	−1.913345	+ 0.395249
1 − 3	+0.000131740	+0.000061991	+0.00847042	+0.00620331	+0.121067	+ 0.146816
1 − 4	−0.000000988	−0.000006126	+0.00006848	−0.00060403	+0.005713	− 0.014632
1 − 5	−0.000000225	+0.000000147	−0.00003190	+0.00001265	−0.001207	+ 0.000159
1 − 6	+0.000000012	+0.000000005	+0.0000014	+0.0000013	+0.000046	+ 0.000059
2 + 4	−0.000000002	0.000000000	−0.00000215	+0.00000020	−0.000043	+ 0.000011
2 + 3	+0.000000066	+0.000000130	+0.00000996	+0.00001301	+0.000381	+ 0.000345
2 + 2	+0.000002326	−0.000002868	+0.00011554	−0.00026211	+0.001659	− 0.006430
2 + 1	−0.000088053	−0.000038336	−0.00479184	+0.00053154	−0.074745	+ 0.016074
2 0	+0.002864107	+0.004044874	+0.04443816	+0.06259484	+0.416081	+ 0.581322
2 − 1	+0.039793422	−0.062478629	+0.39593654	−0.56323564	+2.500797	− 3.415191
2 − 2	−0.235015102	−0.101206109	−1.95113893	−0.82963714	−9.780635	− 4.099003
2 − 3	+0.005066355	−0.003542935	+0.04056762	+0.07642442	+0.134383	+ 1.226405
2 − 4	+0.000069656	−0.000000812	+0.00313416	−0.00412321	+0.089266	− 0.065707
2 − 5	−0.000003104	+0.000000692	−0.00029492	−0.00003828	−0.008006	− 0.003483
2 − 6	+0.000000061	+0.000000099	+0.0000056	+0.0000158	+0.000072	+ 0.000634
2 − 7			+0.0000007	−0.0000011		
3 + 3	+0.000000001	+0.000000008	+0.00000030	+0.00000182	+0.000021	+ 0.000064
3 + 2	+0.000000312	−0.000000155	+0.00002632	−0.00001996	+0.000645	− 0.000645
3 + 1	−0.000005984	−0.000006925	−0.00051581	−0.00022369	−0.010596	− 0.003290
3 0	+0.000040251	+0.000369645	+0.00111299	+0.00916223	+0.015511	+ 0.115941
3 − 1	+0.006132504	−0.004211786	+0.09235900	−0.05316203	+0.800117	− 0.427144
3 − 2	−0.038011415	−0.035554695	−0.43469458	−0.43697708	−2.840628	− 2.974234
3 − 3	−0.066208490	+0.095200620	−0.74160425	+1.08178722	−4.375191	+ 6.484291
3 − 4	−0.002210673	−0.003847996	+0.03442962	−0.03119988	+0.740468	− 0.038081
3 − 5	+0.000053949	−0.000051903	−0.00180789	−0.00160898	−0.034747	− 0.052383
3 − 6	+0.000000564	+0.000002257	−0.0000211	+0.0001442	−0.002090	+ 0.004311
3 − 7	+0.00000003	−0.00000002	+0.0000091	−0.0000039	+0.0002	0.0000
4 + 3	+0.000000008	−0.000000001	+0.00000022	+0.00000016	+0.000002	+ 0.000009
4 + 2	+0.000000024	−0.000000004	+0.00000359	−0.00000060	+0.000108	− 0.000026
4 + 1	−0.000000225	−0.000000835	−0.00003936	−0.00005157	−0.001061	− 0.001118
4 0	−0.000012106	+0.000027374	−0.00040493	+0.00100525	−0.006223	+ 0.016502
4 − 1	+0.000689461	−0.000117837	+0.01440813	−0.00096355	+0.161572	− 0.004719
4 − 2	−0.003158672	−0.006601681	−0.04321484	−0.10925388	−0.335176	− 0.954530
4 − 3	−0.027082107	+0.019492364	−0.40318669	+0.27357050	−3.022802	+ 1.973497
4 − 4	+0.037875691	+0.039559784	+0.54831062	+0.56266530	+3.850885	+ 3.877681
4 − 5	−0.002708439	+0.001068270	−0.02530938	−0.01666339	−0.017672	− 0.434243
4 − 6	−0.000027431	−0.000066585	−0.0007872	+0.0006010	−0.029824	+ 0.017755
4 − 7	+0.00000199	−0.00000026	+0.0000743	+0.0000150	+0.0023	+ 0.0011
4 − 8			−0.000010	−0.000003	0.0000	− 0.0001

Arg	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
5 + 2	+0.000000001	−0.000000011	+0.00000046	−0.00000008	+0.000013	−0.000012
5 + 1	+0.000000006	−0.000000070	−0.00000120	−0.00000730	−0.000051	−0.000201
5 0	−0.000002061	+0.000001483	−0.00009749	+0.00007917	−0.001905	+0.001651
5 − 1	+0.0000611541	+0.0000183402	+0.00167590	+0.00071761	+0.023387	+0.011072
5 − 2	−0.0000440511	−0.0008744402	+0.00088227	−0.01843048	+0.019972	−0.198533
5 − 3	−0.005850270	+0.001589795	−0.10919015	+0.02304507	−0.999876	+0.172078
5 − 4	+0.008451522	+0.018617851	+0.14044483	+0.32787286	+1.118180	+2.719583
5 − 5	+0.022322999	−0.014177514	+0.38572588	−0.25092360	+3.051100	−2.034818
5 − 6	+0.000374093	+0.001776944	−0.0098351	+0.0197303	−0.255592	+0.020664
5 − 7	−0.00005898	+0.00000758	+0.0000239	+0.0003108	+0.0087	+0.0163
5 − 8	+0.00000010	−0.00000178	0.000000	−0.000043	+0.0007	−0.0011
6 + 1	+0.000000023	−0.000000008	−0.00000001	+0.00000015	+0.000004	−0.000028
6 0	−0.000000216	+0.000000033	−0.00001412	+0.00000270	−0.000337	+0.000076
6 − 1	+0.000004134	+0.000004042	+0.00013941	+0.00017120	+0.002336	+0.003077
6 − 2	+0.000034393	−0.000089303	+0.00121750	−0.00228942	+0.018026	−0.029410
6 − 3	−0.000882242	−0.000098703	−0.01984301	−0.00393200	−0.215399	−0.054884
6 − 4	+0.000343775	+0.004551938	+0.00267385	+0.09583348	−0.005696	+0.935477
6 − 5	+0.011872674	−0.002749242	+0.24231846	−0.05190099	+2.218561	−0.448374
6 − 6	−0.004674085	−0.012058875	−0.0989498	−0.2457720	−0.920227	−2.200452
6 − 7	+0.00109630	−0.00003131	+0.0143572	+0.0071733	+0.0245	+0.1557
6 − 8	−0.00000448	+0.00004450	+0.000027	+0.000189	+0.0085	−0.0037
6 − 9	−0.0000017	−0.0000004	−0.000023	−0.000009	−0.0006	−0.0004
7 0	−0.000000020	+0.000000005	−0.00000164	−0.00000032	−0.000043	−0.000009
7 − 1	+0.000000163	+0.000000500	+0.00000510	+0.00002562	+0.000090	+0.000543
7 − 2	+0.000007141	−0.000006850	+0.00027509	−0.00019927	+0.004637	−0.002897
7 − 3	−0.000100096	−0.000055409	−0.00260484	−0.00183584	−0.032469	−0.026305
7 − 4	−0.000226657	+0.000759906	−0.00701279	+0.01854593	−0.090968	+0.208031
7 − 5	+0.003210470	+0.000382706	+0.07571960	+0.01209625	+0.792862	+0.151209
7 − 6	−0.000205770	−0.007123398	−0.0026960	−0.1657167	−0.012445	−1.666873
7 − 7	−0.00627479	+0.00111343	−0.1477090	+0.0285046	−1.4803	+0.3091
7 − 8	+0.00010073	−0.00063987	+0.005769	−0.009714	+0.1000	−0.0236
7 − 9	+0.0000304	+0.0000097	+0.000232	+0.000114	−0.0015	−0.0039
7 − 10	−0.0000005	+0.0000007	−0.000003	+0.000007	−0.0003	+0.0003
8 − 1	+0.000000034	+0.000000062	−0.00000069	+0.00000209	−0.000022	+0.000072
8 − 2	+0.000000944	−0.000000307	+0.00004193	−0.00000687	+0.000801	−0.000064
8 − 3	−0.000008249	−0.000010756	−0.00022931	−0.00039576	−0.003068	−0.006384
8 − 4	−0.000073320	+0.000092680	−0.00241109	+0.00252016	−0.034289	+0.031420
8 − 5	+0.000578366	+0.000298475	+0.01535549	+0.00915939	+0.180088	+0.119135
8 − 6	+0.000679098	−0.002086163	+0.0198473	−0.0546519	+0.241701	−0.614590
8 − 7	−0.00404931	−0.00069304	−0.1059393	−0.0191499	−1.1646	−0.2181
8 − 8	−0.00005190	+0.00315303	+0.000050	+0.084397	+0.0164	+0.9386
8 − 9	−0.0003539	−0.0001270	−0.006098	−0.004642	−0.0187	−0.0676
8 − 10	+0.0000105	−0.0000189	+0.000169	−0.000200	−0.0014	+0.0002
8 − 11	+0.0000007	+0.0000005	+0.000007	+0.000009	+0.0002	+0.0001

Arg.	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
9 — 2	+0.000000105	+0.000000011	+0.00000506	+0.00000147	+0.000102	+0.000028
9 — 3	−0.000000347	−0.000001478	−0.00000497	−0.00006134	+0.000003	−0.001114
9 — 4	−0.000013864	+0.000007724	−0.00050777	+0.00021178	−0.008041	+0.002627
9 — 5	+0.000073128	+0.000082574	+0.00209163	+0.00279303	+0.026442	+0.040181
9 — 6	+0.000311153	−0.000394850	+0.0099776	−0.0113624	+0.133724	−0.139955
9 — 7	−0.00125724	−0.00070623	−0.0362737	−0.0217643	−0.4374	−0.2763
9 — 8	−0.00084109	+0.00218541	−0.024950	+0.063599	−0.3017	+0 7604
9 — 9	+0.0015302	+0.0003202	+0.046043	+0.008720	+0.5640	+0.0960
9 — 10	−0.0001100	+0.0001856	−0.003582	+0.003558	−0.0472	+0.0123
9 — 11	−0.0000106	−0.0000092	−0.000125	−0.000181	0.0000	0.0000
9 — 12	+0.0000006	−0.0000006	+0.000011	−0.000002	+0.0001	−0.0001
10 — 3	+0.000000031	−0.000000166	+0.00000281	−0.00000752	+0.000073	−0.000149
10 — 4	−0.0000019688	+0.0000002034	−0.00008126	−0.00000258	−0.001414	−0.000171
10 — 5	+0.000005589	+0.000015625	+0.00014437	+0.00058416	+0.001554	+0.009252
10 — 6	+0.000081939	−0.000049204	+0.0028987	−0.0014681	+0.042790	−0.018728
10 — 7	−0.00024173	−0.00028135	−0.0074859	−0.0095793	−0.0968	−0.1339
10 — 8	−0.00060228	+0.00070156	−0.019938	+0.022108	−0.2676	+0.2851
10 — 9	+0.0011171	+0.0007077	+0.035851	+0.022907	+0.4639	+0.2976
10 — 10	+0.0002972	−0.0007158	+0.009379	−0.023994	+0.1184	−0.3220
10 — 11	+0.0000916	+0.0000811	+0.001899	+0.002612	+0.0068	+0.0329
10 — 12	−0.0000072	+0.0000052	−0.000158	+0.000073	−0.0004	0.0000
10 — 13	−0.0000005	−0.0000006	+0.000002	−0.000009		
11 — 4	−0.000000226	−0.000000072	−0.00000981	−0.00000619	−0.000176	−0.000117
11 — 5	−0.000000111	+0.000002286	−0.00001586	+0.00009324	−0.000444	+0.001645
11 — 6	+0.000015719	−0.000002670	+0.0006086	−0.0000475	+0.009827	−0.000089
11 — 7	−0.00002700	−0.00007344	−0.0008149	−0.0027374	−0.0101	−0.0418
11 — 8	−0.00022986	+0.00012997	−0.008343	+0.004277	−0.1224	+0.0575
11 — 9	+0.0003583	+0.0004580	+0.012228	+0.016330	+0.1680	+0.2325
11 — 10	+0.0005112	−0.0005361	+0.018019	−0.018847	+0.2512	−0.2628
11 11	−0.0003217	−0.0002097	−0.011933	−0.007396	−0.1746	−0.1032
11 — 12	+0.0000545	−0.0000419	+0.001804	−0.000910	+0.0227	−0.0027
11 — 13	+0.0000021	+0.0000051	+0.000034	+0.000127		
11 — 14	−0.0000006	+0.0000003	−0.000006	−0.000003		
12 — 5	−0.000000129	+0.000000269	−0.00000787	+0.00001157	−0.000143	+0.000232
12 — 6	+0.000002358	+0.000000512	+0.0000984	+0.0000327	+0.001721	+0.000738
12 — 7	+0.00000022	−0.00001435	+0.0000533	−0.0005819	+0.0015	−0.0094
12 — 8	−0.00006043	+0.00000978	−0.002375	+0.000248	−0.0378	+0.0020
12 — 9	+0.0000572	+0.0001738	+0.001950	+0.006724	+0.0267	+0.1039
12 — 10	+0.0003212	−0.0001616	+0.012315	−0.005919	+0.1859	−0.0860
12 — 11	−0.0002373	−0.0003367	−0.009085	−0.012877	−0.1362	−0.1921
12 — 12	−0.0001306	+0.0001378	−0.005084	+0.005635	−0.0773	+0.0897
12 — 13	−0.0000172	−0.0000343	−0.000333	−0.001179	0.0000	−0.0154
12 — 14	+0.0000034	−0.0000004	+0.000095	−0.000008		
12 — 15	+0.0000001	+0.0000005	−0.000004	+0.000004		

Arg.	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
13 — 6	+0.000000279	+0.000000194	+0.0000122	+0.0000107	+0.000213	+0.000241
13 — 7	+0.00000089	—0.00000218	+0.0000515	—0.0000945	+0.0011	—0.0016
13 — 8	—0.00001200	—0.00000243	—0.000499	—0.000140	—0.0085	—0.0029
13 — 9	—0.0000014	+0.0000463	—0.000159	+0.001925	—0.0040	+0.0318
13 — 10	+0.0001225	—0.0000152	+0.005049	—0.000456	+0.0820	—0.0051
13 — 11	—0.0000582	—0.0002114	—0.002249	—0.008688	—0.0341	—0.1389
13 — 12	—0.0002084	+0.0000928	—0.008593	+0.003852	—0.1368	+0.0619
13 — 13	+0.0000557	+0.0000756	+0.002503	+0.003219	+0.0433	+0.0530
13 — 14	—0.0000205	+0.0000058	—0.000731	+0.000039	—0.0101	—0.0015
13 — 15	+0.0000003	—0.0000017	+0.000006	—0.000067		
14 — 7	+0.00000024	—0.00000026	+0.0000141	—0.0000112	+0.0003	—0.0002
14 — 8	—0.00000184	—0.00000118	—0.000075	—0.000057	—0.0015	—0.0014
14 — 9	—0.0000039	+0.0000092	—0.000198	+0.000408	—0.0039	+0.0073
14 — 10	+0.0000322	+0.0000076	+0.001441	+0.000403	+0.0251	+0.0080
14 — 11	+0.0000061	—0.0000815	+0.000375	—0.003575	+0.0078	—0.0609
14 — 12	—0.0001319	+0.0000094	—0.005794	+0.000338	—0.0979	+0.0046
14 — 13	+0.0000279	+0.0001229	+0.001267	+0.005432	+0.0218	+0.0919
14 — 14	+0.0000416	—0.0000207	+0.001927	—0.001021	+0.0343	—0.0191
14 — 15	+0.0000010	+0.0000118	—0.000049	+0.000427	—0.0021	+0.0064
14 — 16	—0.0000011	—0.0000005	—0.000044	—0.000012		
15 — 8	—0.00000021	—0.00000028	—0.000006	—0.000008	—0.0002	—0.0003
15 — 9	—0.0000014	+0.0000015	—0.000068	+0.000064	—0.0014	+0.0012
15 — 10	+0.0000065	+0.0000045	+0.000299	+0.000234	+0.0055	+0.0045
15 — 11	+0.0000099	—0.0000219	+0.000507	—0.001018	+0.0100	—0.0184
15 — 12	—0.0000513	—0.0000143	—0.002384	—0.000732	—0.0426	—0.0142
15 — 13	—0.0000099	+0.0000784	—0.000506	+0.003669	—0.0097	+0.0654
15 — 14	+0.0000695	—0.0000019	+0.003279	—0.000122	+0.0586	—0.0024
15 — 15	—0.0000066	—0.0000220	—0.000360	—0.001105	—0.0073	—0.0211
15 — 16	+0.0000065	+0.0000007	+0.000236	+0.000077	+0.0038	+0.0021
15 — 17	—0.0000005	+0.0000006	—0.000013	+0.000027		
16 — 9	—0.0000003	+0.0000002	—0.000014	+0.000007	—0.0003	+0.0001
16 — 10	+0.0000010	+0.0000016	+0.000048	+0.000076	+0.0008	+0.0014
16 — 11	+0.0000042	—0.0000044	+0.000227	—0.000209	+0.0045	—0.0039
16 — 12	—0.0000138	—0.0000097	—0.000672	—0.000517	—0.0125	—0.0102
16 — 13	—0.0000155	+0.0000305	—0.000779	+0.001493	+0.0158	+0.0281
16 — 14	+0.0000444	+0.0000149	+0.002210	+0.000762	+0.0414	+0.0147
16 — 15	+0.0000064	—0.0000378	+0.000289	—0.001897	+0.0054	—0.0357
16 — 16	—0.0000113	+0.0000014	—0.000611	—0.000089	+0.0125	+0.0020
16 — 17	+0.0000010	—0.0000034	+0.000071	—0.000124	+0.0019	—0.0022
16 — 18	+0.0000002	+0.0000004	+0.000015	+0.000011		

$\left(\frac{a'}{\Delta}\right)^7$

Arg.	cos.	sin.	Arg.	cos.	sin.	Arg.	cos.	sin.
i′ i			i′ i			i′ i		
0 0	30.504		3 − 1	+5.965	−3.074	6 − 6	−6.600	−15.225
0 − 1	−5.696	−17.950	3 − 2	−16.682	−17.875	6 − 7	−0.726	+1.821
0 − 2	−2.405	+1.974	3 − 3	−22.720	+34.220	6 − 8	+0.215	−0.018
0 − 3	+0.335	+0.164	3 − 4	+7.243	+0.818			
0 − 4	−0.001	−0.038	3 − 5	−0.332	−0.820	7 − 2	+0.057	−0.033
0 − 5	−0.005	+0.002	3 − 6	−0.056	+0.069	7 − 3	−0.321	−0.283
			3 − 7	+0.008	+0.002	7 − 4	−0.878	+1.827
1 + 4	−0.004	+0.006				7 − 5	+6.396	+1.367
1 + 3	+0.062	+0.005	4 + 1	−0.014	−0.020	7 − 6	−0.023	−12.726
1 + 2	−0.201	−0.538	4 0	−0.072	+0.196	7 − 7	−11.173	+2.503
1 + 1	−3.129	+2.978	4 − 1	+1.480	−0.024	7 − 8	+1.135	+0.483
1 0	+20.066	+10.186	4 − 2	−2.295	−7.035	7 − 9	+0.003	−0.112
1 − 1	+11.295	−56.854	4 − 3	−19.060	+12.142			
1 − 2	−14.393	+2.242	4 − 4	+22.507	+22.239	8 − 2	+0.010	−0.001
1 − 3	+1.158	+1.786	4 − 5	+1.061	−4.708	8 − 3	−0.038	−0.078
1 − 4	+0.121	−0.206	4 − 6	−0.521	+0.163	8 − 4	−0.365	+0.305
1 − 5	−0.029	−0.002	4 − 7	+0.037	+0.039	8 − 5	+1.617	+1.150
1 − 6	+0.001	+0.004				8 − 6	+2.166	−5.200
			5 0	−0.026	+0.023	8 − 7	−9.498	−1.819
2 + 3	+0.003	−0.006	5 − 1	+0.256	+0.126	8 − 8	+0.264	+7.678
2 + 2	+0.016	−0.096	5 − 2	+0.227	−1.746	8 − 9	+0.295	−0.656
2 + 1	−0.812	+0.207	5 − 3	−7.484	+1.101	8 − 10	+0.058	−0.008
2 0	+3.266	+4.542	5 − 4	+7.280	+18.200			
2 − 1	+14.662	−19.650	5 − 5	+19.255	−13.169	9 − 3	+0.001	−0.016
2 − 2	−46.677	−19.275	5 − 6	−2.953	−0.945	9 − 4	−0.096	+0.025
2 − 3	+0.141	+10.570	5 − 7	+0.007	+0.331	9 − 5	+0.261	+0.429
2 − 4	+1.240	−0.638	5 − 8	+0.037	−0.002	9 − 6	+1.317	−1.292
2 − 5	−0.121	−0.086				9 − 7	−3.845	−2.557
2 − 6	−0.002	+0.014	6 − 1	+0.028	+0.036	9 − 8	−2.657	+6.605
			6 − 2	+0.199	−0.302	9 − 9	+5.006	+0.763
3 + 2	+0.010	−0.012	6 − 3	−1.879	−0.547	9 − 10	−0.379	−0.180
3 + 1	−0.142	−0.036	6 − 4	−0.217	+7.233	9 − 11	−0.007	+0.029
3 0	+0.159	+1.135	6 − 5	+15.843	−3.076			

In order to be serviceable in the method of treating the problem we intend to follow it is necessary to transform the preceding series into others, the arguments of whose terms are of the general form $i'g' + ig$. This is done by means of the **Besselian** functions $J_i^{(k)}$ (we use **Hansen's** notation for these quantities). The following formulæ serve for their computation:*

Derive p_k from

$$\frac{1}{p_k} = r_k - p_{k+1}$$

---

*Auseinandersetzung, Abh. I, s. 173.

where

$$r_k = \frac{k}{l}$$

in which equations we must begin with so large an integer for k that, for the first application, we can put $p_{k+1} = 0$. This integer, in the present case, may be assumed as about 8 or 9. The value of $J_l^{(k)}$ is then

$$J_l^{(k)} = J_l^{(0)} p_1 p_2 p_3 \cdot \cdot \cdot p_k$$

where

$$J_l^{(0)} = 1 - \frac{l^2}{1^2} + \frac{l^4}{1^2 \,.\, 2^2} - \frac{l^6}{1^2 \,.\, 2^2 \,.\, 3^2} + \cdot \cdot \cdot$$

For the present purpose it suffices to suppose in these formulæ l equal in succession to $\frac{e}{2}$, e, $\frac{3}{2}e$, $2e$, etc.; but as we shall hereafter need these functions corresponding to the eccentricity e' of Saturn, the following tables contain the latter quantities also:

i	$\log\left(J^{(0)}_{\frac{ie}{2}} - 1\right)$	$\log \frac{1}{i} J^{(1)}_{\frac{ie}{2}}$	$\log \frac{1}{i} J^{(2)}_{\frac{ie}{2}}$	$\log \frac{1}{i} J^{(3)}_{\frac{ie}{2}}$	$\log \frac{1}{i} J^{(4)}_{\frac{ie}{2}}$	$\log \frac{1}{i} J^{(5)}_{\frac{ie}{2}}$
1	6.76464*n*	8.3822758	6.4636901	4.3689921	2.1493469	
2	7.3666118*n*	8.3818968	6.7644675	4.9708626	3.0522853	
3	7.7184785*n*	8.3812646	6.9401375	5.3227293	3.5803065	1.7409358
4	7.9679134*n*	8.3803790	7.0644861	5.5721642	3.9547687	2.2403957
5	8.1611649*n*	8.3792397	7.1606373	5.7654153	4.2450436	2.6276565
6	8.3188323*n*	8.3778459	7.2388909	5.9230826	4.4820315	2.9439185
7	8.4518964*n*	8.3761967	7.3047404	6.0561538	4.6822141	3.2111575
8	8.5669394*n*	8.3742908	7.3614652	6.1711884	4.8554309	3.4424930
9	8.6681692*n*	8.3721278	7.4111809	6.2724174	5.0080283	3.6463866
10	8.7584745*n*	8.3697058	7.4553308	6.3627288	5.1443388	3.8286154
11	8.8398*n*	8.36701	7.4949	6.4441	5.267	3.994
12	8.9138*n*	8.36404	7.5307	6.5181	5.380	4.145
13	8.9814*n*	8.36078	7.5632	6.5858	5.484	4.284

i'	$\log\left(J^{(0)}_{\frac{i'e'}{2}} - 1\right)$	$\log J^{(1)}_{\frac{i'e'}{2}}$	$\log J^{(2)}_{\frac{i'e'}{2}}$	$\log J^{(3)}_{\frac{i'e'}{2}}$	$\log J^{(4)}_{\frac{i'e'}{2}}$	$\log J^{(5)}_{\frac{i'e'}{2}}$	$\log J^{(6)}_{\frac{i'e'}{2}}$
1	6.8951130*n*	8.4474282	6.5940540	4.5645600	2.4101160	0.1587563	
2	7.4969158*n*	8.7479463	7.1957728	5.4673942	3.6140313	1.6637357	
3	7.8486718*n*	8.9231839	7.5473865	5.9952414	4.3180552	2.5439078	
4	8.0979528*n*	9.0469266	7.7964672	6.3694592	4.8173311	3.1682020	
5	8.2910046*n*	9.1422972	7.9892624	6.6594221	5.2043580	3.6522414	2.0209
6	8.4484286*n*	9.2195944	8.1463714	6.8960265	5.5203318	4.0475219	2.4944
7	8.5812122*n*	9.2843110	8.2787822	7.0957559	5.7872306	4.3815156	2.8961

The transformation we seek to accomplish is arrived at by the aid of the equations

$$\cos(\beta - k\varepsilon) = \sum_{i=-\infty}^{i=+\infty} \frac{k}{i} J_{\frac{ie}{2}}^{(i-k)} \cos(\beta - ig)$$

$$\sin(\beta - k\varepsilon) = \sum_{i=-\infty}^{i=+\infty} \frac{k}{i} J_{\frac{ie}{2}}^{(i-k)} \sin(\beta - ig)$$

where β denotes any arbitrary angle. In employing these equations it must be remembered that

$$J_i^{(-k)} = (-1)^k J_i^{(k)} \qquad J_{-i}^{(k)} = (-1)^k J_i^{(k)} \qquad J_{-i}^{(-k)} = J_i^{(k)}$$

and when $i = 0$ and $k = 1$ or -1 we must suppose that the multiplier

$$\frac{k}{i} J_{\frac{ie}{2}}^{(i-k)} = -\frac{1}{2}e$$

but when k has, in the same case, values different from 1 or -1 this multiplier vanishes.

The developments found for the four odd powers of the reciprocal of the distance of the planets, when the terms are made to take the form

$$C^{(c)} \cos(i'g' + ig) + C^{(s)} \sin(i'g' + ig)$$

are as follows:

Arg.	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
0 0	1.09099923		2.22201791		7.39905	
0 − 1	−0.00960195	−0.00524015	−0.13001896	−0.32304848	−0.908103	− 2.732029
0 − 2	−0.00026172	+0.00027740	−0.01569650	+0.00896318	−0.25240	+ 0.14988
0 − 3	+0.00000505	+0.00001192	+0.00050731	+0.00065823	+0.01419	+ 0.01703
0 − 4	+0.00000029	+0.00000001	+0.00002881	−0.00002901	+0.00094	− 0.00106
0 − 5	0.00000000	0.00000000	−0.00000104	−0.00000126	−0.00007	− 0.00006
1 + 4	+0.00000002	+0.00000001	+0.00000131	+0.00000566		
1 + 3	+0.00000080	−0.00000060	+0.00009035	−0.00003872	+0.00281	− 0.00110
1 + 2	−0.00001498	−0.00002952	−0.00105960	−0.00182006	−0.02186	− 0.03715
1 + 1	−0.00105822	+0.00025165	−0.03306005	+0.02561226	−0.36944	+ 0.33465
1 0	+0.063516727	+0.048362194	+0.52839296	+0.35166657	+3.326835	+ 2.021865
1 − 1	+0.12543117	−0.60717079	+0.65850333	−3.18494695	+2.718265	−13.046252
1 − 2	+0.00641586	−0.00826731	−0.14876566	−0.01247904	−1.854333	+ 0.069633
1 − 3	+0.00040449	−0.00015969	+0.00108483	+0.00657454	+0.03012	+ 0.15512
1 − 4	+0.00002113	−0.00000945	+0.00032505	−0.00012160	+0.01018	− 0.00351
1 − 5	+0.00000088	−0.00000046	−0.00000609	−0.00001649	−0.00036	− 0.00060
1 − 6	+0.00000004	−0.00000001	−0.00000075	+0.00000029	+0.00004	− 0.00004
2 + 4	+0.00000001	0.00000000	−0.00000133	+0.00000056		
2 + 3	+0.00000008	−0.00000002	+0.00001138	+0.00000099	+0.00040	+ 0.00004
2 + 2	−0.00000012	−0.00000319	−0.00000416	−0.00024417	−0.00017	− 0.00605
2 + 1	−0.00009858	−0.00001954	−0.00490064	+0.00071159	−0.07545	+ 0.01738

Arg.		$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
		cos.	sin.	cos.	sin.	cos.	sin.
i'	i						
2	0	+0.00190636	+0.00555286	+0.03500321	+0.07616804	+0.357561	+0.663313
2 —	1	+0.05110921	—0.05756433	+0.48984397	—0.52282903	+2.971190	—3.214450
2 —	2	—0.23387573	—0.10221994	—1.93998606	—0.84681116	—9.707139	—4.260549
2 —	3	—0.00624030	—0.00844844	—0.05348459	+0.03601136	—0.343378	+1.026094
2 —	4	—0.00010949	—0.00049321	+0.00155033	—0.00052144	+0.07647	+0.01401
2 —	5	—0.00000172	—0.00002670	—0.00004175	—0.00020075	+0.00010	—0.00496
2 —	6	+0.00000001	—0.00000134	—0.00000972	—0.00000341	—0.00032	+0.00006
2 —	7	0.00000000	—0.00000006	—0.00000012	—0.00000027		
3 +	3	+0.00000001	0.00000000	+0.00000108	+0.00000068		
3 +	2	+0.00000012	—0.00000027	+0.00001305	—0.00002483	+0.00038	—0.00072
3 +	1	—0.00000760	—0.00000551	—0.00054159	—0.00020514	—0.01084	—0.00313
3	0	—0.00010753	+0.00047141	—0.00110240	+0.01044997	—0.00353	+0.12632
3 —	1	+0.00790442	—0.00241147	+0.11262276	—0.03111197	+0.932828	—0.277793
3 —	2	—0.03299484	—0.04246351	—0.37777494	—0.51550556	—2.496781	—3.446378
3 —	3	—0.06747291	+0.09335838	—0.76188256	+1.05805306	—4.55970	+6.31032
3 —	4	—0.00705343	+0.00296843	—0.02009453	+0.04618699	+0.41605	+0.42870
3 —	5	—0.00045060	—0.00001077	—0.00175177	+0.00005193	+0.01765	—0.02808
3 —	6	—0.00002521	—0.00000696	—0.00018823	+0.00000271	—0.00219	—0.00066
3 —	7	—0.00000133	—0.00000061	—0.00000757	+0.00000181	—0.0003	+0.0002
3 —	8	—0.00000007	—0.00000005				
4 +	2	+0.00000001	—0.00000002	+0.00000251	—0.00000183		
4 +	1	—0.00000042	—0.00000078	—0.00004348	—0.00005072	—0.0011	—0.0011
4	0	—0.00002873	+0.00003023	—0.00075153	+0.00102973	—0.01009	+0.01664
4 —	1	+0.00081742	+0.00021726	+0.01612695	+0.00453965	+0.174970	+0.043006
4 —	2	—0.001089010	—0.007906123	—0.01234856	—0.12748736	—0.103061	—1.086030
4 —	3	—0.03074857	+0.01527037	—0.45602074	+0.21266233	—3.393635	+1.542189
4 —	4	+0.03589089	+0.04045233	+0.51711042	+0.57889510	+3.59852	+4.03385
4 —	5	+0.00084473	+0.00493552	+0.02593559	+0.03856237	+0.3427	—0.0505
4 —	6	—0.00009466	+0.00034150	—0.00010393	+0.00256650	—0.0058	—0.0072
4 —	7	—0.00001245	+0.00001929	—0.00004875	+0.00019287	—0.0005	+0.0010
4 —	8	—0.00000100	+0.00000097	—0.000010	+0.000012	+0.0001	+0.0001
4 —	9	—0.00000006	+0.00000005				
5 +	2			+0.00000041	—0.00000027		
5 +	1			—0.00000171	—0.00000741		
5	0	—0.00000354	+0.00000104	—0.00013788	+0.00006204	—0.0025	+0.0014
5 —	1	+0.00005805	+0.00006172	+0.00153582	+0.00162309	+0.02154	+0.02075
5 —	2	+0.0003989933	—0.0009429136	+0.00912144	—0.01926179	+0.09521	—0.20382
5 —	3	—0.006537862	—0.000314216	—0.12040911	—0.01060942	—1.08796	—0.10894
5 —	4	+0.00527436	+0.02027174	+0.08490433	+0.35670452	+0.66777	+2.95066
5 —	5	+0.02272923	—0.01243714	+0.39452729	—0.21861518	+3.1465	—1.7469
5 —	6	+0.00309749	+0.00017604	+0.0373479	—0.0084018	+0.1198	—0.2063
5 —	7	+0.00022496	+0.00012675	+0.0025727	+0.0007449	+0.0033	—0.0004
5 —	8	+0.00001228	+0.00001409	+0.000158	+0.000106	+0.0009	+0.0005
5 —	9	+0.00000052	+0.00000101	+0.000009	+0.000007		

Arg.		$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
		cos.	sin.	cos.	sin.	cos.	sin.
i'	i						
6	0	−0.00000032	−0.00000007	−0.0000175	−0.0000014		
6	−1	+0.00000170	+0.00000822	+0.0000633	+0.0002772	+0.0012	+0.0045
6	−2	+0.0000984225	−0.0000711576	+0.00264710	−0.00177064	+0.03351	−0.02311
6	−3	−0.00085675	−0.00055099	−0.0188698	−0.0134388	−0.20307	−0.14769
6	−4	−0.00118097	+0.00474846	−0.0285594	+0.0991728	−0.2938	+0.9612
6	−5	+0.01241237	−0.00054155	+0.2533389	−0.0066021	+2.3174	−0.0348
6	−6	−0.00334080	−0.01209833	−0.0703527	−0.2473777	−0.6403	−2.2278
6	−7	+0.00051911	−0.00178506	+0.0022716	−0.0286114	−0.0867	−0.1660
6	−8	+0.00012209	−0.00013046	+0.0012304	−0.0020526	+0.0013	−0.0083
6	−9	+0.00001267	−0.00000623	+0.0001426	−0.0001228	+0.0005	−0.0008
6	−10	+0.00000076	−0.00000013	+0.0000182	−0.0000040		
6	−11	+0.00000005	+0.00000002				
7	0			−0.0000017	−0.0000009		
7	−1	−0.00000027	+0.00000078	−0.0000104	+0.0000334	−0.0001	+0.0006
7	−2	+0.00001367	−0.00000107	+0.0004431	−0.0000231	+0.0067	−0.0005
7	−3	−0.00006343	−0.00012601	−0.0015734	−0.0035468	−0.0198	−0.0454
7	−4	−0.00061694	+0.00065304	−0.0162048	+0.0156279	−0.1874	+0.1744
7	−5	+0.00310773	+0.00148474	+0.0728272	+0.0377925	+0.7594	+0.4114
7	−6	+0.00122992	−0.00711895	+0.0335344	−0.1655894	+0.3306	−1.6647
7	−7	−0.00611335	+0.00019169	−0.1442453	+0.0060336	−1.4512	+0.0688
7	−8	−0.00094401	−0.00053174	−0.0189536	−0.0069320	−0.1484	+0.0048
7	−9	−0.00006403	−0.00009925	−0.0013549	−0.0013939	−0.0093	−0.0048
7	−10	−0.00000770	−0.00001066	−0.0000604	−0.0001589	−0.0007	+0.0010
7	−11	−0.00000051	−0.00000099	−0.0000005	−0.0000141		
7	−12	−0.00000002	−0.00000008				
8	−1			−0.0000029	+0.0000021		
8	−2	+0.0000013	+0.0000007	+0.0000521	+0.0000272	+0.0009	+0.0005
8	−3	+0.00000131	−0.00001807	+0.0000700	−0.0005898	+0.0010	−0.0088
8	−4	−0.0001368	+0.0000409	−0.0040826	+0.0009939	−0.0538	+0.0120
8	−5	+0.0004244	+0.0005943	+0.0109759	+0.0168853	+0.1277	+0.2061
8	−6	+0.0014097	−0.0018469	+0.0389561	−0.0480294	+0.4528	−0.5379
8	−7	−0.0038285	−0.0015701	−0.1000754	−0.0424509	−1.0987	−0.4782
8	−8	−0.0006357	+0.0029199	−0.015920	+0.078347	−0.1698	+0.8739
8	−9	−0.0004220	+0.0004628	−0.007732	+0.011120	−0.0351	+0.1078
8	−10	−0.0000725	+0.0000260	−0.001296	+0.000721	−0.0067	+0.0070
8	−11	−0.0000075	−0.0000006	−0.000139	+0.000012	−0.0008	+0.0003
8	−12	−0.0000004	−0.0000004	−0.000011	−0.000004		

Arg.	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
9 — 2			+0.0000042	+0.0000063		
9 — 3	+0.0000013	-0.0000018	+0.0000518	-0.0000678	+0.0009	-0.0012
9 — 4	-0.00002004	-0.00000492	-0.0006759	-0.0002028	-0.0101	-0.0032
9 — 5	+0.0000137	+0.0001308	+0.0002240	+0.0041555	+0.0017	+0.0568
9 — 6	+0.0005105	-0.0002285	+0.0156959	-0.0062787	+0.2025	-0.0757
9 — 7	-0.0009886	-0.0011518	-0.0282181	-0.0346847	-0.3379	-0.4310
9 — 8	-0.0013426	+0.0019186	-0.039794	+0.055715	-0.4826	+0.6645
9 — 9	+0.0013036	+0.0006601	+0.039393	+0.019050	+0.4842	+0.2267
9 — 10	+0.0002004	+0.0002937	+0.005737	+0.006654	+0.0668	+c.0488
9 — 11	+0.0000052	+0.0000486	+0.000282	+0.001044	+0.0041	+0.0073
9 — 12	-0.0000015	+0.0000047	-0.000012	+0.000111	+0.0001	+0.0007
10 — 3	+0.000000234	-0.000000111	+0.0000108	-0.0000046		
10 — 4	-0.0000019808	-0.0000019503	-0.0000755	-0.0000807	-0.0013	-0.0014
10 — 5	-0.00000860	+0.00001932	-0.0003480	+0.0006794	-0.0055	+0.0104
10 — 6	+0.0001121	+0.0000099	+0.003803	+0.000523	+0.0543	+0.0090
10 — 7	-0.0000885	-0.0003999	-0.002426	-0.013274	-0.0288	-0.1812
10 — 8	-0.0008504	+0.0004610	-0.027830	+0.014248	-0.3692	+0.1812
10 — 9	+0.0008799	+0.0009726	+0.028136	+0.031559	+0.3625	+0.4118
10 — 10	+0.0004779	-0.0005308	+0.015439	-0.017969	+0.2014	-0.2430
10 — 11	+0.0001866	-0.0000712	+0.004959	-0.002484	+0.0463	-0.0354
10 — 12	+0.0000302	+0.0000035	+0.000757	-0.000008	+0.0066	-0.0015
10 — 13	+0.0000026	+0.0000032	+0.000080	+0.000029		
11 — 4	-0.000000096	-0.000000344	-0.000004	-0.000016		
11 — 5	-0.0000026	+0.0000019	-0.000108	+0.000069	-0.0018	+0.0012
11 — 6	+0.0000164	+0.0000115	+0.000606	+0.000468	+0.0095	+0.0077
11 — 7	+0.0000256	-0.0000875	+0.001082	-0.003163	+0.0176	-0.0470
11 — 8	-0.0002897	+0.0000035	-0.010335	-0.000236	-0.1492	-0.0068
11 — 9	+0.0001682	+0.0005804	+0.005485	+0.020559	+0.0727	+0.2907
11 — 10	+0.0006370	-0.0003533	+0.022510	-0.012355	+0.3152	-0.1710
11 — 11	-0.0001860	-0.0002949	-0.007090	-0.010545	-0.1059	-0.1499
11 — 12	-0.0000139	-0.0001101	-0.000743	-0.003345	-0.0146	-0.0369
11 — 13	+0.0000062	-0.0000175	+0.000122	-0.000502		
11 — 14	+0.0000013	-0.0000015	+0.000037	-0.000052		
12 — 5	-0.00000047	+0.00000005	-0.000020	+0.000001		
12 — 6	+0.0000016	+0.0000029	+0.000052	+0.000128	+0.0008	+0.0021
12 — 7	+0.0000126	-0.0000123	+0.000539	-0.000480	+0.0093	-0.0075
12 — 8	-0.0000626	-0.0000332	-0.002394	-0.001405	-0.0375	-0.0234
12 — 9	-0.0000390	+0.0001956	-0.001737	+0.007464	-0.0291	+0.1140
12 — 10	+0.0003701	-0.0000240	+0.014134	-0.000643	+0.2124	-0.0066
12 — 11	-0.0001088	-0.0003860	-0.004134	-0.014787	-0.0613	-0.2210
12 — 12	-0.0001638	+0.0000463	-0.006451	+0.002072	-0.0991	+0.0308
12 — 13	-0.0000609	-0.0000071	-0.002051	-0.000050		
12 — 14	-0.0000094	-0.0000059	-0.000295	-0.000159		
12 — 15	-0.0000006	-0.0000009	-0.000031	-0.000034		

Arg.	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
13 — 6	0.0000000	+0.0000006	−0.00001	+0.00003		
13 — 7	+0.0000031	−0.0000008	+0.00014	−0.00003		
13 — 8	−0.0000083	−0.0000126	−0.00032	−0.00056		
13 — 9	−0.0000336	+0.0000410	−0.00147	+0.00165		
13 — 10	+0.0001226	+0.0000529	+0.00500	+0.00233		
13 — 11	+0.0000349	−0.0002218	+0.00159	−0.00910		
13 — 12	−0.0002197	+0.0000086	−0.00907	+0.00034		
13 — 13	−0.0000022	+0.0000835	+0.00006	+0.00360		
13 — 14	−0.0000119	+0.0000315	−0.00033	+0.00115		
13 — 15	−0.0000045	+0.0000050	−0.00013	+0.00015		
14 — 7	+0.0000004	+0.0000001	+0.00002	+0.00001		
14 — 8	−0.0000002	−0.0000029	0.00000	−0.00013		
14 — 9	−0.0000114	+0.0000044	−0.00052	+0.00018		
14 — 10	+0.0000234	+0.0000300	+0.00102	+0.00137		
14 — 11	+0.0000509	−0.0000713	+0.00233	−0.00309		
14 — 12	−0.0001248	−0.0000499	−0.00545	−0.00227		
14 — 13	−0.0000241	+0.0001177	−0.00105	+0.00521		
14 — 14	+0.0000387	+0.0000139	+0.00185	+0.00055		
14 — 15	+0.0000152	+0.0000107	+0.00062	+0.00036		
14 — 16	+0.0000021	+0.0000032	+0.00008	+0.00011		
15 — 8	+0.0000002	−0.0000005	+0.00001	−0.00001		
15 — 9	−0.0000024	−0.0000004	−0.00012	−0.00002		
15 — 10	+0.0000014	+0.0000094	+0.00006	+0.00045		
15 — 11	+0.0000251	−0.0000122	+0.00119	−0.00055		
15 — 12	−0.0000441	−0.0000421	−0.00175	−0.00202		
15 — 13	−0.0000474	+0.0000655	−0.00217	+0.00306		
15 — 14	+0.0000650	+0.0000283	+0.00280	+0.00132		
15 — 15	+0.0000141	−0.0000159	+0.00061	−0.00083		
15 — 16	+0.0000081	−0.0000066	+0.00030	−0.00029		
15 — 17	+0.0000020	−0.0000006	+0.00008	−0.00002		
16 — 9	−0.0000004	−0.0000003	−0.00001	−0.00002		
16 — 10	−0.0000008	+0.0000022	−0.00003	+0.00011		
16 — 11	+0.0000072	+0.0000001	+0.00037	+0.00002		
16 — 12	−0.0000048	−0.0000182	−0.00022	−0.00093		
16 — 13	−0.0000314	+0.0000178	−0.00158	+0.00085		
16 — 14	+0.0000314	+0.0000347	+0.00156	+0.00173		
16 — 15	+0.0000230	−0.0000274	+0.00114	−0.00132		
16 — 16	−0.0000052	−0.0000092	−0.00032	−0.00061		
16 — 17	−0.0000025	−0.0000051	−0.00013	−0.00025		
16 — 18	−0.0000002	−0.0000013	−0.00001	−0.00007		

$\left(\frac{a'}{\Delta}\right)^7$

Arg.	cos.	sin.	Arg.	cos.	sin.	Arg.	cos.	sin.
i′ i			i′ i			i′ i		
0 0	30.641		3 − 1	+ 6.746	− 2.180	6 − 6	− 4.450	−15.528
0 − 1	− 5.575	−18.040	3 − 2	−14.840	−20.378	6 − 7	− 1.528	− 0.427
0 − 2	− 2.560	+ 1.525	3 − 3	−24.097	+33.095	6 − 8	+ 0.008	+ 0.082
0 − 3	+ 0.212	+ 0.247	3 − 4	+ 5.541	+ 3.332	7 − 2	+ 0.078	− 0.009
0 − 4	+ 0.017	− 0.021	3 − 5	+ 0.274	− 0.593	7 − 3	− 0.204	− 0.452
0 − 5	− 0.004	− 0.001	3 − 6	− 0.054	− 0.015	7 − 4	− 1.658	+ 1.537
1 + 4	0.000	+ 0.005	3 − 7	0.000	+ 0.004	7 − 5	+ 6.108	+ 3.375
1 + 3	+ 0.049	− 0.018	4 + 1	− 0.014	− 0.020	7 − 6	+ 2.618	−12.697
1 + 2	− 0.280	− 0.465	4 0	− 0.107	+ 0.197	7 − 7	−11.012	+ 0.537
1 + 1	− 3.120	+ 3.019	4 − 1	+ 1.573	+ 0.326	7 − 8	− 0.754	+ 0.729
1 0	+19.869	+11.485	4 − 2	− 0.826	− 7.847	7 − 9	+ 0.016	+ 0.017
1 − 1	+11.984	−56.928	4 − 3	−21.229	+ 9.580	8 − 2	+ 0.012	+ 0.006
1 − 2	−14.172	+ 0.738	4 − 4	+20.788	+23.456	8 − 3	+ 0.004	− 0.102
1 − 3	+ 0.457	+ 1.856	4 − 5	+ 3.194	− 2.481	8 − 4	− 0.541	+ 0.121
1 − 4	+ 0.174	− 0.072	4 − 6	− 0.238	− 0.250	8 − 5	+ 1.152	+ 1.887
1 − 5	− 0.015	− 0.014	4 − 7	− 0.017	+ 0.023	8 − 6	+ 3.901	− 4.542
1 − 6	− 0.002	+ 0.003	5 0	− 0.032	+ 0.020	8 − 7	− 8.949	− 3.971
2 + 3	+ 0.003	− 0.011	5 − 1	+ 0.239	+ 0.211	8 − 8	− 1.352	+ 7.163
2 + 2	− 0.004	− 0.091	5 − 2	+ 0.789	− 1.775	8 − 9	+ 0.162	+ 0.783
2 + 1	− 0.817	+ 0.218	5 − 3	− 8.050	− 0.798	8 − 10	+ 0.111	+ 0.026
2 0	+ 2.932	+ 5.011	5 − 4	+ 4.342	+19.680	9 − 3	+ 0.011	− 0.016
2 − 1	+16.904	−18.700	5 − 5	+20.065	−11.090	9 − 4	− 0.117	− 0.035
2 − 2	−46.222	−20.468	5 − 6	− 0.547	− 2.426	9 − 5	+ 0.021	+ 0.582
2 − 3	− 2.212	+ 9.632	5 − 7	− 0.223	+ 0.062	9 − 6	+ 1.919	− 0.696
2 − 4	+ 1.145	+ 0.093	5 − 8	+ 0.010	+ 0.031	9 − 7	− 2.949	− 3.907
2 − 5	− 0.004	− 0.104	6 − 1	+ 0.016	+ 0.050	9 − 8	− 4.245	+ 5.757
2 − 6	− 0.006	+ 0.003	6 − 2	+ 0.334	− 0.243	9 − 9	+ 4.205	+ 1.925
3 + 2	+ 0.007	− 0.013	6 − 3	− 1.768	− 1.265	9 − 10	+ 0.610	+ 0.128
3 + 1	− 0.144	− 0.034	6 − 4	− 2.298	+ 7.387	9 − 11	+ 0.128	+ 0.065
3 0	+ 0.018	+ 1.210	6 − 5	+16.525	− 0.135			

CHAPTER II.

PERTURBATIONS OF JUPITER AND SATURN ARISING FROM THEIR MUTUAL ACTION AND OF THE FIRST ORDER WITH RESPECT TO DISTURBING FORCES.

The next step in arriving at the proper expressions for the forces which the planets exert on each other is to multiply the function $\left(\frac{a'}{\Delta}\right)^3$ in its final form, given in the preceding chapter, severally by each of the four factors

$$\alpha^2\left(\frac{r}{a}\right)^2 \qquad \left(\frac{r'}{a'}\right)^2 \qquad \frac{r'}{a'}\sin(f'+\Pi') \qquad -\frac{r}{a}\sin(f+\Pi)$$

We have*

$$\left(\frac{r}{a}\right)^2 = 1 + \frac{3}{2}e^2 - \frac{4}{1}J^{(1)}_{e}\cos g - \frac{4}{4}J^{(2)}_{2e}\cos 2g - \frac{4}{9}J^{(3)}_{3e}\cos 3g - \ldots$$

with a similar equation for $\left(\frac{r'}{a'}\right)^2$.

In addition †

$$\begin{aligned}\frac{r'}{a'}\sin(f'+\Pi') = & \qquad\qquad -\frac{3}{2}e'\sin\Pi' \\ & + \left[J^{(0)}_{e'} + J^{(2)}_{e'}\right]\cos\varphi'\cos\Pi'\sin g' + \left[J^{(0)}_{e'} - J^{(2)}_{e'}\right]\sin\Pi'\cos g' \\ & + \frac{1}{2}\left[J^{(1)}_{2e'} + J^{(3)}_{2e'}\right]\cos\varphi'\cos\Pi'\sin 2g' + \frac{1}{2}\left[J^{(1)}_{2e'} - J^{(3)}_{2e'}\right]\sin\Pi'\cos 2g' \\ & + \frac{1}{3}\left[J^{(2)}_{3e'} + J^{(4)}_{3e'}\right]\cos\varphi'\cos\Pi'\sin 3g' + \frac{1}{3}\left[J^{(2)}_{3e'} - J^{(4)}_{3e'}\right]\sin\Pi'\cos 3g' \\ & + \ldots\ldots\ldots\ldots\ldots\ldots\end{aligned}$$

with a similar expression for $\frac{r}{a}\sin(f+\Pi)$.

The numerical expressions for these four factors are (the logarithms of the coefficients are given)

$$\begin{aligned}\alpha^2\left(\frac{r}{a}\right)^2 = & \quad [9.4746164] \\ & - 2[8.1564087]\cos g \\ & - 2[6.2375704]\cos 2g \\ & - 2[4.6197410]\cos 3g \\ & - 2[3.1268416]\cos 4g\end{aligned} \qquad\qquad \begin{aligned}\left(\frac{r'}{a'}\right)^2 = & \quad [0.0020422] \\ & - 2[8.7484582]\cos g' \\ & - 2[6.8947428]\cos 2g' \\ & - 2[5.3420289]\cos 3g' \\ & - 2[3.9142411]\cos 4g' \\ & - 2[2.5553314]\cos 5g'\end{aligned}$$

---

*Auseinandersetzung, Abh. I, s. 175.

†Auseinandersetzung, Abh. I, s. 176.

$$\begin{aligned}\frac{r'}{a'}\sin(f'+\Pi') = {} & \qquad\qquad\qquad\qquad + [8.6965298] \\ & + 2[9.6046934]\sin g' - 2[9.4702676]\cos g' \\ & + 2[8.0520079]\sin 2g' - 2[7.9174682]\cos 2g' \\ & + 2[6.6753852]\sin 3g' - 2[6.5407886]\cos 3g' \\ & + 2[5.3725363]\sin 4g' - 2[5.2379055]\cos 4g' \\ & + 2[4.1105366]\sin 5g' - 2[3.9758829]\cos 5g'\end{aligned}$$

$$\begin{aligned}-\frac{r}{a}\sin(f+\Pi) = {} & \qquad\qquad\qquad\qquad - [8.8188230] \\ & + 2[9.3147278]\sin g + 2[9.6578903]\cos g \\ & + 2[7.6969195]\sin 2g + 2[8.0399977]\cos 2g \\ & + 2[6.2551813]\sin 3g + 2[6.5982173]\cos 3g \\ & + 2[4.8872207]\sin 4g + 2[5.2302313]\cos 4g \\ & + 2[3.5601085]\sin 5g + 2[3.9031023]\cos 5g\end{aligned}$$

The required products are:

Arg.	$\alpha^2\left(\frac{r}{a}\right)^2\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{r'}{a'}\right)^2\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^3\frac{r'}{a'}\sin(f'+\Pi')$		$-\left(\frac{a'}{\Delta}\right)^3\frac{r}{a}\sin(f+\Pi)$	
i' i	cos.	sin.	cos.	sin.	sin.	cos.	sin.	cos.
0 0	+0.6646384		+2.20285566			+0.096539		−0.139090
0 − 1	−0.10224078	−0.09654157	−0.16606148	−0.14425645	+0.652427	−1.472529	−0.890963	+2.021227
0 − 2	−0.00359271	+0.00729372	−0.00584431	+0.01027942	+0.092267	+0.044090	−0.142260	−0.075960
0 − 3	+0.00037990	+0.00012406	+0.00050293	+0.00024342	−0.002049	+0.003890	+0.003572	−0.006580
0 − 4	+0.00000398	−0.00001827	+0.00000947	−0.00002311	−0.000137	−0.000129	+0.000233	+0.000204
0 − 5	−0.00000072	−0.00000007	−0.00000066	−0.00000021				
1 + 4	−0.00000066	+0.00000240	−0.00000051	+0.00000398				
1 + 3	+0.00004553	+0.00001192	+0.00006269	+0.00000476	+0.000439	+0.000255	−0.000732	−0.000421
1 + 2	+0.00006254	−0.00095698	−0.00002512	−0.00134143	−0.006697	+0.009449	+0.010197	−0.015371
1 + 1	−0.01753370	+0.00317476	−0.02618290	+0.00508023	−0.163541	−0.133286	+0.235056	+0.192100
1 0	+0.14866616	+0.15018589	+0.27948671	+0.34932553	+1.778166	−1.266003	−1.318411	+0.910778
1 − 1	+0.19097831	−0.95485704	+0.64138181	−3.15251634	−0.159202	−0.154638	+0.224895	+0.204201
1 − 2	−0.05391969	+0.04178008	−0.03958295	+0.03481677	+1.029740	+0.222978	−1.579388	−0.341385
1 − 3	+0.00233553	+0.00269350	+0.00466646	+0.00371574	+0.011210	+0.048408	−0.013547	−0.078501
1 − 4	+0.00010439	−0.00011490	+0.00024243	−0.00014058	−0.001063	+0.000140	+0.001990	−0.000163
1 − 5	−0.00000614	−0.00000384	−0.00000325	−0.00000596	+0.000065	−0.000057	−0.000087	+0.000093
1 − 6	−0.00000017	+0.00000035	−0.00000009	+0.00000044				
2 + 4	−0.00000054	+0.00000019	−0.00000144	+0.00000022				
2 + 3	+0.00000410	+0.00000342	+0.00000596	+0.00000385			−0.000097	+0.000001
2 + 2	+0.00006102	−0.00009392	+0.00007165	−0.00013682	−0.000139	+0.001320	+0.000186	−0.002105
2 + 1	−0.00204003	−0.00078227	−0.00295369	−0.00103270	−0.025483	−0.005037	+0.037735	+0.007827
2 0	+0.00382420	+0.03034997	+0.00212295	+0.05624290	+0.160407	−0.328034	−0.159335	+0.309261
2 − 1	+0.17342674	−0.14490556	+0.44903301	−0.34483157	+1.144372	+1.070503	−0.723589	−0.709105
2 − 2	−0.58491034	−0.24561672	−1.91960384	−0.82112372	+0.206918	−0.144719	−0.277128	+0.211746
2 − 3	+0.01175029	+0.02297816	−0.01074537	−0.02364232	−0.002108	+0.651351	+0.004800	−1.050101
2 − 4	+0.00156295	−0.00052038	+0.00225773	−0.00356737	−0.016195	+0.026941	+0.027428	−0.042361
2 − 5	−0.00001728	−0.00005498	+0.00002732	−0.00022913	−0.000033	+0.000521	+0.000113	−0.000727
2 − 6	−0.00000209	+0.00000191	+0.00000010	−0.00000522	+0.000044	+0.000065	−0.000071	−0.000100
2 − 7	+0.00000011	0.00000000	+0.00000029	−0.00000052				

Arg.	$\alpha^2\left(\frac{r}{a}\right)^2\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{r'}{a'}\right)^2\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^3\frac{r'}{a'}\sin(f'+\Pi')$		$-\left(\frac{a'}{\Delta}\right)^3\frac{r}{a}\sin(f+\Pi)$	
i' i	cos.	sin.	cos.	sin.	sin.	cos.	sin.	cos.
3 + 3	+0.00000020	+0.00000056	+0.00000037	+0.00000066				
3 + 2	+0.00001141	−0.00000609	+0.00001456	−0.00000958	+0.000065	+0.000141	−0.000097	−0.000220
3 + 1	−0.00016372	−0.00020324	−0.00023885	−0.00027297	−0.002959	+0.000917	+0.004484	−0.001309
3 0	−0.00186708	+0.00365057	−0.00353956	+0.00589773	−0.002715	−0.050285	+0.001175	+0.055370
3 − 1	+0.03915569	−0.00222274	+0.08428652	+0.00028893	+0.375924	+0.099078	−0.297611	−0.084841
3 − 2	−0.10336968	−0.16849363	−0.27004696	−0.46331603	−0.515184	+0.848304	+0.321116	−0.495661
3 − 3	−0.22156586	+0.32232369	−0.73682965	+1.04910933	+0.142585	+0.184461	−0.210273	−0.245654
3 − 4	+0.00501813	−0.00130270	−0.04931858	+0.01371360	−0.379900	+0.082387	+0.631353	−0.134324
3 − 5	−0.00009853	−0.00082735	−0.00352623	−0.00192573	−0.024540	+0.002047	+0.040371	−0.002909
3 − 6	−0.00002422	−0.00001228	−0.00021055	−0.00012873	−0.000948	+0.000551	+0.001542	−0.000886
3 − 7	+0.00000092	+0.00000016	−0.00000682	−0.00000892			+0.000096	−0.000109
4 + 2	+0.00000143	0.00000000	+0.00000182	−0.00000021				
4 + 1	−0.00000491	−0.00003014	−0.00000857	−0.00004059	− 0.000249	+0.000267	+0.000393	−0.000390
4 0	−0.00045076	+0.00026387	−0.00072831	+0.00037805	−0.003442	−0.005424	+0.003869	+0.006707
4 − 1	+0.00507472	+0.00312773	+0.00940728	+0.00669380	+0.064903	−0.016600	−0.059831	+0.013548
4 − 2	+0.00253344	−0.04124030	+0.00977433	−0.09745652	−0.019066	+0.333559	+0.018700	−0.239453
4 − 3	−0.14326266	+0.05695325	−0.40867444	+0.15495361	−0.557465	−0.192150	+0.301129	+0.114139
4 − 4	+0.16040847	+0.16908990	+0.51593778	+0.55897005	−0.132922	+0.132894	+0.174298	−0.194550
4 − 5	+0.00040331	+0.00313047	+0.00384834	+0.05099580	−0.095862	−0.204872	+0.159746	+0.348452
4 − 6	−0.00048955	+0.00010905	−0.00213220	+0.00324705	−0.009616	−0.016660	+0.016037	+0.028111
4 − 7	−0.00001948	+0.00001147	−0.00019125	+0.00017428	−0.000969	−0.000748	+0.001616	+0.001256
4 − 8	−0.00000244	+0.00000011	−0.000020	+0.000008	−0.000080	−0.000034	+0.000144	+0.000061
5 + 1	+0.00000118	−0.00000330	+0.00000128	−0.00000449				
5 0	−0.00006421	− 0.00000133	−0.00009593	−0.00000545	−0.000696	−0.000369	+0.000866	+0.000518
5 − 1	+0.00034972	+0.00075972	+0.00053616	+0.00139932	+0.007133	−0.006858	−0.007490	+0.006686
5 − 2	+0.00440850	−0.00567722	+0.01004694	−0.01168643	+0.030550	+0.063918	−0.023671	−0.052800
5 − 3	−0.03733148	−0.00796436	−0.09376594	−0.02265121	−0.252613	+0.037606	+0.169414	−0.021443
5 − 4	+0.02138719	+0.10968672	+0.05795719	+0.32034193	+0.036543	−0.333149	−0.021647	+0.162487
5 − 5	+0.11594525	−0.07019864	+0.38068179	−0.22146658	−0.112988	−0.081190	+0.163516	+0.103628
5 − 6	+0.00543316	+0.00055559	+0.0414448	+0.0054077	+0.101134	−0.080101	−0.175942	+0.135550
5 − 7	+0.00016122	+0.00037740	+0.0025756	+0.0023371	+0.009132	−0.010846	−0.015734	+0.018470
5 − 8	+0.00000202	+0.00002314	+0.000106	+0.000223	+0.000356	−0.001060	−0.000625	+0.001806
5 − 9	−0.00000024	+0.00000050	+0.000002	+0.000015	+0.000007	−0.000089	−0.000009	+0.000144
6 0	−0.0000065	−0.0000040	−0.0000092	−0.0000058	−0.000095	+0.000001	+0.000127	+0.000001
6 − 1	−0.0000154	+0.0001100	−0.0000375	+0.0001833	+0.000375	−0.001292	−0.000480	+0.001392
6 − 2	+0.001063053	−0.000356538	+0.00214318	−0.00058631	+0.009983	+0.007019	−0.008794	−0.006602
6 − 3	−0.0053004	−0.0054027	−0.0117489	−0.0128985	−0.051937	+0.039058	+0.039831	−0.028549
6 − 4	−0.0118680	+0.0299111	−0.0329458	+0.0783205	−0.063277	−0.169741	+0.037431	+0.107611
6 − 5	+0.0769855	+0.0001624	+0.2283160	+0.0034555	+0.182241	−0.022776	−0.075963	+0.009741
6 − 6	−0.0246439	−0.0732984	−0.0746870	−0.2387583	+0.040547	−0.088585	−0.050329	+0.125647
6 − 7	+0.0016248	−0.0049578	+0.0103000	−0.0290922	+0.057162	+0.044418	−0.098086	−0.079475
6 − 8	+0.0003435	−0.0001575	+0.0023028	−0.0017425	+0.009136	+0.003838	−0.015783	−0.006766
6 − 9	+0.0000245	−0.0000012	+0.0002239	−0.0000548	+0.000922	+0.000024	−0.001590	−0.000050
6 − 10	+0.0000032	+0.0000011	+0.0000226	+0.0000042	+0.000075	−0.000025	−0.000128	+0.000035

Arg.	$\alpha^2\left(\frac{r}{a}\right)^2\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{r'}{a'}\right)^2\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^2\frac{r'}{a'}\sin(f'+\Pi')$		$-\left(\frac{a'}{\Delta}\right)^3\frac{r}{a}\sin(f+\Pi)$	
	cos.	sin.	cos.	sin.	sin.	cos.	sin.	cos.
i′ i								
7 0	0.0000005	—0.0000008	—0.0000006	—0.0000008				
7 — 1	—0.0000091	+0.0000108	—0.0000153	+0.0000166	—0.000042	—0.000172	+0.000036	+0.000201
7 — 2	+0.0001574	+0.0000405	+0.0002875	+0.0000928	+0.001858	+0.000173	—0.001804	—0.000243
7 — 3	—0.0002561	—0.0012874	—0.0004222	—0.0027746	—0.005250	+0.011804	+0.004611	—0.009643
7 — 4	—0.0058602	+0.0041990	—0.0145296	+0.0097964	—0.040593	—0.035851	+0.028230	+0.026086
7 — 5	+0.0214993	+0.0134218	+0.0580492	+0.0375619	+0.102594	—0.065760	—0.061125	+0.038835
7 — 6	+0.0110324	—0.0500209	+0.0354100	—0.1498052	+0.035375	+0.090744	—0.013879	—0.028417
7 — 7	—0.0432461	+0.0042664	—0.1394246	+0.0100710	+0.063106	+0.015946	—0.089666	—0.015636
7 — 8	—0.0035723	—0.0021057	—0.0181881	—0.0112840	—0.015975	+0.036724	+0.030078	—0.063868
7 — 9	—0.0001067	—0.0003143	—0.0009675	—0.0020323	—0.000775	+0.006593	+0.001481	—0.011523
7 — 10	+0.0000053	—0.0000260	+0.0000061	—0.0002046	+0.000188	+0.000681	—0.000301	—0.001198
7 — 11	+0.0000011	—0.0000017	+0.0000070	—0.0000156	+0.000037	+0.000054	—0.000067	—0.000093
8 — 1	—0.0000016	+0.0000003	—0.0000023	0.0000000	—0.00002	—0.00002		
8 — 2	+0.0000152	+0.0000163	+0.0000250	+0.0000301	+0.00024	—0.00011	—0.00026	+0.00010
8 — 3	+0.0000766	—0.0001932	+0.0001734	—0.0003796	+0.00021	+0.00221	—0.00012	—0.00198
8 — 4	—0.0013823	+0.0000713	—0.0031357	+0.0000484	—0.01198	—0.00269	+0.00923	+0.00232
8 — 5	+0.0027912	+0.0057178	+0.0067270	+0.0146239	+0.02084	—0.03661	—0.01457	+0.02435
8 — 6	+0.0129004	—0.0139732	+0.0364325	—0.0384313	+0.05677	+0.05485	—0.03057	—0.03146
8 — 7	—0.0301808	—0.0131014	—0.0908832	—0.0410126	—0.03994	+0.02974	+0.00548	—0.00895
8 — 8	—0.0032098	+0.0238263	—0.012682	+0.075973	—0.00189	—0.04217	—0.00107	—0.05887
8 — 9	—0.0020423	+0.0021910	—0.009922	+0.010158	—0.02164	—0.00326	+0.03826	+0.00739
8 — 10	—0.0002706	+0.0000423	—0.001633	+0.000375	—0.00425	+0.00066	+0.00752	—0.00104
8 — 11	—0.0000213	—0.0000089	—0.000160	—0.000043	—0.00044	+0.00026	+0.00079	—0.00045
8 — 12	—0.0000010	—0.0000015	—0.000011	—0.000010			+0.00005	—0.00007
9 — 2	+0.0000007	+0.0000029	+0.0000009	+0.0000048	+0.00002	—0.00003		
9 — 3	+0.0000249	—0.0000181	+0.0000492	—0.0000316	+0.00020	+0.00028	—0.00018	—0.00028
9 — 4	—0.0002080	—0.0001179	—0.0004329	—0.0002698	—0.00226	+0.00067	+0.00190	—0.00050
9 — 5	—0.0001434	+0.0013382	—0.0004333	+0.0031614	+0.00022	—0.01076	—0.00035	+0.00791
9 — 6	+0.0050898	—0.0014450	+0.0133486	—0.0035112	+0.02965	+0.00942	—0.01891	—0.00646
9 — 7	—0.0080781	—0.0110583	—0.0224950	—0.0317272	—0.02622	+0.04170	+0.01377	—0.02153
9 — 8	—0.0120336	+0.0168420	—0.037508	+0.050795	—0.01982	—0.01454	+0.00313	—0.00321
9 — 9	+0.0122431	+0.0047938	+0.038433	+0.016733	—0.02633	+0.00438	+0.03628	—0.00844
9 — 10	+0.0011494	+0.0016872	+0.004953	+0.007662	—0.00147	—0.01174	+0.00143	+0.02121
9 — 11	—0.0000046	+0.0002109	+0.000019	+0.001196	—0.00110	—0.00248	+0.00186	+0.00446
9 — 12	—0.0000087	+0.0000169	—0.000052	+0.000115	—0.00026	—0.00025	+0.00045	+0.00046
10 — 3	+0.0000044	—0.0000003	+0.0000079	—0.0000002	+0.00005	+0.00002		
10 — 4	—0.0000184	—0.0000337	—0.0000342	—0.0000700	—0.00028	+0.00029	+0.00026	—0.00025
10 — 5	—0.0001567	+0.0001985	—0.0003664	+0.0004318	—0.00107	—0.00202	+0.00080	+0.00162
10 — 6	+0.0011787	+0.0003340	+0.002876	+0.000893	+0.00873	—0.00162	—0.00616	+0.00099
10 — 7	—0.0003842	—0.0041766	—0.000836	—0.011183	—0.00212	+0.02192	+0.00155	—0.01344
10 — 8	—0.0086730	+0.0039907	—0.025138	+0.011146	—0.02833	—0.00983	+0.01355	+0.00458
10 — 9	+0.0085693	+0.0094695	+0.025762	+0.029474	+0.00330	—0.01121	+0.00502	—0.00086
10 — 10	+0.0041353	—0.0057791	+0.013920	—0.017735	—0.00614	—0.01534	+0.00992	+0.02083
10 — 11	+0.0012421	—0.0004888	+0.005366	—0.001951	+0.00582	—0.00254	—0.01083	+0.00379
10 — 12	+0.0001508	+0.0000358	+0.000809	+0.000168	+0.00129	—0.00105	—0.00242	+0.00179

Arg.	$\alpha^2\left(\frac{r}{a}\right)^2 \left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{r'}{a'}\right)^2 \left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^3 \frac{r'}{a'} \sin(f'+\Pi')$		$-\left(\frac{'}{\Delta}\right)^3 \frac{r}{a} \sin(f+\Pi)$	
	cos.	sin.	cos.	sin.	sin.	cos.	sin.	cos.
i' i								
11 — 4	+0.0000002	—0.0000059	+0.000001	—0.000012	0.0000	+0.0001	0.0000	—0.0001
11 — 5	—0.0000410	+0.0000146	—0.000089	+0.000028	—0.0004	—0.0002	+0.0003	+0.0002
11 — 6	+0.0001686	+0.0001838	+0.000380	+0.000440	+0.0016	—0.0013	—0.0012	+0.0010
11 — 7	+0.0004611	—0.0009502	+0.001217	—0.002379	+0.0026	+0.0064	—0.0017	—0.0044
11 — 8	—0.0031808	—0.0003171	—0.008660	—0.001002	—0.0150	+0.0018	+0.0088	—0.0009
11 — 9	+0.0014625	+0.0063151	+0.004001	+0.018454	+0.0018	—0.0178	—0.0005	+0.0077
11 — 10	+0.0067393	—0.0038281	+0.020951	—0.011377	+0.0054	—0.0008	+0.0028	+0.0041
11 — 11	—0.0024282	—0.0029259	—0.007171	—0.009626	+0.0082	—0.0055	—0.0110	+0.0087
11 — 12	—0.0001255	—0.0008371	—0.000410	—0.003438	+0.0023	+0.0024	—0.0034	—0.0052
12 — 5	—0.000007	—0.000002	—0.000014	—0.000004	—0.0001	0.0000		
12 — 6	+0.000008	+0.000045	+0.000015	+0.000101	+0.0002	—0.0004	—0.0001	+0.0003
12 — 7	+0.000194	—0.000126	+0.000476	—0.000294	+0.0014	+0.0012	—0.0010	—0.0008
12 — 8	—0.000699	—0.000519	—0.001785	—0.001380	—0.0046	+0.0031	+0.0029	—0.0019
12 — 9	—0.000686	+0.002258	—0.001993	+0.006230	—0.0034	—0.0106	+0.0018	+0.0053
12 — 10	+0.004300	—0.000088	+0.012646	—0.000076	+0.0130	—0.0009	—0.0038	+0.0009
12 — 11	—0.001344	—0.004433	—0.003837	—0.013705	—0.0004	+0.0061	—0.0027	+0.0032
12 — 12	—0.001838	+0.000830	—0.006034	+0.001897	+0.0008	+0.0012	—0.0060	—0.0043

In order to get the developments of the disturbing forces it is necessary to have the expressions for the action of each disturbing planet on the Sun. In the case of Jupiter disturbed by Saturn this is proportional to the function $a'\frac{r}{r'^2}H$, and, in the case of Saturn disturbed by Jupiter, to the function $a'\frac{r'}{r^2}H$ (the signification of H has been given, page 22). To obtain the periodic developments of these quantities in terms of the mean anomalies of the two planets we employ some of the auxiliary constants of page 22, as well as the Besselian functions. We compute*

$$h = \alpha k \cos K \qquad\qquad l = \frac{1}{2}p \cos P$$

$$h_1 = \frac{1}{2}v \cos V \qquad\qquad l_1 = \frac{1}{2}v \sin V$$

$$P_i = \frac{1}{i}\left[J^{(i-1)}_{\frac{ie}{2}} - J^{(i+1)}_{\frac{ie}{2}}\right] \qquad\qquad Q_i = \frac{1}{i}\left[J^{(i-1)}_{\frac{ie}{2}} + J^{(i+1)}_{\frac{ie}{2}}\right]$$

$$P_i' = \frac{1}{i}\left[J^{(i-1)}_{\frac{ie'}{2}} - J^{(i+1)}_{\frac{ie'}{2}}\right] \qquad\qquad Q_i' = \frac{1}{i}\left[J^{(i-1)}_{\frac{ie'}{2}} + J^{(i+1)}_{\frac{ie'}{2}}\right]$$

notice being taken that

$$P_0 = -3e \qquad\qquad Q_0 = 0 \qquad\qquad P_0' = -3e' \qquad\qquad Q_0' = 0$$

Then

$$a'\frac{r}{r'^2}H = \frac{1}{2}i'^2\left[hP_iP_{i'}' \pm h_1Q_iQ_{i'}'\right]\cos(\pm i'g' - ig) - \frac{1}{2}i'^2\left[lQ_iP_{i'}' \pm l_1P_iQ_{i'}'\right]\sin(\pm i'g' - ig)$$

$$a'\frac{r'}{r^2}H = \frac{1}{2}\frac{i^2}{\alpha^3}\left[hP_iP_{i'}' \pm h_1Q_iQ_{i'}'\right]\cos(\pm i'g' - ig) - \frac{1}{2}\frac{i^2}{\alpha^3}\left[lQ_iP_{i'}' \pm l_1P_iQ_{i'}'\right]\sin(\pm i'g' - ig)$$

in which one attributes all positive integral values to i and i', and takes the double sign in both significations.

---

*Auseinandersetzung, Abh. I, s. 177, and Gegenseitige Störungen des Jupiter und Saturn, s. 46.

The numerical values being substituted, we have

$\log h = 9.0480387$ $\log l = 9.7266962$
$\log h_1 = 9.0472930$ $\log l_1 = 9.7264665$

$\log P_0 = 9.1605535n$ $\log P_0' = 9.2257502n$
$\log P_1 = 9.9996208$ $\log Q_1 = 9.9998736$ $\log P_1' = 9.9994880$ $\log Q_1' = 9.9998294$
$\log P_2 = 8.3817282$ $\log Q_2 = 8.3820653$ $\log P_2' = 8.4466886$ $\log Q_2' = 8.4471439$
$\log P_3 = 6.9399478$ $\log Q_3 = 6.9403271$ $\log P_3' = 7.0700090$ $\log Q_3' = 7.0705212$
$\log P_4 = 5.5719618$ $\log Q_4 = 5.5723665$ $\log P_4' = 5.7671259$ $\log Q_4' = 5.7676723$
$\log P_5 = 4.2448328$ $\log Q_5 = 4.2452543$ $\log P_5' = 4.5051033$ $\log Q_5' = 4.5056725$
$\log P_6 = 2.9437$ $\log Q_6 = 2.9441$ $\log P_6' = 3.2690$ $\log Q_6' = 3.2696$

Arg.	$-a'\frac{r}{r'^2}H$		$-a'\frac{r'}{r^2}H$	
	cos.	sin.	cos.	sin.
i' i				
1 0	+0.00807328	−0.03853181		
2 0	+0.00090430	−0.00431712		
3 0	+0.00008547	−0.00040808		
4 0	+0.00000756	−0.00003612		
5 0	+0.00000065	−0.00000309		
6 0	+0.00000005	−0.00000026		
−3 −1	+0.00000003	−0.00000019	+0.00000002	−0.00000013
−2 −1	−0.00000054	+0.00000187	−0.00000083	+0.00000288
−1 −1	−0.00001944	+0.00008644	−0.00011995	+0.00053340
0 −1			+0.05790623	−0.27646210
1 −1	−0.11144785	+0.53209260	−0.68772890	+3.28346820
2 −1	−0.01248502	+0.05960803	−0.01926083	+0.09195819
3 −1	−0.00118010	+0.00563425	−0.00080914	+0.00386313
4 −1	−0.00010445	+0.00049871	−0.00004029	+0.00019234
5 −1	−0.00000893	+0.00004262	−0.00000220	+0.00001052
6 −1	−0.00000075	+0.00000356	−0.00000013	+0.00000061
7 −1	−0.00000006	+0.00000030	−0.00000001	+0.00000004
−3 −2			+0.00000001	+0.00000002
−2 −2	+0.00000002	+0.00000018	+0.00000010	+0.00000114
−1 −2	−0.00000021	+0.00000333	−0.00000513	+0.00008217
0 −2			+0.00558332	−0.02666167
1 −2	−0.00268671	+0.01282735	−0.06631731	+0.31662297
2 −2	−0.00030098	+0.00143699	−0.00185731	+0.00886747
3 −2	−0.00002845	+0.00013583	−0.00007803	+0.00037253
4 −2	−0.000002518	+0.000012023	−0.000003885	+0.000018548
5 −2	−0.0000002152	+0.0000010276	−0.0000002125	+0.0000010145
6 −2	−0.0000000180	+0.0000000859	−0.00000001	+0.00000006
−2 −3			+0.00000001	+0.00000010
−1 −3	0.00000000	+0.00000014	−0.00000016	+0.00000794
0 −3			+0.00045426	−0.00216936
1 −3	−0.00009715	+0.00046385	−0.00539573	+0.02576117
2 −3	−0.00001088	+0.00005196	−0.00015112	+0.00072147
3 −3	−0.00000103	+0.00000491	−0.00000635	+0.00003031
4 −3	−0.00000009	+0.00000043	−0.00000032	+0.00000151
5 −3	−0.000000008	+0.000000037	−0.00000002	+0.00000008

Arg.	$-a'\frac{r}{r'^2}H$		$-a'\frac{r'}{r^2}H$	
	cos.	sin.	cos.	sin.
−2−4			0.00000000	+0.00000001
−1−4			0.00000000	+0.00000053
0−4			+0.00003461	−0.00016530
1−4	−0.00000416	+0.00001988	−0.00041111	+0.00196280
2−4	−0.00000047	+0.00000223	−0.00001151	+0.00005497
3−4	−0.00000004	+0.00000021	−0.00000048	+0.00000231
4−4			−0.00000002	+0.00000009
−1−5			0.00000000	+0.00000004
0−5			+0.00000255	−0.00001216
1−5	−0.00000020	+0.00000094	−0.00003025	+0.00014440
2−5	−0.00000002	+0.00000010	−0.00000085	+0.00000404
3−5			−0.00000003	+0.00000013
0−6			+0.00000018	−0.00000088
1−6			−0.00000218	+0.00001040
2−6			−0.00000006	+0.00000029

On account of the action of the disturbing planets on the Sun it is necessary to include in the disturbing forces perpendicular to the planes of the orbits, respectively in the motions of Jupiter and Saturn, terms which are proportional to

$$-\left(\frac{a'}{r'}\right)^2 \sin(f' + \Pi') \text{ and } \frac{1}{\alpha^3}\left(\frac{a}{r}\right)^2 \sin(f + \Pi)$$

We have*

$$\begin{aligned}\left(\frac{a}{r}\right)^2 \sin(f+\Pi) = {} & \left[J^{(0)}_{\frac{e}{2}} + J^{(2)}_{\frac{e}{2}}\right] \cos\varphi \cos\Pi \sin g + \left[J^{(0)}_{\frac{e}{2}} - J^{(2)}_{\frac{e}{2}}\right] \sin\Pi \cos g \\ & + 2\left[J^{(1)}_{e} + J^{(3)}_{e}\right] \cos\varphi \cos\Pi \sin 2g + 2\left[J^{(1)}_{e} - J^{(3)}_{e}\right] \sin\Pi \cos 2g \\ & + 3\left[J^{(2)}_{\frac{3}{2}e} + J^{(4)}_{\frac{3}{2}e}\right] \cos\varphi \cos\Pi \sin 3g + 3\left[J^{(2)}_{\frac{3}{2}e} - J^{(4)}_{\frac{3}{2}e}\right] \sin\Pi \cos 3g \\ & + \ldots\ldots\ldots\ldots\ldots\ldots\end{aligned}$$

with a similar formula for $\left(\frac{a'}{r'}\right)^2 \sin(f' + \Pi')$. The numerical values being substituted, we get:

Arg.	$-\left(\frac{a'}{r'}\right)^2 \sin(f'+\Pi')$		Arg.	$\frac{1}{\alpha^3}\left(\frac{a}{r}\right)^2 \sin(f+\Pi)$	
	sin.	cos.		sin.	cos.
i' i			i' i		
1 0	−0.804866	+0.590606	0−1	+2.547437	−5.613916
2 0	−0.090177	+0.066154	0−2	+0.245672	−0.541294
3 0	−0.008524	+0.006253	0−3	+0.019989	−0.044039
4 0	−0.000755	+0.000554	0−4	+0.001523	−0.003355
5 0	−0.000064	+0.000047	0−5	+0.000112	−0.000247
6 0	−0.000005	+0.000004	0−6	+0.000008	−0.000018

---

*Auseinandersetzung, Abh. I, s. 178.

Certain factors dependent upon the masses are here necessary. We put

$$\mu = \frac{m'}{1+m}\alpha\frac{a}{\mathrm{a}} \qquad \mu' = \frac{m}{1+m'}\frac{a'}{\mathrm{a}'}$$

These factors being expressed in seconds of arc by multiplying by the radius in seconds, we have for Jupiter

$$\log\ \mu = 1.5063000 \qquad \log\,(\mu\alpha\sin\mathrm{J}) = 9.5836560$$

and for Saturn

$$\log\ \mu' = 2.2938045 \qquad \log\,(\mu'\alpha\sin\mathrm{J}) = 0.3711605$$

For Jupiter, then, we compute the three functions

$$a\Omega = \mu\left[\frac{\mathrm{a}'}{\triangle} - \mathrm{a}'\frac{r}{r'^2}\mathrm{H}\right]$$

$$ar\frac{d\Omega}{dr} = \mu\left[-\frac{1}{2}\left(\frac{\mathrm{a}'}{\triangle}\right)^3\left(\alpha^2\frac{r^2}{\mathrm{a}^2} - \frac{r'^2}{\mathrm{a}'^2}\right) - \frac{1}{2}\frac{\mathrm{a}'}{\triangle} - \mathrm{a}'\frac{r}{r'^2}\mathrm{H}\right]$$

$$a^2\frac{d\Omega}{dZ} = \mu\alpha\sin\mathrm{J}\left[\left(\frac{\mathrm{a}'}{\triangle}\right)^3\frac{r'}{\mathrm{a}'}\sin(f'+\Pi') - \left(\frac{\mathrm{a}'}{r'}\right)^2\sin(f'+\Pi')\right]$$

and for Saturn the three functions

$$a'\Omega' = \mu'\left[\frac{\mathrm{a}'}{\triangle} - \mathrm{a}'\frac{r'}{r^2}\mathrm{H}\right]$$

$$a'r'\frac{d\Omega'}{dr'} = \mu'\left[\frac{1}{2}\left(\frac{\mathrm{a}'}{\triangle}\right)^3\left(\alpha^2\frac{r^2}{\mathrm{a}^2} - \frac{r'^2}{\mathrm{a}'^2}\right) - \frac{1}{2}\frac{\mathrm{a}'}{\triangle} - \mathrm{a}'\frac{r'}{r^2}\mathrm{H}\right]$$

$$a'^2\frac{d\Omega'}{dZ'} = \mu'\alpha\sin\mathrm{J}\left[-\left(\frac{\mathrm{a}'}{\triangle}\right)^3\frac{r}{\mathrm{a}}\sin(f+\Pi) + \frac{1}{\alpha^3}\left(\frac{\mathrm{a}}{r}\right)^2\sin(f+\Pi)\right]$$

The quantities $a\Omega$ and $a'\Omega'$ are, moreover, differentiated severally with respect to g and g'. For Jupiter the three functions are:

Arg.	$a\frac{d\Omega}{dg}$		$ar\frac{d\Omega}{dr}$		$a^2\frac{d\Omega}{dZ}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	″	″	″	″	″	″
0 0			+7.174462			+0.037014
0 − 1	−0.3080771	+0.1681294	−0.8698000	−0.6813978	+0.2501428	−0.5645727
0 − 2	−0.016795	−0.017801	−0.031923	+0.043448	+0.035375	+0.016904
0 − 3	+0.000486	−0.001147	+0.001893	+0.001551	−0.00079	+0.00149
0 − 4	+0.000037	−0.000001	+0.000083	−0.000078	−0.00005	−0.00005
1 + 4			+0.000002	+0.000025		
1 + 3	−0.000077	−0.000071	+0.000262	−0.000110	+0.00017	+0.00010
1 + 2	+0.000975	−0.002108	−0.001173	−0.000580	−0.00257	+0.00362
1 + 1	+0.034578	+0.005301	−0.122401	+0.023759	−0.06270	−0.05110
1 0			+1.338747	+1.182548	+0.37316	−0.25895
1 − 1	+0.448653	+2.408872	+1.637557	−8.443179	−0.06104	−0.05929
1 − 2	+0.239298	−0.292616	+0.040867	+0.432481	+0.39480	+0.08549
1 − 3	+0.029583	−0.029277	+0.027787	+0.033842	+0.00430	+0.01856
1 − 4	+0.002181	−0.001338	+0.001742	+0.000378	−0.00041	+0.00005
1 − 5	+0.000109	−0.000077	+0.000026	+0.000004	+0.00002	−0.00002

Arg.	$a\frac{d\Omega}{dg}$		$ar\frac{d\Omega}{dr}$		$a^2\frac{d\Omega}{dZ}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i′ i	″	″	″	″	″	″
2 + 4			− 0.000014	0.000000		
2 + 3			+ 0.000029	+ 0.000007		
2 + 2	+ 0.000006	−0.000216	+ 0.000173	− 0.000643	−0.00005	+0.00051
2 + 1	+ 0.003180	−0.000687	− 0.013093	− 0.003764	−0.00977	−0.00193
2 0			− 0.028860	+ 0.187790	+0.02692	−0.10041
2 − 1	+ 1.239251	−0.065572	+ 3.200897	− 0.371312	+0.43875	+0.41043
2 − 2	−15.027048	+6.467210	−17.669445	− 7.546567	+0.07933	−0.05549
2 − 3	− 0.601705	+0.808199	− 0.261124	− 0.610705	−0.00081	+0.24973
2 − 4	− 0.014112	+0.063011	+ 0.012887	− 0.040897	−0.00621	+0.01033
2 − 5	− 0.000279	+0.004268	+ 0.000742	− 0.002362	−0.00001	+0.00020
2 − 6	+ 0.000002	+0.000258	+ 0.000035	− 0.000093	+0.00002	+0.00002
3 + 2			+ 0.000048	− 0.000052	+0.00002	+0.00005
3 + 1	+ 0.000243	−0.000171	− 0.001082	− 0.001024	−0.00113	+0.00035
3 0			− 0.022363	+ 0.015394	−0.00431	−0.01688
3 − 1	+ 0.215749	−0.103402	+ 0.559338	+ 0.259753	+0.14413	+0.03798
3 − 2	− 2.119094	+2.716155	− 2.145503	− 4.044090	−0.19752	+0.32525
3 − 3	− 6.494673	−8.986642	− 7.183684	+10.161867	+0.05467	+0.07072
3 − 4	− 0.905238	−0.380993	− 0.758540	+ 0.193284	−0.14566	+0.03159
3 − 5	− 0.072287	+0.001728	− 0.047760	− 0.017444	−0.00941	+0.00078
3 − 6	− 0.004853	+0.001340	− 0.002585	− 0.001756	−0.00036	+0.00021
3 − 7	− 0.000299	+0.000137	− 0.000103	− 0.000136		
4 + 1	+ 0.000013	−0.000025	− 0.000052	− 0.000155	−0.00010	+0.00010
4 0			− 0.003749	+ 0.000188	−0.00161	−0.00187
4 − 1	+ 0.022875	−0.022972	+ 0.053040	+ 0.069724	+0.02488	−0.00636
4 − 2	− 0.0700420	+0.5065620	+ 0.133551	− 0.774625	−0.00731	+0.12789
4 − 3	− 2.959698	−1.469884	− 3.764570	+ 1.327203	−0.21374	−0.07367
4 − 4	+ 4.606216	−5.191627	+ 5.127778	+ 5.605672	−0.05096	+0.05095
4 − 5	+ 0.135515	−0.791777	+ 0.041715	+ 0.688698	−0.03675	−0.07855
4 − 6	− 0.018223	−0.065741	− 0.024833	+ 0.044863	−0.00369	−0.00639
4 − 7	− 0.002796	−0.004332	− 0.002556	+ 0.002303	−0.00037	−0.00029
4 − 8	− 0.000257	−0.000249	− 0.000311	+ 0.000111	−0.00003	−0.00001
5 0			− 0.000431	− 0.000182	−0.00029	−0.00012
5 − 1	+ 0.001576	−0.003348	+ 0.001773	+ 0.010638	+0.002735	−0.002629
5 − 2	+ 0.02558947	+0.06044055	+ 0.0840463	− 0.0812427	+0.01171	+0.02451
5 − 3	− 0.6292999	+0.0302412	− 0.800463	− 0.230570	−0.096852	+0.014418
5 − 4	+ 0.676908	−2.601663	+ 0.502058	+ 3.054213	+0.01401	−0.12773
5 − 5	+ 3.646320	+1.995219	+ 3.822384	− 2.227183	−0.04332	−0.03113
5 − 6	+ 0.596295	−0.033889	+ 0.528020	+ 0.075013	+0.03877	−0.03071
5 − 7	+ 0.050524	−0.028467	+ 0.035123	+ 0.029403	+0.00350	−0.00416
5 − 8	+ 0.003152	−0.003617	+ 0.001476	+ 0.002984	+0.00014	−0.00041
5 − 9	+ 0.000150	−0.000292	0.000000	+ 0.000225	0.00000	−0.00003
6 0			− 0.000038	− 0.000035	−0.00004	0.00000
6 − 1	+ 0.000030	−0.000378	− 0.000404	+ 0.001158	+0.00014	−0.00050
6 − 2	+ 0.00631401	+0.00456065	+ 0.0157483	− 0.0025418	+0.00383	+0.00269

Arg.	$a\frac{d\Omega}{dg}$		$ar\frac{d\Omega}{dr}$		$a^2\frac{d\Omega}{dZ}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	″	″	″	″	″	″
6 − 3	−0.082465	+0.053035	−0.089706	−0.111411	−0.01991	+0.01497
6 − 4	−0.151565	−0.609415	−0.319198	+0.700425	−0.02426	−0.06508
6 − 5	+1.991245	+0.086878	+2.228580	+0.061513	+0.06987	−0.00873
6 − 6	−0.643134	+2.329039	−0.749217	−2.460292	+0.01555	−0.03396
6 − 7	+0.116590	+0.400914	+0.130843	−0.358535	+0.02192	+0.01703
6 − 8	+0.031338	+0.033486	+0.02947	−0.02333	+0.00350	+0.00147
6 − 9	+0.003659	+0.001799	+0.00299	−0.00076	+0.00035	+0.00001
6 − 10	+0.000244	+0.000042	+0.00030	+0.00005	+0.00003	−0.00001
7 − 1	−0.000011	−0.000025	−0.000096	+0.000090	−0.00002	−0.00007
7 − 2	+0.000877	+0.000069	+0.001867	+0.000857	+0.00071	+0.00007
7 − 3	−0.006105	+0.012129	−0.001646	−0.021836	−0.00201	+0.00452
7 − 4	−0.079178	−0.083811	−0.129178	+0.079320	−0.01556	−0.01375
7 − 5	+0.498555	−0.238188	+0.536494	+0.363447	+0.03934	−0.02521
7 − 6	+0.236771	+1.370463	+0.371344	−1.486577	+0.01357	+0.03479
7 − 7	−1.373021	−0.043052	−1.444864	+0.090045	+0.02419	+0.00611
7 − 8	−0.242307	+0.136486	−0.21932	−0.13871	−0.00612	+0.01408
7 − 9	−0.01848	+0.02865	−0.01278	−0.02596	−0.00030	+0.00253
7 − 10	−0.00247	+0.00343	+0.00014	−0.00269	+0.00007	+0.00026
7 − 11	−0.00018	+0.00035	+0.00010	−0.00021	+0.00001	+0.00002
8 − 2	+0.00008	−0.00004	+0.00013	+0.00021	+0.00009	−0.00004
8 − 3	+0.000126	+0.001740	+0.001530	−0.002698	+0.00008	+0.00085
8 − 4	−0.01756	−0.00525	−0.02593	−0.00102	−0.00459	−0.00103
8 − 5	+0.06808	−0.09534	+0.05633	+0.13332	+0.00799	−0.01404
8 − 6	+0.27138	+0.35555	+0.35490	−0.36274	+0.02176	+0.02103
8 − 7	−0.85986	+0.35263	−0.91239	−0.42257	−0.01531	+0.01140
8 − 8	−0.16317	−0.74948	−0.14176	+0.78972	−0.00073	−0.01617
8 − 9	−0.12186	−0.13364	−0.11965	+0.12039	−0.00830	−0.00125
8 − 10	−0.02319	−0.00837	−0.02070	+0.00492	−0.00163	+0.00025
8 − 11	−0.00265	+0.00021	−0.00210	−0.00050	−0.00017	+0.00010
8 − 12	−0.00023	+0.00019	−0.00014	−0.00009		
9 − 3	+0.00013	+0.00017	+0.00037	−0.00019	+0.00008	+0.00011
9 − 4	−0.00257	+0.00063	−0.00329	−0.00236	−0.00087	+0.00026
9 − 5	+0.00220	−0.02099	−0.00487	+0.02715	+0.00008	−0.00413
9 − 6	+0.09828	+0.04399	+0.12430	−0.02948	+0.01137	+0.00361
9 − 7	−0.22203	+0.25869	−0.21542	−0.31311	−0.01005	+0.01599
9 − 8	−0.34462	−0.49246	−0.38713	+0.51385	−0.00760	−0.00557
9 − 9	+0.37643	−0.19061	+0.39923	+0.18094	−0.01010	+0.00168
9 − 10	+0.06430	−0.09423	+0.05780	+0.09114	−0.00056	−0.00450
9 − 11	+0.00191	−0.01708	+0.00029	+0.01502	−0.00042	−0.00095
9 − 12	−0.00058	−0.00181	−0.00069	+0.00149	−0.00010	−0.00010
10 − 3	+0.000023	+0.000011	+0.000052	+0.000003	+0.000018	+0.000007
10 − 4	−0.0002542	+0.0002503	−0.000221	−0.000552	−0.00011	+0.00011
10 − 5	−0.001380	−0.003099	−0.003224	+0.003433	−0.000409	−0.000776
10 − 6	+0.02158	−0.00191	+0.02541	+0.00879	+0.00335	−0.00062
10 − 7	−0.01987	+0.08981	−0.00584	−0.10598	−0.00081	+0.00841

Arg.	$a\frac{d\Omega}{dg}$		$ar\frac{d\Omega}{dr}$		$a^2\frac{d\Omega}{dZ}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	"	"	"	"	"	"
10 — 8	—0.21828	—0.11833	—0.2505	+0.1074	—0.01086	—0.00377
10 — 9	+0.25409	—0.28085	+0.2617	+0.3053	+0.00126	—0.00430
10 — 10	+0.15333	+0.17031	+0.1493	—0.1833	—0.00235	—0.00588
10 — 11	+0.06586	+0.02513	+0.0632	—0.0223	+0.00223	—0.00097
10 — 12	+0.01155	—0.00127	+0.0100	+0.0021	+0.00050	—0.00040
11 — 4	+0.000012	—0.000044	0.000000	+0.000081	—0.00001	+0.00002
11 — 5	—0.000417	—0.000304	—0.000706	+0.000160	—0.00014	—0.00009
11 — 6	+0.00316	—0.00221	+0.00311	+0.00391	+0.00062	—0.00050
11 — 7	+0.00575	+0.01965	+0.01171	—0.02153	+0.00101	—0.00247
11 — 8	—0.07436	—0.00090	—0.0832	—0.0110	—0.00574	+0.00068
11 — 9	+0.04857	—0.16760	+0.0380	+0.1854	+0.00069	—0.00681
11 — 10	+0.20438	+0.11335	+0.2178	—0.1154	+0.00208	—0.00029
11 — 11	—0.06568	+0.10415	—0.0731	—0.1029	+0.00315	—0.00213
11 — 12	—0.00512	+0.04181	—0.0045	—0.0397	+0.00087	+0.00091
12 — 5	—0.000075	—0.000008	—0.000106	—0.000030	—0.00003	0.00000
12 — 6	+0.000308	—0.000558	+0.00006	+0.00083	+0.00006	—0.00015
12 — 7	+0.00283	+0.00276	+0.00443	—0.00250	+0.00055	+0.00045
12 — 8	—0.01607	+0.00852	—0.0167	—0.0138	—0.00178	+0.00118
12 — 9	—0.01126	—0.05648	—0.0217	+0.0621	—0.00131	—0.00406
12 — 10	+0.11875	+0.00770	+0.1324	+0.0027	+0.00498	—0.00035
12 — 11	—0.03815	+0.13574	—0.0369	—0.1510	—0.00015	+0.00235
12 — 12	—0.06514	—0.01386	—0.0719	+0.0184	+0.00030	+0.00047

The similar quantities for Saturn are:

Arg.	$a'\frac{d\Omega'}{dg'}$		$a'r'\frac{d\Omega'}{dr'}$		$a'^2\frac{d\Omega'}{dZ'}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	"	"	"	"	"	"
0 0			—258.5835			—0.32693
1 0	— 12.493747	+ 9.512847	— 19.11307	— 24.34181	—3.098926	+2.140785
2 0	— 0.749962	+ 2.184496	— 0.02017	— 3.09269	—0.37452	+0.72692
3 0	+ 0.063454	+ 0.278183	+ 0.17506	— 0.26737	+0.00276	+0.13015
4 0	+ 0.02260	+ 0.02378	+ 0.03012	— 0.01420	+0.00909	+0.01576
5 0	+ 0.00348	+ 0.00102	+ 0.00347	+ 0.00030	+0.00204	+0.00122
6 0	+ 0.00038	— 0.00008	+ 0.00030	+ 0.00018	+0.00030	0.00000
—4 — 1	— 0.00033	— 0.00061	+ 0.00040	— 0.00110	—0.00092	—0.00092
—3 — 1	— 0.00447	— 0.00317	+ 0.00814	— 0.00742	—0.01054	—0.00308
—2 — 1	— 0.03910	— 0.00882	+ 0.09939	— 0.02598	—0.08870	+0.01840
—1 — 1	— 0.23174	— 0.05542	+ 0.93113	+ 0.31707	—0.55250	+0.45153
0 — 1			+ 18.61128	— 49.17198	+3.89354	—8.44462
1 — 1	+110.60400	+526.42790	—191.90960	+921.71340	+0.52862	+0.47997

Arg.	$a'\frac{d\Omega'}{dg'}$		$a'r'\frac{d\Omega'}{dr'}$		$a'^2\frac{d\Omega'}{dZ'}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	″	″	″	″	″	″
2 − 1	−12.52916	+13.53055	− 35.92108	+43.41236	−1.70080	−1.66675
3 − 1	− 4.18693	+ 0.85662	− 5.37518	+ 0.75002	−0.69954	−0.19941
4 − 1	− 0.61143	+ 0.32227	− 0.51443	− 0.33426	−0.14063	+0.03184
5 − 1	− 0.05492	+ 0.07103	− 0.02448	− 0.06690	−0.01762	+0.01572
6 − 1	− 0.00185	+ 0.01042	+ 0.00199	− 0.00789	−0.00113	+0.00327
7 − 1	+ 0.00039	+ 0.00113	+ 0.00063	− 0.00065	+0.00008	+0.00047
− 3 − 2			− 0.00032	− 0.00037	+0.00023	−0.00052
− 2 − 2	− 0.00001	− 0.00170	− 0.00101	− 0.00431	−0.00044	−0.00495
− 1 − 2	− 0.00396	− 0.02197	+ 0.00908	− 0.02455	−0.02397	−0.03613
0 − 2			+ 1.34542	− 5.56528	+0.24307	−1.45086
1 − 2	+11.78262	+60.65358	− 15.08564	+63.77769	−3.71235	−0.80243
2 − 2	+92.73743	−36.72487	+153.90350	+68.39870	−0.65139	+0.49771
3 − 2	+19.51631	−24.83790	+ 19.62241	+33.24536	+0.75478	−1.16505
4 − 2	+ 0.859890	− 6.205946	− 0.605801	+ 6.310084	+0.043954	−0.562835
5 − 2	− 0.3922011	− 0.9263581	− 0.59382	+ 0.68394	−0.05564	−0.12411
6 − 2	− 0.1161438	− 0.0839107	− 0.115913	+ 0.029608	−0.02067	−0.01552
7 − 2	− 0.01882	− 0.00147	− 0.01414	− 0.00503	−0.00424	−0.00057
8 − 2	− 0.00205	+ 0.00110	− 0.00108	− 0.00142	−0.00060	+0.00024
− 2 − 3			− 0.00019	+ 0.00006	+0.00023	0.00000
− 1 − 3	+ 0.00013	− 0.00168	− 0.00180	+ 0.00080	+0.00172	−0.00099
0 − 3			+ 0.07676	− 0.43962	+0.05525	−0.11898
1 − 3	+ 0.98178	+ 5.03581	− 1.33037	+ 4.98239	−0.03184	−0.18451
2 − 3	+ 2.51439	− 3.03979	+ 2.79646	+ 5.55794	+0.01128	−2.46826
3 − 3	+39.81953	+55.10867	+ 57.31092	−80.65523	−0.49425	−0.57741
4 − 3	+24.19323	+12.01592	+ 29.12732	−11.13988	+0.70780	+0.26828
5 − 3	+ 6.43001	− 0.30895	+ 6.19333	+ 1.47537	+0.39821	−0.05040
6 − 3	+ 1.01114	− 0.65028	+ 0.71845	+ 0.79138	+0.09362	−0.06710
7 − 3	+ 0.08734	− 0.17350	+ 0.02256	+ 0.15867	+0.01084	−0.02267
8 − 3	− 0.00206	− 0.02844	− 0.00964	+ 0.02010	−0.00029	−0.00465
9 − 3	− 0.00230	− 0.00319	− 0.00252	+ 0.00149	−0.00042	−0.00065
10 − 3	+ 0.00046	− 0.00022	− 0.00037	0.00000		
− 1 − 4			− 0.00002	+ 0.00026		
0 − 4			+ 0.00624	− 0.03204	+0.00413	−0.00741
1 − 4	+ 0.07671	+ 0.38422	− 0.09652	+ 0.38954	+0.00468	−0.00038
2 − 4	+ 0.04760	− 0.17240	− 0.05983	+ 0.35900	+0.06447	−0.09957
3 − 4	+ 4.16251	+ 1.75303	+ 6.03763	− 1.76834	+1.48400	−0.31573
4 − 4	−28.23897	+31.82790	− 38.49617	−42.32321	+0.40969	−0.45729
5 − 4	− 5.18733	+19.93726	− 4.11539	−22.71167	−0.05088	+0.38193
6 − 4	+ 1.39378	+ 5.60414	+ 2.18915	− 5.22807	+0.08798	+0.25294
7 − 4	+ 0.84946	+ 0.89916	+ 0.91330	− 0.61473	+0.06635	+0.06132
8 − 4	+ 0.21528	+ 0.06436	+ 0.18587	− 0.00177	+0.02170	+0.00546
9 − 4	+ 0.03548	− 0.00871	+ 0.024077	+ 0.015421	+0.00446	−0.00118
10 − 4	+ 0.0038962	− 0.0038362	+ 0.00175	+ 0.00376	+0.00061	−0.00058
11 − 4	+ 0.000208	− 0.000745	+ 0.000019	− 0.000429		

Arg.	$a'\frac{d\Omega'}{dg'}$		$a'r\frac{d\Omega'}{dr'}$		$a'^2\frac{d\Omega'}{dZ'}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i′ i	″	″	″	″	″	″
0 − 5			+ 0.00050	− 0.00238	+0.00026	−0.00058
1 − 5	+ 0.00578	+ 0.02831	− 0.00632	+ 0.02866	−0.00020	+0.00022
2 − 5	+ 0.00101	− 0.00891	− 0.00438	+ 0.02055	+0.00027	−0.00171
3 − 5	+ 0.26592	− 0.00628	+ 0.38143	+ 0.10911	+0.09487	−0.00684
4 − 5	− 0.66461	+ 3.88327	− 0.42190	− 5.19296	+0.37548	+0.81904
5 − 5	−22.35421	−12.23193	−28.27227	+16.10040	+0.38434	+0.24358
6 − 5	−14.64908	− 0.63903	−16.10411	− 0.27062	−0.17855	+0.02454
7 − 5	− 4.27903	+ 2.04434	− 3.90033	− 2.52020	−0.14367	+0.09128
8 − 5	− 0.66790	+ 0.93520	− 0.42883	− 0.93435	−0.03424	+0.05723
9 − 5	− 0.02425	+ 0.23150	+ 0.02716	− 0.19218	−0.00082	+0.01860
10 − 5	+ 0.01692	+ 0.03800	+ 0.02146	− 0.02484	+0.00188	+0.00381
11 − 5	+ 0.00563	+ 0.00411	+ 0.00492	− 0.00138	+0.00069	+0.00048
12 − 5	+ 0.00111	+ 0.00012	+ 0.00059	+ 0.00020		
1 − 6	+ 0.00042	+ 0.00204	− 0.00044	+ 0.00204		
2 − 6	0.00000	− 0.00053	− 0.00023	+ 0.00089	−0.00017	−0.00024
3 − 6	+ 0.01488	− 0.00411	+ 0.02080	+ 0.01214	+0.00363	−0.00208
4 − 6	+ 0.07448	+ 0.26869	+ 0.17086	− 0.34221	+0.03769	+0.06608
5 − 6	− 3.04638	+ 0.17313	− 3.84637	− 0.49450	−0.41355	+0.31861
6 − 6	+ 3.94281	−14.27845	+ 5.25030	+17.46285	−0.11830	+0.29533
7 − 6	− 1.69348	− 9.80208	− 2.51849	+10.51393	−0.03262	−0.06679
8 − 6	− 2.21830	− 2.90630	− 2.45303	+ 2.58710	−0.07187	−0.07395
9 − 6	− 0.90365	− 0.40439	− 0.86244	+ 0.22567	−0.04446	−0.01518
10 − 6	− 0.22050	+ 0.01947	− 0.17783	− 0.05586	−0.01448	+0.00234
11 − 6	− 0.03548	+ 0.02488	− 0.02223	− 0.02616	−0.00291	+0.00225
12 − 6	− 0.00378	+ 0.00684	− 0.00079	− 0.00570	−0.00033	+0.00073
3 − 7	+ 0.0008	− 0.0004	+ 0.0009	+ 0.0010	+0.0002	−0.0003
4 − 7	+ 0.0098	+ 0.0152	+ 0.0181	− 0.0179	+0.0038	+0.0030
5 − 7	− 0.2212	+ 0.1247	− 0.2596	− 0.2052	−0.0370	+0.0434
6 − 7	− 0.6127	− 2 1067	− 0.9042	+ 2.5492	−0.2305	−0.1868
7 − 7	+ 8.4175	+ 0.2639	+10.0604	− 0.5897	−0.2108	−0.0367
8 − 7	+ 6.0245	− 2.4707	+ 6.3466	+ 2.8995	+0.0129	−0.0210
9 − 7	+ 1.7498	− 2.0390	+ 1.5151	+ 2.1461	+0.0324	−0.0506
10 − 7	+ 0.1741	− 0.7867	+ 0.0531	+ 0.7285	+0.0036	−0.0316
11 − 7	− 0.0554	− 0.1893	− 0.0769	+ 0.1491	−0.0040	−0.0103
12 − 7	− 0.0297	− 0.0290	− 0.0297	+ 0.0177	−0.0023	−0.0020
4 − 8	+ 0.0008	+ 0.0008	+ 0.0021	− 0.0009	+0.0003	+0.0001
5 − 8	− 0.0121	+ 0.0139	− 0.0114	− 0.0210	−0.0015	+0.0042
6 − 8	− 0.1441	− 0.1540	− 0.2047	+ 0.1687	−0.0371	−0.0159
7 − 8	+ 1.2998	− 0.7321	+ 1.5303	+ 0.9550	+0.0707	−0.1501
8 − 8	+ 1.0003	+ 4.5946	+ 0.9941	− 5.4158	−0.0025	−0.1384
9 − 8	+ 2.3768	+ 3.3965	+ 2.6374	− 3.5276	+0.0074	−0.0076
10 − 8	+ 1.6726	+ 0.9068	+ 1.7026	− 0.7490	+0.0319	+0.0108
11 − 8	+ 0.6269	+ 0.0076	+ 0.5673	+ 0.0671	+0.0207	−0.0021
12 − 8	+ 0.1478	− 0.0784	+ 0.1147	+ 0.0909	+0.0067	−0.0044

Arg.	$a'\frac{d\Omega'}{dg'}$		$a'r\frac{d\Omega'}{dr'}$		$a'^2\frac{d\Omega'}{dZ'}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	″	″	″	″	″	″
5 — 9	—0.0005	+0.0010	—0.0002	—0.0014	0.0000	+0.0003
6 — 9	—0.0150	—0.0073	—0.0208	+0.0059	—0.0037	—0.0001
7 — 9	+0.0881	—0.1366	+0.0909	+0.1787	+0.0035	—0.0271
8 — 9	+0.6640	+0.7283	+0.8165	—0.8291	+0.0899	+0.0174
9 — 9	—2.3077	+1.1684	—2.7039	—1.2391	+0.0853	—0.0198
10 — 9	—1.7310	+1.9130	—1.7775	—2.0630	+0.0118	—0.0020
11 — 9	—0.3639	+1.2557	—0.2661	—1.2511	—0.0012	+0.0180
12 — 9	+0.0921	+0.4618	+0.1405	—0.4190	+0.0042	+0.0125
6 — 10	—0.0009	—0.0002	—0.0020	—0.0003	—0.0003	+0.0001
7 — 10	+0.0106	—0.0147	+0.0007	+0.0186	—0.0007	—0.0028
8 — 10	+0.1138	+0.0411	+0.1411	—0.0353	+0.0177	—0.0024
9 — 10	—0.3547	+0.5199	—0.3938	—0.6165	+0.0034	+0.0498
10 — 10	—0.9402	—1.0440	—1.0093	+1.2280	+0.0233	+0.0490
11 — 10	—1.3782	—0.7645	—1.4608	+0.7770	+0.0065	+0.0096
12 — 10	—0.8736	—0.0566	—0.8847	—0.0118	—0.0089	+0.0021
7 — 11	+0.0007	—0.0014	—0.0005	+0.0015	—0.0002	—0.0002
8 — 11	+0.0118	—0.0009	+0.0143	+0.0032	+0.0019	—0.0011
9 — 11	—0.0096	+0.0857	—0.0028	—0.1016	+0.0044	+0.0105
10 — 11	—0.3670	—0.1401	—0.4238	+0.1507	—0.0255	+0.0089
11 — 11	+0.4026	—0.6386	+0.4847	+0.6888	—0.0259	+0.0204
12 — 11	+0.2552	—0.9078	+0.2474	+1.0020	—0.0064	+0.0076
8 — 12	+0.0009	—0.0008	+0.0010	+0.0006	+0.0001	—0.0002
9 — 12	+0.0027	+0.0083	+0.0045	—0.0101	+0.0011	+0.0011
10 — 12	—0.0590	+0.0065	—0.0675	—0.0132	—0.0057	+0.0042
11 — 12	+0.0288	—0.2349	+0.0299	+0.2649	—0.0079	—0.0121
12 — 12	+0.3994	+0.0850	+0.4738	—0.1202	—0.0142	—0.0102

Following HANSEN'S method of integration we now introduce a set of symbols, τ, γ, ρ, and φ, which are, respectively, the equivalents of t, g, r, and f, except that they are regarded as constant whenever we integrate. When the integration is accomplished the original symbols are restored. The next step in deriving the perturbations is to obtain the development for Jupiter of the functions*

$$T = \frac{1}{n}\frac{dW}{dt} = Aa\frac{d\Omega}{dg} + Bar\frac{d\Omega}{dr} \qquad \frac{1}{n}\frac{dR}{dt} = Ca^2\frac{d\Omega}{dZ}$$

and for Saturn the similar functions

$$T' = \frac{1}{n'}\frac{dW'}{dt} = A'a'\frac{d\Omega'}{dg'} + B'a'r'\frac{d\Omega'}{dr'} \qquad \frac{1}{n'}\frac{dR'}{dt} = C'a'^2\frac{d\Omega'}{dZ'}$$

---

[*] Gegenseitige Störungen des Jupiter und Saturn, s. 9, and Störungen der grossen Planeten, insbesondere des Jupiter, ss. 37, 50.

The expressions for A, B, and C are

$$\mathrm{A} = -3 + \frac{1}{1-e^2}\left\{\left(2\frac{\rho}{\mathrm{a}}\cos\varphi + 3e\right)\frac{\mathrm{a}^2(1-e^2)-r^2}{\mathrm{a}^2 e} + 2\frac{\rho\sin\varphi}{\mathrm{a}\sqrt{1-e^2}}\int\left(2\frac{r}{\mathrm{a}}\cos f + 3e\right)dg\right\}$$

$$\mathrm{B} = -\frac{1}{1-e^2}\left\{\left(2\frac{\rho}{\mathrm{a}}\cos\varphi + 3e\right)\frac{r\sin f}{\mathrm{a}\sqrt{1-e^2}} - 2\frac{\rho\sin\varphi}{\mathrm{a}\sqrt{1-e^2}}\left(\frac{r}{\mathrm{a}}\cos f + 2e\right)\right\}$$

$$\mathrm{C} = \frac{1}{\sqrt{1-e^2}}\frac{r}{\mathrm{a}}\frac{\rho}{\mathrm{a}}\sin(\varphi - f)$$

The expressions for A′, B′, and C′ are obtained by accenting all the symbols contained in these equations.

In developing T and $\frac{1}{n}\frac{d\mathrm{R}}{dt}$ in a series of periodic terms whose arguments are composed of integral multiples of γ, g', and g we need compute directly only the terms which involve $\pm\gamma$, and in the case of the first function those which are independent of γ. The rest are readily supplied after integration in the functions W and R. This simplification is available, not only for the first order approximation, but for all succeeding orders. In deriving the proper developments of the multipliers A, B, and C we can employ the quantities P_i and Q_i of page 63. Thus it suffices to put

$$2\frac{\rho}{\mathrm{a}}\cos\varphi + 3e = 2\mathrm{P}_1\cos\gamma + \ldots \qquad 2\frac{\rho\sin\varphi}{\mathrm{a}\sqrt{1-e^2}} = 2\mathrm{Q}_1\sin\gamma + \ldots$$

$$\frac{\mathrm{a}^2(1-e^2)-r^2}{\mathrm{a}^2 e} = -\frac{5}{2}e + \frac{2}{1}\mathrm{Q}_1\cos g + \frac{2}{2}\mathrm{Q}_2\cos 2g + \frac{2}{3}\mathrm{Q}_3\cos 3g + \ldots$$

$$\int\left(2\frac{r}{\mathrm{a}}\cos f + 3e\right)dg = \frac{2}{1}\mathrm{P}_1\sin g + \frac{2}{2}\mathrm{P}_2\sin 2g + \frac{2}{3}\mathrm{P}_3\sin 3g + \ldots$$

$$\frac{r\sin f}{\mathrm{a}\sqrt{1-e^2}} = \mathrm{Q}_1\sin g + \mathrm{Q}_2\sin 2g + \mathrm{Q}_3\sin 3g + \ldots$$

$$\frac{r}{\mathrm{a}}\cos f + 2e = \frac{1}{2}e + \mathrm{P}_1\cos g + \mathrm{P}_2\cos 2g + \mathrm{P}_3\cos 3g + \ldots$$

The portion of C which involves the single multiple of γ is

$$\mathrm{C} = \Sigma_i \,.\, \frac{1}{2}(\mathrm{P}_i\mathrm{Q}_1 \pm \mathrm{P}_1\mathrm{Q}_i)\sin(\gamma \mp ig)$$

the double sign being taken both ways.

With the understood restriction the three multipliers for Jupiter are

$$\begin{array}{lll}
\mathrm{A} = -3 & & \\
\quad + 2[0.3015363]\cos(\gamma - g) & \mathrm{B} = -2[0.0005063]\sin(\gamma - g) & \mathrm{C} = 2[9.6984644]\sin(\gamma - g) \\
\quad - 2[9.0820049]\cos\gamma & \quad - 2[8.3832877]\sin\gamma & \quad - 2[8.5583671]\sin\gamma \\
\quad + 2[8.3826558]\cos(\gamma - 2g) & \quad - 2[8.3826558]\sin(\gamma - 2g) & \quad + 2[8.0806139]\sin(\gamma - 2g) \\
\quad + 2[6.7648052]\cos(\gamma - 3g) & \quad - 2[6.9408965]\sin(\gamma - 3g) & \quad + 2[6.6388546]\sin(\gamma - 3g) \\
\quad + 2[5.2718932]\cos(\gamma - 4g) & \quad - 2[5.5729232]\sin(\gamma - 4g) & \quad + 2[5.2708813]\sin(\gamma - 4g) \\
\quad + 2[4.3697]\cos(\gamma + 2g) & \quad + 2[4.3697]\sin(\gamma + 2g) & \quad - 2[4.0677]\sin(\gamma + 2g) \\
\quad + 2[3.8479]\cos(\gamma - 5g) & \quad - 2[4.2458]\sin(\gamma - 5g) & \quad + 2[3.94376]\sin(\gamma - 5g) \\
\quad + 2[2.9281]\cos(\gamma + 3g) & \quad + 2[3.1042]\sin(\gamma + 3g) &
\end{array}$$

The corresponding quantities for Saturn are

$$
\begin{array}{llllll}
A' = -3 & & & & & \\
& +2[0.3017143]\cos(\gamma'-g') & B' = & -2[0.0006843]\sin(\gamma'-g') & C' = & 2[9.6982874]\sin(\gamma'-g') \\
& -2[9.1474238]\cos\gamma' & & -2[8.4487952]\sin\gamma' & & -2[8.6235196]\sin\gamma' \\
& +2[8.4479419]\cos(\gamma'-2g') & & -2[8.4479419]\sin(\gamma'-2g') & & +2[8.1455450]\sin(\gamma'-2g') \\
& +2[6.8951994]\cos(\gamma'-3g') & & -2[7.0712907]\sin(\gamma'-3g') & & +2[6.7688938]\sin(\gamma'-3g') \\
& +2[5.4673946]\cos(\gamma'-4g') & & -2[5.7684247]\sin(\gamma'-4g') & & +2[5.4660278]\sin(\gamma'-4g') \\
& +2[4.5657\quad]\cos(\gamma'+2g') & & +2[4.5657\quad]\sin(\gamma'+2g') & & -2[4.2633\quad]\sin(\gamma'+2g') \\
& +2[4.1085\quad]\cos(\gamma'-5g') & & -2[4.5064\quad]\sin(\gamma'-5g') & & +2[4.2040\quad]\sin(\gamma'-5g') \\
& +2[3.1892\quad]\cos(\gamma'+3g') & & +2[3.3650\quad]\sin(\gamma'+3g') & &
\end{array}
$$

The terms of W or R, which involve other multiples of γ than ± 1, are obtained in the following way:*

Let

$$
\mathrm{W \text{ or } R} = \Sigma \alpha^{(i)} \frac{\sin}{\cos}\left(i\gamma + \beta t\right)
$$

then

$$
\alpha^{(\pm i)} = \eta^{(i)}\alpha^{(\pm 1)} + \theta^{(i)}\alpha^{(\mp 1)}
$$

where

$$
\eta^{(i)} = \frac{1}{2}\frac{P_i}{P_1} + \frac{1}{2i}\frac{J^{(i)}_{\frac{ie}{2}}}{J^{(1)}_{\frac{e}{2}}} \qquad\qquad \theta^{(i)} = \frac{1}{2}\frac{P_i}{P_1} - \frac{1}{2i}\frac{J^{(i)}_{\frac{ie}{2}}}{J^{(1)}_{\frac{e}{2}}}
$$

In the case of R we have, in addition,

$$
\alpha^{(0)} = \eta^{(0)}(\alpha^{(1)} + \alpha^{(-1)})
$$

where

$$
\eta^{(0)} = \frac{1}{2}\frac{P_0}{P_1}
$$

For Jupiter

$$
\begin{array}{ll}
\log \eta^{(0)} = 8.8599027_n & \\
\log \eta^{(2)} = 8.3821495 & \log \theta^{(2)} = 4.3692_n \\
\log \eta^{(3)} = 6.9403902 & \log \theta^{(3)} = 3.1037_n \\
\log \eta^{(4)} = 5.5724169 & \log \theta^{(4)} = 1.8152_n \\
\log \eta^{(5)} = 4.2453 & \\
\log \eta^{(6)} = 2.9442 &
\end{array}
$$

For Saturn

$$
\begin{array}{ll}
\log \eta^{(0)} = 8.9252322_n & \\
\log \eta^{(2)} = 8.4472576 & \log \theta^{(2)} = 4.5653_n \\
\log \eta^{(3)} = 7.0706064 & \log \theta^{(3)} = 3.3645_n \\
\log \eta^{(4)} = 5.7677404 & \log \theta^{(4)} = 2.1407_n \\
\log \eta^{(5)} = 4.5057 & \\
\log \eta^{(6)} = 3.2696 &
\end{array}
$$

The developments of T and $\frac{1}{n}\frac{dR}{dt}$ follow; they have the form

$$
A \frac{\sin}{\cos}\left(\varkappa\gamma + i'g' + ig\right)
$$

---

* Gegenseitige Störungen des Jupiter und Saturn, ss. 25–29, and Störungen der grossen Planeten, insbesondere des Jupiter, ss. 37–39.

Arg.			T		$\frac{1}{n}\frac{dR}{dt}$	
			sin.	cos.	cos.	sin.
κ	i	$'i$	″	″	″	″
1	0	−1	−14.27897	− 0.0163215	+0.0120	+0.0506
−1	0	0	− 1.1420391	+ 1.0173637	+0.1253514	+0.2844315
0	0	−1	+ 0.9242313	− 0.5043882		
1	0	−2	− 0.08864	− 0.341651	−0.1235	−0.2819
−1	0	−1	− 0.04934	− 0.08305	+0.0087	−0.0288
0	0	−2	+ 0.050385	+ 0.053403		
1	0	−3	− 0.00074	− 0.00437	−0.0207	+0.0015
−1	0	−2	+ 0.00412	− 0.00275	−0.0017	−0.0001
0	0	−3	− 0.00146	+ 0.00344		
1	0	−4	− 0.00052	− 0.00062	−0.0001	+0.0007
−1	0	−3	+ 0.00017	+ 0.00022		
0	0	−4	− 0.00011	0.00000		
1	0	−5	− 0.00006	− 0.00008		
1	1	+3	+ 0.00001	− 0.00011		
0	1	+3	+ 0.00023	+ 0.00021		
1	1	+2	− 0.00050	− 0.00144	−0.0002	−0.0001
−1	1	+3	− 0.00010	+ 0.00045	−0.0019	−0.0011
0	1	+2	− 0.00292	+ 0.00632		
1	1	+1	+ 0.00190	− 0.01006	−0.0010	+0.0036
−1	1	+2	− 0.01946	− 0.03253	−0.0267	+0.0287
0	1	+1	− 0.10373	− 0.01590		
1	1	0	+ 0.15948	+ 0.06278	+0.0448	−0.0161
−1	1	+1	+ 1.38370	− 0.92376	+0.1882	+0.1282
1	1	−1	− 1.43030	+ 0.68960	−0.1878	−0.1278
−1	1	0	+ 2.57698	+13.23026	−0.0392	+0.0192
0	1	−1	− 1.34596	− 7.22662		
1	1	−2	− 0.80316	− 3.55532	+0.0403	−0.0358
−1	1	−1	+ 0.50684	− 1.10731	+0.1994	−0.0450
0	1	−2	-- 0.71789	+ 0.87785		
1	1	−3	+ 0.40413	− 0.29319	−0.1966	+0.0412
−1	1	−2	+ 0.05922	− 0.06763	−0.0123	−0.0062
0	1	−3	− 0.08875	+ 0.08783		
1	1	−4	+ 0.03467	− 0.02711	−0.0068	+0.0103
--1	1	−3	+ 0.00321	− 0.00034	−0.0003	+0.0007
0	1	−4	− 0.00654	+ 0.00401		
1	1	−5	+ 0.00270	− 0.00225		
−1	1	−4	+ 0.00005	0.00000		
0	1	−5	− 0.00033	+ 0.00023		
1	1	−6	+ 0.00020	− 0.00019		
1	2	+2	− 0.00003	+ 0.00012		
−1	2	+3	+ 0.00005	+ 0.00013		
0	2	+2	− 0.00002	+ 0.00065		
1	2	+1	− 0.00022	− 0.00108		
−1	2	+2	− 0.00487	− 0.00141	−0.0044	+0.0020
0	2	+1	− 0.00954	+ 0.00206		

Arg.			T		$\frac{1}{n}\frac{dR}{dt}$	
			sin.	cos.	cos.	sin.
κ	i′	i	″	″	″	″
1	2	0	+ 0.02018	− 0.00063	+0.0059	+0.0026
−1	2	+1	+ 0.05341	− 0.17006	+0.0191	+0.0451
1	2	−1	− 0.19776	+ 0.18685	+0.0026	−0.0649
−1	2	0	+ 4.89560	+ 0.57514	+0.2191	−0.2081
0	2	−1	− 3.717753	+ 0.196716		
1	2	−2	+ 1.51956	− 1.46206	−0.2165	+0.2058
−1	2	−1	−47.87233	+20.55610	+0.0237	+0.0395
0	2	−2	+45.08114	−19.40163		
1	2	−3	−12.36743	+ 5.27139	−0.0449	−0.0318
−1	2	−2	− 0.07834	+ 1.63350	−0.0034	−0.1268
0	2	−3	+ 1.80511	− 2.42460		
1	2	−4	− 0.88028	+ 0.97188	−0.0010	+0.1239
−1	2	−3	+ 0.05101	+ 0.08440	−0.0031	+0.0038
0	2	−4	+ 0.04234	− 0.18903		
1	2	−5	− 0.04279	+ 0.08655	+0.0031	+0.0082
−1	2	−4	+ 0.00219	+ 0.00428	+0.0002	+0.0003
0	2	−5	+ 0.00084	− 0.01280		
1	2	−6	− 0.00169	+ 0.00646		
−1	2	−5	+ 0.00008	+ 0.00015		
0	2	−6	− 0.00001	− 0.00077		
1	2	−7	− 0.00008	+ 0.00043		
1	3	+1	− 0.00005	− 0.00005		
−1	3	+2	− 0.00063	+ 0.00021		
0	3	+1	− 0.00073	+ 0.00051		
1	3	0	+ 0.00211	− 0.00098	+0.0004	+0.0008
−1	3	+1	− 0.00723	− 0.01958	−0.0006	+0.0075
1	3	−1	− 0.01716	+ 0.03416	+0.0074	−0.0095
−1	3	0	+ 0.87841	− 0.31841	+0.0698	−0.0235
0	3	−1	− 0.647247	+ 0.310206		
1	3	−2	+ 0.18035	− 0.37244	−0.0790	+0.0070
−1	3	−1	− 6.73500	+ 9.03111	−0.1032	−0.1619
0	3	−2	+ 6.357282	− 8.148465		
1	3	−3	− 1.14531	+ 2.72466	+0.0989	+0.1603
−1	3	−2	−20.03269	−28.41215	+0.0326	−0.0239
0	3	−3	+19.48402	+26.95993		
1	3	−4	− 5.68448	− 7.80172	−0.0303	+0.0381
−1	3	−3	− 1.96410	− 0.11612	−0.0748	−0.0132
0	3	−4	+ 2.71571	+ 1.14298		
1	3	−5	− 1.02603	− 0.54359	+0.0718	+0.0168
−1	3	−4	− 0.10172	+ 0.06235	+0.0006	+0.0007
0	3	−5	+ 0.21686	− 0.00518		
1	3	−6	− 0.09729	− 0.01520	+0.0065	+0.0008
−1	3	−5	− 0.00474	+ 0.00465		
0	3	−6	+ 0.01456	− 0.00402		
1	3	−7	− 0.00741	+ 0.00068		

Arg.			T		$\frac{1}{n}\frac{dR}{dt}$	
			sin.	cos.	cos.	sin.
κ	i'	i	"	"	"	"
−1	3	−6	− 0.00017	+ 0.00029		
0	3	−7	+ 0.00090	− 0.00041		
1	3	−8	− 0.00055	+ 0.00011		
−1	4	+2	− 0.00005	+ 0.00008		
0	4	+1	− 0.00004	+ 0.00007		
1	4	0	+ 0.00017	− 0.00021		
−1	4	+1	− 0.00203	− 0.00153		
1	4	−1	− 0.00029	+ 0.00465	+0.0017	−0.0007
−1	4	0	+ 0.09561	− 0.08720	+0.0123	+0.0016
0	4	−1	− 0.068625	+ 0.068916		
1	4	−2	− 0.00198	− 0.05611	−0.0127	−0.0078
−1	4	−1	− 0.16313	+ 1.71546	−0.0072	−0.0632
0	4	−2	+ 0.210126	− 1.519686		
1	4	−3	+ 0.17380	+ 0.44953	−0.0043	+0.0665
−1	4	−2	− 9.44851	− 4.57606	−0.1070	+0.0408
0	4	−3	+ 8.87909	+ 4.40965		
1	4	−4	− 2.84258	− 0.85831	+0.1050	−0.0371
−1	4	−3	+14.62768	−15.89793	−0.0182	−0.0273
0	4	−4	−13.81865	+15.57488		
1	4	−5	+ 4.09131	− 4.67472	+0.0268	+0.0275
−1	4	−4	− 0.12034	− 1.78601	−0.0166	+0.0411
0	4	−5	− 0.40655	+ 2.37533		
1	4	−6	+ 0.22138	− 0.87661	+0.0190	−0.0384
−1	4	−5	− 0.07683	− 0.09770	−0.0005	+0.0004
0	4	−6	+ 0.05467	+ 0.19722		
1	4	−7	− 0.01066	− 0.08671	+0.0022	−0.0041
−1	4	−6	− 0.00656	− 0.00412		
0	4	−7	+ 0.00839	+ 0.01300		
1	4	−8	− 0.00291	− 0.00659		
−1	4	−7	− 0.00054	− 0.00015		
0	4	−8	+ 0.00077	+ 0.00075		
1	4	−9	− 0.00020	− 0.00042		
−1	5	+1	− 0.00031	− 0.00004		
1	5	−1	+ 0.00020	+ 0.00048	+0.0002	0.0000
−1	5	0	+ 0.00653	− 0.01387	+0.0015	+0.0010
0	5	−1	− 0.004728	+ 0.010044		
1	5	−2	− 0.00372	− 0.00531	−0.00095	−0.00220
−1	5	−1	+ 0.10176	+ 0.20473	+0.0045	−0.0125
0	5	−2	− 0.07676841	− 0.18132165		
1	5	−3	+ 0.06246	+ 0.03064	−0.0093	+0.0117
−1	5	−2	− 2.02853	+ 0.15265	−0.04865	−0.00476
0	5	−3	+ 1.887900	− 0.090724		
1	5	−4	− 0.55397	+ 0.21727	+0.0488	+0.0121
−1	5	−3	+ 2.09722	− 8.16344	+0.0100	+0.0647
0	5	−4	− 2.03072	+ 7.80499		

Arg.			T		$\frac{1}{n}\frac{dR}{dt}$	
			sin.	cos.	cos.	sin.
κ	i′	i	″	″	″	″
1	5	— 5	+ 0.32258	—2.45130	—0.0074	—0.0625
—1	5	— 4	+11.14562	+6.46261	—0.0216	+0.0113
0	5	— 5	—10.93896	—5.98566		
1	5	— 6	+ 3.33401	+1.78195	+0.0228	—0.0159
—1	5	— 5	+ 1.37812	—0.33152	+0.0210	+0.0143
0	5	— 6	— 1.78888	+0.10167		
1	5	— 7	+ 0.65267	+0.00695	—0.0188	—0.0156
—1	5	— 6	+ 0.07719	—0.08431	+0.0003	+0.0010
0	5	— 7	— 0.15157	+0.08540		
1	5	— 8	+ 0.06594	—0.02677	—0.0022	—0.0025
—1	5	— 7	+ 0.00254	—0.00743		
0	5	— 8	— 0.00946	+0.01085		
1	5	— 9	+ 0.00500	—0.00425		
—1	5	— 8	— 0.00004	—0.00021		
0	5	— 9	— 0.00045	+0.00088		
1	5	— 10	+ 0.00033	—0.00059		
1	6	— 1	0.00000	+0.00008		
—1	6	0	+ 0.00005	—0.00165		
0	6	— 1	— 0.00009	+0.00113		
1	6	— 2	— 0.00068	—0.00021	0.0000	—0.0003
—1	6	— 1	+ 0.02401	+0.01471	+0.0017	—0.0015
0	6	— 2	— 0.0189420	—0.0136819		
1	6	— 3	+ 0.00900	—0.00249	—0.0026	+0.0008
—1	6	— 2	— 0.26360	+0.18577	—0.0103	—0.0066
0	6	— 3	+ 0.247395	—0.159105		
1	6	— 4	— 0.04954	+0.08524	+0.0090	+0.0099
—1	6	— 3	— 0.51443	—1.92108	—0.0106	+0.0331
0	6	— 4	+ 0.45469	+1.82824		
1	6	— 5	— 0.27814	—0.52941	+0.0148	—0.0320
—1	6	— 4	+ 6.19549	+0.28513	+0.0360	+0.0024
0	6	— 5	— 5.97373	—0.26063		
1	6	— 6	+ 1.85582	—0.10314	—0.0340	—0.0040
—1	6	— 5	— 2.21848	+7.13306	+0.0056	+0.0165
0	6	— 6	+ 1.92940	—6.98712		
1	6	— 7	— 0.56043	+2.14709	—0.0078	—0.0177
—1	6	— 6	+ 0.42549	+0.94126	+0.0103	—0.0097
0	6	— 7	— 0.34977	—1.20274		
1	6	— 8	+ 0.09974	+0.43618	—0.0110	+0.0080
—1	6	— 7	+ 0.08149	+0.05070	+0.0009	—0.0001
0	6	— 8	— 0.09401	—0.10046		
1	6	— 9	+ 0.03261	+0.04370	—0.0020	+0.0009
—1	6	— 8	+ 0.00724	+0.00088		
0	6	— 9	— 0.01098	—0.00540		
1	6	— 10	+ 0.00432	+0.00295		
—1	6	— 9	+ 0.00042	—0.00017		

Arg.			T		$\frac{1}{n}\frac{dR}{dt}$	
			sin.	cos.	cos.	sin.
κ	i′	i	″	″	″	″
0	6	— 10	—0.00073	—0.00013		
1	6	— 11	+0.00020	+0.00014		
— 1	7	0	—0.00005	—0.00013		
0	7	— 1	+0.00003	+0.00007		
1	7	— 2	—0.00008	+0.00004		
— 1	7	— 1	+0.00330	—0.00003		
0	7	— 2	—0.00263	—0.00021		
1	7	— 3	+0.00067	—0.00099	—0.0005	—0.0002
— 1	7	— 2	—0.01820	+0.04180	—0.0012	—0.0021
0	7	— 3	+0.018315	—0.036387		
1	7	— 4	+0.00204	+0.01448	+0.0004	+0.0028
— 1	7	— 3	—0.26181	—0.26059	—0.0072	+0.0074
0	7	— 4	+0.23753	+0.25143		
1	7	— 5	—0.10250	—0.05109	+0.0092	—0.0059
— 1	7	— 4	+1.55438	—0.76375	+0.0204	+0.0117
0	7	— 5	—1.49566	+0.71456		
1	7	— 6	+0.42479	—0.31465	—0.0189	—0.0141
— 1	7	— 5	+0.73029	+4.24941	+0.0057	—0.0184
0	7	— 6	—0.71031	—4.11139		
1	7	— 7	+0.30222	+1.26623	—0.0064	+0.0169
— 1	7	— 6	—4.22658	—0.29928	+0.0115	—0.0020
0	7	— 7	+4.11906	+0.12916		
1	7	— 8	—1.27156	—0.01850	—0.0125	+0.0030
— 1	7	— 7	—0.57462	+0.41645	—0.0040	—0.0068
0	7	— 8	+0.72692	—0.40946		
1	7	— 9	—0.26152	+0.13102	+0.0028	+0.0070
— 1	7	— 8	—0.02589	+0.07035	+0.0001	—0.0008
0	7	— 9	+0.05544	—0.08595		
1	7	— 10	—0.02401	+0.03087	+0.0002	+0.0015
— 1	7	— 9	—0.00289	+0.00674		
0	7	— 10	+0.00741	—0.01029		
1	7	— 11	—0.00514	+0.00415		
— 1	7	— 10	+0.00005	+0.00057		
0	7	— 11	+0.00054	—0.00105		
1	7	— 12	—0.00051	+0.00052		
— 1	8	— 1	+0.00031	—0.00020		
0	8	— 2	—0.00024	+0.00012		
1	8	— 3	—0.00002	—0.00015		
— 1	8	— 2	+0.00082	+0.00592	0.0000	—0.0004
0	8	— 3	—0.000378	—0.005220		
1	8	— 4	+0.00147	+0.00139	—0.0002	+0.0004
— 1	8	— 3	—0.05766	—0.01461	—0.0022	+0.0007
0	8	— 4	+0.05268	+0.01575		
1	8	— 5	—0.01882	+0.00319	+0.0026	0.0000
— 1	8	— 4	+0.20803	—0.30585	+0.0045	+0.0067

Arg.			T		$\frac{1}{n}\frac{dR}{dt}$	
			sin.	cos.	cos.	sin.
$\varkappa$	i'	i	"	"	"	"
0	8 —	5	—0. 20424	+0. 28602		
1	8 —	6	+0. 03879	—0. 10928	—0. 0031	—0. 0078
—1	8 —	5	+0. 84886	+1. 10096	+0. 0104	—0. 0111
0	8 —	6	—0. 81414	—1. 06665		
1	8 —	7	+0. 31429	+0. 29688	—0. 0116	+0. 0099
—1	8 —	6	—2. 66695	+1. 05766	—0. 0084	—0. 0047
0	8 —	7	+2. 57958	—1. 05789		
1	8 —	8	—0. 78720	+0. 39256	+0. 0073	+0. 0066
—1	8 —	7	—0. 39269	—2. 32986	+0. 0001	+0. 0085
0	8 —	8	+0. 48951	+2. 24844		
1	8 —	9	—0. 16606	—0. 69278	+0. 0003	—0. 0080
—1	8 —	8	—0. 34858	—0. 31699	—0. 0041	0. 0000
0	8 —	9	+0. 36558	+0. 40092		
1	8 —	10	—0. 12114	—0. 14513	+0. 0040	—0. 0008
—1	8 —	9	—0. 05544	—0. 00845	—0. 0005	—0. 0001
0	8 —	10	+0. 06957	+0. 02511		
1	8 —	11	—0. 02535	—0. 01199	+0. 0009	+0. 0001
—1	8 —	10	—0. 00510	+0. 00181		
0	8 —	11	+0. 00795	—0. 00063		
1	8 —	12	—0. 00320	—0. 00012		
—1	8 —	11	—0. 00033	+0. 00046		
0	8 —	12	+0. 00069	—0. 00057		
1	8 —	13	—0. 00031	+0. 00029		
—1	9 —	2	+0. 00048	+0. 00059		
0	9 —	3	—0. 00039	—0. 00051		
1	9 —	4	+0. 00027	+0. 00002		
—1	9 —	3	—0. 00836	+0. 00250		
0	9 —	4	+0. 00771	—0. 00189		
1	9 —	5	—0. 00201	+0. 00209	+0. 0005	+0. 0002
—1	9 —	4	+0. 00479	—0. 06705	0. 0000	+0. 0021
0	9 —	5	—0. 00660	+0. 06297		
1	9 —	6	—0. 00557	—0. 02090	+0. 0003	—0. 0021
—1	9 —	5	+0. 30975	+0. 13250	+0. 0056	—0. 0021
0	9 —	6	—0. 29484	—0. 13197		
1	9 —	7	+0. 10452	+0. 01988	—0. 0061	+0. 0012
—1	9 —	6	—0. 68623	+0. 80230	—0. 0054	—0. 0079
0	9 —	7	+0. 66609	—0. 77607		
1	9 —	8	—0. 17856	+0. 27679	+0. 0046	+0. 0082
—1	9 —	7	—1. 03723	—1. 53330	—0. 0035	+0. 0034
0	9 —	8	+1. 03386	+1. 47738		
1	9 —	9	—0. 35780	—0. 44556	+0. 0035	—0. 0027
—1	9 —	8	+1. 18867	—0. 52024	—0. 0047	—0. 0009
0	9 —	9	—1. 12929	+0. 57183		
1	9 —	10	+0. 34595	—0. 18655	+0. 0051	+0. 0009
—1	9 —	9	+0. 15083	—0. 26204	+0. 0001	+0. 0023

Arg.		T		$\frac{1}{n}\frac{dR}{dt}$	
		sin.	cos.	cos.	sin.
κ	i′ i	″	″	″	″
0	9 — 10	—0.19290	+0.28269		
1	9 — 11	+0.07024	—0.09511	+0.0004	—0.0022
—1	9 — 10	—0.00227	—0.04014	—0.0002	+0.0003
0	9 — 11	—0.00573	+0.05124		
1	9 — 12	+0.00365	—0.01890		
—1	9 — 11	—0.00208	—0.00341		
0	9 — 12	+0.00174	+0.00543		
1	9 — 13	—0.00045	-0.00215		
0	10 — 3	—0.00007	—0.00003		
1	10 — 4	+0.00003	—0.00004		
—1	10 — 3	—0.00081	+0.00090	—0.0001	—0.0001
0	10 — 4	+0.0007626	—0.0007509		
1	10 — 5	—0.00004	+0.00040	+0.0001	+0.0001
—1	10 — 4	—0.00484	—0.00979	—0.00016	+0.00040
0	10 — 5	+0.00414	+0.00930		
1	10 — 6	—0.00276	—0.00233	+0.0003	—0.0004
—1	10 — 5	+0.06777	—0.00776	+0.0017	+0.0002
0	10 — 6	—0.06474	+0.00573		
1	10 — 7	+0.02036	—0.00842	—0.0017	—0.0006
—1	10 — 6	—0.05858	+0.28008	—0.0006	—0.0042
0	10 — 7	+0.05961	—0.26943		
1	10 — 8	—0.00162	+0.09079	0.0000	+0.0043
—1	10 — 7	—0.67294	—0.36663	—0.0054	+0.0023
0	10 — 8	+0.65484	+0.35499		
1	10 — 9	—0.22364	—0.08854	+0.0054	—0.0016
—1	10 — 8	+0.79847	—0.84775	+0.0010	+0.0021
0	10 — 9	—0.76227	+0.84255		
1	10 — 10	+0.22542	—0.28203	—0.0006	—0.0019
—1	10 — 9	+0.43528	+0.55215	—0.0012	+0.0027
0	10 — 10	—0.45999	—0.51093		
1	10 — 11	+0.14795	+0.15461	+0.0013	—0.0030
—1	10 — 10	+0.18069	+0.05642	+0.0012	+0.0003
0	10 — 11	—0.19758	—0.07539		
1	10 — 12	+0.06700	+0.02798	—0.0011	—0.0005
—1	10 — 11	+0.02676	—0.00710	+0.0001	+0.0002
0	10 — 12	—0.03465	+0.00381		
1	10 — 13	+0.01311	—0.00047		
0	11 — 4	—0.000036	+0.000132		
1	11 — 5	+0.00007	+0.00004		
—1	11 — 4	—0.00137	—0.00089		
0	11 — 5	+0.00125	+0.00091		
1	11 — 6	—0.00058	—0.00008		
—1	11 — 5	+0.00978	—0.00732	+0.0003	+0.0003
0	11 — 6	—0.00948	+0.00663		

Arg.		T		$\frac{1}{n}\frac{dR}{di}$	
		sin.	cos.	cos.	sin.
κ	i′ i	″	″	″	″
1	11 — 7	+0.00223	—0.00340	—0.0003	—0.0002
—1	11 — 6	+0.01919	+0.06105	+0.0004	+0.0012
0	11 — 7	—0.01725	—0.05895		
1	11 — 8	+0.01078	+0.01769	—0.0007	—0.0012
—1	11 — 7	—0.23021	—0.00103	—0.0029	—0.0003
0	11 — 8	+0.22308	+0.00270		
1	11 — 9	—0.07247	+0.01189	+0.0029	+0.0005
—1	11 — 8	+0.15237	—0.51520	+0.0005	+0.0034
0	11 — 9	—0.14571	+0.50280		
1	11 — 10	+0.02946	—0.16672	—0.0001	—0.0034
—1	11 — 9	+0.61899	+0.36725	+0.0010	—0.0001
0	11 — 10	—0.61314	—0.34005		
1	11 — 11	+0.20117	+0.09678	—0.0009	—0.0001
—1	11 — 10	—0.22432	+0.30259	+0.0015	+0.0011
0	11 — 11	+0.19704	—0.31245		
1	11 — 12	—0.05793	+0.09947	—0.0016	—0.0011
—1	11 — 11	—0.00855	+0.11339	+0.0003	—0.0005
0	11 — 12	+0.01536	—0.12543		
1	11 — 13	—0.00569	+0.04396		
0	12 — 5	+0.000225	+0.000024		
1	12 — 6	—0.00004	+0.00009		
—1	12 — 5	+0.00085	—0.00182		
0	12 — 6	—0.00092	+0.00167		
1	12 — 7	+0.00011	—0.00067		
—1	12 — 6	+0.00925	+0.00855	+0.0003	—0.0002
0	12 — 7	—0.00849	—0.00828		
1	12 — 8	+0.00358	+0.00167	—0.0004	+0.0002
—1	12 — 7	—0.04976	+0.02772	—0.0009	—0.0006
0	12 — 8	+0.04821	—0.02556		
1	12 — 9	—0.01355	+0.01160	+0.0009	+0.0007
—1	12 — 8	—0.03667	—0.17558	—0.0005	+0.0020
0	12 — 9	+0.03378	+0.16944		
1	12 — 10	—0.01840	—0.05193	+0.0009	—0.0020
—1	12 — 9	+0.36928	+0.02495	+0.0025	+0.0001
0	12 — 10	—0.35625	—0.02310		
1	12 — 11	+0.11091	—0.00180	—0.0025	—0.0003
—1	12 — 10	—0.12776	+0.42118	—0.0003	—0.0012
0	12 — 11	+0.11445	—0.40722		
1	12 — 12	—0.03014	+0.12294	0.0000	+0.0012
—1	12 — 11	—0.19867	—0.05897		
0	12 — 12	+0.19542	+0.04158		
1	12 — 13	—0.05857	—0.00965		

The developments of T′ and $\frac{1}{n'}\frac{dR'}{dt}$, which follow, have the form

$$A \begin{smallmatrix}\sin\\ \cos\end{smallmatrix}\left(\varkappa\gamma' + i'g' + ig\right)$$

Arg.			T′		$\frac{1}{n'}\frac{dR'}{dt}$	
			sin.	cos.	cos.	sin.
$\varkappa$	i'	i	″	″	″	″
−1	1	0	−516.95043	− 1.069530	+0.1737	+0.3861
−1	2	0	− 58.58491	+ 43.19474	−1.5295	−1.0304
0	1	0	+ 37.48124	− 28.53854		
1	0	0	+ 8.631067	− 5.350080	+1.552265	+1.106428
−1	3	0	− 3.02325	+ 8.39040	−0.2303	−0.3870
0	2	0	+ 2.24989	− 6.55349		
1	1	0	+ 0.80623	− 0.74136	+0.0568	+0.2747
−1	4	0	+ 0.21579	+ 1.00619	−0.0060	−0.0758
0	3	0	− 0.19036	− 0.83455		
1	2	0	+ 0.05554	− 0.10394	−0.0172	+0.0347
−1	5	0	+ 0.07799	+ 0.08407	+0.0041	−0.0101
0	4	0	− 0.06780	− 0.07134		
1	3	0	+ 0.00109	− 0.01303	−0.0044	+0.0024
−1	6	0	+ 0.01205	+ 0.00368	+0.0011	−0.0009
0	5	0	− 0.01044	− 0.00306		
1	4	0	− 0.00051	− 0.00137	−0.0006	−0.0001
0	6	0	− 0.00114	+ 0.00024		
1	5	0	− 0.00013	− 0.00012		
−1	−3	−1	+ 0.00060	+ 0.00053	−0.0001	+0.0006
0	−4	−1	+ 0.00099	+ 0.00183		
−1	−2	−1	+ 0.00745	+ 0.00309	−0.0016	+0.0023
0	−3	−1	+ 0.01341	+ 0.00951		
1	−4	−1	− 0.02257	− 0.01363	+0.0067	−0.0010
−1	−1	−1	+ 0.08115	+ 0.01269	−0.0212	+0.0098
0	−2	−1	+ 0.11730	+ 0.02646		
1	−3	−1	− 0.21738	− 0.02439	+0.0493	+0.0104
−1	0	−1	+ 0.99324	+ 0.95414	−0.4406	−0.5806
0	−1	−1	+ 0.69522	+ 0.16626		
1	−2	−1	− 1.60135	+ 0.33043	+0.2174	+0.1068
−1	1	−1	− 2.26425	− 50.58646	+1.9137	+4.2296
1	−1	−1	− 10.11588	− 8.54976	−1.9733	−4.2290
−1	2	−1	+ 30.6172	+ 129.6082	+0.3895	−0.1918
0	1	−1	−331.8120	−1579.2837		
1	0	−1	+413.9052	+1977.8997	−0.0761	+0.5711
−1	3	−1	− 62.89740	− 27.54848	−0.8100	+0.8220
0	2	−1	+ 37.58748	− 40.59165		
1	1	−1	+ 0.77630	+ 22.61485	+0.8812	−0.8551
−1	4	−1	− 15.19654	− 0.57900	−0.3667	+0.1240
0	3	−1	+ 12.56079	− 2.56986		
1	2	−1	− 0.23716	+ 1.78715	+0.2797	−0.0291

Arg.			T′		$\frac{1}{n'}\frac{dR'}{dt}$	
			sin	cos.	cos.	sin.
κ	i′	i	″	″	″	″
−1	5 −	1	− 2.06132	+ 0.89623	−0.0803	−0.0114
0	4 −	1	+ 1.83429	− 0.96681		
1	3 −	1	+ 0.02828	+ 0.21024	+0.0410	+0.0245
−1	6 −	1	− 0.17839	+ 0.22186	−0.0112	−0.0080
0	5 −	1	+ 0.16476	− 0.21309		
1	4 −	1	+ 0.01448	+ 0.02047	+0.0029	+0.0065
−1	7 −	1	− 0.00563	+ 0.03301	−0.0003	−0.0002
0	6 −	1	+ 0.00555	− 0.03126		
1	5 −	1	+ 0.00264	+ 0.00113	−0.0001	+0.0009
−1	8 −	1	+ 0.00124	+ 0.00347		
0	7 −	1	− 0.00117	− 0.00339		
1	6 −	1	+ 0.00033	− 0.00018		
−1	−2 −	2	− 0.00003	+ 0.00035		
−1	−1 −	2	− 0.00019	+ 0.00514	+0.0008	+0.0010
0	−2 −	2	+ 0.00003	+ 0.00510		
1	−3 −	2	− 0.00021	− 0.00985	+0.0005	−0.0039
−1	0 −	2	+ 0.03874	+ 0.13724	−0.0222	−0.0429
0	−1 −	2	+ 0.01188	+ 0.06591		
1	−2 −	2	− 0.03406	− 0.09873	+0.0108	−0.0386
−1	1 −	2	− 0.73080	− 4.73524	+0.2770	+0.6911
1	−1 −	2	− 0.70265	− 2.02830	−0.0700	−0.7337
−1	2 −	2	− 0.16574	+ 61.01074	−1.8224	+0.4418
0	1 −	2	− 35.34786	−181.96074		
1	0 −	2	+ 36.95055	+186.12850	+1.8721	−0.3333
−1	3 −	2	+337.63347	−139.59991	−0.4087	−0.2854
0	2 −	2	−278.21229	+110.17461		
1	1 −	2	+ 30.38905	− 11.54497	+0.1586	+0.2656
−1	4 −	2	+ 65.51974	− 85.33376	+0.3637	+0.5514
0	3 −	2	− 58.54893	+ 74.51370		
1	2 −	2	+ 2.13473	− 9.37408	−0.4048	−0.6105
−1	5 −	2	+ 2.50567	− 20.38129	+0.03434	+0.29178
0	4 −	2	− 2.57967	+ 18.61784		
1	3 −	2	− 0.95705	− 1.69618	+0.0106	−0.2337
−1	6 −	2	− 1.31008	− 2.94476	−0.0258	+0.0699
0	5 −	2	+ 1.1766033	+ 2.7790743		
1	4 −	2	− 0.293699	− 0.123765	+0.0298	−0.0385
−1	7 −	2	− 0.37201	− 0.25777	−0.0109	+0.0097
0	6 −	2	+ 0.3484314	+ 0.2517321		
1	5 −	2	− 0.04472	+ 0.01049	+0.00804	−0.00254
−1	8 −	2	− 0.05907	− 0.00340	−0.0024	+0.0006
0	7 −	2	+ 0.05646	+ 0.00441		
1	6 −	2	− 0.00400	+ 0.00456	+0.0012	+0.0004
−1	9 −	2	− 0.00639	+ 0.00354		
0	8 −	2	+ 0.00615	− 0.00330		
1	7 −	2	+ 0.00002	+ 0.00086		

Arg.	T′		$\frac{1}{n'}\frac{dR'}{dt}$	
	sin.	cos.	cos.	sin.
$\varkappa$ i' i	″	″	″	″
—1 —1— 3	— 0.00007	+ 0.00026		
—1 0— 3	+ 0.00059	+ 0.00814	—0.0014	—0.0045
0 —1— 3	— 0.00039	+ 0.00504		
1 —2— 3	+ 0.00197	— 0.00509	—0.0017	—0.0024
—1 1— 3	— 0.09853	— 0.40700	+0.0289	+0.0516
1 —1— 3	— 0.01560	— 0.15803	—0.0271	—0.0634
—1 2— 3	+ 0.36186	+ 5.38023	—0.0156	—0.0099
0 1— 3	— 2.94534	— 15.10743		
1 0— 3	+ 3.25176	+ 15.08398	+0.0183	—0.1219
—1 3— 3	+ 3.84726	— 17.12524	+0.0260	+1.2106
0 2— 3	— 7.54317	+ 9.11937		
1 1— 3	+ 1.62923	— 1.80976	—0.0004	—1.2323
—1 4— 3	+134.73558	+189.55633	—0.2762	+0.3342
0 3— 3	—119.45859	—165.32601		
1 2— 3	+ 21.79101	+ 30.21788	+0.2371	—0.1808
—1 5— 3	+ 79.63698	+ 39.02827	+0.3297	—0.1265
0 4— 3	— 72.57969	— 36.04776		
1 3— 3	+ 12.09393	+ 2.94027	—0.3798	+0.1575
—1 6— 3	+ 20.55631	— 1.24009	+0.2045	+0.0190
0 5— 3	— 19.29003	+ 0.92685		
1 4— 3	+ 2.46966	— 1.13752	—0.1704	—0.0374
—1 7— 3	+ 3.14534	— 2.09648	+0.0522	+0.0330
0 6— 3	— 3.03342	+ 1.95084		
1 5— 3	+ 0.23109	— 0.42504	—0.0302	—0.0317
—1 8— 3	+ 0.26103	— 0.54421	+0.0069	+0.0120
0 7— 3	— 0.26202	+ 0.52050		
1 6— 3	— 0.00941	— 0.07521	—0.0015	—0.0086
—1 9— 3	— 0.00817	— 0.08750	+0.0002	+0.0026
0 8— 3	+ 0.00618	+ 0.08532		
1 7— 3	— 0.00731	— 0.00807	+0.0006	—0.0013
—1 10— 3	— 0.00727	— 0.00964		
0 9— 3	+ 0.00690	+ 0.00957		
1 8— 3	— 0.00153	— 0.00033		
—1 11— 3	— 0.00014	— 0.00018		
0 10— 3	— 0.00138	+ 0.00066		
1 9— 3	+ 0.00039	+ 0.00049		
—1 0— 4	+ 0.00018	+ 0.00090		
—1 1— 4	— 0.00724	— 0.03283	+0.0019	+0.0037
1 —1— 4	— 0.00139	— 0.01029	—0.0022	—0.0038
—1 2— 4	+ 0.04884	+ 0.39446	—0.0003	—0.0039
0 1— 4	— 0.23013	— 1.15266		
1 0— 4	+ 0.25078	+ 1.16180	—0.0039	—0.0015
—1 3— 4	— 0.37993	— 0.90147	—0.0301	+0.0364
0 2— 4	— 0.14280	+ 0.51720		
1 1— 4	+ 0.11784	— 0.05486	—0.0529	—0.0544
—1 4— 4	+ 17.26822	+ 1.98797	+0.7245	+0.1398

Arg.			T'		$\frac{1}{n'}\frac{dR}{dt}$	
			sin.	cos.	cos.	sin.
κ	i'	i	"	"	"	"
0	3	—4	—12.48753	— 5.25909		
1	2	—4	+ 2.57442	+ 1.46913	—0.7438	—0.1596
—1	5	—4	—94.22508	+104.08309	+0.2273	+0.2489
0	4	—4	+84.71691	— 95.48370		
1	3	—4	—18.79588	+ 20.99078	—0.1415	—0.2096
—1	6	—4	—16.50867	+ 64.12834	—0.0225	—0.1735
0	5	—4	+15.56199	— 59.81178		
1	4	—4	— 1.24466	+ 11.54170	+0.0414	+0.2134
—1	7	—4	+ 4.56295	+ 17.62466	+0.0406	—0.1287
0	6	—4	— 4.18134	— 16.81242		
1	5	—4	+ 1.44161	+ 2.55979	—0.0469	+0.1111
—1	8	—4	+ 2.67978	+ 2.75756	+0.0334	—0.0341
0	7	—4	— 2.54838	— 2.69748		
1	6	—4	+ 0.53021	+ 0.25365	—0.0297	+0.0201
—1	9	—4	+ 0.66570	+ 0.18655	+0.0116	—0.0037
0	8	—4	— 0.64584	— 0.19308		
1	7	—4	+ 0.10036	— 0.01604	—0.0081	+0.0001
—1	10	—4	+ 0.10783	— 0.02865	+0.00257	+0.00047
0	9	—4	— 0.10644	+ 0.02613		
1	8	—4	+ 0.01155	— 0.01104	—0.0013	—0.0008
—1	11	—4	+ 0.01170	— 0.01199	+0.0004	+0.0003
0	10	—4	— 0.0116886	+ 0.0115086		
1	9	—4	+ 0.00039	— 0.00227	—0.0001	—0.0003
0	11	—4	— 0.00062	+ 0.00223		
1	10	—4	— 0.00060	+ 0.00065		
—1	1	—5	— 0.00049	— 0.00241		
1	—1	—5	— 0.00017	— 0.00078		
—1	2	—5	+ 0.00500	+ 0.02873		
0	1	—5	— 0.01734	— 0.08493		
1	0	—5	+ 0.01789	+ 0.08566		
—1	3	—5	— 0.02898	— 0.04060	—0.0041	+0.0006
0	2	—5	— 0.00303	+ 0.02673		
1	1	—5	+ 0.00352	— 0.00005	—0.0014	—0.0005
—1	4	—5	+ 0.99604	— 0.52220	+0.0316	+0.0378
0	3	—5	— 0.79776	+ 0.01884		
1	2	—5	+ 0.16006	+ 0.07095	—0.0528	+0.0080
—1	5	—5	+ 0.60860	+ 14.24175	+0.1725	—0.3986
0	4	—5	+ 1.99383	— 11.64981		
1	3	—5	— 0.78322	+ 2.68920	—0.1886	+0.4126
—1	6	—5	—71.52119	— 40.27642	+0.2047	—0.1320
0	5	—5	+67.06263	+ 36.69579		
1	4	—5	—16.31481	— 9.09478	—0.1735	+0.0876
—1	7	—5	—46.40372	— 2.01085	—0.0776	—0.0115
0	6	—5	+43.94724	+ 1.91709		
1	5	—5	— 9.29195	+ 0.60525	+0.1073	+0.0025
—1	8	—5	—13.30987	+ 6.47574	—0.0726	—0.0436

Arg.			T′		$\frac{1}{n'}\frac{dR'}{dt}$	
			sin.	cos.	cos.	sin.
κ	i′	i	″	″	″	″
0	7−	5	+12.83709	− 6.13302		
1	6−	5	− 2.16221	+ 1.65307	+0.0647	+0.0454
−1	9−	5	− 2.02539	+ 2.90856	−0.0192	−0.0291
0	8−	5	+ 2.00370	− 2.80560		
1	7−	5	− 0.19946	+ 0.58058	+0.0111	+0.0251
−1	10−	5	− 0.06333	+ 0.70862	−0.0011	−0.0100
0	9−	5	+ 0.07275	− 0.69450		
1	8−	5	+ 0.02979	+ 0.11411	−0.0010	+0.0070
−1	11−	5	+ 0.05337	+ 0.11438	+0.0009	−0.0022
0	10−	5	− 0.05076	− 0.11400		
1	9−	5	+ 0.01501	+ 0.01342	−0.0009	+0.0011
−1	12−	5	+ 0.01710	+ 0.01185	+0.0003	−0.0003
0	11−	5	− 0.01689	− 0.01233		
1	10−	5	+ 0.00338	+ 0.00084	−0.0002	0.0000
−1	13−	5	+ 0.00282	+ 0.00050		
0	12−	5	− 0.00333	− 0.00036		
1	11−	5	+ 0.00037	+ 0.00005		
0	1−	6	− 0.00126	− 0.00612		
−1	3−	6	− 0.00174	− 0.00171		
0	2−	6	0.00000	+ 0.00159		
1	1−	6	+ 0.00004	− 0.00007		
−1	4−	6	+ 0.04495	− 0.04847	+0.0002	+0.0038
0	3−	6	− 0.04464	+ 0.01233		
1	2−	6	+ 0.00823	+ 0.00215	−0.0021	+0.0001
−1	5−	6	+ 0.64099	+ 0.87014	+0.0363	−0.0196
0	4−	6	− 0.22344	− 0.80607		
1	3−	6	− 0.00516	+ 0.19707	−0.0127	+0.0378
−1	6−	6	−10.35397	+ 2.37338	−0.2010	−0.1476
0	5−	6	+ 9.13914	− 0.51939		
1	4−	6	− 2.30015	− 0.10173	+0.2098	+0.1604
−1	7−	6	+13.13057	−44.99210	−0.0635	−0.1547
0	6−	6	−11.82843	+42.83535		
1	5−	6	+ 3.19966	−11.12902	+0.0422	+0.1331
−1	8−	6	− 5.42124	−30.71983	−0.0152	+0.0259
0	7−	6	+ 5.08044	+29.40624		
1	6−	6	− 1.56414	− 6.61780	+0.0123	−0.0467
−1	9−	6	− 6.90697	− 8.96409	−0.0346	+0.0370
0	8−	6	+ 6.65490	+ 8.71890		
1	7−	6	− 1.67922	− 1.56369	+0.0351	−0.0343
−1	10−	6	− 2.78286	− 1.21282	−0.0226	+0.0087
0	9−	6	+ 2.71095	+ 1.21317		
1	8−	6	− 0.56704	− 0.10429	+0.0194	−0.0045
−1	11−	6	− 0.66978	+ 0.06823	−0.0077	−0.0009
0	10−	6	+ 0.66150	− 0.05841		
1	9−	6	− 0.11282	+ 0.04610	+0.0053	+0.0018
−1	12−	6	− 0.10593	+ 0.07651	−0.0017	−0.0011

Arg.			T′		$\frac{1}{n'}\frac{dR'}{dt}$	
			sin.	cos.	cos.	sin.
ϰ	i′	i	″	″	″	″
0	11—	6	+ 0.10644	−0.07464		
1	10—	6	− 0.01295	+0.01935	+0.0009	+0.0010
−1	13—	6	− 0.01044	+0.02091		
0	12—	6	+ 0.01134	−0.02052		
1	11—	6	− 0.00117	+0.00377		
−1	14—	6	− 0.00020	+0.00040		
1	12—	6	+ 0.00055	−0.00112		
−1	4—	7	+ 0.00160	−0.00329		
0	3—	7	− 0.0024	+0.0012		
1	2—	7	+ 0.00058	+0.00011		
−1	5—	7	+ 0.06154	+0.03657	+0.0036	+0.0003
0	4—	7	− 0.0294	−0.0456		
1	3—	7	+ 0.00263	+0.01163	−0.0013	+0.0020
−1	6—	7	− 0.64180	+0.68033	−0.0087	−0.0295
0	5—	7	+ 0.6636	−0.3741		
1	4—	7	− 0.18233	+0.05361	+0.0220	+0.0190
−1	7—	7	− 3.04559	−6.78447	−0.1067	+0.0912
0	6—	7	+ 1.8381	+6.3201		
1	5—	7	− 0.33211	−1.69777	+0.1164	−0.0956
−1	8—	7	+26.22716	+1.25454	−0.1089	+0.0200
0	7—	7	−25.2525	−0.7917		
1	6—	7	+ 6.88722	+0.31847	+0.0953	−0.0108
−1	9—	7	+18.73810	−7.60789	+0.0020	+0.0090
0	8—	7	−18.0735	+7.4121		
1	7—	7	+ 4.25317	−2.09548	−0.0158	−0.0097
−1	10—	7	+ 5.36501	−6.29392	+0.0161	+0.0243
0	9—	7	− 5.2494	+6.1170		
1	8—	7	+ 0.96673	−1.50824	−0.0158	−0.0248
−1	11—	7	+ 0.51209	−2.40578	+0.0025	+0.0161
0	10—	7	− 0.5223	+2.3601		
1	9—	7	+ 0.00785	−0.50066	−0.0003	−0.0138
−1	12—	7	− 0.17451	−0.57187	−0.0020	+0.0054
0	11—	7	+ 0.1662	+0.5679		
1	10—	7	− 0.05982	−0.09927	+0.0022	−0.0038
−1	13—	7	− 0.09269	−0.08706	−0.0012	+0.0011
0	12—	7	+ 0.0891	+0.0870		
1	11—	7	− 0.01989	−0.00967	+0.0009	−0.0006
−1	14—	7	− 0.00179	−0.00171		
1	12—	7	+ 0.00501	+0.00458		
−1	5—	8	+ 0.00510	+0.00104		
0	4—	8	− 0.0024	−0.0024		
1	3—	8	− 0.00051	+0.00056		
−1	6—	8	− 0.02106	+0.06573	+0.0009	−0.0028
0	5—	8	+ 0.0363	−0.0417		
1	4—	8	− 0.01205	+0.00730	+0.0012	+0.0018
−1	7—	8	− 0.63382	−0.40047	−0.0215	+0.0015

Arg.			T'		$\frac{1}{n'}\frac{dR'}{dt}$	
			sin	cos.	cos.	sin.
$\varkappa$	i'	i	"	"	"	"
0	6—	8	+0.4323	+ 0.4620		
1	5—	8	—0.08852	— 0.13862	+0.0174	—0.0103
—1	8—	8	+4.01398	— 2.92508	+0.0349	+0.0693
0	7—	8	—3.8994	+ 2.1963		
1	6—	8	+1.09582	— 0.50831	—0.0369	—0.0761
—1	9—	8	+2.81889	+14.20255	—0.0006	+0.0709
0	8—	8	—3.0009	—13.7838		
1	7—	8	+0.77454	+ 3.90507	+0.0042	—0.0629
—1	10—	8	+7.27448	+10.50958	+0.0024	+0.0063
0	9—	8	—7.1304	—10.1895		
1	8—	8	+1.95009	+ 2.47755	—0.0042	+0.0022
—1	11—	8	+5.12638	+ 2.76778	+0.0151	—0.0053
0	10—	8	—5.0178	— 2.7204		
1	9—	8	+1.23890	+ 0.49213	—0.0159	+0.0057
—1	12—	8	+1.90645	+ 0.01015	+0.0104	+0.0007
0	11—	8	—1.8807	— 0.0238		
1	10—	8	+0.40603	— 0.06564	—0.0091	—0.0017
—1	13—	8	+0.44791	— 0.24774	+0.0037	+0.0022
0	12—	8	—0.4434	+ 0.2352		
1	11—	8	+0.07719	— 0.06511	—0.0025	—0.0021
—1	14—	8	+0.00868	— 0.00475		
1	12—	8	—0.02396	+ 0.01355		
—1	6—	9	+0.00029	+ 0.00423		
0	5—	9	+0.0015	— 0.0030		
1	4—	9	—0.00072	+ 0.00061		
—1	7—	9	—0.06068	— 0.00639	—0.0020	—0.0010
0	6—	9	+0.0450	+ 0.0219		
1	5—	9	—0.00942	— 0.00833	+0.0018	—0.0005
—1	8—	9	+0.19633	— 0.53191	—0.0022	+0.0142
0	7—	9	—0.2643	+ 0.4098		
1	6—	9	+0.08526	— 0.09689	—0.0033	—0.0133
—1	9—	9	+2.4010	+ 2.1511	+0.0413	—0.0091
0	8—	9	—1.9920	— 2.1849		
1	7—	9	+0.5093	+ 0.6496	—0.0460	+0.0095
—1	10—	9	—7.0961	+ 3.4142	+0.0434	+0.0096
0	9—	9	+6.9231	— 3.5052		
1	8—	9	—2.0293	+ 0.9691	—0.0390	—0.0106
—1	11—	9	—5.3431	+ 5.8259	+0.0072	+0.0021
0	10—	9	+5.1930	— 5.7390		
1	9—	9	—1.2898	+ 1.5666	—0.0023	+0.0001
—1	12—	9	—1.1080	+ 3.8293	—0.0005	—0.0085
0	11—	9	+1.0917	— 3.7671		
1	10—	9	—0.1705	+ 0.9368	+0.0010	+0.0093
—1	13—	9	+0.3036	+ 1.4189	+0.0021	—0.0065
0	12—	9	—0.2763	— 1.3854		
1	11—	9	+0.1023	+ 0.2936	—0.0021	+0.0054

Arg.		T′		$\frac{1}{n'}\frac{dR'}{dt}$	
		sin.	cos.	cos.	sin.
κ	i′ i	″	″	″	″
—1	14— 9	+0.0057	+0.0274		
1	12— 9	—0.0168	—0.0766		
—1	7—10	—0.0053	+0.0016		
0	6—10	+0.0027	+0.0006		
1	5—10	+0.0004	—0.0005		
—1	8—10	+0.0098	—0.0529	—0.0011	+0.0013
0	7—10	—0.0318	+0.0441		
1	6—10	+0.0202	—0.0109	+0.0002	—0.0014
—1	9—10	+0.4082	+0.0610	+0.0087	+0.0033
0	8—10	—0.3414	—0.1233		
1	7—10	+0.0868	+0.0475	—0.0088	—0.0004
—1	10—10	—0.9941	+1.7732	+0.0009	—0.0228
0	9—10	+1.0641	—1.5597		
1	8—10	—0.3335	+0.4226	—0.0013	+0.0257
—1	11—10	—2.7624	—3.2038	+0.0113	—0.0247
0	10—10	+2.8206	+3.1320		
1	9—10	—0.8087	—0.9511	—0.0116	+0.0224
—1	12—10	—4.1813	—2.3639	+0.0040	—0.0054
0	11—10	+4.1346	+2.2935		
1	10—10	—1.1369	—0.5739	—0.0022	+0.0027
—1	13—10	—2.7176	—0.1471	—0.0043	—0.0011
0	12—10	+2.6208	+0.1698		
1	11—10	—0.6290	+0.0038	+0.0047	+0.0006
—1	14—10	—0.0522	—0.0029		
1	12—10	+0.1476	+0.0077		
—1	8—11	—0.0004	—0.0043		
0	7—11	—0.0021	+0.0042		
1	6—11	+0.0018	—0.0012		
—1	9—11	+0.0391	—0.0142	+0.0007	+0.0009
0	8—11	—0.0354	+0.0027		
1	7—11	+0.0092	+0.0011	—0.0010	—0.0004
—1	10—11	+0.0184	+0.2888	+0.0033	—0.0048
0	9—11	+0.0288	—0.2571		
1	8—11	—0.0171	+0.0706	—0.0017	+0.0053
—1	11—11	—1.2030	—0.3558	—0.0115	—0.0036
0	10—11	+1.1010	+0.4203		
1	9—11	—0.3117	—0.1426	+0.0133	+0.0043
—1	12—11	+1.2409	—1.8777	—0.0130	—0.0100
0	11—11	—1.2078	+1.9158		
1	10—11	+0.3848	—0.5626	+0.0119	+0.0099
—1	13—11	+0.7830	—2.8595	—0.0036	—0.0041
0	12—11	—0.7656	+2.7234		
1	11—11	+0.1932	—0.7059	+0.0021	+0.0029
—1	14—11	+0.0150	—0.0549		
1	12—11	—0.0428	+0.1557		

Arg.		T'		$\frac{1}{n'}\frac{dR'}{dt}$	
		sin.	cos.	cos.	sin.
$\varkappa$	i' i	″	″	″	″
−1	9−12	+0.0026	−0.0031		
0	8−12	−0.0027	+0.0024		
1	7−12	+0.0010	−0.0011		
−1	10−12	+0.0162	+0.0263	+0.0007	−0.0003
0	9−12	−0.0081	−0.0249		
1	8−12	+0.0009	+0.0066	−0.0004	+0.0006
−1	11−12	−0.1888	+0.0523	−0.0025	−0.0026
0	10−12	+0.1770	−0.0195		
1	9−12	−0.0514	−0.0009	+0.0029	+0.0019
−1	12−12	+0.0411	−0.7440	−0.0034	+0.0056
0	11−12	−0.0864	+0.7047		
1	10−12	+0.0358	−0.2076	+0.0039	−0.0064
−1	13−12	+1.2761	+0.2766	−0.0072	+0.0053
0	12−12	−1.1982	−0.2550		
1	11−12	+0.3206	+0.0902	+0.0068	−0.0046
−1	14−12	+0.0245	+0.0053		
1	12−12	−0.0694	−0.0153		

The preceding expressions have to be integrated, consequently we give the logarithms of the factors proper to each argument:

Logarithms of the integrating factors for Jupiter.

Arg.	log $\frac{n}{i'n'+in}$	Arg.	log $\frac{n}{i'n'+in}$	Arg.	log $\frac{n}{i'n'+in}$	Arg.	log $\frac{n}{i'n'+in}$
i' i		i' i		i' i		i' i	
0 — 1	0.0000000n	3 — 5	9.4211383n	6 — 6	9.4456460n	9 — 9	9.26955n
0 — 2	9.6989700n	3 — 6	9.31949n	6 — 7	9.3387663n	9 — 10	9.19546n
0 — 3	9.5228787n	3 — 7	9.23718n	6 — 8	9.2530636n	9 — 11	9.13219n
0 — 4	9.39794n	3 — 8	9.1680n	6 — 9	9.18152n	9 — 12	9.07697n
0 — 5	9.3010n	3 — 9	9.1084n	6 — 10	9.12011n	9 — 13	9.02799n
0 — 6	9.2218n	4 + 2	9.4424	6 — 11	9.0663n	10 — 3	9.9885
1 + 5	9.2674	4 + 1	9.58324	7 0	9.5499	10 — 4	1.57090
1 + 4	9.3563	4 0	9.7929735	7 — 1	9.74021	10 — 5	0.01182n
1 + 3	9.46818	4 — 1	0.2141409	7 — 2	0.08682	10 — 6	9.70484n
1 + 2	9.61930	4 — 2	0.4097644n	7 — 3	0.7418456n	10 — 7	9.52678n
1 + 1	9.8530395	4 — 3	9.8572177n	7 — 4	9.9276772n	10 — 8	9.40087n
1 0	0.3950335	4 — 4	9.6217373n	7 — 5	9.6613048n	10 — 9	9.30337n
1 — 1	0.2237972n	4 — 5	9.4698956n	7 — 6	9.4974092n	10 — 10	9.22380n
1 — 2	9.7966097n	4 — 6	9.3576090n	7 — 7	9.3786993n	10 — 11	9.15657n
1 — 3	9.5854756n	4 — 7	9.26847n	7 — 8	9.28557n	10 — 12	9.09837n
1 — 4	9.4440216n	4 — 8	9.19455n	7 — 9	9.20893n	10 — 13	9.0471n
1 — 5	9.33750n	4 — 9	9.1314n	7 — 10	9.14380n	10 — 14	9.0012n
1 — 6	9.2520n	4 — 10	9.0763n	7 — 11	9.08718n	11 — 4	0.36699
1 — 7	9.1806n	5 + 2	9.3965	7 — 12	9.0371n	11 — 5	0.24378n
2 + 4	9.3183	5 + 1	9.5209	8 — 1	9.6534	11 — 6	9.80397n
2 + 3	9.4196	5 0	9.69606	8 — 2	9.91311	11 — 7	9.58999n
2 + 2	9.55201	5 — 1	9.9942063	8 — 3	0.6546508	11 — 8	9.44728n
2 + 1	9.7434333	5 — 2	1.8719342	8 — 4	0.1087344n	11 — 9	9.34004n
2 0	0.0940035	5 — 3	0.0058720n	8 — 5	9.7499431n	11 — 10	9.25411n
2 — 1	0.7107944n	5 — 4	9.7018961n	8 — 6	9.5561876n	11 — 11	9.18240n
2 — 2	9.9227672n	5 — 5	9.5248272n	8 — 7	9.4226792n	11 — 12	9.12088n
2 — 3	9.6586391n	5 — 6	9.3994005n	8 — 8	9.3207072n	11 — 13	9.0670n
2 — 4	9.4955797n	5 — 7	9.3021981n	8 — 9	9.2381840n	11 — 14	9.0191n
2 — 5	9.37731n	5 — 8	9.22282n	8 — 10	9.16887n	12 — 5	0.77529n
2 — 6	9.28445n	5 — 9	9.15574n	8 — 11	9.10910n	12 — 6	9.93264n
2 — 7	9.2080n	5 — 10	9.0976n	8 — 12	9.05658n	12 — 7	9.66399n
2 — 8	9.1430n	5 — 11	9.0464n	8 — 13	9.0097n	12 — 8	9.49925n
3 + 3	9.3759	6 + 1	9.4665	9 — 2	9.7894	12 — 9	9.38010n
3 + 2	9.4938	6 0	9.6169	9 — 3	0.20469	12 — 10	9.28670n
3 + 1	9.65599	6 — 1	9.84890	9 — 4	0.42501n	12 — 11	9.20987n
3 0	9.9179122	6 — 2	0.3807859	9 — 5	9.86144n	12 — 12	9.14462n
3 — 1	0.6818160	6 — 3	0.2336731n	9 — 6	9.62419n	12 — 13	9.0879n
3 — 2	0.1013065n	6 — 4	9.8002765n	9 — 7	9.47162n	12 — 14	9.0377n
3 — 3	9.7466760n	6 — 5	9.5877269n	9 — 8	9.35894n	12 — 15	8.9928n
3 — 4	9.5540936n						

Logarithms of the integrating factors for Saturn.

Arg.	$\log \frac{n'}{i'n'+in}$	Arg.	$\log \frac{n'}{i'n'+in}$	Arg.	$\log \frac{n'}{i'n'+in}$	Arg.	$\log \frac{n'}{i'n'+in}$
i' i		i' i		i' i		i' i	
1 0	0.0000000	6—3	9.8386396n	5—6	9.0043670n	7—9	8.81389n
2 0	9.6989700	7—3	0.3468120n	6—6	9.0506124n	8—9	8.84315n
3 0	9.5228787	8—3	0.2596173	7—6	9.1023757n	9—9	8.87452n
4 0	9.3979400	9—3	9.8096612	8—6	9.1611541n	10—9	8.90834n
5 0	9.30103	10—3	9.59346	9—6	9.2291518n	11—9	8.94501n
6 0	9.2218	11—3	9.44977	10—6	9.3098085n	12—9	8.98506n
7 0	9.1549			11—6	9.40894n	13—9	9.02919n
		−1—4	8.96125n	12—6	9.53761n	14—9	9.0783n
−4—1	9.18820n	0—4	9.00291n	13—6	9.72126n		
−3—1	9.26095n	1—4	9.0489881n	14—6	0.0458n	4—10	8.6812n
−2—1	9.34840n	2—4	9.1005462n			5—10	8.7026n
−1—1	9.4580060n	3—4	9.1590601n	1—7	8.7856n	6—10	8.7251n
0—1	9.6049665n	4—4	9.2267037n	2—7	8.8130n	7—10	8.74877n
1—1	9.8287637n	5—4	9.3068626n	3—7	8.84214n	8—10	8.77383n
2—1	0.3157609n	6—4	9.4052430n	4—7	8.87344n	9—10	8.80043n
3—1	0.2867824	7—4	9.5326437n	5—7	8.90716n	10—10	8.82876n
4—1	9.8191074	8—4	9.7137009n	6—7	8.9437329n	11—10	8.85908n
5—1	9.5991729	9—4	0.0299790n	7—7	8.9836657n	12—10	8.89166n
6—1	9.4538678	10—4	1.1758707	8—7	9.0276457n	13—10	8.92690n
7—1	9.3451812	11—4	9.9719575	9—7	9.0765864n	14—10	8.9652n
8—1	9.25832	12—4	9.68472	10—7	9.1317510n		
				11—7	9.19496n	5—11	8.6514n
−3—2	9.09872n	−1—5	8.87236n	12—7	9.26895n	6—11	8.6713n
−2—2	9.15698n	0—5	8.90600n	13—7	9.3582n	7—11	8.6921n
−1—2	9.2242695n	1—5	8.9424624n	14—7	9.4707n	8—11	8.71407n
0—2	9.3039365n	2—5	8.9822730n			9—11	8.73716n
1—2	9.4015761n	3—5	9.0261048n	2—8	8.74796n	10—11	8.76154n
2—2	9.5277337n	4—5	9.0748621n	3—8	8.77297n	11—11	8.78737n
3—2	9.7062730n	5—5	9.1297937n	4—8	8.79952n	12—11	8.81484n
4—2	0.0147309n	6—5	9.1926934n	5—8	8.82779n	13—11	8.84416n
5—2	1.4769007	7—5	9.2662713n	6—8	8.85803n	14—11	8.8756n
6—2	9.9857524	8—5	9.3549096n	7—8	8.8905362n		
7—2	9.6917878	9—5	9.4664028n	8—8	8.9256737n	6—12	8.6234n
8—2	9.51808	10—5	9.6167909n	9—8	8.9639064n	7—12	8.6421n
9—2	9.39433	11—5	9.84875n	10—8	9.0058326n	8—12	8.6615n
		12—5	0.38026n	11—8	9.0522430n	9—12	8.68194n
−2—3	9.02457n	13—5	0.23405	12—8	9.10421n	10—12	8.70334n
−1—3	9.07314n	14—5	9.80042	13—8	9.16326n	11—12	8.72584n
0—3	9.1278452n			14—8	9.2316n	12—12	8.74958n
1—3	9.1904421n	0—6	8.82682n			13—12	8.77469n
2—3	9.2636055n	1—6	8.85699n	3—9	8.7133n	14—12	8.8013n
3—3	9.3516425n	2—6	8.88941n	4—9	8.7364n		
4—3	9.4621841n	3—6	8.9244550n	5—9	8.7607n		
5—3	9.6108385n	4—6	8.9625755n	6—9	8.78648n		

The functions W and R are obtained by means of the formulæ

$$\mathrm{W}=\int \mathrm{T}n\,dt \qquad\qquad \mathrm{R}=\int\left(\frac{1}{n}\frac{d\mathrm{R}}{dt}\right)n\,dt$$

The portions involving other multiples of γ than the single result from employing the system of multipliers η and θ, as has been explained. In most cases the terms arising from the θ multipliers can be neglected. As an illustration of this process, and on account of its interest, the terms which make up the great inequalities of the two planets may be given. They are as follows:

Arg.			W		Arg.			W′	
			cos.	sin.				cos.	sin.
$\varkappa$	i'	i	″	″	$\varkappa$	i'	i	″	″
−2	5	0	−0.00008	− 0.00017	−2	7	−2	+ 0.00512	− 0.00355
−1	5	−1	−0.10041	+ 0.20202	−1	6	−2	+ 1.26780	− 2.84972
0	5	−2	+5.71632	−13.50156	0	5	−2	−35.28018	+83.32990
1	5	−3	+0.06331	− 0.03106	1	4	−2	− 0.30383	+ 0.12803
2	5	−4	−0.00672	− 0.00264	2	3	−2	− 0.01426	+ 0.02389
3	5	−5	+0.00009	+ 0.00072	3	2	−2	+ 0.00085	+ 0.00372
4	5	−6	+0.00003	− 0.00002	4	1	−2	+ 0.00045	+ 0.00017
					5	0	−2	+ 0.00002	− 0.00012
			+5.67254	−13.33271				−34.32403	+80.63232

W and R must be completed by the addition of functions of τ, having the forms

$$\mathrm{W}=k_0+k_1\left(\frac{\rho}{\mathrm{a}}\cos\varphi+\tfrac{3}{2}e\right)+k_2\frac{\rho}{\mathrm{a}}\sin\varphi \qquad\qquad \mathrm{R}=k_3\frac{\rho}{\mathrm{a}}\cos\varphi+k_4\frac{\rho}{\mathrm{a}}\sin\varphi$$

the k being constants. It is, however, more commodious to alter the signification of the k so that we may write in the case of Jupiter

$$\begin{aligned}
\mathrm{W}=k_0&+k_1\cos\gamma+k_2\sin\gamma\\
&+[8.3821074]\,k_1\cos 2\gamma+[8.3821917]\,k_2\sin 2\gamma\\
&+[6.9403270]\,k_1\cos 3\gamma+[6.9404535]\,k_2\sin 3\gamma\\
&+[5.5723410]\,k_1\cos 4\gamma+[5.5724929]\,k_2\sin 4\gamma\\
&+[4.2452120]\,k_1\cos 5\gamma+[4.2453807]\,k_2\sin 5\gamma
\end{aligned}$$

$$\begin{aligned}
\mathrm{R}=&-[8.8599027]\,k_3\\
&+k_3\cos\gamma+k_4\sin\gamma\\
&+[8.3821074]\,k_3\cos 2\gamma+[8.3821917]\,k_4\sin 2\gamma\\
&+\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}$$

and in the case of Saturn

$$
\begin{aligned}
W' = k_0 + {} & \qquad\qquad\quad k_1 \cos \gamma' + \qquad\qquad\quad k_2 \sin \gamma' \\
& + [8.4472006]\, k_1 \cos 2\gamma' + [8.4473145]\, k_2 \sin 2\gamma' \\
& + [7.0705210]\, k_1 \cos 3\gamma' + [7.0706918]\, k_2 \sin 3\gamma' \\
& + [5.7676379]\, k_1 \cos 4\gamma' + [5.7678429]\, k_2 \sin 4\gamma' \\
& + [4.5056153]\, k_1 \cos 5\gamma' + [4.5058431]\, k_2 \sin 5\gamma
\end{aligned}
$$

$$
\begin{aligned}
R' = {} & - [8.9252322]\, k_3 \\
& + \qquad\qquad\quad k_3 \cos \gamma' + k_4 \sin \gamma' \\
& + \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.
\end{aligned}
$$

The perturbations of the fundamental argument, the residual perturbations of the natural logarithm of the radius vector, and the perturbations perpendicular to the plane of the orbit, so far as they depend on the first power of the disturbing force, are given by the formulæ

$$
\frac{d\delta z}{dt} = \overline{W} \qquad\qquad \frac{1}{n}\frac{d\nu}{dt} = -\frac{1}{2}\left(\frac{d\overline{W}}{d\gamma}\right) \qquad\qquad \frac{u}{\cos i} = \overline{R}
$$

where the dash above the quantities denotes that τ has been changed into t or, which is the same thing, γ into g.

The arbitrary constants k have been so assumed that the expressions for the perturbations may be simplified as much as possible. We take k_0, so that δz may contain no term proportional to t, k_1, and k_2, so that the terms having the argument g may disappear In like manner k_3 and k_4 are determined so that $\frac{u}{\cos i}$ may be free from terms having the argument g. In the case of Jupiter this has led us to put

$$
\begin{aligned}
&k_0 = +\ 14''.2801 \qquad && k_1 = -\ 1''.6777 \qquad && k_2 = +\ 1''.6921 \\
&k_3 = +\ \ 0''.0636 && k_4 = +\ 0''.1414 &&
\end{aligned}
$$

and in the case of Saturn

$$
\begin{aligned}
&k_0 = -517''.7721 \qquad && k_1 = -\ 14''.9831 \qquad && k_2 = +\ 1''.5080 \\
&k_3 = -\ \ \ 0''.5642 && k_4 = +\ \ 0''.7901 &&
\end{aligned}
$$

The expressions for $\frac{d\delta z}{dt}$ and $\frac{1}{n}\frac{dv}{dt}$ follow:

Arg.	$\frac{d\delta z}{dt}$		$\frac{1}{n}\frac{dv}{dt}$	
	cos.	sin.	sin.	cos.
i' i	"	"	"	"
0 0				− 0.0122808
0 − 1	− 1.1420	− 1.0174	+ 1.1610	− 0.9308
	+ 1.0173636nt	− 1.1420391nt	− 0.5086818nt	− 0.5710195nt
0 − 2	− 0.0771	+ 0.0170	+ 0.0835	0.0000
	+ 0.0245236nt	− 0.0275342nt	− 0.0245236nt	− 0.0275342nt
0 − 3	− 0.0017	+ 0.0010	+ 0.0033	+ 0.0004
	+ 0.0008868nt	− 0.0009957nt	− 0.0013302nt	− 0.0014935nt
1 + 3	− 0.00032	− 0.00061	− 0.0006	+ 0.0004
1 + 2	− 0.0118	− 0.0017	− 0.0133	+ 0.0013
1 + 1	− 0.3720	+ 0.1052	− 0.2603	− 0.0597
1 0	− 3.3928	− 1.7597	− 0.7161	+ 0.1939
1 − 1	− 9.1757	+ 47.1662	+ 2.9759	+ 15.2960
1 − 2	+ 0.3997	+ 2.2087	− 0.1906	+ 1.6614
1 − 3	+ 0.0274	+ 0.0893	− 0.0258	+ 0.1050
1 − 4	+ 0.0014	+ 0.0033	− 0.0019	+ 0.0057
2 + 2	− 0.0014	− 0.0012	− 0.0019	+ 0.0016
2 + 1	− 0.0417	− 0.0223	− 0.0366	+ 0.0218
2 0	− 1.0199	− 1.0268	− 0.4698	+ 0.4065
2 − 1	− 24.04532	+ 0.86691	+ 3.53984	− 0.19888
2 − 2	− 214.0201	− 91.7688	+ 120.3064	− 51.5832
2 − 3	− 5.4539	− 3.1129	+ 5.8327	− 3.0764
2 − 4	− 0.1899	− 0.1250	+ 0.3069	− 0.1800
2 − 5	− 0.0081	− 0.0054	+ 0.0174	− 0.0106
3 + 1	+ 0.0010	+ 0.0039	+ 0.0013	− 0.0042
3 0	+ 0.0906	+ 0.1654	+ 0.0440	− 0.0960
3 − 1	+ 2.5944	+ 1.6632	+ 0.4591	− 0.3340
3 − 2	+ 39.6924	+ 52.2365	− 16.5368	+ 22.3896
3 − 3	− 15.6852	+ 24.6755	+ 10.8438	+ 17.5836
3 − 4	− 0.9761	+ 0.7016	+ 0.9797	+ 0.8822
3 − 5	− 0.0467	+ 0.0168	+ 0.0651	+ 0.0390
3 − 6	− 0.0022	+ 0.0005	+ 0.0041	+ 0.0020
4 0	+ 0.0013	+ 0.0102	− 0.0001	− 0.0072
4 − 1	+ 0.0498	+ 0.1955	+ 0.0288	− 0.0919
4 − 2	+ 0.9030	+ 6.3979	− 0.0967	+ 1.5544
4 − 3	− 19.0362	+ 9.0422	+ 11.5647	+ 5.7324
4 − 4	+ 5.3687	+ 6.5947	− 4.0749	+ 5.3143
4 − 5	+ 0.1126	+ 0.5330	− 0.1716	+ 0.5649
4 − 6	− 0.0051	+ 0.0284	− 0.0005	+ 0.0402
4 − 7	− 0.0005	+ 0.0013	+ 0.0003	+ 0.0025
5 0	+ 0.0067	− 0.0090	+ 0.0066	+ 0.0093
5 − 1	+ 0.2796	− 0.3932	+ 0.1413	+ 0.1951

Arg.	$\frac{d\delta z}{dt}$		$\frac{1}{n}\frac{dv}{dt}$	
	cos.	sin.	sin.	cos.
i' i	$''$	$''$	$''$	$''$
5— 2	+ 5.67254	—13.33271	+ 0.07542	+0.11796
5— 3	+152.6838	+11.3735	—75.6574	+5.7237
5— 4	+ 4.8731	+ 5.4299	— 4.6297	+4.0120
5— 5	+ 2.9701	— 1.4865	— 2.6327	—1.1887
5— 6	+ 0.2863	+ 0.0133	— 0.3146	—0.0105
5— 7	+ 0.0164	+ 0.0087	— 0.0236	+0.0074
5— 8	+ 0.0007	+ 0.0008	— 0.0014	+0.0008
6— 1	+ 0.0021	— 0.0003	+ 0.0012	0.0000
6— 2	+ 0.0430	— 0.0193	+ 0.0153	+0.0041
6— 3	+ 1.0234	+ 0.6703	— 0.33389	+0.24544
6— 4	— 0.6738	+ 2.3519	+ 0.3837	+1.5534
6— 5	+ 2.0940	+ 0.0180	— 1.6795	—0.0131
6— 6	— 0.3486	— 1.2827	+ 0.2755	—1.1442
6— 7	+ 0.0431	— 0.1452	— 0.0346	—0.1587
6— 8	+ 0.0083	— 0.0084	— 0.0085	—0.0121
6— 9	+ 0.0007	— 0.0004	— 0.0008	—0.0007
7— 2	+ 0.0051	+ 0.0049	+ 0.0027	—0.0024
7— 3	+ 0.1240	+ 0.2402	— 0.0112	+0.0309
7— 4	— 1.2870	+ 1.2519	+ 0.7017	+0.7065
7— 5	+ 0.7304	+ 0.4453	— 0.5550	+0.3159
7— 6	+ 0.2083	— 0.9416	— 0.1670	—0.8053
7— 7	— 0.5807	+ 0.0198	+ 0.5308	—0.0003
7— 8	— 0.0711	— 0.0412	+ 0.0792	—0.0394
7— 9	— 0.0038	— 0.0063	+ 0.0060	—0.0070
7—10	— 0.0003	— 0.0006	+ 0.0002	—0.0006
8— 2	+ 0.0002	— 0.0007	+ 0.0001	+0.0002
8— 3	+ 0.0026	— 0.0206	+ 0.0009	+0.0033
8— 4	+ 0.3177	— 0.0869	— 0.1351	—0.0330
8— 5	+ 0.1746	+ 0.2678	— 0.1311	+0.1772
8— 6	+ 0.2701	— 0.3063	— 0.2077	—0.2588
8— 7	— 0.4311	— 0.1945	+ 0.3854	—0.1666
8— 8	— 0.0533	+ 0.2568	+ 0.0597	+0.2379
8— 9	— 0.0308	+ 0.0329	+ 0.0312	+0.0368
8—10	— 0.0045	+ 0.0014	+ 0.0051	+0.0024
8—11	— 0.0004	— 0.0002	+ 0.0004	—0.0001
9— 3	+ 0.0010	— 0.0005	+ 0.0005	+0.0002
9— 4	+ 0.0323	+ 0.0077	— 0.0075	+0.0025
9— 5	+ 0.0066	+ 0.1417	— 0.0071	+0.0850
9— 6	+ 0.1313	— 0.0439	— 0.0983	—0.0394
9— 7	— 0.1285	— 0.1712	+ 0.1170	—0.1412
9— 8	— 0.1431	+ 0.1919	+ 0.1283	+0.1767
9— 9	+ 0.1083	+ 0.0527	— 0.1007	+0.0550
9—10	+ 0.0135	+ 0.0206	— 0.0154	+0.0214
9—11	+ 0.0001	+ 0.0030	— 0.0006	+0.0034
9—12	— 0.0001	+ 0.0003	+ 0.0001	+0.0003

Arg.	$\frac{d\delta z}{dt}$		$\frac{1}{n}\frac{dv}{dt}$	
	cos.	sin.	cos.	sin.
i' i	″	″	″	″
10 − 4	−0.02767	−0.02746	−0.00044	+0.00061
10 − 5	+0.1833	−0.3728	−0.0906	−0.1829
10 − 6	+0.0479	−0.0014	−0.0357	−0.0057
10 − 7	−0.0092	−0.0739	+0.0116	−0.0602
10 − 8	−0.1064	+0.0495	+0.0923	+0.0482
10 − 9	+0.0803	+0.0935	−0.0756	+0.0863
10 − 10	+0.0365	−0.0427	−0.0374	−0.0390
10 − 11	+0.0126	−0.0046	−0.0133	−0.0054
10 − 12	+0.0017	+0.0004	−0.0019	+0.0003
11 − 5	+0.0050	−0.0036	−0.0018	−0.0010
11 − 6	+0.0121	+0.0098	−0.0080	+0.0059
11 − 7	+0.0085	−0.0208	−0.0054	−0.0165
11 − 8	−0.0426	−0.0032	+0.0367	−0.0001
11 − 9	+0.0146	+0.0638	−0.0158	+0.0576
11 − 10	+0.0568	−0.0309	−0.0536	−0.0291
11 − 11	−0.0147	−0.0218	+0.0130	−0.0225
11 − 12	−0.0010	−0.0071	+0.0013	−0.0074
12 − 6	+0.0044	+0.0097	−0.0025	+0.0052
12 − 7	+0.0052	−0.0037	−0.0035	−0.0031
12 − 8	−0.0110	−0.0075	+0.0096	−0.0054
12 − 9	−0.0072	−0.0244	+0.0049	+0.0224
12 − 10	+0.0373	−0.0003	−0.0351	−0.0014
12 − 11	−0.0082	−0.0328	+0.0081	−0.0322
12 − 12	−0.0127	+0.0030	+0.0131	+0.0022

The corresponding quantities for Saturn are :

Arg.	$\frac{d\delta z'}{dt}$		$\frac{1}{n'}\frac{dv'}{dt}$	
	cos.	sin.	sin.	cos.
i' i	″	″	″	″
0 0				+ 0.075066
1 0	−8.6311	−5.3501	−7.6361	+10.1319
	$-5.350080n't$	$+8.631067n't$	$-2.675040n't$	$-4.315533n't$
2 0	−0.7346	−1.1722	−0.4093	+ 1.7346
	$-0.149816n't$	$+0.241756n't$	$-0.149816n't$	$-0.241756n't$
3 0	−0.0303	−0.0972	+0.0233	+ 0.1703
	$-0.006294n't$	$+0.010158n't$	$-0.009441n't$	$-0.015237n't$
4 0	−0.0002	−0.0077	+0.0078	+ 0.0132
	$-0.000312n't$	$+0.000504n't$	$-0.000624n't$	$-0.001020n't$
5 0	+0.0002	−0.0005	+0.0010	+ 0.0006
−4 − 1	+0.0003	−0.0003	0.0000	+ 0.0002
−3 − 1	+0.0019	+0.0004	−0.0042	0.0000

Arg.	$\frac{d\delta z'}{dt}$		$\frac{1}{n'}\frac{d\nu'}{dt}$	
	cos.	sin.	sin.	cos.
i' i	″	″	″	″
−2— 1	+0.0227	+0.0114	−0.0478	+0.0163
−1— 1	+0.2802	+0.1311	−0.4629	+0.3204
0— 1	−2.5227	+28.9829	−2.6889	+8.2224
1— 1	+9.6656	−1.3654	+48.1544	+262.5939
2— 1	+204.9729	−6.8944	−56.2251	+3.2555
3— 1	−14.5483	−10.4073	−4.9697	+3.5005
4— 1	−0.4471	−0.0399	−0.4316	+0.1989
5— 1	−0.0263	−0.0040	−0.0316	+0.0280
6— 1	−0.0018	−0.0005	−0.0015	+0.0027
7— 1	−0.0001	−0.0002	+0.0001	+0.0003
−2— 2	+0.0006	−0.0006	−0.0014	−0.0015
−1— 2	+0.0099	−0.0108	−0.0217	−0.0146
0— 2	−0.0979	+1.0448	−0.2758	−0.0188
1— 2	+3.3526	−10.1030	−1.1744	+10.5178
2— 2	+87.5789	+37.4603	−83.5668	+36.4790
3— 2	+36.8496	+36.4676	−31.2005	+25.4230
4— 2	−78.2220	−629.5161	+37.3212	−306.1645
5— 2	−34.32403	+80.63232	−0.80297	−1.52185
6— 2	+1.1778	+0.4359	+0.5690	−0.2259
7— 2	+0.0331	+0.0168	+0.0295	−0.0142
8— 2	+0.0013	+0.0008	+0.0017	−0.0006
−1— 3	+0.0002	−0.0007	0.0000	−0.0018
0— 3	−0.0118	+0.0554	−0.0015	−0.0052
1— 3	+0.1182	−0.6269	+0.0881	+0.5273
2— 3	+0.8781	+0.8416	−1.4465	+0.2740
3— 3	+17.1428	−23.7761	−18.4562	−25.1481
4— 3	+14.7028	−6.2679	−15.1906	−7.4477
5— 3	+7.2953	+0.9109	−6.8424	+0.4197
6— 3	+5.0024	+3.4675	−3.4094	+2.2076
7— 3	−1.0596	−2.0905	+0.2382	−0.5277
8— 3	−0.0221	+0.1181	−0.0109	−0.0391
9— 3	+0.0007	+0.0024	−0.0005	−0.0022
0— 4	−0.0007	+0.0032	+0.0001	−0.0001
1— 4	+0.0064	−0.0358	+0.0081	+0.0341
2— 4	−0.0001	+0.0324	−0.0127	+0.0167
3— 4	+0.8945	−0.3721	−0.7506	−0.6953
4— 4	−7.6391	−8.5017	+8.3181	−9.4975
5— 4	−1.2828	−6.3801	+1.8724	−7.2651
6— 4	+0.8151	−2.3512	−0.6822	−2.7251
7— 4	+0.6780	−0.5943	−0.6353	−0.6689
8— 4	+0.3723	−0.1089	−0.2900	−0.1119
9— 4	−1.7237	−0.4520	+0.8129	−0.2179
10— 4	+0.16489	+0.16391	+0.00591	−0.00701
11— 4	+0.0084	+0.0077	+0.0045	−0.0050
1— 5	−0.001	−0.003	0.000	+0.002

Arg.	$\frac{d\delta z'}{dt}$		$\frac{1}{n'}\frac{dv'}{dt}$	
	cos.	sin.	sin.	cos.
i′ i	″	″	″	″
2— 5	−0.001	+0.001	0.000	−0.001
3— 5	+0.036	+0.006	−0.033	−0.010
4— 5	−0.086	−0.646	+0.245	−0.641
5— 5	−4.288	+2.410	+4.848	+2.613
6— 5	−3.1103	−0.0213	+3.6879	+0.1539
7— 5	−1.0347	−0.6167	+1.3164	−0.6281
8— 5	−0.1878	−0.3388	+0.2670	−0.3732
9— 5	+0.0014	−0.1213	+0.0135	−0.1326
10— 5	+0.0224	−0.0390	−0.0175	−0.0384
11— 5	+0.0305	−0.0201	−0.0196	−0.0142
12— 5	−0.0113	+0.0026	+0.0025	+0.0004
3— 6	−0.002	+0.001	−0.003	−0.000
4— 6	+0.014	−0.031	−0.002	−0.032
5— 6	−0.405	−0.041	+0.431	+0.030
6— 6	+0.627	+2.133	−0.652	+2.413
7— 6	−0.3429	+1.5477	+0.3477	+1.8657
8— 6	−0.4394	+0.4827	+0.4909	+0.6449
9— 6	−0.2016	+0.0630	+0.2413	+0.1087
10— 6	−0.0594	−0.0135	+0.0746	−0.0062
11— 6	−0.0126	−0.0117	+0.0165	−0.0114
12— 6	−0.0020	−0.0050	+0.0025	−0.0049
4— 7	+0.001	−0.002	0.000	−0.002
5— 7	−0.021	−0.016	+0.025	−0.010
6— 7	−0.078	+0.232	+0.051	+0.256
7— 7	+1.031	−0.056	−1.161	−0.031
8— 7	+0.756	+0.342	−0.915	+0.377
9— 7	+0.218	+0.301	−0.302	+0.351
10— 7	+0.012	+0.125	−0.036	+0.156
11— 7	−0.015	+0.033	+0.013	+0.046
12— 7	−0.007	+0.006	+0.009	+0.009
5— 8	−0.001	−0.001	0.000	−0.001
6— 8	−0.015	+0.012	+0.012	+0.015
7— 8	+0.122	+0.074	−0.138	+0.066
8— 8	+0.090	−0.478	−0.120	−0.532
9— 8	+0.263	−0.353	−0.300	−0.429
10— 8	+0.196	−0.087	−0.234	−0.128
11— 8	+0.076	+0.009	−0.100	−0.002
12— 8	+0.019	+0.013	−0.027	+0.015
6— 9	−0.002	0.000	+0.001	+0.001
7— 9	+0.006	+0.011	−0.009	+0.010
8— 9	+0.057	−0.060	−0.056	−0.069
9— 9	−0.209	−0.096	+0.231	−0.117
10— 9	−0.155	−0.177	+0.186	−0.206
11— 9	−0.027	−0.121	+0.042	−0.148
12— 9	+0.014	−0.046	−0.012	−0.061

Arg.	$\frac{d\delta z'}{dt}$		$\frac{1}{n'}\frac{d\nu'}{dt}$	
	cos.	sin.	sin.	cos.
i' i	″	″	″	″
7—10	+0.001	+0.002	0.000	0.000
8—10	+0.009	—0.003	—0.009	—0.004
9—10	—0.026	—0.041	+0.029	—0.042
10—10	—0.071	+0.085	+0.083	+0.092
11—10	—0.111	+0.059	+0.130	+0.071
12—10	—0.073	0.000	+0.091	+0.005
9—11	0.000	—0.006	+0.001	—0.005
10—11	—0.025	+0.009	+0.027	+0.010
11—11	+0.031	+0.044	—0.031	+0.051
12—11	+0.018	+0.066	—0.020	+0.078
10—12	—0.003	—0.001	+0.004	0.000
11—12	+0.001	+0.015	—0.002	+0.016
12—12	+0.026	—0.007	—0.030	—0.006

The integration of the preceding expressions gives those for δz and ν, except that we do not thus obtain the constant term of the latter quantity. This, however, is known to be

$$-\frac{1}{6}\left[k_0+\frac{3}{2}\frac{e}{P_1}k_1\right]$$

where P_1 has the signification attributed to it at page 63.*

This formula gives us in the case of Jupiter as the constant term of ν $-2''.3598$, and in the case of Saturn as the constant term of ν' $+86''.5056$.

With regard to the terms of the perturbations factored by t it may be noted that if we denote the terms in $n\delta z$, having the argument g, by

$$k_1nt\sin g+k_2nt\cos g$$

and those in $\frac{u}{\cos i}$ by

$$k_3nt\sin g+k_4nt\cos g$$

the complete expressions for these parts of the perturbations will be

$$\begin{aligned}
n\delta z = {} & k_1nt\left[\sin g+\frac{1}{2}\frac{P_2}{P_1}\sin 2g+\frac{1}{3}\frac{P_3}{P_1}\sin 3g+\ \cdot\ \cdot\ \cdot\ \right]\\
& + k_2nt\left[\cos g+\frac{1}{2}\frac{Q_2}{Q_1}\cos 2g+\frac{1}{3}\frac{Q_3}{Q_1}\cos 3g+\ \cdot\ \cdot\ \cdot\ \right]\\
\nu = {} & -\frac{1}{2}k_1nt\left[\frac{1}{2}\frac{e}{P_1}+\cos g+\frac{P_2}{P_1}\cos 2g+\frac{P_3}{P_1}\cos 3g+\ \cdot\ \cdot\ \cdot\ \right]\\
& +\frac{1}{2}k_2nt\left[\qquad\sin g+\frac{Q_2}{Q_1}\sin 2g+\frac{Q_3}{Q_1}\sin 3g+\ \cdot\ \cdot\ \cdot\ \right]\\
\frac{u}{\cos i} = {} & k_3nt\left[\qquad\sin g+\frac{Q_2}{Q_1}\sin 2g+\frac{Q_3}{Q_1}\sin 3g+\ \cdot\ \cdot\ \cdot\ \right]\\
& + k_4nt\left[-\frac{3}{2}\frac{e}{P_1}+\cos g+\frac{P_2}{P_1}\cos 2g+\frac{P_3}{P_1}\cos 3g+\ \cdot\ \cdot\ \cdot\ \right]
\end{aligned}$$

The P and Q have been defined (page 63).

---

*American Journal of Mathematics, Vol. IV, p. 258.

In the case of Jupiter these equations give

$$\begin{aligned}
n\delta z = {} & k_1nt \sin(-g) + k_2nt \cos(-g) \\
& + [8.0810774]\, k_1nt \sin(-2g) + [8.0811617]\, k_2nt \cos(-2g) \\
& + [6.4632057]\, k_1nt \sin(-3g) + [6.4633322]\, k_2nt \cos(-3g) \\
& + [4.9702810]\, k_1nt \sin(-4g) + [4.9704329]\, k_2nt \cos(-4g) \\
& + [3.5462420]\, k_1nt \sin(-5g) + [3.5464107]\, k_2nt \cos(-5g)
\end{aligned}$$

$$\begin{aligned}
\nu = {} & [8.0817514]\, k_1nt \\
& + \frac{1}{2}k_1nt \cos(-g) - \frac{1}{2}k_2nt \sin(-g) \\
& + [8.0810774]\, k_1nt \cos(-2g) - [8.0811617]\, k_2nt \sin(-2g) \\
& + [6.6392970]\, k_1nt \cos(-3g) - [6.6394235]\, k_2nt \sin(-3g) \\
& + [5.2713110]\, k_1nt \cos(-4g) - [5.2714629]\, k_2nt \sin(-4g) \\
& + [3.9441820]\, k_1nt \cos(-5g) - [3.9443507]\, k_2nt \sin(-5g)
\end{aligned}$$

$$\begin{aligned}
\frac{u}{\cos i} = {} & - [8.8599027]\, k_4nt \\
& + k_3nt \sin(-g) + k_4nt \cos(-g) \\
& + [8.3821917]\, k_3nt \sin(-2g) + [8.3821074]\, k_4nt \cos(-2g) \\
& + [6.9404535]\, k_3nt \sin(-3g) + [6.9403270]\, k_4nt \cos(-3g) \\
& + [5.5724929]\, k_3nt \sin(-4g) + [5.5723410]\, k_4nt \cos(-4g) \\
& + [4.2453807]\, k_3nt \sin(-5g) + [4.2452120]\, k_4nt \cos(-5g)
\end{aligned}$$

And in the case of Saturn

$$\begin{aligned}
n'\delta z' = {} & k'_1n't \sin g' + k'_2n't \cos g' \\
& + [8.1461706]\, k'_1n't \sin 2g' + [8.1462845]\, k'_2n't \cos 2g' \\
& + [6.5933997]\, k'_1n't \sin 3g' + [6.5935705]\, k'_2n't \cos 3g' \\
& + [5.1655779]\, k'_1n't \sin 4g' + [5.1657829]\, k'_2n't \cos 4g' \\
& + [3.8066453]\, k'_1n't \sin 5g' + [3.8068731]\, k'_2n't \cos 5g'
\end{aligned}$$

$$\begin{aligned}
\nu' = {} & - [8.1470809]\, k'_1n't \\
& - \frac{1}{2}k'_1n't \cos g' + \frac{1}{2}k'_2n't \sin g' \\
& - [8.1461706]\, k'_1n't \cos 2g' + [8.1462845]\, k'_2n't \sin 2g' \\
& - [6.7694910]\, k'_1n't \cos 3g' + [6.7696618]\, k'_2n't \sin 3g' \\
& - [5.4666079]\, k'_1n't \cos 4g' + [5.4668129]\, k'_2n't \sin 4g' \\
& - [4.2045853]\, k'_1n't \cos 5g' + [4.2048131]\, k'_2n't \sin 5g'
\end{aligned}$$

$$\begin{aligned}
\frac{u'}{\cos i} = {} & - [8.9252322]\, k'_4n't \\
& + k'_3n't \sin g' + k'_4n't \cos g' \\
& + [8.4473145]\, k'_3n't \sin 2g' + [8.4472006]\, k'_4n't \cos 2g' \\
& + [7.0706918]\, k'_3n't \sin 3g' + [7.0705210]\, k'_4n't \cos 3g' \\
& + [5.7678429]\, k'_3n't \sin 4g' + [5.7676379]\, k'_4n't \cos 4g' \\
& + [4.5058431]\, k'_3n't \sin 5g' + [4.5056153]\, k'_4n't \cos 5g'
\end{aligned}$$

The expressions for the perturbations of Jupiter and Saturn, arising from their mutual action, and of the first order with respect to disturbing forces, are as follows:

Arg= $i'g' + ig$	$n\delta z$		v		$\frac{u}{\cos i}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	″	″	″	″	″	″
0 0			— 2.3598			+0.0369
			— 0.0122808*nt*			—0.0090789*nt*
0 — 1	0.0000	0.0000	+ 0.5900	+ 0.4221	0.0000	0.0000
	— 1.0173636*nt*	— 1.1420391*nt*	— 0.5086818*nt*	+ 0.5710195*nt*	+0.2844315*nt*	+0.1253514*nt*
0 — 2	+ 0.0316	+ 0.0146	+ 0.0348	— 0.0061	—0.0079	—0.0147
	— 0.0122618*nt*	— 0.0137671*nt*	— 0.0122618*nt*	+ 0.0137671*nt*	+0.0068576*nt*	+0.0030216*nt*
0 — 3	+ 0.0005	+ 0.0004	+ 0.0009	— 0.0003	0.0000	—0.0004
	— 0.0002956*nt*	— 0.0003319*nt*	— 0.0004434*nt*	+ 0.0004979*nt*	+0.0002480*nt*	+0.0001118*nt*
0 — 4	— 0.0000095*nt*	— 0.0000107*nt*	— 0.0000190*nt*	+ 0.0000213*nt*	+0.0000106*nt*	+0.0000047*nt*
0 — 5	— 0.0000004*nt*	— 0.0000004*nt*	— 0.0000009*nt*	+ 0.0000010*nt*	+0.0000005*nt*	+0.0000002*nt*
1 + 3	— 0.0001	+ 0.0002	+ 0.0002	+ 0.0001		
1 + 2	— 0.0049	+ 0.0007	+ 0.0055	+ 0.0005	+0.0025	—0.0006
1 + 1	— 0.2652	— 0.0750	+ 0.1856	— 0.0426	+0.0983	+0.0297
1 0	— 8.4254	+ 4.3699	+ 1.7783	+ 0.4815	+0.4468	—0.3056
1 — 1	+ 15.3616	+78.9638	+ 4.9822	—25.6080	—0.1161	—0.0509
1 — 2	— 0.2503	+ 1.3828	— 0.1193	— 1.0401	—0.2590	—0.0586
1 — 3	— 0.0106	+ 0.0344	— 0.0099	— 0.0404	—0.0040	—0.0039
1 — 4	— 0.0004	+ 0.0009	— 0.0005	— 0.0016	—0.0001	+0.0001
2 + 2	— 0.0005	+ 0.0004	+ 0.0007	+ 0.0006		
2 + 1	— 0.0231	+ 0.0124	+ 0.0203	+ 0.0121	+0.0049	—0.0099
2 0	— 1.2664	+ 1.2749	+ 0.5833	+ 0.5047	—0.0186	—0.3728
2 — 1	+123.5450	+ 4.4541	+ 18.1877	+ 1.0218	+0.4637	+0.4391
2 — 2	+179.1521	—76.8179	+100.7061	+43.1791	—0.1080	+0.1908
2 — 3	+ 2.4851	— 1.4184	+ 2.6577	+ 1.4018	—0.0014	—0.0615
2 — 4	+ 0 0595	— 0.0391	+ 0.0961	+ 0.0563	+0.0007	- 0.0015
2 — 5	+ 0.0019	— 0.0013	+ 0.0041	+ 0.0025	+0.0002	—0.0001
3 + 1	+ 0.0005	— 0.0018	— 0.0006	— 0.0019	+0.0013	+0.0005
3 0	+ 0.0750	— 0.1369	— 0.0364	— 0.0795	+0.0335	+0.0412
3 — 1	+ 12.4697	— 7.9940	— 2.2066	— 1.6053	+0.1895	—0.0292
3 — 2	— 50.1203	+65.9600	— 20.8813	—28.2718	—0.5536	+0.8698
3 — 3	+ 8.7532	+13.7703	+ 6.0514	— 9.8126	—0.0420	—0.0038
3 — 4	+ 0.3496	+ 0.2513	+ 0.3509	— 0.3160	+0.0206	—0.0040
3 — 5	+ 0.0123	+ 0.0045	+ 0.0172	— 0.0103	+0.0008	0.0000
3 — 6	+ 0.0005	+ 0.0001	+ 0.0009	— 0.0004		
4 0	+ 0.0008	— 0.0063	+ 0.0001	— 0.0045	+0.0031	+0.0007
4 — 1	+ 0.0815	— 0.3201	— 0.0472	— 0.1504	+0.0410	—0.0274
4 — 2	— 2.3198	+16.4362	— 0.2484	— 3.9932	—0.0319	+0.1450
4 — 3	+ 13.7024	+ 6.5087	+ 8.3244	— 4.1262	+0.2293	+0.0900
4 — 4	— 2.2470	+ 2.7601	— 1.7055	— 2.2242	+0.0144	—0.0092
4 — 5	— 0.0332	+ 0.1572	— 0.0506	— 0.1667	+0.0037	+0.0075
4 — 6	+ 0.0012	+ 0.0065	— 0.0001	— 0.0092	+0.0002	+0.0003
4 — 7	+ 0.0001	+ 0.0002	+ 0.0001	— 0.0005		

Arg= $i'g' + ig$	$n\delta z$		v		$\frac{u}{\cos i}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	″	″	″	″	″	″
5 0	+0.0033	+0.0045	−0.0033	+0.0046	−0.0016	+0.0039
5 −1	+0.2759	+0.3880	−0.1394	+0.1924	−0.0701	+0.1627
5 −2	+422.3883	+992.7791	−5.6159	+8.7834	+0.2807	−0.0133
5 −3	−154.7622	+11.5283	−76.6873	−5.8016	−3.6469	+0.3547
5 −4	−2.4531	+2.7333	−2.3305	−2.0196	−0.0940	+0.0523
5 −5	−0.9945	−0.4977	−0.8815	+0.3980	+0.0022	+0.0047
5 −6	−0.0718	+0.0033	−0.0789	+0.0026	−0.0025	+0.0022
5 −7	−0.0033	+0.0017	−0.0047	−0.0015	−0.0002	+0.0002
5 −8	−0.0001	+0.0001	−0.0002	−0.0001		
6 −1	+0.0015	+0.0002	−0.0008	0.0000	0.0000	+0.0007
6 −2	+0.1034	+0.0464	−0.0368	+0.0099	+0.0073	+0.0013
6 −3	−1.7528	+1.1480	−0.5718	−0.4205	−0.0322	+0.0175
6 −4	+0.4254	+1.4849	+0.2423	−0.9807	+0.0141	+0.0441
6 −5	−0.8104	+0.0070	−0.6500	+0.0051	−0.0122	+0.0021
6 −6	+0.0972	−0.3579	+0.0769	+0.3193	−0.0015	+0.0028
6 −7	−0.0094	−0.0317	−0.0075	+0.0346	−0.0011	−0.0009
6 −8	−0.0015	−0.0015	−0.0015	+0.0022	0.0000	0.0000
6 −9	−0.0001	−0.0001	−0.0001	+0.0001		
7 −2	+0.0062	−0.0060	−0.0033	−0.0029	+0.0029	−0.0012
7 −3	−0.6842	+1.3256	−0.0618	−0.1705	−0.0050	+0.0020
7 −4	+1.0896	+1.0599	+0.5940	−0.5981	+0.0368	+0.0372
7 −5	−0.3348	+0.2042	−0.2545	−0.1448	−0.0099	+0.0074
7 −6	−0.0655	−0.2960	−0.0525	+0.2531	−0.0015	−0.0039
7 −7	+0.1389	+0.0047	+0.1269	+0.0001	−0.0016	−0.0004
7 −8	+0.0137	−0.0080	+0.0153	+0.0076	+0.0002	−0.0005
7 −9	+0.0006	−0.0010	+0.0010	+0.0011		
7 −10	0.0000	−0.0001	0.0000	+0.0001		
8 −2	+0.0002	+0.0006	−0.0001	+0.0002		
8 −3	+0.0117	+0.0930	−0.0041	+0.0149	+0.0010	+0.0010
8 −4	−0.4080	−0.1116	−0.1735	+0.0424	−0.0110	−0.0039
8 −5	−0.0981	+0.1506	−0.0737	−0.0996	−0.0043	+0.0063
8 −6	−0.0972	−0.1102	−0.0748	+0.0932	−0.0031	−0.0031
8 −7	+0.1141	−0.0515	+0.1020	+0.0441	+0.0012	−0.0009
8 −8	+0.0112	+0.0537	+0.0125	−0.0498	0.0000	+0.0007
8 −9	+0.0053	+0.0057	+0.0054	−0.0064	+0.0003	+0.0001
8 −10	+0.0007	+0.0002	+0.0008	−0.0004		
8 −11	+0.0001	0.0000	+0.0001	0.0000		
9 −3	+0.0016	+0.0008	−0.0008	+0.0003		
9 −4	−0.0859	+0.0205	−0.0200	−0.0067	−0.0004	−0.0003
9 −5	−0.0048	+0.1030	−0.0052	−0.0618	+0.0002	+0.0048
9 −6	−0.0553	−0.0185	−0.0414	+0.0166	−0.0025	−0.0008
9 −7	+0.0381	−0.0507	+0.0347	+0.0418	+0.0009	−0.0015
9 −8	+0.0327	+0.0438	+0.0293	−0.0404	+0.0004	+0.0003
9 −9	−0.0201	+0.0098	−0.0187	−0.0102	+0.0002	−0.0001

Arg= $i'g' + ig$	$n\delta z$		ν		$\frac{u}{\cos i}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	″	″	″	″	″	″
9—10	—0.0021	+0.0032	—0.0024	—0.0034		
9—11	0.0000	+0.0004	—0.0001	—0.0005		
10— 4	—1.0301	+1.0224	+0.0164	+0.0227	+0.0003	+0.0012
10— 5	—0.1884	—0.3832	—0.0931	+0.1880	—0.0061	—0.0151
10— 6	—0.0243	—0.0007	—0.0181	+0.0029	—0.0012	—0.0003
10— 7	+0.0031	—0.0249	+0.0039	+0.0202	+0.0002	—0.0011
10— 8	+0.0268	+0.0125	+0.0232	—0.0121	+0.0007	+0.0003
10— 9	—0.0161	+0.0188	—0.0152	—0.0174		
10—10	—0.0061	—0.0071	—0.0063	+0.0065		
10—11	—0.0018	—0.0007	—0.0019	+0.0008		
10—12	—0.0002	+0.0001	—0.0002	0.0000		
11— 5	—0.0088	—0.0063	—0.0032	+0.0018		
11— 6	—0.0077	+0.0062	—0.0051	—0.0038	—0.0004	+0.0003
11— 7	—0.0033	—0.0081	—0.0021	+0.0064	—0.0002	+0.0005
11— 8	+0.0119	—0.0009	+0.0103	0.0000	+0.0005	—0.0001
11— 9	—0.0032	+0.0140	—0.0035	—0.0126	—0.0001	+0.0004
11—10	—0.0102	—0.0055	—0.0096	+0.0052		
11—11	+0.0022	—0.0033	+0.0020	+0.0034		
11—12	+0.0001	—0.0009	+0.0002	+0.0010		
12— 5	—0.0068	—0.0017				
12— 6	—0.0038	+0.0083	—0.0021	—0.0045		
12— 7	—0.0024	—0.0017	—0.0016	+0.0014		
12— 8	+0.0035	—0.0024	+0.0030	+0.0017		
12— 9	+0.0017	+0.0059	+0.0012	—0.0054		
12—10	—0.0072	—0.0001	—0.0068	+0.0003		
12—11	+0.0013	—0.0053	+0.0013	+0.0052		
12—12	+0.0018	+0.0004	+0.0018	—0.0003		

Arg=$i'g+ig$		$n'\delta z'$		ν'		$\dfrac{u'}{\cos i'}$	
		sin.	cos.	cos.	sin.	sin.	cos.
i'	i	″	″	″	″	″	″
0	0			+ 86.5056			−0.3240
				$+ 0.075066n't$			$-0.130677n't$
1	0	0.0000	0.0000	+ 3.3206	+ 7.4569	0.0000	0.0000
		$- 5.350080n't$	$- 8.631067n't$	$+ 2.675040n't$	$- 4.315533n't$	$+1.106428n't$	$+1.552265n't$
2	0	− 0.3069	+ 0.5487	+ 0.1442	+ 0.8298	+0.0670	−0.2034
		$- 0.074908n't$	$- 0.120878n't$	$+ 0.074908n't$	$- 0.120878n't$	$+0.030991n't$	$+0.043468n't$
3	0	− 0.0090	+ 0.0317	− 0.0095	+ 0.0557	−0.0010	−0.0176
		$- 0.002098n't$	$- 0.003386n't$	$+ 0.003147n't$	$- 0.005078n't$	$+0.001302n't$	$+0.001826n't$
4	0	0.0000	+ 0.0019	− 0.0021	+ 0.0033	−0.0008	−0.0009
		$- 0.000078n't$	$- 0.000126n't$	$+ 0.000157n't$	$- 0.000253n't$	$+0.000065n't$	$+0.000091n't$
5	0	0.0000	+ 0.0001	− 0.0002	+ 0.0001		
		$- 0.000003n't$	$- 0.000006n't$	$+ 0.000009n't$	$- 0.000014n't$	$+0.000004n't$	$+0.000005n't$
−3	−1	− 0.0003	+ 0.0001	− 0.0008	0.0000	+0.0005	+0.0001
−2	−1	− 0.0051	+ 0.0025	− 0.0107	− 0.0036	+0.0046	−0.0005
−1	−1	− 0.0804	+ 0.0376	− 0.1329	− 0.0920	+0.0433	−0.0286
0	−1	+ 1.0159	+ 11.6710	− 1.0828	− 3.3110	−0.7669	+1.6253
1	−1	− 6.5162	− 0.9205	+ 32.4638	−177.0306	−0.6452	−0.4370
2	−1	− 424.0893	− 14.2645	−116.3299	− 6.7355	−2.0505	−2.1264
3	−1	− 28.1575	+ 20.1428	+ 9.6186	+ 6.7750	−0.7127	−0.0198
4	−1	− 0.2948	+ 0.0263	+ 0.2846	+ 0.1311	+0.0507	−0.0380
5	−1	− 0.0105	+ 0.0016	+ 0.0126	+ 0.0111	+0.0029	−0.0038
6	−1	− 0.0005	+ 0.0001	+ 0.0004	+ 0.0008	+0.0005	−0.0007
−2	−2	− 0.0001	− 0.0001	− 0.0002	+ 0.0002	−0.0001	+0.0002
−1	−2	− 0.0017	− 0.0018	− 0.0036	+ 0.0024	+0.0006	+0.0013
0	−2	+ 0.0197	+ 0.2104	− 0.0555	+ 0.0038	−0.0091	+0.0615
1	−2	− 0.8452	− 2.5470	− 0.2961	− 2.6516	+0.2524	+0.0542
2	−2	− 29.5211	+ 12.6271	− 28.1687	− 12.2964	+0.0847	−0.0696
3	−2	− 18.7373	+ 18.5430	− 15.8648	− 12.9271	−0.2295	+0.1436
4	−2	+ 80.9207	− 651.2349	+ 38.6088	+316.7274	+1.0617	−8.9202
5	−2	−1029.1981	−2417.7411	+ 24.0769	− 45.6323	−0.1630	+0.6190
6	−2	+ 1.1398	− 0.4218	− 0.5506	− 0.2186	+0.2368	+0.0758
7	−2	+ 0.0163	− 0.0083	− 0.0145	− 0.0070	+0.0077	+0.0019
8	−2	+ 0.0004	− 0.0003	− 0.0006	− 0.0002		
−1	−3	0.0000	− 0.0001	0.0000	+ 0.0002	0.0000	−0.0001
0	−3	+ 0.0016	+ 0.0074	− 0.0002	+ 0.0007	−0.0010	+0.0021
1	−3	− 0.0183	− 0.0972	+ 0.0137	− 0.0817	+0.0008	+0.0047
2	−3	− 0.1611	+ 0.1544	− 0.2654	− 0.0503	−0.0004	+0.0859
3	−3	− 3.8523	− 5.3430	− 4.1475	+ 5.6513	+0.0259	+0.0311
4	−3	− 4.2617	− 1.8168	− 4.4031	+ 2.1588	−0.0652	−0.0240
5	−3	− 2.9777	+ 0.3718	− 2.7928	− 0.1713	−0.0822	+0.0108
6	−3	− 3.4500	+ 2.3914	− 2.3514	− 1.5225	−0.0900	+0.0589
7	−3	+ 2.3548	− 4.6458	+ 0.5294	+ 1.1726	+0.0238	−0.0340
8	−3	− 0.0402	− 0.2147	+ 0.0198	− 0.0711	−0.0023	−0.0030
9	−3	+ 0.0005	− 0.0015	+ 0.0003	− 0.0014		

Arg=$i'g+ig$	$n'\delta z'$		v'		$\frac{u'}{\cos i'}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	″	″	″	″	″	″
0— 4	+0.0001	+0.0003	0.0000	0.0000		
1— 4	—0.0007	—0.0040	+0.0009	—0.0038	—0.0001	—0.0001
2— 4	0.0000	+0.0041	—0.0016	—0.0021	—0.0012	+0.0016
3— 4	—0.1290	—0.0537	—0.1083	+0.1003	—0.0315	+0.0067
4— 4	+1.2875	—1.4328	+1.4019	+1.6007	—0.0120	+0.0134
5— 4	+0.2600	—1.2933	+0.3795	+1.4727	+0.0021	—0.0164
6— 4	—0.2072	—0.5978	—0.1734	+0.6928	—0.0061	—0.0176
7— 4	—0.2311	—0.2026	—0.2166	+0.2280	—0.0088	—0.0083
8— 4	—0.1926	—0.0563	—0.1500	+0.0579	—0.0069	—0.0025
9— 4	+1.8469	—0.4843	+0.8710	+0.2335	+0.0403	—0.0070
10— 4	+2.4721	—2.4574	—0.0886	—0.1051	—0.0027	0.0000
11— 4	+0.0084	—0.0092	—0.0042	—0.0047		
1— 5	+0.0001	—0.0003	0.0000	—0.0002		
2— 5	+0.0001	+0.0001	0.0000	—0.0001		
3— 5	—0.0038	+0.0006	—0.0035	+0.0011	—0.0011	+0.0001
4— 5	+0.0102	—0.0767	+0.0291	+0.0762	—0.0055	—0.0118
5— 5	+0.5781	+0.3249	+0.6537	—0.3523	—0.0071	—0.0046
6— 5	+0.4847	—0.0033	+0.5747	—0.0240	+0.0045	—0.0005
7— 5	+0.1910	—0.1139	+0.2430	+0.1159	+0.0051	—0.0032
8— 5	+0.0425	—0.0767	+0.0605	+0.0845	+0.0019	—0.0031
9— 5	—0.0004	—0.0355	+0.0040	+0.0388	+0.0001	- 0.0017
10— 5	—0.0093	—0.0161	—0.0072	+0.0159	—0.0003	—0.0009
11— 5	—0.0215	—0.0142	—0.0138	+0.0100	—0.0007	—0.0007
12— 5	+0.0271	+0.0062	+0.0060	—0.0010		
3— 6	+0.0002	+0.0001	—0.0003	0.0000		
4— 6	—0.0013	—0.0028	—0.0002	+0.0029	—0.0004	—0.0006
5— 6	+0.0409	—0.0041	+0.0435	—0.0030	+0.0043	—0.0033
6— 6	—0.0704	+0.2397	—0.0732	—0.2711	+0.0015	—0.0038
7— 6	+0.0434	+0.1959	+0.0440	—0.2362	+0.0006	+0.0012
8— 6	+0.0637	+0.0699	+0.0711	—0.0935	+0.0017	+0.0016
9— 6	+0.0342	+0.0107	+0.0409	—0.0184	+0.0014	+0.0004
10— 6	+0.0121	—0.0028	+0.0152	+0.0013	+0.0006	—0.0001
11— 6	+0.0032	—0.0030	+0.0042	+0.0029	+0.0002	—0.0002
12— 6	+0.0007	—0.0017	+0.0009	+0.0017		
4— 7	—0.0001	—0.0001	0.0000	+0.0001		
5— 7	+0.0017	—0.0013	+0.0020	+0.0009	+0.0003	—0.0003
6— 7	+0.0069	+0.0204	+0.0045	—0.0225	+0.0019	+0.0015
7— 7	—0.0993	—0.0054	—0.1118	+0.0030	+0.0019	+0.0003
8— 7	—0.0806	+0.0368	—0.0975	—0.0402	—0.0001	+0.0003
9— 7	—0.0260	+0.0359	—0.0360	—0.0419	—0.0005	+0.0008
10— 7	—0.0016	+0.0169	—0.0049	—0.0211	—0.0002	+0.0006
11— 7	+0.0023	+0.0052	+0.0020	—0.0072	+0.0001	+0.0005
12— 7	+0.0013	+0.0011	+0.0017	—0.0017	+0.0002	+0.0001

Arg=$i'g+ig$	$n'\delta z'$		ν'		$\frac{u'}{\cos i'}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	″	″	″	″	″	″
6— 8	+0.0011	+0.0009	+0.0009	−0.0011	+0.0002	+0.0001
7— 8	−0.0095	+0.0057	−0.0107	−0.0051	−0.0003	+0.0009
8— 8	−0.0076	−0.0403	−0.0101	+0.0448	+0.0001	+0.0009
9— 8	−0.0242	−0.0325	−0.0276	+0.0395		
10— 8	−0.0199	−0.0088	−0.0237	+0.0130		
11— 8	−0.0086	+0.0010	−0.0113	+0.0002		
12— 8	−0.0024	+0.0017	−0.0034	−0.0019		
7— 9	−0.0004	+0.0007	−0.0006	−0.0007		
8— 9	−0.0040	−0.0042	−0.0039	+0.0048	−0.0004	−0.0001
9— 9	+0.0157	−0.0072	+0.0173	+0.0088	−0.0007	+0.0001
10— 9	+0.0125	−0.0143	+0.0151	+0.0167		
11— 9	+0.0024	−0.0107	+0.0037	+0.0130		
12— 9	−0.0014	−0.0044	−0.0012	+0.0059		
8—10	−0.0005	−0.0002	−0.0005	+0.0002		
9—10	+0.0016	−0.0026	+0.0018	+0.0027		
10—10	+0.0048	+0.0057	+0.0056	−0.0062		
11—10	+0.0080	+0.0043	+0.0094	−0.0051		
12—10	+0.0057	0.0000	+0.0071	−0.0004		
9—11	0.0000	−0.0003	+0.0001	+0.0003		
10—11	+0.0014	+0.0005	+0.0016	−0.0006		
11—11	−0.0019	+0.0027	−0.0019	−0.0031		
12—11	−0.0012	+0.0043	−0.0013	−0.0051		
10—12	+0.0002	−0.0001	+0.0002	0.0000		
11—12	−0.0001	+0.0008	−0.0001	−0.0009		
12—12	−0.0015	−0.0004	−0.0017	+0.0003		

CHAPTER III.

PERTURBATIONS OF SATURN BY URANUS OF THE FIRST ORDER WITH RESPECT TO THE DISTURBING FORCE.

The method to be followed in this chapter is almost precisely identical with that of the preceding chapters. This will relieve us from the necessity of restating formulæ. The single point of difference is that here it will suffice to divide the circumference, with reference to the mean anomaly of Uranus, into twelve parts, instead of the sixteen which were employed for Saturn in Chapter I. As here the elements of Saturn take the place of those of Jupiter, and the elements of Uranus the place of those of Saturn, for convenience we will denote the former without accents and the latter with a single accent, reserving to ourselves the liberty of denoting, at the end, the mean anomaly of Uranus as g''.

The elements of Uranus adopted are as follows:*

Epoch, 1850, Jan. 0.0, Greenwich M. T.

$$\begin{aligned}
L' &= 28^\circ\ 25'\ 17.''05 \\
\pi' &= 168^\circ\ 15'\ 6.''7 \\
\theta' &= 73^\circ\ 14'\ 8.''0 \\
i' &= 0^\circ\ 46'\ 20.''54 \\
e' &= 0.0469236 \\
n' &= 15425.''752 \\
m' &= \frac{1}{21000} \\
\log \mathrm{a}' &= 1.2831044
\end{aligned}$$

These elements include the effect of the 4000 year inequality produced by Neptune. It seems better to do this than to take mean elements, for the reason that, in the latter way, it would be necessary to consider terms proportional to the product of the masses of Uranus and Neptune and involving the anomalies of all three planets. Log a' includes the constant term of the perturbations of the logarithm of the radius vector, which is $+0.0001972$. For a like reason, adding to log a of Saturn $+0.0001854$, we have $\log \mathrm{a} = 0.9796819$, which gives $\log \alpha = 9.6965775$.

The coefficients of the terms of the developments of the reciprocal of the distance between Saturn and Uranus $\frac{\mathrm{a}'}{\Delta}$ and its odd powers, as periodic functions of the two mean anomalies, are then functions of the following six elements:

$$\begin{array}{rlcrrr}
 & & & {}^\circ & {}' & {}'' \\
\log \alpha &= 9.6965775 & \qquad J = & 1 & 57 & 24.44 \\
e &= 0.05605688 & \qquad \Pi = & 143 & 20 & 42.93 \\
e' &= 0.0469236 & \qquad \Pi' = & 221 & 29 & 41.56
\end{array}$$

---

*An Investigation of the Orbit of Uranus, with General Tables of its Motion. By Prof. S. Newcomb, p. 181.

Π and Π' are measured from the ascending node of the orbit of Uranus on that of Saturn. In developing these quantities it is preferable to proceed as if Uranus were the disturbed planet, as in this way γ_2 is smaller.

The values of the auxilliary constants, entirely similar to those of Chapter I, are

$\log k = 9.9999098$ $\quad K = 78^\circ\ 8'\ 0''.98$

$\log k_1 = 9.9998370$ $\quad K_1 = 78^\circ\ 9'\ 56''.24$

$\log p = 0.0193100$ $\quad P = 21^\circ\ 41'\ 9''.00$

$\log v = 9.9969581$ $\quad V = 78^\circ\ 11'\ 3''.42$

$\log w = 9.9966348$ $\quad W = 56^\circ\ 28'\ 12''.76$

$\log w_1 = 9.9971648$ $\quad W_1 = 56^\circ\ 27'\ 26''.24$

$\log \gamma_2 = 6.8904128$ $\quad \log (e' \text{ in}'') = 3.9858164$

$$\gamma_0 = 1.2457067 - [8.9724213] \cos \varepsilon' + [7.3427826] \cos^2 \varepsilon' + [8.7486289] f \cos F$$

We compute the values of ε' corresponding to the five following values of g':

g'	ε'		
30°	31°	24′	2.″80
60	62	22	55. 89
90	92	41	8. 06
120	122	16	23. 44
150	151	17	29. 20

The values for the seven remaining points of division of the circumference are either known or readily deducible from these. By substituting them in the equations which give the values of γ_0, f, and F we get the following table:

g'	γ_0	$\log f$	$F-g'$		
°			°	′	″
0	1. 1665387	9. 9787586	76	28	56. 89
30	1. 1496185	9. 9746368	79	25	47. 29
60	1. 1607263	9. 9776345	81	59	12. 98
90	1. 1961616	9. 9862614	83	21	40. 55
120	1. 2452725	9. 9972220	83	15	36. 05
150	1. 2945026	0. 0073536	81	51	46. 09
180	1. 3313106	0. 0145142	79	37	5. 58
210	1. 3468479	0. 0175338	77	4	25. 21
240	1. 3374098	0. 0159203	74	47	12. 93
270	1. 3050141	0. 0097458	73	16	49. 81
300	1. 2572822	9. 9998678	72	58	59. 03
330	1. 2063954	9. 9883863	74	6	34. 67
S	7. 4985401	9. 9839171	469	7	3. 50
S′	7. 4985401	9. 9839171	469	7	3. 54

In the next step we obtain

g'	C	log q	$Q-g'$		
			°	′	″
(0)	1.1672734	9.9791451	76	30	30.41
(1)	1.1503098	9.9749759	79	23	37.58
(2)	1.1610217	9.9775301	81	55	52.67
(3)	1.1961720	9.9858422	83	20	53.54
(4)	1.2453941	9.9969293	83	18	3.05
(5)	1.2949840	0.0074544	81	55	0.82
(6)	1.3320624	0.0149069	79	38	16.18
(7)	1.3475581	0.0178806	77	2	33.62
(8)	1.3378019	0.0159238	74	43	53.70
(9)	1.3050786	0.0093943	73	14	59.36
(10)	1.2573216	9.9994858	73	0	27.55
(11)	1.2067726	9.9883736	74	9	58.57
S	7.5008751	9.9839210	469	7	3.56
S′	7.5008751	9.9839210	469	7	3.49

And in fine

g'	log N	log a	log b
(0)	0.0179713	9.7140577	6.61024
(1)	0.0219611	9.7178680	6.61441
(2)	0.0194011	9.7153021	6.61185
(3)	0.0112542	9.7073206	6.60354
(4)	0.0003161	9.6965315	6.59245
(5)	9.9897914	9.6860072	6.58193
(6)	9.9822142	9.6783051	6.57448
(7)	9.9791056	9.6750617	6.57150
(8)	9.9810313	9.6769564	6.57346
(9)	9.9876671	9.6836984	6.57999
(10)	9.9977306	9.6939169	6.58990
(11)	0.0088850	9.7051136	6.60101
S	9.9986646	8.1750697	89.55238
S′	9.9986644	8.1750695	89.55239

The values of the $b_s^{(i)}$ are taken from Runkle's Tables.* We get

	$\log b_{\frac{1}{2}}^{(0)}$	$\log b_{\frac{1}{2}}^{(1)}$	$\log b_{\frac{1}{2}}^{(2)}$	$\log b_{\frac{1}{2}}^{(3)}$	$\log b_{\frac{1}{2}}^{(4)}$	$\log b_{\frac{1}{2}}^{(5)}$	$\log b_{\frac{1}{2}}^{(6)}$
(0)	0.3342404	9.7638610	9.3586856	8.9964864	8.654338	8.323844	8.00097
(1)	0.3349185	9.7686880	9.3674426	9.0091166	8.670818	8.344162	8.02511
(2)	0.3344602	9.7654350	9.3615427	9.0006082	8.659717	8.330477	8.00885
(3)	0.3330791	9.7553834	9.3432656	8.9742217	8.625270	8.287993	7.95834
(4)	0.3313146	9.7419491	9.3187311	8.9387355	8.578894	8.230757	7.89028
(5)	0.3296984	9.7290018	9.2949759	8.9043078	8.533850	8.175127	7.82409
(6)	0.3285771	9.7196184	9.2776942	8.8792213	8.500998	8.134529	7.77576
(7)	0.3281197	9.7156892	9.2704416	8.8686836	8.487192	8.117462	7.75541
(8)	0.3283857	9.7179830	9.2746766	8.8748376	8.495256	8.127431	7.76730
(9)	0.3293570	9.7261811	9.2897866	8.8967784	8.523993	8.162948	7.80959
(10)	0.3309036	9.7387185	9.3128136	8.9301657	8.567687	8.216919	7.87380
(11)	0.3327087	9.7526214	9.3382313	8.9669462	8.615766	8.276267	7.94440
S	1.9878816	8.4475650	5.9041438	3.6200547	1.456890	89.363957	87.31696
S′	1.9878814	8.4475649	5.9041436	3.6200543	1.456889	89.363959	87.31694

	$\log b_{\frac{1}{2}}^{(7)}$	$\log b_{\frac{1}{2}}^{(8)}$	$\log b_{\frac{1}{2}}^{(9)}$	$\log b_{\frac{1}{2}}^{(10)}$	$\log b_{\frac{3}{2}}^{(0)}$	$\log b_{\frac{3}{2}}^{(1)}$
(0)	7.68353	7.37005	7.05989	6.7527	0.6006482	0.4467923
(1)	7.71150	7.40184	7.09551	6.7922	0.6067952	0.4559624
(2)	7.69265	7.38043	7.07152	6.7656	0.6026394	0.4497729
(3)	7.63414	7.31389	6.99698	6.6830	0.5901237	0.4308901
(4)	7.55523	7.22416	6.89642	6.5717	0.5741411	0.4062060
(5)	7.47848	7.13684	6.79854	6.4633	0.5595140	0.3829910
(6)	7.42242	7.07306	6.72703	6.3841	0.5493706	0.3665031
(7)	7.39880	7.04620	6.69693	6.3507	0.5452339	0.3596800
(8)	7.41259	7.06189	6.71452	6.3703	0.5476408	0.3636573
(9)	7.46166	7.11771	6.77709	6.4396	0.5564251	0.3780055
(10)	7.53614	7.20245	6.87208	6.5448	0.5704212	0.4003617
(11)	7.61798	7.29552	6.97638	6.6602	0.5867688	0.4257641
S	85.30256	83.31204	81.34146	79.3892	3.4448613	1.6332933
S′	85.30256	83.31200	81.34143	79.3892	3.4448607	1.6332931

---

*New Tables of the Coefficients of the Perturbative Function.

	log $b_{\frac{3}{2}}^{(2)}$	log $b_{\frac{3}{2}}^{(3)}$	log $b_{\frac{3}{2}}^{(4)}$	log $b_{\frac{3}{2}}^{(5)}$	log $b_{\frac{3}{2}}^{(6)}$	log $b_{\frac{3}{2}}^{(7)}$	log $b_{\frac{3}{2}}^{(8)}$
(0)	0.2415444	0.0140476	9.7740086	9.5258905	9.272126	9.01418	8.75302
(1)	0.254[illegible]033	0.0303401	9.7939980	9.5496131	9.299601	9.04543	8.78804
(2)	0.2456651	0.0193555	9.7805243	9.5336254	9.281087	9.02437	8.76445
(3)	0.2194566	9.9855253	9.7389410	9.4842189	9.223814	8.95921	8.69138
(4)	0.1848239	9.9405628	9.6834802	9.4181700	9.147123	8.87184	8.59331
(5)	0.1518569	9.8974915	9.6301498	9.3544982	9.073061	8.78736	8.49839
(6)	0.1282042	9.8664278	9.5915693	9.3083396	9.019292	8.72597	8.42934
(7)	0.1183573	9.8534560	9.5754269	9.2890071	8.996753	8.70021	8.40036
(8)	0.1241015	9.8610258	9.5848450	9.3002930	9.009899	8.71522	8.41726
(9)	0.1447261	9.8881404	9.6185459	9.3406239	9.056907	8.76892	8.47765
(10)	0.1765611	9.9297923	9.6701626	9.4022843	9.128657	8.85079	8.56967
(11)	0.2122997	9.9762577	9.7275277	9.4706408	9.208060	8.94128	8.67127
S	1.1009002	9.6312118	8.0845910	6.4886028	4.858184	3.20237	1.52705
S′	1.1008999	9.6312110	8.0845901	6.4886020	4.858196	3.20241	1.52709

	log $b_{\frac{3}{2}}^{(9)}$	log $b_{\frac{3}{2}}^{(10)}$	log $b_{\frac{5}{2}}^{(0)}$	log $b_{\frac{5}{2}}^{(1)}$	log $b_{\frac{5}{2}}^{(2)}$	log $b_{\frac{5}{2}}^{(3)}$
(0)	8.48933	8.2234	1.05063	0.99609	0.88273	0.73389
(1)	8.52813	8.2660	1.06489	1.01189	0.90113	0.75536
(2)	8.50199	8.2373	1.05526	1.00122	0.88872	0.74086
(3)	8.42099	8.1484	1.02616	0.96885	0.85085	0.69654
(4)	8.31221	8.0285	0.98880	0.92691	0.80132	0.63817
(5)	8.20683	7.9126	0.95443	0.88790	0.75479	0.58291
(6)	8.13012	7.8283	0.93049	0.86045	0.72176	0.54344
(7)	8.09792	7.7929	0.92069	0.84914	0.70804	0.52700
(8)	8.11671	7.8137	0.92640	0.85573	0.71606	0.53660
(9)	8.18380	7.8875	0.94717	0.87959	0.74483	0.57102
(10)	8.28596	7.9997	0.98008	0.91705	0.78962	0.62430
(11)	8.39867	8.1235	1.01833	0.96010	0.84055	0.68444
S	89.83632	88.1309	5.93166	5.55745	4.80021	3.81726
S′	89.83634	88.1309	5.93167	5.55747	4.80019	3.81727

	$\log b_{\frac{1}{2}}^{(4)}$	$\log b_{\frac{1}{2}}^{(5)}$	$\log b_{\frac{1}{2}}^{(6)}$	$\log b_{\frac{1}{2}}^{(7)}$	$\log b_{\frac{1}{2}}^{(8)}$	$\log b_{\frac{1}{2}}^{(9)}$
(0)	0.56148	0.37240	0.17082	9.95968	9.7411	9.5165
(1)	0.58627	0.40065	0.20263	9.99512	9.7801	9.5591
(2)	0.56956	0.38170	0.18119	9.97108	9.7538	9.5304
(3)	0.51826	0.32305	0.11522	9.89771	9.6725	9.4411
(4)	0.45037	0.24528	0.02727	9.79946	9.5639	9.3220
(5)	0.38580	0.17103	9.94318	9.70534	9.4595	9.2072
(6)	0.33948	0.11762	9.88259	9.63746	9.3843	9.1245
(7)	0.32005	0.09520	9.85684	9.60864	9.3523	9.0894
(8)	0.33144	0.10832	9.87204	9.62560	9.3711	9.1100
(9)	0.37190	0.15500	9.92513	9.68507	9.4369	9.1821
(10)	0.43423	0.22672	0.00637	9.77600	9.5376	9.2927
(11)	0.50420	0.30697	0.09701	9.87738	9.6501	9.4168
S	2.68656	1.45204	0.14028	8.76928	7.3518	5.8961
S′	2.68648	1.45190	0.14001	8.76926	7.3514	5.8957

The quantities $\delta \log k_i$ and K_i were computed as in Chapter I, but we pass over them to the coefficients A.* In the development of $\frac{a'}{\triangle}$ the latter are:

	$A_0^{(c)}$	$A_1^{(c)}$	$A_1^{(s)}$	$A_2^{(c)}$	$A_2^{(s)}$	$A_3^{(c)}$	$A_3^{(s)}$
	7	8	8	8	8	8	8
(0)	1874943	+1177066	− 4899385	−1766623	− 899917	− 559069	+ 654946
(1)	1895225	944398	5055152	1903671	738274	471567	760405
(2)	1882312	709523	5028771	1924952	556101	357552	796140
(3)	1841607	563723	4840437	1835652	433858	275136	757973
(4)	1788462	539609	4574377	1690549	403260	249140	680311
(5)	1738996	616009	4316660	1541518	447582	268384	595211
(6)	1704404	754842	4122426	1417124	536430	312651	518401
(7)	1690485	923128	4019319	1331006	646312	368098	456851
(8)	1699164	1094561	4020736	1293582	762323	428572	417018
(9)	1729327	1241964	4132273	1317235	871408	491164	408667
(10)	1776200	1329741	4346774	1413165	953038	548722	444557
(11)	1829843	+1316124	− 4628633	−1576929	− 973637	− 581363	+ 532293
S	1.0725485	+5605342	−26992469	−9505995	−4111069	−2455706	+3511373
S′	1.0725483	+5605346	−26992474	−9506011	−4111071	−2455712	+3511400

---

* These coefficients have all been divided by 12 in order to save the division by 6 and afterwards by 2, which otherwise would have to be performed in the following process of mechanical quadratures.

	$A_4^{(c)}$	$A_4^{(s)}$	$A_5^{(c)}$	$A_5^{(s)}$	$A_6^{(c)}$	$A_6^{(s)}$	$A_7^{(c)}$
	s	s	s	s	s	s	s
(0)	+ 230353	+ 316767	+169020	— 70126	— 13674	— 85928	— 41765
(1)	303146	276930	154603	116421	41240	83172	43395
(2)	336667	212463	120708	142210	59060	66506	35797
(3)	322830	161679	91066	138875	59632	49871	26755
(4)	282354	142699	78353	118332	49485	41832	21876
(5)	235041	148889	79004	92715	35955	40665	20448
(6)	189918	167766	85665	67368	22265	42194	20173
(7)	150672	191591	94119	44339	— 9604	44169	19884
(8)	120769	218345	103940	25296	+ 1296	46664	19740
(9)	105863	249265	117227	12860	9517	51397	20858
(10)	114928	284148	136144	11982	12856	60675	24940
(11)	+ 157470	+ 313849	+157834	— 30142	+ 6551	— 74548	— 32981
S	+1274989	+1342188	+693830	—435314	—130332	—343800	—164291
S′	+1275022	+1342203	+693853	—435352	—130363	—343822	—164321

	$A_7^{(s)}$	$A_8^{(c)}$	$A_8^{(s)}$	$A_9^{(c)}$	$A_9^{(s)}$	$A_{10}^{(c)}$	$A_{10}^{(s)}$
	s	s	s	s	s	s	s
(0)	— 3250	— 6270	+19364	+ 8503	+5197	+3468	— 3479
(1)	+12253	+ 1988	22013	10868	+1036	+1501	5218
(2)	23726	8998	18893	9805	—3072	— 824	5012
(3)	25338	10556	14116	7348	4266	1643	3782
(4)	20466	8293	11252	5709	3256	1216	2863
(5)	13482	4769	10082	4888	—1516	— 376	2335
(6)	+ 6344	+ 1169	9388	4258	+ 243	+ 456	1881
(7)	— 240	— 2083	8580	3532	1769	1134	1372
(8)	5982	4892	7790	2798	3043	1662	860
(9)	10738	7380	7645	2370	4231	2176	483
(10)	13799	9500	9191	2811	5501	2863	508
(11)	—12564	—10036	+13469	+ 4901	+6393	+3615	— 1435
S	+27505	— 2202	+75878	+33884	+7656	+6409	—14603
S′	+27531	— 2186	+75905	+33907	+7647	+6407	—14625

In the development of $\left(\frac{a'}{\Delta}\right)^3$ the coefficients are:

	$A_0^{(c)}$	$A_1^{(c)}$	$A_1^{(s)}$	$A_2^{(c)}$	$A_2^{(s)}$	$A_3^{(c)}$	$A_3^{(s)}$
	7	7	7	7	7	7	7
(0)	3758796	+ 615689	— 2563681	—1464338	— 746092	— 632006	+ 740184
(1)	3919241	508729	2721311	1621970	628809	547365	883001
(2)	3817256	375589	2658486	1612294	465458	408028	909249
(3)	3507985	281454	2415836	1454794	343773	297590	820013
(4)	3134340	249239	2115153	1244371	297026	250852	684493
(5)	2815968	264622	1856488	1057581	307310	252204	558847
(6)	2609347	308240	1683942	924813	350156	279553	463397
(7)	2529922	369410	1607757	851309	413247	322526	400448
(8)	2579589	443847	1629379	838178	493695	380301	370301
(9)	2757523	526661	1751727	891710	589751	455135	378832
(10)	3053091	603826	1974374	1022472	689690	543002	439795
(11)	3421787	+ 645563	— 2271919	—1229155	— 759248	— 618961	+ 566361
S	1.8952419	+2596430	—1.2625015	—7106466	—3042117	—2493742	+3607419
S′	1.8952426	+2596439	—1.2625038	—7106519	—3042138	—2493781	+3607502

	$A_4^{(c)}$	$A_4^{(s)}$	$A_5^{(c)}$	$A_5^{(s)}$	$A_6^{(c)}$	$A_6^{(s)}$	$A_7^{(c)}$
	7	7	7	7	7	7	7
(0)	+ 329286	+ 452971	+ 292157	—121152	— 27677	—174135	— 97057
(1)	445195	406510	274288	206655	85834	172991	103502
(2)	486402	306719	210659	248380	120958	136095	84010
(3)	442114	221371	150799	230015	115846	96865	59613
(4)	359918	182025	120937	182524	89559	75757	45462
(5)	279792	177387	113954	133616	60814	68842	39732
(6)	215365	190299	117692	92525	35900	68059	37359
(7)	167613	213033	126788	59767	— 15213	69877	36125
(8)	136120	245866	141794	34575	+ 2041	74756	36316
(9)	124495	292961	166853	18344	15881	85888	40018
(10)	144048	356320	206619	18145	22917	108059	50971
(11)	+ 212039	+ 423003	+ 257269	— 49022	+ 12585	—142516	— 72322
S	+1671139	+1734200	+1089858	—697301	—249136	—636861	—351175
S′	+1671248	+1734265	+1089951	—697419	—249241	—636979	—351312

	$A_7^{(s)}$	$A_8^{(c)}$	$A_8^{(s)}$	$A_9^{(c)}$	$A_9^{(s)}$	$A_{10}^{(c)}$	$A_{10}^{(s)}$
	7	7	7	7	7	7	7
(0)	− 75[illegible]2	−16446	+ 50757	+24804	+15167	+11130	−11156
(1)	+29256	+ 5367	59220	32538	+ 3092	+ 4936	17179
(2)	55736	23851	50019	28891	− 9068	− 2674	16240
(3)	56466	26545	35488	20563	11941	5062	11641
(4)	42501	19435	26386	14905	8493	3483	8217
(5)	26172	10448	22114	11940	− 3697	− 1006	6268
(6)	+11742	+ 2440	19629	9915	+ 568	+ 1168	4816
(7)	− 425	− 4269	17602	8072	4040	2848	3448
(8)	10987	10152	16185	6477	7036	4228	2189
(9)	20589	15980	16562	5721	10205	5772	1281
(10)	28215	21922	21200	7216	14133	8085	1433
(11)	−27589	−24851	+ 33322	+13493	+17616	+10947	− 4338
S	+63205	− 2794	+184176	+92208	+19343	+18454	−44051
S′	+63291	− 2740	+184308	+92327	+19315	+18435	−44155

In the development of $\left(\frac{a'}{\triangle}\right)^5$ the coefficients are:

	$A_0^{(c)}$	$A_1^{(c)}$	$A_1^{(s)}$	$A_2^{(c)}$	$A_2^{(s)}$	$A_3^{(c)}$	$A_3^{(s)}$	$A_4^{(c)}$	$A_4^{(s)}$
		5	5	5	5	5	5	5	5
(0)	1. 14989	+23672	− 98590	− 69581	− 35452	− 35995	+ 42155	+ 21912	+ 30146
(1)	1. 24434	20243	108237	79550	30841	32133	51840	30516	27861
(2)	1. 18376	14635	103505	77514	22377	23498	52370	32729	20634
(3)	1. 00912	10239	87870	65616	15505	16122	44427	28033	14036
(4)	81594	8276	70291	51559	12307	12529	34185	21083	10665
(5)	66690	8066	56640	40435	11749	11661	25836	15204	9642
(6)	57781	8848	48352	33401	12646	12236	20281	11097	9806
(7)	54516	10353	45044	30042	14583	13806	17142	8451	10740
(8)	56541	12635	46360	30016	17680	16510	16077	6961	12570
(9)	64100	15806	52558	33568	22200	20730	17256	6670	15695
(10)	77652	19645	64250	41538	28019	26611	21552	8285	20495
(11)	96292	+ 23006	− 81006	− 54405	− 33605	− 32935	+ 30132	+ 13206	+ 26352
S	5. 06933	+ 87711	−4. 31348	−3. 03609	−1. 28481	−1. 27379	+1. 86620	+1. 02067	+1. 04316
S′	5. 06944	+ 87713	−4. 31355	−3. 03616	−1. 28483	−1. 27387	+1. 86633	+1. 02080	+1. 04326

	$A_5^{(c)}$	$A_5^{(s)}$	$A_6^{(c)}$	$A_6^{(s)}$	$A_7^{(c)}$	$A_7^{(s)}$	$A_8^{(c)}$	$A_8^{(s)}$	$A_9^{(c)}$	$A_9^{(s)}$
	5	5	5	5	5	5	5	5	5	5
(0)	+22274	− 9235	− 2378	−14972	− 9294	− 726	−1738	+ 5361	+2867	+1753
(1)	21522	16219	7594	15302	10193	+2883	+ 583	6430	3864	+ 367
(2)	16235	19148	10513	11824	8126	5393	2545	5337	3373	−1059
(3)	10965	16726	9508	7950	5453	5165	2679	3581	2270	1318
(4)	8140	12281	6808	5762	3857	3604	1820	2471	1527	870
(5)	7126	8353	4301	4871	3139	2066	911	1929	1140	− 353
(6)	6985	5491	2412	4575	2806	+ 882	+ 202	1629	901	+ 52
(7)	7366	3473	− 1001	4596	2656	− 31	− 347	1430	718	360
(8)	8351	2038	+ 135	4987	2708	818	837	1333	584	635
(9)	10287	1132	1108	5994	3119	1604	1376	1426	539	961
(10)	13657	1198	1713	8074	4248	2352	2016	1950	726	1421
(11)	+18387	− 3500	+ 1018	−11497	− 6503	−2483	−2466	+ 3306	+1465	+1913
S	+75642	−49391	−20263	−50194	−31039	+5983	− 24	+18081	+9978	+1932
S′	+75653	−49403	−20278	−50210	−31063	+5996	− 16	+18102	+9996	+1930

The formulæ for mechanical quadratures, when the circumference is divided into twelve parts, are the following:*

Let

$$(0.6) = Y_0 + Y_6 \qquad \left(\frac{0}{6}\right) = Y_0 - Y_6$$

$$(1.7) = Y_1 + Y_7 \qquad \left(\frac{1}{7}\right) = Y_1 - Y_7$$

$$(2.8) = Y_2 + Y_8 \qquad \left(\frac{2}{8}\right) = Y_2 - Y_8$$

$$\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot \qquad \cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot$$

$$(5.11) = Y_5 + Y_{11} \qquad \left(\frac{5}{11}\right) = Y_5 - Y_{11}$$

Then will

$$6\,(c_0 + c_6) = (0.6) + (2.8) + (4.10)$$
$$6\,(c_0 - c_6) = (1.7) + (3.9) + (5.11)$$
$$3\,(c_2 + c_4) = (0.6) - \{(2.8) + (4.10)\} \sin 30^\circ$$
$$3\,(c_2 - c_4) = \{(1.7) + (5.11)\} \sin 30^\circ - (3.9)$$
$$3\,(s_2 + s_4) = \{(1.7) - (5.11)\} \cos 30^\circ$$
$$3\,(s_2 - s_4) = \{(2.8) - (4.10)\} \cos 30^\circ$$
$$3\,(c_1 + c_5) = \left(\tfrac{0}{6}\right) + \left\{\left(\tfrac{2}{8}\right) - \left(\tfrac{4}{10}\right)\right\} \sin 30^\circ$$
$$3\,(c_1 - c_5) = \left\{ \left(\tfrac{1}{7}\right) - \left(\tfrac{5}{11}\right) \right\} \cos 30^\circ$$
$$6c_3 = \left(\tfrac{0}{6}\right) - \left(\tfrac{2}{8}\right) + \left(\tfrac{4}{10}\right)$$
$$3\,(s_1 + s_5) = \left\{ \left(\tfrac{1}{7}\right) + \left(\tfrac{5}{11}\right) \right\} \sin 30^\circ + \left(\tfrac{3}{9}\right)$$
$$3\,(s_1 - s_5) = \left\{ \left(\tfrac{2}{8}\right) + \left(\tfrac{4}{10}\right) \right\} \cos 30^\circ$$
$$6s_3 = \left(\tfrac{1}{7}\right) - \left(\tfrac{3}{9}\right) + \left(\tfrac{5}{11}\right)$$

---

*Auseinandersetzung, Abh. I, ss. 159, 160.

The developments of the reciprocal of the distance between Saturn and Uranus, in terms of the eccentric anomaly of the former and the mean anomaly of the latter, having the form

$$\Sigma \, . \, C \, {}^{\cos}_{\sin}\left(i' g' + i \varepsilon\right)$$

follow:

Arg.	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
0 0	1.0725484		1.8952422		5.06938	
0 − 1	−0.00890679	−0.00299019	−0.1125150	−0.1915831	−0.66434	−1.35412
0 − 2	+0.00018228	+0.00003470	−0.0018907	+0.0076481	−0.05529	+0.09684
0 − 3	−0.00000690	−0.00000305	+0.0001282	−0.0001836	+0.00542	−0.00125
0 − 4	+0.00000019	+0.00000023	−0.0000048	+0.0000074	−0.00018	0.00000
0 − 5			−0.0000010	+0.0000007	0.00000	+0.00003
1 + 4	−0.00000001	+0.00000002	−0.0000005	+0.0000004	−0.00003	+0.00006
1 + 3	−0.00000055	−0.00000029	−0.0000085	+0.0000139	+0.00022	+0.00037
1 + 2	+0.00001030	+0.00001094	+0.0003090	−0.0004494	+0.00200	−0.01093
1 + 1	−0.00033698	−0.00038819	−0.0171818	+0.0063225	−0.17097	+0.08798
1 0	+0.0511968	+0.0338456	+0.3455498	+0.2284427	+1.72340	+1.13934
1 − 1	+0.11210688	−0.53984943	+0.5192869	−2.5250053	+1.75424	--8.62703
1 − 2	+0.00256211	+0.00503565	−0.0902955	+0.0568980	−0.90859	+0.33612
1 − 3	−0.00003945	−0.00008823	+0.0033231	+0.0001257	+0.05206	+0.03051
1 − 4	−0.00000072	+0.00000356	−0.0001040	−0.0000192	−0.00085	−0.00254
1 − 5	+0.00000010	−0.00000012	+0.0000052	+0.0000014	+0.00005	+0.00013
2 + 3	0.00000000	0.00000000	−0.0000011	+0.0000005	+0.00003	+0.00002
2 + 2	−0.00000022	+0.00000132	+0.0000357	+0.0000007	+0.00069	−0.00054
2 + 1	+0.00001434	−0.00004077	−0.0012894	−0.0005004	−0.02060	−0.00349
2 0	+0.0012618	+0.0029172	+0.0153956	+0.0364443	+0.11643	+0.27401
2 − 1	+0.03389697	−0.04369799	+0.2927492	−0.3380255	+1.52456	−1.67824
2 − 2	−0.19012006	−0.08222140	−1.4212985	−0.6084255	−6.07225	−2.56964
2 − 3	+0.00343686	−0.00225108	+0.0332629	+0.0390821	+0.16153	+0.55248
2 − 4	−0.00004179	+0.00005201	−0.0003964	−0.0014002	+0.01492	−0.02746
2 − 5	+0.00000156	−0.00000009	+0.0000107	+0.0000560	−0.00117	+0.00066
2 − 6	−0.00000003	--0.00000011	−0.0000001	−0.0000031	+0.00004	−0.00010
3 + 2	−0.00000003	+0.00000007	+0.0000021	+0.0000024	+0.00011	−0.00002
3 + 1	+0.00000338	−0.00000119	−0.0000380	−0.0000875	−0.00133	−0.00144
3 0	−0.0000347	+0.0001613	−0.0006969	+0.0033038	−0.00685	+0.03504
3 − 1	+0.00412364	−0.00186127	+0.0519270	−0.0176977	+0.36525	−0.10750
3 − 2	−0.02362413	−0.02644531	−0.2360309	−0.2881422	−1.28925	−1.65646
3 − 3	−0.04911418	+0.07022773	−0.4987523	+0.7214921	−2.54766	+3.73253
3 − 4	−0.00125449	−0.00230408	+0.0166941	−0.0220076	+0.31695	−0.08208
3 − 5	+0.00004566	+0.00002260	−0.0005478	+0.0004550	−0.01447	−0.00645
3 − 6	−0.00000040	−0.00000056	+0.0000283	−0.0000142	+0.00037	+0.00048
3 − 7	−0.00000006	+0.00000001	−0.0000014	+0.0000010	−0.00004	+0.00001
4 + 1	+0.00000033	+0.00000005	+0.0000036	−0.0000061	0.00000	−0.00016
4 0	−0.0000062	+0.0000050	−0.0001946	+0.0001727	−0.00265	+0.00257

Arg.	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i'						
4— 1	+0.00032826	+0.00002869	+0.0055134	+0.0012174	+0.04970	+0.01383
4— 2	—0.00110824	—0.00396710	—0.0109087	—0.0564675	—0.05873	—0.40866
4— 3	—0.01786394	+0.01058360	—0.2382085	+0.1305823	—1.53333	+0.79638
4— 4	+0.02550011	+0.02684391	+0.3342387	+0.3468465	+2.04147	+2.08642
4— 5	—0.00145411	+0.00054586	—0.0151338	—0.0078763	—0.04808	—0.17767
4— 6	+0.00001511	—0.00003423	+0.0003827	+0.0001791	—0.00226	+0.00762
4— 7	—0.00000008	+0.00000044	—0.0000108	—0.0000129	+0.00015	—0.00024
4— 8	+0.00000002	0.00000000	+0.0000003	+0.0000005	—0.00005	+0.00003
5 0	—0.0000004	—0.0000004	—0.0000182	—0.0000003	—0.00032	+0.00006
5— 1	+0.00001760	+0.00001261	+0.0003644	+0.0003619	+0.00405	+0.00448
5— 2	+0.00007944	—0.00038768	+0.0022294	—0.0067512	+0.02394	—0.05887
5— 3	—0.00312321	+0.00028971	—0.0509773	+0.0013033	—0.39350	—0.00779
5— 4	+0.00384527	+0.01097506	+0.0558747	+0.1741100	+0.37661	+1.25378
5— 5	+0.01387683	—0.00870666	+0.2179809	—0.1394720	+1.51295	—0.98794
5— 6	+0.00017098	+0.00086282	—0.0045475	+0.0101665	—0.10047	+0.03192
5— 7	—0.00002302	—0.00001163	+0.0000332	—0.0002946	+0.00404	+0.00039
5— 8	+0.00000039	—0.00000006	—0.0000048	+0.0000070	—0.00018	—0.00003
5— 9	—0.00000001	—0.00000001	+0.0000004	—0.0000002	+0.00002	+0.00005
6— 1	+0.00000033	+0.00000123	+0.0000074	+0.0000413	+0.00010	+0.00060
6— 2	+0.00002126	—0.00002574	+0.0005661	—0.0005055	+0.00657	—0.00500
6— 3	—0.00035022	—0.00013341	—0.0066434	—0.0033090	—0.05920	—0.03447
6— 4	—0.00021055	+0.00215581	—0.0061802	+0.0400880	—0.06562	+0.33418
6— 5	+0.00627977	—0.00087136	+0.1158520	—0.0135815	+0.92810	—0.09417
6— 6	—0.00260695	—0.00687621	—0.0498377	—0.1273840	—0.40541	—1.00404
6— 7	+0.00048380	—0.00000971	+0.0064929	+0.0031357	+0.02204	+0.05876
6— 8	—0.00000904	+0.00001420	—0.0002185	+0.0000129	—0.00033	—0.00215
6— 9	—0.00000009	—0.00000033	+0.0000049	+0.0000006	—0.00002	+0.00014
6—10	+0.00000002	—0.00000001	—0.0000007	—0.0000011		
7— 2	+0.00000237	—0.00000087	+0.0000711	—0.0000130	+0.00101	—0.00008
7— 3	—0.00002587	—0.00002841	—0.0005245	—0.0007503	—0.00506	—0.00859
7— 4	—0.00016002	+0.00026512	—0.0039942	+0.0055240	—0.04225	+0.05146
7— 5	+0.00134634	+0.00040426	+0.0282162	+0.0099378	+0.25479	+0.10029
7— 6	+0.00020896	—0.00338676	+0.0056211	—0.0713663	+0.05783	—0.63154
7— 7	—0.00328612	+0.00055036	—0.0702487	+0.0126496	—0.62102	+0.11979
7— 8	+0.00004209	—0.00025749	+0.0023135	—0.0039054	+0.03598	—0.01463
7— 9	+0.00000813	+0.00000679	+0.0000190	+0.0001576	—0.00118	+0.00055
7—10	—0.00000028	+0.00000004	—0.0000005	—0.0000042		
8— 3	—0.00000088	—0.00000336	—0.0000097	—0.0000991	+0.00001	—0.00128
8— 4	—0.00003137	+0.00002019	—0.0008558	+0.0004268	—0.00992	+0.00408
8— 5	+0.00017436	+0.00015621	+0.0039729	+0.0041038	+0.03893	+0.04500
8— 6	+0.00040985	—0.00077163	+0.0105231	—0.0180233	+0.11040	—0.17588
8— 7	—0.00173114	—0.00046247	—0.0410675	—0.0115685	—0.39870	—0.11670
8— 8	—0.00004388	+0.00151783	—0.0005534	+0.0368484	—0.00040	+0.36183
8— 9	—0.00013034	—0.00004727	—0.0022034	—0.0016872	—0.00894	—0.02283
8—10	+0.00000478	—0.00000431	+0.0001091	—0.0000119		

Arg.	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
9 — 4	—0.00000393	+0.00000040	—0.0001175	—0.0000036	—0.00153	—0.00016
9 — 5	+0.00001242	+0.00002990	+0.0002644	+0.0008598	+0.00235	+0.01027
9 — 6	+0.00013252	—0.00010042	+0.0037209	—0.0024728	+0.04285	—0.02544
9 — 7	—0.00040704	—0.00033329	—0.0104824	—0.0092088	—0.11012	—0.10241
9 — 8	—0.00041581	+0.00083877	—0.0112711	+0.0221456	—0.12170	+0.23453
9 — 9	+0.00067791	+0.00015303	+0.0184535	+0.0038658	+0.19974	+0.03862
9 — 10	—0.00003725	+0.00006270	—0.0011778	+0.0011569		
10 — 5	—0.00000025	+0.00000388	—0.0000220	+0.0001211	—0.00046	+0.00153
10 — 6	+0.00002535	—0.00000539	+0.0007751	—0.0000971	+0.00966	—0.00060
10 — 7	—0.00004934	—0.00010118	—0.0012882	—0.0030513	—0.01373	—0.03702
10 — 8	—0.00023910	+0.00019580	—0.0071473	+0.0055011	—0.08475	+0.06185
10 — 9	+0.00038360	+0.00029707	+0.0111602	+0.0087864	+0.12826	+0.10217
10 — 10	+0.00012816	—0.00029228	+0.0036889	—0.0088206		
11 — 6	+0.00000338	+0.00000084	+0.0001107	+0.0000397	+0.00151	+0.00068
11 — 7	—0.00000036	—0.00001953	+0.0000358	—0.0006364	+0.00088	—0.00827
11 — 8	—0.00007111	+0.00001874	—0.0023016	+0.0004890	—0.02954	+0.00501
11 — 9	+0.00008339	+0.00015737	+0.0025234	+0.0050808	+0.03012	+0.06429
11 — 10	+0.00018889	—0.00016364	+0.0060688	—0.0051955		
12 — 7	+0.00000120	—0.00000269	+0.0000525	—0.0000911	+0.00086	—0.00132
12 — 8	—0.00001388	—0.00000244	—0.0004793	—0.0001181	—0.00655	—0.00193
12 — 9	+0.00000281	+0.00004657	+0.0000279	+0.0016148	—0.00042	+0.02194
12 — 10	+0.00009694	—0.00002877	+0.0033653	—0.0009135		
13 — 8	—0.00000187	—0.00000135	—0.0000677	—0.0000568	—0.00096	—0.00091
13 — 9	—0.00000351	+0.00000913	—0.0001528	+0.0003342	—0.00244	+0.00479
13 — 10	+0.00002866	+0.00000394	+0.0010597	+0.0001896		
14 — 9	—0.00000126	+0.00000121	—0.0000548	+0.0000542	—0.00090	+0.00063
14 — 10	+0.00000554	+0.00000353	+0.0002143	+0.0001533		
15 — 10	+0.00000071	+0.00000110	+0.0000282	+0.0000477		

These expressions are now changed to the form

$$\Sigma . C \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} \left(i'g' + ig \right)$$

The data and formulæ for this operation have already been given (pp. 52, 53). The resulting expressions are

Arg.	$\frac{s'}{\Delta}$		$\left(\frac{s'}{\Delta}\right)^3$		$\left(\frac{s'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
0 0	1.0727980		1.8983958		5.0880	
0 − 1	−0.00890652	−0.00299095	−0.1122762	−0.1919365	−0.6604	−1.3589
0 − 2	−0.00006683	−0.00004887	−0.0050426	+0.0022754	−0.0742	+0.0587
0 − 3	−0.00000716	−0.00000466	−0.0001100	+0.0000188	+0.0015	+0.0026
0 − 4	−0.00000034	−0.00000009	−0.0000064	+0.0000046		
1 + 3	−0.00000045	+0.00000023	−0.0000117	−0.0000021		
1 + 2	−0.00000075	+0.00000799	−0.0001798	−0.0002353	−0.0028	−0.0084
1 + 1	−0.00038134	−0.00017655	−0.0173889	+0.0073338	−0.1717	+0.0919
1 0	+0.0480640	+0.0489876	+0.3314766	+0.2990375	+1.6788	+1.3786
1 − 1	+0.11187535	−0.53970754	+0.5239494	−2.5262128	+1.8039	−8.6390
1 − 2	+0.00569463	−0.01008010	−0.0757592	−0.0139522	−0.8612	+0.0910
1 − 3	+0.00023582	−0.00044138	−0.0011221	+0.0003367	+0.0031	+0.0393
1 − 4	+0.00001054	−0.00001967	−0.0000779	+0.0000215	+0.0008	+0.0006
1 − 5	+0.00000062	−0.00000105	−0.0000017	+0.0000023		
1 − 6	+0.00000003	−0.00000006	+0.0000006	+0.0000003		
2 + 2	−0.00000024	+0.00000085	−0.0000041	−0.0000080	+0.0001	−0.0006
2 + 1	+0.00000242	−0.00002305	−0.0013949	−0.0003628	−0.0212	−0.0028
2 0	+0.0003113	+0.0041432	+0.0072264	+0.0459326	+0.0743	+0.3211
2 − 1	+0.04452774	−0.03905904	+0.3722012	−0.3036208	+1.8638	−1.5323
2 − 2	−0.18886318	−0.08299694	−1.4114371	−0.6192600	−6.0240	−2.6550
2 − 3	−0.00716277	−0.00688517	−0.0459755	+0.0045796	−0.1787	+0.4062
2 − 4	−0.00034727	−0.00039658	−0.0020440	−0.0000486	+0.0095	+0.0108
2 − 5	−0.00001759	−0.00002263	−0.0000974	+0.0000165	+0.0004	+0.0003
2 − 6	−0.00000090	−0.00000144	−0.0000048	+0.0000002		
2 − 7	−0.00000005	−0.00000010	−0.0000001	+0.0000002		
3 + 2	+0.00000002	+0.00000008	+0.0000003	+0.0000004		
3 + 1	+0.00000193	−0.00000026	−0.0000568	−0.0000786	−0.0014	−0.0014
3 0	−0.0001504	+0.0002135	−0.0021512	+0.0038023	−0.0170	+0.0380
3 − 1	+0.00538633	−0.00029518	+0.0645244	−0.0006875	+0.4341	−0.0102
3 − 2	−0.01931519	−0.03231744	−0.1919124	−0.3483735	−1.0601	−1.9680
3 − 3	−0.04994189	+0.06851021	−0.5102235	+0.7027453	−2.6368	+3.6228
3 − 4	−0.00542270	+0.00350685	−0.0258488	+0.0375883	+0.0980	+0.2265
3 − 5	−0.00038622	+0.00017269	−0.0016509	+0.0021759	+0.0058	+0.0060
3 − 6	−0.00002537	+0.00000841	−0.0000909	+0.0001241	+0.0004	+0.0003
3 − 7	−0.00000171	+0.00000041	−0.0000055	+0.0000079		
3 − 8	−0.00000013	+0.00000002	−0.0000003	+0.0000005		
4 − 0	−0.0000154	+0.0000042	−0.000350	+0.000139	−0.0040	+0.0022
4 − 1	+0.0003687	+0.0002631	+0.005834	+0.004529	+0.0512	+0.0376
4 − 2	+0.0004843	−0.0047580	+0.010327	−0.066129	+0.0780	−0.4674
4 − 3	−0.0206566	+0.0072916	−0.274560	+0.087712	−1.7541	+0.5345
4 − 4	+0.0238870	+0.0273031	+0.312224	+0.354333	+1.8943	+2.1504
4 − 5	+0.0012979	+0.0035822	+0.020807	+0.031507	+0.1708	+0.0607

Arg.	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
4—6	+0.0000449	+0.0002970	+0.001310	+0.002378	+0.0097	+0.0028
4—7	+0.0000004	+0.0000217	+0.000079	+0.000160	+0.0006	+0.0002
4—8	−0.0000001	+0.0000015	+0.000005	+0.000010		
5—0	−0.0000009	−0.0000008	−0.000028	−0.000010	−0.0004	0.0000
5—1	+0.0000093	+0.0000344	+0.000180	+0.000739	+0.0022	+0.0078
5—2	+0.0003529	−0.0003754	+0.006671	−0.006273	+0.0580	−0.0539
5—3	−0.0034448	−0.0010114	−0.055452	−0.019356	−0.4226	−0.1569
5—4	+0.0016055	+0.0120812	+0.020523	+0.191535	+0.1275	+1.3752
5—5	+0.0139854	−0.0074603	+0.220383	- 0.119107	+1.5396	−0.8347
5—6	+0.0021227	−0.0002596	+0.026196	−0.007706	+0.1138	−0.0939
5—7	+0.0001969	+0.0000207	+0.002281	−0.000378	+0.0080	−0.0068
5—8	+0.0000155	+0.0000042	+0.000173	−0.000014	+0.0005	−0.0003
5—9	+0.0000010	+0.0000003	+0.000012	+0.000001		
6—1	−0.0000013	+0.0000024	−0.000033	+0.000064	−0.0004	+0.0009
6—2	+0.0000494	−0.0000075	+0.001093	−0.000097	+0.0113	−0.0011
6—3	−0.0002856	−0.0003784	−0.005183	−0.007847	−0.0455	−0.0722
6—4	−0.0011363	+0.0021744	−0.023261	+0.040002	−0.2027	+0.3308
6—5	+0.0065724	+0.0005291	+0.121250	+0.012370	+0.9702	+0.1125
6—6	−0.0017619	−0.0067813	−0.033752	−0.125917	−0.2707	−0.9971
6—7	+0.0001224	−0.0011570	−0.000360	−0.018156	−0.0330	−0.1096
6—8	+0.0000429	−0.0001167	+0.000244	−0.001764	−0.0025	−0.0095
6—9	+0.0000056	−0.0000096	+0.000039	−0.000145	−0.0002	−0.0008
6—10	+0.0000005	−0.0000007	+0.000002	−0.000012		
7—2	+0.0000040	+0.0000023	+0.000100	+0.000066	+0.0013	+0.0008
7—3	+0.0000002	−0.0000547	+0.000095	−0.001291	+0.0012	−0.0135
7—4	−0.0003444	+0.0001711	−0.007835	+0.003333	−0.0768	+0.0303
7—5	+0.0012225	+0.0009977	+0.025322	+0.022413	+0.2271	+0.2107
7—6	+0.0010241	−0.0033446	+0.022957	−0.070456	+0.2118	−0.6229
7—7	−0.0031172	+0.0000333	−0.066772	+0.001398	−0.5922	+0.0156
7—8	−0.0005851	−0.0002034	−0.011109	−0.002641	−0.0829	−0.0027
7—9	−0.0000623	−0.0000421	−0.001169	−0.000517	−0.0082	−0.0009
7—10	−0.0000053	−0.0000054	−0.000100	−0.000065	−0.0008	−0.0001
7—11	−0.0000005	−0.0000006	−0.000008	−0.000006		
8—3	+0.0000035	−0.0000046	+0.000109	−0.000118	+0.0013	−0.0015
8—4	−0.0000509	−0.0000091	−0.001283	−0.000321	−0.0141	−0.0041
8—5	+0.0000758	+0.0002764	+0.001487	+0.006887	+0.0132	+0.0720
8—6	+0.0007564	−0.0006102	+0.018706	−0.014018	+0.1897	−0.1354
8—7	−0.0015886	−0.0009052	−0.037644	−0.022189	−0.3649	−0.2209
8—8	−0.0003329	+0.0013513	−0.007637	+0.032892	−0.0725	+0.3239
8—9	−0.0001741	+0.0002739	−0.003192	+0.006112	−0.0179	+0.0541
8—10	−0.0000328	+0.0000297	−0.000551	+0.000676	−0.0032	+0.0053
8—11	−0.0000040	+0.0000023	−0.000061	+0.000061	−0.0004	+0.0005
9—4	−0.0000042	−0.0000046	−0.000114	−0.000143	−0.0014	−0.0018
9—5	−0.0000155	+0.0000409	−0.000508	+0.001114	−0.0064	+0.0127

Arg.	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
9 — 6	+0.0002006	—0.0000135	+0.005449	—0.000095	+0.0607	+0.0009
9 — 7	—0.0002612	—0.0005177	—0.006532	—0.014033	—0.0671	—0.1532
9 — 8	—0.0006385	+0.0006958	—0.017222	+0.018323	—0.1851	+0.1933
9 — 9	+0.0005456	+0.0003003	+0.014921	+0.007893	+0.1582	+0.0846
9 — 10	+0.0001164	+0.0001199	+0.003018	+0.002659	+0.0444	+0.0163
9 — 11	+0.0000130	+0.0000253	+0.000318	+0.000526	+0.0068	+0.0022
10 — 5	—0.0000049	+0.0000032	—0.000164	+0.000090		
10 — 6	+0.0000289	+0.0000182	+0.000853	+0.000606		
10 — 7	+0.0000186	—0.0001334	+0.000727	—0.003928		
10 — 8	—0.0003265	+0.0000845	—0.009655	+0.002208		
10 — 9	+0.0002717	+0.0003978	+0.007878	+0.011751		
10 — 10	+0.0002039	—0.0001921	+0.005889	—0.005852		
10 — 11	+0.0000493	—0.0000680	+0.001382	—0.002065		
11 — 6	+0.0000020	+0.0000047	+0.000055	+0.000165		
11 — 7	+0.0000175	—0.0000187	+0.000610	—0.000580		
11 — 8	—0.0000823	—0.0000297	—0.002612	—0.001067		
11 — 9	+0.0000115	+0.0001955	+0.000219	+0.006264		
11 — 10	+0.0001924	—0.0001122	+0.006144	—0.003546		
11 — 11	+0.0000541	—0.0000377	+0.001705	—0.001225		

By the method previously given (p. 52), we compute the Besselian functions corresponding to various multiples of half the eccentricity of Uranus, and find

$$\log J_{\frac{e'}{2}}^{(0)} = 9.9997609 \qquad \log J_{\frac{e'}{2}}^{(1)} = 8.3702418 \qquad \log J_{\frac{e'}{2}}^{(2)} = 6.4396129$$

$$\log J_{e'}^{(0)} = 9.9990432 \qquad \log J_{e'}^{(1)} = 8.6709131 \qquad \log J_{e'}^{(2)} = 7.0414338$$

$$\log J_{e'}^{(3)} = 5.2357835 \qquad \log J_{\frac{3}{2}e'}^{(2)} = 7.3932179 \qquad \log J_{\frac{3}{2}e'}^{(3)} = 5.7637586$$

$$\log J_{\frac{3}{2}e'}^{(4)} = 4.0092888 \qquad \log J_{2e'}^{(3)} = 6.1381561 \qquad \log J_{2e'}^{(4)} = 4.5087087$$

$$\log J_{2e'}^{(5)} = 2.7822875 \qquad \log J_{\frac{5}{2}e'}^{(4)} = 4.8959184 \qquad \log J_{\frac{5}{2}e'}^{(5)} = 3.2664790$$

$$\log J_{\frac{5}{2}e'}^{(6)} = 1.5578013$$

The expressions for the three multipliers of $\left(\frac{a'}{\Delta}\right)^3$ are

$$\alpha^2\left(\frac{r}{a}\right)^2 = [9.3951972] - 2[8.1416132]\cos g - 2[6.28790]\cos 2g - 2[4.7352]\cos 3g - 2[3.31]\cos 4g$$

$$\left(\frac{r'}{a'}\right)^2 = 1 + [7.51887] - 2[8.6712719]\cos g' - 2[6.74040]\cos 2g' - 2[5.1105]\cos 3g' - 2[3.61]\cos 4g'$$

$$\begin{aligned}\frac{r'}{a'}\sin(f' + \Pi') = {} & \qquad\qquad + [8.6687033] \\ & - 2[9.5728624]\sin g' - 2[9.5198320]\cos g' \\ & - 2[7.94302\ \ ]\sin 2g' - 2[7.88991\ \ ]\cos 2g' \\ & - 2[6.4893\ \ ]\sin 3g' - 2[6.4361\ \ ]\cos 3g' \\ & - 2[5.1092\ \ ]\sin 4g' - 2[5.0561\ \ ]\cos 4g'\end{aligned}$$

The multiplication being performed, we obtain

Arg $= i'g + ig$	$\alpha^2 \frac{r^2}{a^2}\left(\frac{a'}{\Delta}\right)^3$		$\frac{r'^2}{a'^2}\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^3 \frac{r'}{a'} \sin (f'+\Pi')$	
	cos.	sin.	cos.	sin.	sin.	cos.
i' i						
0 0	0.4731677		1.8891119			−0.133487
0 − 1	−0.0804060	−0.0477507	−0.1366148	−0.0735532	+1.037767	+0.768901
0 − 2	−0.0004317	+0.0032233	−0.0007182	+0.0032715	−0.031155	+0.046753
0 − 3	+0.0000439	+0.0000103	−0.0000254	−0.0000086	−0.001320	+0.000486
0 − 4	+0.0000007	+0.0000014	−0.0000013	+0.0000031	−0.000072	+0.000020
1 + 3	+0.0000011	+0.0000003	−0.0000051	−0.0000006	+0.000090	+0.000043
1 + 2	+0.0001293	−0.0002043	+0.0001163	−0.0001448	+0.003449	+0.002106
1 + 1	−0.0090114	−0.0018278	−0.0124069	−0.0030223	−0.046000	+0.126816
1 0	+0.0753436	+0.1091910	+0.1539410	+0.2980334	−1.419151	−1.266090
1 − 1	+0.1266233	−0.6315286	+0.5134607	−2.5113038	+0.228194	−0.020260
1 − 2	−0.0261287	+0.0314724	−0.0094558	+0.0151373	−0.321366	+0.702339
1 − 3	+0.0006687	+0.0007652	+0.0013203	−0.0002655	−0.028698	+0.011342
1 − 4	+0.0000080	+0.0000169	+0.0000282	−0.0000016	−0.001269	+0.000370
1 − 5	+0.0000012	+0.0000008	+0.0000038	+0.0000003	−0.000071	+0.000015
2 + 2	+0.0000152	−0.0000042	+0.0000087	+0.0000038	+0.000237	+0.000040
2 + 1	−0.0005110	−0.0006641	−0.0005263	−0.0008426	+0.002698	+0.011672
2 0	−0.0030683	+0.0157428	−0.0102913	+0.0318817	−0.256196	−0.027903
2 − 1	+0.1119294	−0.0674846	+0.3458836	−0.1859817	+0.652930	−1.122850
2 − 2	−0.3551586	−0.1497059	−1.4035449	−0.6042733	+0.048245	+0.148410
2 − 3	+0.0080904	+0.0097770	−0.0219881	−0.0284357	−0.426008	−0.094198
2 − 4	+0.0004023	+0.0000461	−0.0010063	−0.0020105	−0.022147	−0.011159
2 − 5	+0.0000207	+0.0000074	−0.0000344	−0.0001014	−0.001295	−0.000732
2 − 6	+0.0000011	−0.0000001	−0.0000012	−0.0000069	−0.000070	−0.000050
3 + 1	+0.0000043	−0.0000703	+0.0000192	−0.0000694	+0.000681	+0.000638
3 0	−0.0013876	+0.0010190	−0.0027123	+0.0014893	−0.024306	+0.013756
3 − 1	+0.0188174	+0.0044666	+0.0467168	+0.0147328	−0.022958	−0.263625
3 − 2	−0.0414954	−0.0962803	−0.1267809	−0.3173609	+0.743227	+0.248427
3 − 3	−0.1237478	+0.1788859	−0.4968410	+0.7007472	−0.083595	+0.051828
3 − 4	+0.0007075	−0.0003612	−0.0404963	+0.0209869	+0.000692	−0.238255
3 − 5	+0.0000494	−0.0001165	−0.0027506	+0.0007696	+0.000382	−0.019411
3 − 6	+0.0000082	−0.0000105	−0.0001665	+0.0000188	+0.000026	−0.001411
3 − 7	+0.0000004	−0.0000004	−0.0000105	+0.0000003	−0.000035	−0.000094
4 0	−0.000169	−0.000015	−0.000259	−0.000068	−0.00111	+0.00242
4 − 1	+0.001362	+0.002020	+0.002605	+0.004741	−0.02422	−0.02818
4 − 2	+0.006227	−0.017775	+0.019827	−0.049370	+0.20580	−0.06062
4 − 3	−0.072682	+0.017790	−0.248903	+0.055945	−0.05166	+0.44501
4 − 4	+0.081079	+0.086388	+0.313519	+0.344733	−0.04247	−0.04139
4 − 5	+0.000878	+0.002868	+0.010548	+0.037089	+0.12416	−0.02714
4 − 6	−0.000024	+0.000084	+0.000108	+0.002812	+0.01316	−0.00427
4 − 7	−0.000005	−0.000002	−0.000029	+0.000189	+0.00111	−0.00043
4 − 8	0.000000	+0.000001	−0.000003	+0.000012		
5 0	−0.000011	−0.000012	−0.000011	−0.000019	+0.00006	+0.00023
5 − 1	−0.000032	+0.000275	−0.000131	+0.000531	−0.00425	−0.00098

Arg $=i'g'+ig$	$\alpha^2 \frac{r^2}{a^2}\left(\frac{a'}{\Delta}\right)^3$		$\frac{r'^2}{a'^2}\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^3 \frac{r'}{a'} \sin (f'+\Pi')$	
	cos.	sin.	cos.	sin.	sin.	cos.
i' i						
5—2	+0.002417	−0.001337	+0.006281	−0.002987	+0.02316	−0.02952
5—3	−0.014195	−0.007353	−0.042230	−0.023552	+0.07245	+0.13588
5—4	+0.002807	+0.049502	+0.007053	+0.173647	−0.24722	+0.02343
5—5	+0.054113	−0.032132	+0.214434	−0.121571	+0.01754	−0.02996
5—6	+0.003417	−0.000297	+0.027792	−0.001897	+0.02818	+0.06037
5—7	+0.000159	+0.000035	+0.002339	+0.000464	+0.00518	+0.00758
5—8	+0.000005	+0.000004	+0.000169	+0.000070	+0.00058	+0.00069
5—9	+0.000001	0.000000	+0.000011	+0.000008		
6—1	−0.000022	+0.000019	−0.000046	+0.000026		
6—2	+0.000347	+0.000076	+0.000777	+0.000234	+0.00021	−0.00527
6—3	−0.001006	−0.002505	−0.002445	−0.006961	+0.02895	+0.01458
6—4	−0.007380	+0.009900	−0.024104	+0.030798	−0.07873	+0.06582
6—5	+0.030913	+0.004270	+0.110113	+0.016926	−0.04083	−0.12855
6—6	−0.010056	−0.031208	−0.036180	−0.122658	+0.01903	+0.00561
6—7	+0.000352	−0.002743	+0.002685	−0.018252	−0.02716	+0.02115
6—8	+0.000072	−0.000160	+0.000762	−0.001663	−0.00375	+0.00438
6—9	+0.000006	−0.000008	+0.000095	−0.000124	−0.00036	+0.00055
6—10	0.000000	0.000000	+0.000007	−0.000009		
7—2	+0.000026	+0.000033	+0.000045	+0.000075	−0.00038	−0.00053
7—3	+0.000127	−0.000372	+0.000367	−0.000912	+0.00526	−0.00083
7—4	−0.002302	+0.000550	−0.006725	+0.001373	−0.00662	+0.02437
7—5	+0.006095	+0.006498	+0.019527	+0.021650	−0.05115	−0.04013
7—6	+0.006281	−0.017832	+0.023722	−0.064120	+0.06252	−0.03610
7—7	−0.016757	+0.001356	−0.065207	+0.003304	−0.00047	+0.01113
7—8	−0.001823	−0.000654	−0.010789	−0.004120	−0.01367	−0.01106
7—9	−0.000122	−0.000090	−0.001033	−0.000803	−0.00312	−0.00153
7—10	−0.000007	−0.000008	−0.000076	−0.000097	−0.00042	−0.00011
7—11	−0.000001	−0.000001	−0.000005	−0.000009		
8—3	+0.000045	−0.000026	+0.000109	−0.000053	+0.00052	−0.00053
8—4	−0.000346	−0.000170	−0.000901	−0.000495	+0.00165	+0.00445
8—5	+0.000135	+0.001913	+0.000258	+0.005802	−0.01833	−0.00111
8—6	+0.005149	−0.003280	+0.017454	−0.010686	+0.01743	−0.03572
8—7	−0.009504	−0.005776	−0.034330	−0.021658	+0.02586	+0.02813
8—8	−0.001335	+0.008396	−0.006328	+0.032264	−0.00605	+0.00121
8—9	−0.000672	+0.001058	−0.003852	+0.005780	+0.00385	−0.00800
8—10	−0.000091	+0.000076	−0.000693	+0.000559	+0.00044	−0.00201
8—11	−0.000006	+0.000005	−0.000077	+0.000037	−0.00002	−0.00030
9—4	−0.000022	−0.000051	−0.000050	−0.000131		
9—5	−0.000198	+0.000283	−0.000588	+0.000779		
9—6	+0.001454	+0.000151	+0.004537	+0.000576		
9—7	−0.001462	−0.003741	−0.004785	−0.012855		
9—8	−0.004397	+0.004636	−0.016461	+0.016739		
9—9	+0.003905	+0.001672	+0.014751	+0.007078		
9—10	+0.000542	+0.000541	+0.002775	+0.002913		
9—11	+0.000034	+0.000092	+0.000256	+0.000623		

Arg=$i'g'+ig$	$\alpha^2\frac{r^2}{a^2}\left(\frac{a'}{\Delta}\right)^3$		$\frac{r'^2}{a'^2}\left(\frac{a'}{\Delta}\right)^3$		$-\left(\frac{a'}{\Delta}\right)^3\frac{r'}{a'}\sin(f'+\Pi')$	
	cos.	sin.	cos.	sin.	sin.	cos.
i' i						
10— 5	—0.000053	+0.000015	—0.000142	+0.000034		
10— 6	+0.000206	+0.000204	+0.000587	+0.000613		
10— 7	+0.000301	—0.001017	+0.001026	—0.003244		
10— 8	—0.002518	+0.000440	—0.008752	+0.001387		
10— 9	+0.002010	+0.002970	+0.007196	+0.011123		
10—10	+0.001334	—0.001588	+0.005478	—0.005830		
10—11	+0.000260	—0.000434	+0.001292	—0.002040		
11— 6	+0.000006	+0.000049	+0.000012	+0.000138		
11— 7	+0.000187	—0.000132	+0.000582	—0.000390		
11— 8	—0.000662	—0.000343	—0.002158	—0.001185		
11— 9	+0.000005	+0.001620	—0.000158	+0.005730		
11—10	+0.001501	—0.000951	+0.005886	—0.003284		
11—11	+0.000339	—0.000256	+0.001646	—0.001132		

In deriving the portion of the perturbative function, which arises from the action of Uranus on the Sun, we have

$\log h = 9.0095743$ $\log h_1 = 9.0071829$ $\log l = 9.6864004$ $\log l_1 = 9.6866271$

And the expression of this portion is

Arg=$i'g'+ig$	$-a'\frac{r}{r'^2}H$		Arg=$i'g'+ig$	$-a'\frac{r}{r'^2}H$	
	cos.	sin.		cos.	sin.
i' i			i' i		
1 + 3	—0.00000022	—0.00000003	2 — 3	—0.00001124	+0.00005354
1 + 2	—0.00000557	+0.00000017	2 — 4	—0.00000056	+0.00000267
1 + 1	—0.00021225	+0.00006955	3 + 1	—0.00000147	+0.00000105
1 0	+0.00858887	—0.04085337	3 0	+0.00006376	—0.00030330
1 — 1	—0.10181181	+0.48521410	3 — 1	—0.00075575	+0.00360178
1 — 2	—0.00285138	+0.01358911	3 — 2	—0.00002116	+0.00010087
1 — 3	—0.00011978	+0.00057087	3 — 3	—0.00000090	+0.00000424
1 — 4	—0.00000596	+0.00002843	4 0	+0.0000047	—0.0000225
1 — 5	—0.00000032	+0.00000155	4 — 1	—0.0000560	+0.0002669
2 + 2	—0.00000050	+0.00000013	4 — 2	—0.0000016	+0.0000075
2 + 1	—0.00001903	+0.00001070	5 0	+0.0000003	—0.0000016
2 0	+0.00080552	—0.00383222	5 — 1	—0.0000040	+0.0000191
2 — 1	—0.00954950	+0.04551090	5 — 2	—0.0000001	+0.0000005
2 — 2	—0.00026745	+0.00127460			

For the component of this action perpendicular to the plane of Saturn's orbit, we have

$$
\begin{aligned}
-\left(\frac{a'}{r'}\right)^2 \sin(f' + \Pi') = &+ 0.747984 \sin g' + 0.662006 \cos g' \\
&+ 0.070164 \sin 2g' + 0.062088 \cos 2g' \\
&+ 0.005553 \sin 3g' + 0.004913 \cos 3g' \\
&+ 0.00041 \sin 4g' + 0.00036 \cos 4g' \\
&+ 0.00003 \sin 5g' + 0.00003 \cos 5g'
\end{aligned}
$$

For the factors proportional to the mass of Uranus (in seconds of arc), we have

$$\log \mu = 0.6884739 \qquad \log (\mu a \sin J) = 8.9183882$$

The expressions for the forces are

Arg$=i'g'+ig$	$a\frac{d\Omega}{dg}$		$ar\frac{d\Omega}{dr}$		$a^2\frac{d\Omega}{dZ}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	″	″	″	″	″	″
0 0			+0.837381			−0.011062
0 − 1	−0.04346924	+0.01459765	−0.1154317	−0.0556672	+0.0859979	+0.0637175
0 − 2	−0.0006524	+0.0004770	−0.0005359	+0.0002367	−0.002582	+0.003874
0 − 3	−0.0001048	+0.0000682	−0.0001513	−0.0000347	−0.000109	+0.000040
0 − 4	−0.0000066	+0.0000017	−0.0000039	+0.0000044	−0.000006	+0.000002
1 + 3	+0.000010	+0.000003	−0.000015	−0.000002		
1 + 2	+0.000062	+0.000080	−0.000057	+0.000127	+0.000286	+0.000175
1 + 1	+0.002891	−0.000522	−0.008391	−0.002145	−0.003812	+0.010509
1 0			+0.116430	+0.141898	−0.055619	−0.050060
1 − 1	+0.049116	+0.265961	+0.174087	−0.902032	+0.018910	−0.001679
1 − 2	+0.027754	−0.034252	+0.012873	+0.051059	−0.026631	+0.058201
1 − 3	+0.001699	−0.001896	+0.000435	+0.001349	−0.002378	+0.000940
1 − 4	+0.000089	−0.000171	−0.000006	+0.000142	−0.000105	+0.000031
1 − 5	+0.000007	−0.000012	+0.000003	+0.000008	−0.000006	+0.000001
2 + 2	+0.000007	+0.000010	−0.000018	+0.000018	+0.000019	+0.000003
2 + 1	+0.000081	−0.000060	−0.000136	−0.000327	+0.000214	+0.000967
2 0			−0.014454	+0.010570	−0.015416	+0.002833
2 − 1	+0.170715	−0.031489	+0.415651	+0.028267	+0.054106	−0.093048
2 − 2	−1.846145	+0.797709	−2.098802	−0.900524	+0.003997	+0.012298
2 − 3	−0.105041	+0.100028	−0.055976	−0.076187	−0.035303	−0.007805
2 − 4	−0.006790	+0.007690	−0.002593	−0.004031	−0.001835	−0.000925
2 − 5	−0.000430	+0.000549	−0.000091	−0.000210	−0.000107	−0.000061
2 − 6	−0.000026	+0.000042	−0.000003	−0.000013	−0.000006	−0.000004
3 + 1	−0.000002	+0.000004	+0.000024	+0.000008	+0.000056	+0.000053
3 0			−0.002554	−0.000853	−0.001554	+0.001547
3 − 1	+0.0226000	−0.0161382	+0.051250	+0.043351	−0.001902	−0.021846
3 − 2	−0.188746	+0.314473	−0.161091	−0.460147	+0.061590	+0.020587
3 − 3	−0.731254	−1.003177	−0.788591	+1.106335	−0.006928	+0.004294
3 − 4	−0.105865	−0.068466	−0.087316	+0.043537	+0.000057	−0.019744
3 − 5	−0.009425	−0.004214	−0.005890	+0.001741	+0.000032	−0.001609

Arg=$i'g'+ig$	$a\frac{d\Omega}{dg}$		$ar\frac{d\Omega}{dr}$		$a^2\frac{d\Omega}{dZ}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	″	″	″	″	″	″
3— 6	—0. 000743	—0. 000246	—0. 000364	+0. 000051	0. 000000	—0. 000117
3— 7	—0. 000058	—0. 000014	—0. 000024	0. 000000	0. 000000	—0. 000008
4 0			—0. 000156	—0. 000244	—0. 000058	+0. 000230
4— 1	+0. 001526	—0. 002587	+0. 001860	+0. 007302	—0. 002007	—0. 002335
4— 2	+0. 004711	+0. 046370	+0. 031996	—0. 065453	+0. 017054	—0. 005023
4— 3	—0. 302450	—0. 106765	—0. 379621	+0. 075313	—0. 004281	+0. 036877
4— 4	+0. 466332	—0. 533023	+0. 508930	+0. 563813	—0. 003520	—0. 003430
4— 5	+0. 031672	—0. 087416	+0. 020431	+0. 074765	+0. 010290	—0. 002249
4— 6	+0. 001315	—0. 008698	+0. 000210	+0. 005930	+0. 001091	—0. 000354
4— 7	+0. 000013	—0. 000741	—0. 000059	+0. 000410	+0. 000091	—0. 000035
4— 8	—0. 000004	—0. 000059	—0. 000005	+0. 000025		
5 0			0. 000000	—0. 000025		
5— 1	+0. 000026	—0. 000261	—0. 000283	+0. 000634	—0. 00035	—0. 00008
5— 2	+0. 003443	+0. 003659	+0. 008564	—0. 003109	+0. 00192	—0. 00245
5— 3	—0. 050438	+0. 014809	—0. 060007	—0. 037059	+0. 00600	+0. 01126
5— 4	+0. 03134	—0. 23585	+0. 00644	+0. 27347	—0. 02049	+0. 00194
5— 5	+0. 34128	+0. 18205	+0. 35710	—0. 20005	+0. 00145	—0. 00248
5— 6	+0. 06216	+0. 00760	+0. 05430	—0. 00327	+0. 00233	+0. 00500
5— 7	+0. 00673	—0. 00071	+0. 00484	+0. 00100	+0. 00043	+0. 00063
5— 8	+0. 00061	—0. 00016	+0. 00036	+0. 00015	+0. 00005	+0. 00006
5— 9	+0. 00004	—0. 00001	+0. 00001	+0. 00002		
6— 2	+0. 000482	+0. 000073	+0. 00093	+0. 00037	+0. 00002	—0. 00044
6— 3	—0. 004182	+0. 005540	—0. 00282	—0. 00995	+0. 00240	+0. 00121
6— 4	—0. 022183	—0. 042449	—0. 03804	+0. 04570	—0. 00652	+0. 00545
6— 5	+0. 16039	—0. 01291	+0. 17724	+0. 02959	—0. 00338	—0. 01065
6— 6	—0. 05159	+0. 19858	—0. 05945	—0. 20661	+0. 00158	+0. 00046
6— 7	+0. 00418	+0. 03953	+0. 00539	—0. 03502	—0. 00225	+0. 00175
6— 8	+0. 00168	+0. 00456	+0. 00158	—0. 00338	—0. 00031	+0. 00036
6— 9	+0. 00025	+0. 00042	+0. 00020	—0. 00026	—0. 00003	+0. 00005
7— 2	+0. 000039	—0. 000022	+0. 00003	+0. 00010	—0. 00003	—0. 00004
7— 3	+0. 000003	+0. 000801	+0. 00059	—0. 00119	+0. 00044	—0. 00007
7— 4	—0. 006723	—0. 003340	—0. 00995	+0. 00159	—0. 00055	+0. 00202
7— 5	+0. 02983	—0. 02435	+0. 02979	+0. 03454	—0. 00424	—0. 00333
7— 6	+0. 02999	+0. 09794	+0. 04006	—0. 10480	+0. 00518	—0. 00299
7— 7	—0. 10650	—0. 00114	—0. 11062	+0. 00467	—0. 00004	+0. 00092
7— 8	—0. 02285	+0. 00799	—0. 02045	—0. 00796	—0. 00113	—0. 00092
7— 9	—0. 00274	+0. 00185	—0. 00207	—0. 00163	—0. 00026	—0. 00013
7—10	—0. 00026	+0. 00026	—0. 00016	—0. 00020		
8— 3	+0. 000051	+0. 000067	+0. 00015	—0. 00005	+0. 00004	—0. 00004
8— 4	—0. 000995	+0. 000178	—0. 00123	—0. 00077	+0. 00014	+0. 00037
8— 5	+0. 00185	—0. 00674	+0. 00012	+0. 00881	—0. 00152	—0. 00009
8— 6	+0. 02214	+0. 01786	+0. 02819	—0. 01659	+0. 00144	—0. 00296
8— 7	—0. 05427	+0. 03092	—0. 05670	—0. 03655	+0. 00214	+0. 00233
8— 8	—0. 01300	—0. 05276	—0. 01137	+0. 05494	—0. 00050	+0. 00010
8— 9	—0. 00764	—0. 01203	—0. 00734	+0. 01086	+0. 00032	—0. 00066

Arg=$i'g'+ig$	$a\frac{d\Omega}{dg}$		$ar\frac{d\Omega}{dr}$		$a^2\frac{d\Omega}{dZ}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	″	″	″	″	″	″
8—10	—0.00160	—0.00145	—0.00139	+0.00110	+0.00004	—0.00017
8—11	—0.00021	—0.00012	—0.00016	+0.00007		
9— 4	—0.00008	+0.00009	—0.00006	—0.00018		
9— 5	—0.00038	—0.00100	—0.00091	+0.00111		
9— 6	+0.00587	+0.00040	+0.00703	+0.00107		
9— 7	—0.00892	+0.01769	—0.00747	—0.02098		
9— 8	—0.02493	—0.02717	—0.02788	+0.02784		
9— 9	+0.02397	—0.01319	+0.02513	+0.01246		
9—10	+0.00568	—0.00585	+0.00516	+0.00549		
9—11	+0.00070	—0.00136	+0.00051	+0.00123		
10— 5	—0.00012	—0.00008	—0.00020	+0.00004		
10— 6	+0.00085	—0.00053	+0.00086	+0.00095		
10— 7	+0.00064	+0.00456	+0.00172	—0.00511		
10— 8	—0.01275	—0.00330	—0.01442	+0.00210		
10— 9	+0.01194	—0.01748	+0.01199	+0.01893		
10—10	+0.00995	+0.00938	+0.00962	—0.00988		
10—11	+0.00265	+0.00365	+0.00240	—0.00375		
11— 6	+0.00006	—0.00014	+0.00001	+0.00020		
11— 7	+0.00060	+0.00064	+0.00092	—0.00059		
11— 8	—0.00321	+0.00116	—0.00345	—0.00198		
11— 9	+0.00051	—0.00859	—0.00042	+0.00096		
11—10	+0.00939	+0.00548	+0.01023	—0.00542		
11—11	+0.00290	+0.00202	+0.00306	—0.00204		

The expressions for the multipliers to be used in obtaining T and $\frac{1}{n}\frac{dR}{dt}$ from the preceding quantities have been given (p. 74). There is then obtained

Arg=$\varkappa y+i'g'+ig$	T		Arg=$\varkappa y+i'g'+ig$	T	
	sin.	cos.		sin.	cos.
$\varkappa$ i' i	″	″	$\varkappa$ i' i	″	″
1 0— 1	—1.66360	—0.00164	—1 1+ 4	+0.00001	0.00000
—1 0 0	—0.1556525	+0.0850031	0 1+ 3	—0.00003	—0.00001
0 0— 1	+0.1304077	—0.0437929	1 1+ 2	+0.00002	0.00000
1 0— 2	—0.01818	—0.02650	—1 1+ 3	+0.00005	—0.00003
—1 0— 1	+0.00101	+0.00023	0 1+ 2	—0.00019	—0.00024
0 0— 2	+0.00196	—0.00143	1 1+ 1	+0.00001	+0.00030
1 0— 3	—0.00070	+0.00004	—1 1+ 2	+0.00090	—0.00162
—1 0— 2	—0.00028	+0.00009	0 1+ 1	—0.00867	+0.00157
0 0— 3	+0.00031	—0.00020	1 1 0	+0.01092	+0.00079
1 0— 4	—0.00005	+0.00007	—1 1+ 1	+0.12225	—0.10932
—1 0— 3	0.00000	—0.00001	1 1— 1	—0.12808	+0.07935
0 0— 4	+0.00002	—0.00001	—1 1 0	+0.27716	+1.42983
1 0— 5	0.00000	0.00000	0 1— 1	—0.14735	—0.79788

Arg=$\varkappa\gamma+i'g'+ig$			T		Arg=$\varkappa\gamma+i'g'+ig$			T	
			sin.	cos.				sin.	cos.
$\varkappa$	i'	i	″	″	$\varkappa$	i'	i	″	″
1	1—	2	—0.08349	—0.36046	—1	3—	1	—0.58397	+1.03259
—1	1—	1	+0.06653	—0.13184	0	3—	2	+0.56624	—0.94342
0	1—	2	—0.08326	+0.10276	1	3—	3	—0.09272	+0.34179
1	1—	3	+0.03881	—0.03483	—1	3—	2	—2.23812	—3.15197
—1	1—	2	+0.00030	—0.00178	0	3—	3	+2.19376	+3.00953
0	1—	3	—0.00510	+0.00569	1	3—	4	—0.65846	—0.89466
1	1—	4	+0.00319	—0.00280	—1	3—	3	—0.21942	—0.07115
—1	1—	3	—0.00006	—0.00025	0	3—	4	+0.31759	+0.20540
0	1—	4	—0.00027	+0.00051	1	3—	5	—0.12147	—0.09031
1	1—	5	+0.00023	—0.00022	—1	3—	4	—0.01239	—0.00180
—1	1—	4	0.00000	—0.00001	0	3—	5	+0.02827	+0.01264
0	1—	5	—0.00002	+0.00004	1	3—	6	—0.01303	—0.00688
1	1—	6	+0.00001	—0.00001	—1	3—	5	—0.00070	0.00000
0	2+	2	—0.00002	—0.00003	0	3—	6	+0.00223	+0.00074
1	2+	1	+0.00002	+0.00004	1	3—	7	—0.00116	—0.00048
—1	2+	2	+0.00006	—0.00007	—1	3—	6	—0.00005	0.00000
0	2+	1	—0.00024	+0.00018	0	3—	7	+0.00017	+0.00004
1	2	0	+0.00071	—0.00015	1	3—	8	—0.00011	—0.00004
—1	2+	1	—0.00196	—0.01055	—1	4+	1	—0.00006	+0.00009
1	2—	1	—0.02117	+0.01579	1	4—	1	—0.00010	+0.00033
—1	2	0	+0.64706	—0.04388	—1	4	0	+0.00530	—0.00956
0	2—	1	—0.51214	+0.12447	0	4—	1	—0.00458	+0.00776
1	2—	2	+0.24430	—0.17179	1	4—	2	—0.00036	—0.00623
—1	2—	1	—5.81704	+2.50846	—1	4—	1	+0.02317	+0.15242
0	2—	2	+5.53843	—2.39313	0	4—	2	—0.01413	—0.13911
1	2—	3	—1.58653	+0.67972	1	4—	3	+0.03051	+0.04457
—1	2—	2	—0.06648	+0.19031	—1	4—	2	—0.95843	—0.32489
0	2—	3	+0.31512	—0.30008	0	4—	3	+0.90735	+0.32029
1	2—	4	—0.14661	+0.12000	1	4—	4	—0.30618	—0.04826
—1	2—	3	—0.00303	+0.00756	—1	4—	3	+1.47714	—1.62413
0	2—	4	+0.02037	—0.02307	0	4—	4	—1.39900	+1.59907
1	2—	5	—0.01131	+0.01151	1	4—	5	+0.42152	—0.48958
—1	2—	4	—0.00007	+0.00034	—1	4—	4	+0.03277	—0.19139
0	2—	5	+0.00129	—0.00165	0	4—	5	—0.09502	+0.26225
1	2—	6	—0.00083	+0.00095	1	4—	6	+0.04180	—0.09795
—1	2—	5	+0.00001	+0.00002	—1	4—	5	—0.00105	—0.01322
0	2—	6	+0.00008	—0.00013	0	4—	6	—0.00394	+0.02609
1	2—	7	—0.00007	+0.00009	1	4—	7	+0.00252	—0.01148
—1	3+	2	+0.00001	0.00000	—1	4—	6	-0.00020	—0.00085
0	3+	1	+0.00001	—0.00001	0	4—	7	—0.00004	+0.00222
1	3	0	+0.00005	0.00000	1	4—	8	+0.00010	—0.00111
—1	3+	1	—0.00090	—0.00012	—1	4—	7	—0.00001	—0.00005
1	3—	1	—0.00205	+0.00264	0	4—	8	+0.00001	+0.00018
—1	3	0	+0.085207	—0.056091	1	4—	9	—0.00001	—0.00011
0	3—	1	—0.0678000	+0.0484146	—1	5	0	0.00000	—0.00095
1	3—	2	+0.025043	—0.046023	0	5—	1	—0.00008	+0.00078

Arg $=\varkappa\gamma+i'g'+ig$			T	
			sin.	cos.
$\varkappa$	i'	i	″	″
1	5	−2	−0.00039	−0.00048
−1	5	−1	+0.01244	+0.01143
0	5	−2	−0.01033	−0.01098
1	5	−3	+0.00710	+0.00111
−1	5	−2	−0.15962	+0.05245
0	5	−3	+0.15131	−0.04443
1	5	−4	−0.04565	+0.03336
−1	5	−3	+0.09432	−0.73665
0	5	−4	−0.09402	+0.70755
1	5	−5	−0.00137	−0.23034
−1	5	−4	+1.04036	+0.59076
0	5	−5	−1.02384	−0.54615
1	5	−6	+0.31645	+0.16417
−1	5	−5	+0.14136	−0.00148
0	5	−6	−0.18648	−0.02280
1	5	−7	+0.06861	+0.01171
−1	5	−6	+0.01116	−0.00340
0	5	−7	−0.02019	+0.00213
1	5	−8	+0.00861	−0.00037
−1	5	−7	+0.00077	−0.00041
0	5	−8	−0.00183	+0.00048
1	5	−9	+0.00087	−0.00016
−1	5	−8	+0.00003	−0.00003
0	5	−9	−0.00012	+0.00003
1	5	−10	+0.00009	−0.00001
−1	6	−1	+0.001647	+0.000127
0	6	−2	−0.001446	−0.000219
1	6	−3	+0.000703	−0.000544
−1	6	−2	−0.012600	+0.018545
0	6	−3	+0.012546	−0.016620
1	6	−4	−0.00139	+0.00838
−1	6	−3	−0.07267	−0.13209
0	6	−4	+0.06655	+0.12735
1	6	−5	−0.03388	−0.03674
−1	6	−4	+0.49771	−0.03938
0	6	−5	−0.48117	+0.03873
1	6	−6	+0.15312	−0.02983
−1	6	−5	−0.18016	+0.60780
0	6	−6	+0.15477	−0.59574
1	6	−7	−0.04500	+0.18481
−1	6	−6	+0.01944	+0.09241
0	6	−7	−0.01254	−0.11859
1	6	−8	+0.00284	+0.04315
−1	6	−7	+0.00452	+0.00797
0	6	−8	−0.00504	−0.01368
1	6	−9	+0.00173	+0.00572
−1	6	−8	+0.00050	+0.00056
0	6	−9	−0.00075	−0.00126
1	6	−10	+0.00028	+0.00061
−1	7	−1	+0.00011	−0.00009
0	7	−2	−0.00012	+0.00007
1	7	−3	+0.00003	−0.00008
−1	7	−2	+0.00017	+0.00261
0	7	−3	−0.00001	−0.00240
1	7	−4	+0.00064	+0.00092
−1	7	−3	−0.02168	−0.00981
0	7	−4	+0.02017	+0.01002
1	7	−5	−0.00855	−0.00072
−1	7	−4	+0.09201	−0.07725
0	7	−5	−0.08949	+0.07305
1	7	−6	+0.02467	−0.03093
−1	7	−5	+0.09072	+0.30347
0	7	−6	−0.08997	−0.29382
1	7	−7	+0.03800	+0.09182
−1	7	−6	−0.32841	−0.01732
0	7	−7	+0.31950	+0.00342
1	7	−8	−0.09905	+0.00089
−1	7	−7	−0.05455	+0.02401
0	7	−8	+0.06855	−0.02397
1	7	−9	−0.02477	+0.00769
−1	7	−8	−0.00492	+0.00446
0	7	−9	+0.00822	−0.00555
1	7	−10	−0.00339	+0.00201
−1	7	−9	−0.00036	+0.00053
0	7	−10	+0.00078	−0.00078
1	7	−11	−0.00037	+0.00032
−1	8	−2	+0.00019	+0.00018
0	8	−3	−0.00015	−0.00020
1	8	−4	+0.00012	+0.00004
−1	8	−3	−0.00313	+0.00071
0	8	−4	+0.00298	−0.00053
1	8	−5	−0.00102	+0.00079
−1	8	−4	+0.00523	−0.02131
0	8	−5	−0.00555	+0.02022
1	8	−6	−0.00032	−0.00768
−1	8	−5	+0.06920	+0.05488
0	8	−6	−0.06642	−0.05358
1	8	−7	+0.02539	+0.01386
−1	8	−6	−0.16854	+0.09347
0	8	−7	+0.16281	−0.09276
1	8	−8	−0.04995	+0.03431

Arg=$\varkappa\gamma+i'g'+ig$			T	
			sin.	cos.
$\varkappa$	i'	i	″	″
−1	8	−7	−0.03182	−0.16467
0	8	−8	+0.03900	+0.15828
1	8	−9	−0.01331	−0.04886
−1	8	−8	−0.02124	−0.02917
0	8	−9	+0.02292	+0.03609
1	8	−10	−0.00771	−0.01296
−1	8	−9	−0.00375	−0.00262
0	8	−10	+0.00480	+0.00435
1	8	−11	−0.00179	−0.00180
−1	8	−10	−0.00041	−0.00015
0	8	−11	+0.00063	+0.00036
1	8	−12	−0.00027	−0.00019
−1	9	−3	−0.00025	+0.00030
0	9	−4	+0.00024	−0.00027
1	9	−5	−0.00002	+0.00017
−1	9	−4	−0.00132	−0.00310
0	9	−5	+0.00114	+0.00300
1	9	−6	−0.00087	−0.00093
−1	9	−5	+0.01831	+0.00087
0	9	−6	−0.01761	−0.00120
1	9	−7	+0.00620	−0.00121
−1	9	−6	−0.02740	+0.05479
0	9	−7	+0.02676	−0.05307
1	9	−8	−0.00615	+0.01905
−1	9	−7	−0.07543	−0.08493
0	9	−8	+0.07479	+0.08151
1	9	−9	−0.02614	−0.02442
−1	9	−8	+0.07621	−0.03618
0	9	−9	−0.07191	+0.03957
1	9	−10	+0.02196	−0.01296
−1	9	−9	+0.01392	−0.01579
0	9	−10	−0.01704	+0.01755
1	9	−11	+0.00608	−0.00601
−1	9	−10	+0.00126	−0.00329
0	9	−11	−0.00210	+0.00408
1	9	−12	+0.00090	−0.00149
−1	10	−4	−0.00041	−0.00024
0	10	−5	+0.00036	+0.00024
1	10	−6	−0.00017	−0.00002
−1	10	−5	+0.00262	−0.00174
0	10	−6	−0.00255	+0.00159
1	10	−7	+0.00071	−0.00090
−1	10	−6	+0.00216	+0.01411
0	10	−7	−0.00192	−0.01368
1	10	−8	+0.00174	+0.00455
−1	10	−7	−0.03933	−0.01022
0	10	−8	+0.03825	+0.00990
1	10	−9	−0.01315	−0.00153
−1	10	−8	+0.03785	−0.05302
0	10	−9	−0.03582	+0.05244
1	10	−10	+0.01027	−0.01767
−1	10	−9	+0.02836	+0.03081
0	10	−10	−0.02985	−0.02814
1	10	−11	+0.00986	+0.00830
−1	10	−10	+0.00657	+0.01003
0	10	−11	−0.00795	−0.01095
1	10	−12	+0.00291	+0.00354
−1	11	−5	+0.00016	−0.00045
0	11	−6	−0.00018	+0.00042
1	11	−7	+0.00002	−0.00018
−1	11	−6	+0.00192	+0.00194
0	11	−7	−0.00180	−0.00192
1	11	−8	+0.00083	+0.00048
−1	11	−7	−0.00993	+0.00373
0	11	−8	+0.00963	−0.00348
1	11	−9	−0.00305	+0.00182
−1	11	−8	+0.00148	−0.02657
0	11	−9	−0.00153	+0.02577
1	11	−10	−0.00016	−0.00857
−1	11	−9	+0.02915	+0.01746
0	11	−10	−0.02817	−0.01644
1	11	−11	+0.00808	+0.00523
−1	11	−10	+0.00785	+0.00548
0	11	−11	−0.00870	−0.00606
1	11	−12	+0.00273	+0.00200

Arg=$\kappa\gamma+i'g'+ig$			$\frac{1}{n}\frac{dR}{dt}$		Arg=$\kappa\gamma+i'g'+ig$			$\frac{1}{n}\frac{dR}{dt}$	
			cos.	sin.				cos.	sin.
κ	i'	i	″	″	κ	i'	i	″	″
1	0	—1	+0.00481	—0.01283	—1	3	—3	+0.00032	+0.01006
—1	0	0	+0.0428953	—0.0327927	1	3	—5	+0.00003	—0.00972
1	0	—2	—0.04299	+0.03138	—1	3	—4	+0.00002	—0.00003
—1	0	—1	—0.00490	+0.00075	1	3	—6	—0.00002	—0.00108
1	0	—3	+0.00009	+0.00281	—1	3	—5	0.00000	—0.00001
—1	0	—2	+0.00006	+0.00014	1	3	—7	0.00000	—0.00009
1	0	—4	+0.00004	+0.00011	—1	4	+1	—0.00005	—0.00008
—1	1	+3	+0.00006	—0.00021	1	4	—1	—0.00006	+0.00021
1	1	+1	—0.00030	—0.00035	—1	4	0	—0.00076	+0.00123
—1	1	+2	—0.00268	—0.00454	1	4	—2	+0.00172	—0.00096
1	1	0	—0.00044	+0.00735	—1	4	—1	+0.00854	+0.00189
—1	1	+1	—0.02737	+0.02542	1	4	—3	—0.00866	—0.00409
1	1	—1	+0.02861	—0.02477	—1	4	—2	—0.00290	—0.01857
—1	1	0	+0.01141	—0.00207	1	4	—4	+0.00175	+0.01848
1	1	—2	—0.00978	—0.00398	—1	4	—3	—0.00144	+0.00329
—1	1	—1	—0.01411	—0.02913	1	4	—5	+0.00224	—0.00109
1	1	—3	+0.01296	+0.02896	—1	4	—4	+0.00531	+0.00098
—1	1	—2	—0.00007	+0.00198	1	4	—6	—0.00504	—0.00114
1	1	—4	+0.00155	+0.00128	—1	4	—5	+0.00011	+0.00008
—1	1	—3	+0.00005	+0.00004	1	4	—7	—0.00068	—0.00021
1	1	—5	+0.00010	+0.00004	—1	4	—6	0.00000	+0.00001
—1	2	+2	—0.00008	—0.00047	1	4	—8	—0.00008	—0.00002
1	2	0	—0.00076	+0.00036	—1	5	0	—0.00015	+0.00006
—1	2	+1	—0.00695	—0.00008	1	5	—2	+0.00024	+0.00006
1	2	—1	+0.00997	+0.00533	—1	5	—1	+0.00104	+0.00106
—1	2	0	+0.02770	+0.04640	1	5	—3	—0.00071	—0.00169
1	2	—2	—0.02662	—0.04693	—1	5	—2	+0.00265	—0.00575
—1	2	—1	—0.00076	—0.00994	1	5	—4	—0.00389	+0.00551
1	2	—3	—0.00423	+0.00517	—1	5	—3	—0.01046	—0.00047
—1	2	—2	—0.01782	+0.00443	1	5	—5	+0.01021	+0.00123
1	2	—4	+0.01745	—0.00374	—1	5	—4	+0.00162	+0.00125
—1	2	—3	+0.00056	+0.00013	1	5	—6	—0.00034	—0.00141
1	2	—5	+0.00141	—0.00056	—1	5	—5	+0.00112	—0.00261
—1	2	—4	+0.00003	—0.00001	1	5	—7	—0.00116	+0.00244
1	2	—6	+0.00010	—0.00004	—1	5	—6	+0.00011	—0.00010
—1	3	+2	+0.00001	—0.00004	1	5	—8	—0.00024	+0.00038
1	3	0	—0.00010	—0.00004	—1	5	—7	0.00000	0.00000
—1	3	+1	—0.00077	—0.00047	1	5	—9	—0.00003	+0.00004
1	3	—1	+0.000696	+0.001690	—1	6	—1	+0.00004	+0.00020
—1	3	0	—0.00002	+0.01069	1	6	—3	+0.00009	—0.00027
1	3	—2	+0.00356	—0.01176	—1	6	—2	+0.001107	—0.000691
—1	3	—1	+0.030730	—0.011244	1	6	—4	—0.00147	+0.00036
1	3	—3	—0.03101	+0.00979	—1	6	—3	—0.00341	—0.00252
—1	3	—2	—0.00605	—0.00099	1	6	—5	+0.00309	+0.00319
1	3	—4	+0.00260	+0.00325	—1	6	—4	—0.00140	+0.00554

Arg=$\varkappa\gamma+i'g'+ig$		$\frac{1}{n}\frac{dR}{dt}$		Arg=$\varkappa\gamma+i'g'+ig$		$\frac{1}{n}\frac{dR}{dt}$	
		cos.	sin.			cos.	sin.
$\varkappa$	i' i	″	″	$\varkappa$	i' i	″	″
1	6—6	+0.00185	—0.00526	—1	7—4	—0.00203	+0.00179
—1	6—5	+0.00090	—0.00070	1	7—6	+0.00235	—0.00150
1	6—7	—0.00084	+0.00001	—1	7—5	+0.00277	+0.00134
—1	6—6	—0.00119	—0.00085	1	7—7	—0.00253	—0.00158
1	6—8	+0.00109	+0.00085	—1	7—6	—0.00026	—0.00058
—1	6—7	—0.00006	—0.00011	1	7—8	—0.00010	+0.00046
1	6—9	+0.00019	+0.00020	—1	7—7	—0.00057	+0.00050
—1	7—2	+0.00021	0.00000	1	7—9	+0.00056	—0.00044
1	7—4	—0.00024	—0.00012	—1	7—8	—0.00008	+0.00002
—1	7—3	—0.00035	—0.00096	1	7—10	+0.00015	—0.00007
1	7—5	+0.00008	+0.00115				

The logarithms of the integrating factors are contained in the following table:

Arg.	$\log\frac{n}{i'n'+in}$	Arg.	$\log\frac{n}{i'n'+in}$	Arg.	$\log\frac{n}{i'n'+in}$	Arg.	$\log\frac{n}{i'n'+in}$
i' i		i' i		i' i		i' i	
0—1	0.0000000*n*	3—5	9.40360*n*	6—4	9.72209*n*	9—3	0.8081
0—2	9.6989700*n*	3—6	9.3056*n*	6—5	9.53816*n*	9—4	0.0734*n*
0—3	9.5228787*n*	3—7	9.2256*n*	6—6	9.40935*n*	9—5	9.7341*n*
0—4	9.39794*n*	3—8	9.1581*n*	6—7	9.31013*n*	9—6	9.5460*n*
1+3	9.4749	4+1	9.6194	6—8	9.22942*n*	9—7	9.4152*n*
1+2	9.6288	4 0	9.8531	6—9	9.1614*n*	9—8	9.3148*n*
1+1	9.86947	4—1	0.39527	6—10	9.1026*n*	9—9	9.2333*n*
1 0	0.4551677	4—2	0.22364*n*	7—1	9.8374	9—10	9.1647*n*
1—1	0.1874988*n*	4—3	9.79655*n*	7—2	0.3426	9—11	9.1054*n*
1—2	9.78268*n*	4—4	9.58544*n*	7—3	0.2631*n*	9—12	9.0533*n*
1—3	9.57685*n*	4—5	9.44399*n*	7—4	9.81088*n*	10—4	0.3064*n*
1—4	9.4378*n*	4—6	9.3375*n*	7—5	9.59420*n*	10—5	9.8257*n*
1—5	9.3326*n*	4—7	9.2520*n*	7—6	9.45030*n*	10—6	9.6031*n*
2+2	9.5684	4—8	9.1806*n*	7—7	9.34240*n*	10—7	9.4567*n*
2+1	9.7692	4—9	9.1193*n*	7—8	9.25604*n*	10—8	9.3474*n*
2 0	0.15414	5 0	9.7562	7—9	9.1840*n*	10—9	9.2601*n*
2—1	0.5246674*n*	5—1	0.1232	7—10	9.1223*n*	10—10	9.1875*n*
2—2	9.8864688*n*	5—2	0.60745*n*	7—11	9.0683*n*	10—11	9.1253*n*
2—3	9.6385050*n*	5—3	9.90416*n*	8—2	0.0943	10—12	9.0709*n*
2—4	9.48165*n*	5—4	9.64841*n*	8—3	0.7098*n*	11—4	0.8440*n*
2—5	9.3667*n*	5—5	9.48853*n*	8—4	9.9226*n*	11—5	9.9418*n*
2—6	9.2758*n*	5—6	9.37193*n*	8—5	9.6586*n*	11—6	9.6689*n*
3+1	9.6879	5—7	9.28009*n*	8—6	9.4955*n*	11—7	9.5026*n*
3 0	9.9780464	5—8	9.2043*n*	8—7	9.3773*n*	11—8	9.3827*n*
3—1	1.2852549	5—9	9.1398*n*	8—8	9.2844*n*	11—9	9.2888*n*
3—2	0.0231228*n*	6—1	9.9572	8—9	9.2080*n*	11—10	9.2116*n*
3—3	9.7103776*n*	6—2	0.98422	8—10	9.1430*n*	11—11	9.1461*n*
3—4	9.53045*n*	6—3	0.04755*n*	8—11	9.0864*n*	11—12	9.0892*n*
				8—12	9.0364*n*		

In making the integrations, for a like purpose, as in Chapter II, we put

$k_0 = +1''.6639 \qquad k_1 = -0''.2304 \qquad k_2 = -0''.1420 \qquad k_3 = -0''.0216 \qquad k_4 = -0''.0163$

The values of $\frac{d\delta z}{dt}$ and $\frac{dv}{dt}$ obtained are

Arg.	$\frac{d\delta z}{dt}$		$\frac{1}{n}\frac{dv}{dt}$	
	cos.	sin.	sin.	cos.
i' i	″	″	″	″
0— 0				— 0.001193
0— 1	— 0.1557	— 0.0850	+0.1573	— 0.0776
	+ 0.0850031*nt*	— 0.1556525*nt*	—0.042501*nt*	— 0.077826*nt*
0— 2	— 0.0067	— 0.0035	+0.0089	— 0.0041
	+ 0.002382*nt*	— 0.004362*nt*	—0.002382*nt*	— 0.004362*nt*
0— 3	— 0.0004	— 0.0002	+0.0006	— 0.0003
0— 3	+ 0.000100*nt*	— 0.000183*nt*	—0.000150*nt*	— 0.000275*nt*
1+ 2	— 0.0010	+ 0.0001	—0.0012	— 0.0001
1+ 1	— 0.0307	— 0.0003	—0.0209	+ 0.0016
1 0	— 0.28914	— 0.19699	—0.05475	+ 0.01448
1— 1	— 1.0701	+ 5.5233	+0.3728	+ 1.9271
1— 2	+ 0.0444	+ 0.2679	—0.0216	+ 0.2090
1— 3	+ 0.0012	+ 0.0103	—0.0012	+ 0.0130
1— 4	0.0000	+ 0.0004	—0.0001	+ 0.0009
2+ 1	— 0.0027	— 0.0014	—0.0022	+ 0.0014
2 0	— 0.0652	— 0.0557	—0.0319	+ 0.0201
2— 1	— 2.46819	— 0.25498	+0.53598	— 0.08925
2— 2	—15.9230	— 6.8519	+9.4146	— 4.0510
2— 3	— 0.5050	— 0.2877	+0.5502	— 0.2902
2— 4	— 0.0220	— 0.0130	+0.0352	— 0.0192
2— 5	— 0.0011	— 0.0007	+0.0023	— 0.0014
3+ 1	+ 0.0011	+ 0.0014	+0.0011	— 0.0014
3 0	+ 0.0406	+ 0.0522	+0.0202	— 0.0266
3— 1	+ 1.25145	+ 0.92442	+0.05199	— 0.04655
3— 2	+11.8038	+20.7417	—5.6592	+10.0352
3— 3	— 1.1433	+ 2.6413	+0.7523	+ 2.0675
3— 4	— 0.0887	+ 0.1062	+0.0870	+ 0.1351
3— 5	— 0.0049	+ 0.0049	+0.0069	+ 0.0088
3— 6	— 0.0002	+ 0.0002	+0.0006	+ 0.0005
4 0	+ 0.0003	+ 0.0011	+0.0001	— 0.0007
4— 1	+ 0.0074	+ 0.0221	+0.0019	— 0.0078
4— 2	— 0.0654	+ 0.5841	+0.0353	+ 0.2024
4— 3	— 1.1522	+ 0.3762	+0.7480	+ 0.2693
4— 4	+ 0.4585	+ 0.5534	—0.3584	+ 0.4556
4— 5	+ 0.0193	+ 0.0513	—0.0249	+ 0.0545
4— 6	+ 0.0007	+ 0.0034	—0.0015	+ 0.0054
5— 1	— 0.0013	+ 0.0024	—0.0006	— 0.0013
5— 2	— 0.05323	+ 0.05842	+0.01054	+ 0.00834
5— 3	— 0.5458	— 0.1892	+0.3136	— 0.1003
5— 4	+ 0.0174	+ 0.3398	—0.0178	+ 0.2551
5— 5	+ 0.2239	— 0.1172	—0.1948	— 0.0960
5— 6	+ 0.0258	— 0.0030	—0.0284	— 0.0051
5— 7	+ 0.0020	+ 0,0002	—0.0027	— 0.0001

Arg.	$\frac{d\delta z}{dt}$		$\frac{1}{n}\frac{dv}{dt}$	
	cos.	sin.	sin.	cos.
i' i	″	″	″	″
6—2	+0.01321	−0.00150	+0.00110	−0.00014
6—3	+0.1346	+0.1932	−0.0614	+0.0913
6—4	−0.0532	+0.0981	+0.0324	+0.0722
6—5	+0.1332	+0.0183	−0.1097	+0.0120
6—6	−0.0244	−0.0940	+0.0194	−0.0849
6—7	+0.0015	−0.0127	−0.0011	−0.0141
6—8	+0.0003	−0.0010	−0.0004	−0.0015
7—3	−0.0001	+0.0095	+0.0003	+0.0032
7—4	−0.0299	+0.0122	+0.0184	+0.0089
7—5	+0.0304	+0.0299	−0.0250	+0.0217
7—6	+0.0198	−0.0561	−0.0158	−0.0481
7—7	−0.0393	+0.0007	+0.0363	−0.0008
7—8	−0.0060	−0.0021	+0.0067	−0.0019
8—3	−0.00094	+0.00122	+0.00016	+0.00014
8—4	−0.0140	−0.0034	+0.0078	−0.0017
8—5	+0.0016	+0.0108	−0.0019	+0.0077
8—6	+0.0167	−0.0112	−0.0131	−0.0101
8—7	−0.0232	−0.0142	+0.0206	−0.0122
8—8	−0.0036	+0.0158	+0.0042	+0.0149
8—9	−0.0017	+0.0027	+0.0017	+0.0030
9—4	+0.0019	+0.0021	−0.0008	+0.0010
9—5	−0.0007	+0.0024	+0.0010	+0.0016
9—6	+0.0053	+0.0002	−0.0042	−0.0004
9—7	−0.0036	−0.0094	+0.0039	−0.0076
9—8	−0.0090	+0.0090	+0.0079	+0.0084
9—9	+0.0061	+0.0032	−0.0058	+0.0034
9—10	+0.0011	+0.0011	−0.0012	+0.0012
10—5	−0.0007	+0.0003	+0.0004	+0.0002
10—6	+0.0010	+0.0009	−0.0008	+0.0005
10—7	+0.0008	−0.0028	−0.0002	−0.0023
10—8	−0.0052	+0.0008	+0.0044	+0.0012
10—9	+0.0032	+0.0051	−0.0031	+0.0046
10—10	+0.0021	−0.0021	−0.0021	−0.0019
10—11	+0.0003	−0.0006	−0.0004	−0.0008
11—6	0.0000	+0.0003	−0.0001	+0.0002
11—7	+0.0005	−0.0002	−0.0003	−0.0005
11—8	−0.0015	−0.0008	+0.0013	−0.0004
11—9	+0.0001	+0.0028	−0.0002	+0.0025
11—10	+0.0022	−0.0012	−0.0022	−0.0011
11—11	+0.0006	−0.0004	−0.0006	−0.0004

Integrating again we obtain $n\delta z$ and ν. The constant term of the latter quantity is obtained in the way mentioned at the end of Chapter II. As the quantities now given appertain to Saturn, we will restore to the symbols their accent, and the mean anomaly of Uranus will be denoted as g''.

Arg=$i''g''+i'g'$	$n'\delta z'$		ν'		$\frac{u'}{\cos i'}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i'' i'	″	″	″	″	″	″
0 0			−0.2741			−0.0110
			−0.001193$n't$			−0.003611$n't$
0 − 1	0.0000	0.0000	+0.0795	+0.0351	0.0000	0.0000
	−0.085000$n't$	−0.155652$n't$	−0.042500$n't$	+0.077826$n't$	−0.032793$n't$	+0.042895$n't$
0 − 2	+0.0022	−0 0011	+0.0033	+0.0014	+0.0025	−0.0002
	−0.001191$n't$	−0.002181$n't$	−0.001191$n't$	+0.002181$n't$	−0.000918$n't$	+0.001201$n't$
0 − 3	+0.0001	−0.0001	+0.0001	+0.0001		
	−0.000033$n't$	−0.000061$n't$	−0.000050$n't$	+0.000092$n't$	−0.000039$n't$	+0.000050$n't$
1 + 2	−0.0004	0.0000	+0.0005	+0.0000		
1 + 1	−0.0227	+0.0002	+0.0155	+0.0012	−0.0019	−0.0186
1 0	−0.8247	+0.5618	+0.1562	+0.0413	−0.0668	−0.0556
1 − 1	+1.6479	+8.5055	+0.5741	−2.9676	+0.0396	+0.0103
1 − 2	−0.0269	+0.1624	−0.0131	−0.1267	+0.0172	−0.0337
1 − 3	−0.0005	+0.0039	−0.0005	−0.0049	+0.0006	−0.0006
1 − 4	0.0000	+0.0001	0.0000	−0.0002		
2 + 1	−0.0016	+0.0008	+0.0013	+0.0008	−0.0017	+0.0002
2 0	−0.0930	+0.0794	+0.0455	+0.0287	−0.0403	+0.0224
2 − 1	+8.2613	−0.8535	+1.7940	+0.2987	+0.0626	−0.1009
2 − 2	+12.2601	−5.2757	+7.2489	+3.1191	+0.0024	−0.0302
2 − 3	+0.2197	−0.1251	+0.2394	+0.1262	+0.0084	+0.0011
2 − 4	+0.0067	−0.0039	+0.0107	+0.0058		
2 − 5	+0.0003	−0.0002	+0.0005	+0.0003		
3 + 1	+0.0005	−0.0007	−0.0005	−0.0007	+0.0004	−0.0009
3 0	+0.0386	−0.0496	−0.0192	−0.0253	+0.0129	−0.0318
3 − 1	+24.1362	−17.8289	−1.0026	−0.8978	−0.0544	−0.0380
3 − 2	−12.4493	+21.8760	−5.9688	−10.5840	+0.6084	+0.2227
3 − 3	+0.5869	+1.3558	+0.3862	−1.0612	+0.0208	+0.0053
3 − 4	+0.0301	+0.0360	+0.0295	−0.0458	+0.0008	+0.0029
3 − 5	+0.0012	+0.0012	+0.0017	−0.0022		
4 0	+0.0002	−0.0008	−0.0001	−0.0005		
4 − 1	+0.0184	−0.0549	−0.0047	−0.0194	−0.0050	−0.0022
4 − 2	+0.1094	+0.9775	+0.0591	−0.3387	+0.0264	−0.0043
4 − 3	+0.7212	+0.2354	+0.4682	−0.1686	+0.0043	−0.0241
4 − 4	−0.1765	+0.2130	−0.1380	−0.1754	+0.0006	+0.0003
4 − 5	−0.0054	+0.0143	−0.0069	−0.0151	−0.0008	+0.0002
4 − 6	−0.0002	+0.0007	−0.0003	−0.0012		
5 − 1	−0.0017	−0.0032	+0.0008	−0.0017	−0.0012	+0.0003
5 − 2	+0.2155	+0.2366	+0.0427	−0.0338	+0.0030	−0.0008
5 − 3	+0.4377	−0.1517	+0.2514	+0.0804	−0.0099	−0.0207

Arg $=i''g''+i'g'$	$n'\delta z'$		ν'		$\frac{u'}{\cos i'}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i'' i'	″	″	″	″	″	″
5— 4	−0.0077	+0.1512	−0.0079	−0.1135	+0.0049	−0.0010
5— 5	−0.0689	−0.0361	−0.0600	+0.0296		
5— 6	−0.0061	−0.0007	−0.0067	+0.0012		
5— 7	−0.0004	0.0000	−0.0005	0.0000		
6— 2	+0.1274	+0.0145	−0.0106	−0.0014	−0.0010	−0.0011
6— 3	−0.1501	+0.2155	−0.0685	−0.1019	+0.0112	+0.0072
6— 4	+0.0281	+0.0517	+0.0171	−0.0381	+0.0029	−0.0018
6— 5	−0.0460	+0.0063	−0.0379	−0.0041	+0.0004	+0.0014
6— 6	+0.0063	−0.0241	+0.0050	+0.0218		
6— 7	−0.0003	−0.0026	−0.0002	+0.0029		
6— 8	−0.0001	−0.0002	−0.0001	+0.0003		
7— 3	+0.0002	+0.0174	+0.0006	−0.0059	+0.0006	0.0000
7— 4	+0.0193	+0.0079	+0.0119	−0.0058	+0.0005	−0.0015
7— 5	−0.0119	+0.0117	−0.0098	−0.0085	+0.0007	+0.0007
7— 6	−0.0056	−0.0158	−0.0045	+0.0136	−0.0005	+0.0002
7— 7	+0.0086	+0.0002	+0.0080	+0.0002		
7— 8	+0.0011	−0.0004	+0.0012	+0.0003		
8— 3	+0.0048	+0.0063	+0.0008	−0.0007		
8— 4	+0.0117	−0.0028	+0.0065	+0.0014		
8— 5	−0.0007	+0.0049	−0.0009	−0.0035		
8— 6	−0.0052	−0.0035	−0.0041	+0.0032		
8— 7	+0.0054	−0.0034	+0.0049	+0.0029		
8— 8	+0.0007	+0.0030	+0.0008	−0.0029		
8— 9	+0.0003	+0.0004	+0.0003	−0.0005		
9— 4	−0.0022	+0.0025	−0.0009	−0.0012		
9— 5	+0.0004	+0.0013	+0.0005	−0.0009		
9— 6	−0.0019	+0.0001	−0.0015	+0.0001		
9— 7	+0.0009	−0.0024	+0.0010	+0.0020		
9— 8	+0.0019	+0.0019	+0.0016	−0.0017		
9— 9	−0.0010	+0.0005	−0.0010	−0.0006		
9—10	−0.0002	+0.0002	−0.0002	−0.0002		
10— 5	+0.0005	+0.0002	+0.0003	−0.0001		
10— 6	−0.0004	+0.0004	−0.0003	−0.0002		
10— 7	−0.0002	−0.0008	−0.0001	+0.0005		
10— 8	+0.0012	+0.0002	+0.0010	−0.0003		
10— 9	−0.0006	+0.0009	−0.0006	−0.0008		
10—10	−0.0003	−0.0003	−0.0003	+0.0003		
11— 7	−0.0002	−0.0001	−0.0001	+0.0001		
11— 8	+0.0004	−0.0002	+0.0003	+0.0001		
11— 9	0.0000	+0.0005	0.0000	−0.0005		
11—10	−0.0004	−0.0002	−0.0004	+0.0002		

CHAPTER IV.

PERTURBATIONS OF JUPITER BY URANUS OF THE FIRST ORDER WITH RESPECT TO DISTURBING FORCES.

For a like reason as in the preceding chapter we here denote the quantities pertaining to Jupiter without accents and those pertaining to Uranus with a single accent.

The elements of the two planets being the same as those which have already been given (pages 19, 109), we have the corrected $\log \mathrm{a} = 0.7162333$ and corrected $\log \mathrm{a}' = 1.2831044$. Whence $\log \alpha = 9.4331289$. The coefficients of the terms of the developments of the reciprocal of the distance between Jupiter and Uranus $\frac{\mathrm{a}'}{\Delta}$ and its odd powers are functions of the six following elements:

$$\begin{array}{rlcrrr}
 & & \qquad & & {}^{\circ} \quad {}' & {}'' \\
\log \alpha & =9.4331289 & & \mathrm{J} = & 0 \quad 42 & 3.44 \\
e & =0.04824277 & & \Pi = & 64 \quad 26 & 56.50 \\
e' & =0.0469236 & & \Pi' = & 220 \quad 46 & 7.70
\end{array}$$

Π and Π' are measured from the ascending node of the orbit of Uranus on that of Jupiter. In developing these functions it is better to take the eccentric anomaly of Jupiter, as, in this way, the quantity γ_2 is smaller.

The values of the auxiliary constants, entirely similar to those of Chapter I, are

$$\begin{array}{rlcrrr}
 & & \qquad & & {}^{\circ} \quad {}' & {}'' \\
\log k & =9.9999735 & & \mathrm{K} = & 156 \quad 19 & 17.20 \\
\log k_1 & =9.9999939 & & \mathrm{K}_1 = & 156 \quad 19 & 5.20 \\
\log p & =9.8345468 & & \mathrm{P} = & 71 \quad 26 & 4.40 \\
\log v & =9.7332373 & & \mathrm{V} = & 156 \quad 17 & 50.42 \\
\log w & =9.7340868 & & \mathrm{W} = & 84 \quad 52 & 22.26 \\
\log w_1 & =9.7332136 & & \mathrm{W}_1 = & 84 \quad 53 & 51.63 \\
\log \frac{1}{2}\gamma_2 & =5.9320922 & & & &
\end{array}$$

On account of the smallness of α and the consequent smallness of $\frac{1}{2}\gamma_2$, we shall not employ the transformation involving the quantities we have denoted as N, **a** and **b**, and the resulting $\delta \log k$ and K, but proceed by the method of Hansen,* where he puts $\mathrm{D} = \gamma_0 + \frac{1}{2}\gamma_2$, and we have

$$\left(\frac{\Delta}{\mathrm{a}'}\right)^2 = \mathrm{D} - f \cos (\varepsilon - \mathrm{F}) + \frac{1}{2}\gamma_2 \cos 2\varepsilon$$

Here we have

$$\mathrm{D} = 1.07323843 - [8.9724213] \cos \varepsilon' + [7.3427826] \cos^2 \varepsilon' + [8.6834322] f \cos \mathrm{F}$$

---

* Auseinandersetzung, Abh. I, s. 167.

As in the preceding chapter, we divide the circumference into twelve parts, with reference to g' the mean anomaly of Uranus. The values of ε', for these points, have already been given (page 110), and we obtain:

g'	D	$\log f$	$F - g'$		
			°	′	″
(0)	0.9591055	9.7077032	156	1	21.54
(1)	0.9702936	9.7104830	159	15	53.22
(2)	1.0112715	9.7199745	161	35	17.38
(3)	1.0697477	9.7328869	162	23	4.63
(4)	1.1293612	9.7454930	161	34	9.60
(5)	1.1746782	9.7548019	159	27	53.89
(6)	1.1947071	9.7589259	156	37	34.52
(7)	1.1847586	9.7570545	153	42	35.70
(8)	1.1471045	9.7494304	151	23	25.00
(9)	1.0907100	9.7374805	150	16	27.65
(10)	1.0298657	9.7239800	150	45	52.98
(11)	0.9812273	9.7128003	152	51	45.93
S	6.4714155	8.4055070	937	57	41.02
S′	6.4714154	8.4055071	937	57	41.02

We propose now to develop the quantity $[D - f\cos(\varepsilon - F)]^{-\frac{n}{2}}$, n being a positive odd integer; afterwards passing thence to the developments of $\frac{a'}{\Delta}$, $\left(\frac{a'}{\Delta}\right)^3$, etc., by multiplying by the very small factor $\frac{1}{2}\gamma_2 \cos 2\varepsilon$, and applying the resulting corrections. With HANSEN* we put

$$[D - f\cos(\varepsilon - F)]^{-\frac{n}{2}} = \alpha_0^{(n)} + 2\alpha_1^{(n)}\cos(\varepsilon - F) + 2\alpha_2^{(n)}\cos 2(\varepsilon - F) + \cdot \cdot \cdot \cdot \cdot$$

By the method given by HANSEN the values of $\log \frac{1}{6}\alpha_i^{(n)}$ have been computed. The division by six is employed for the purpose of saving the constant division by this integer, which occurs in the following process of mechanical quadratures.

---

[*] Auseinandersetzung, Abh. I, s. 150.

	$\log \frac{1}{6} \alpha_0^{(1)}$	$\log \frac{1}{6} \alpha_1^{(1)}$	$\log \frac{1}{6} \alpha_2^{(1)}$	$\log \frac{1}{6} \alpha_3^{(1)}$	$\log \frac{1}{6} \alpha_4^{(1)}$	$\log \frac{1}{6} \alpha_5^{(1)}$	$\log \frac{1}{6} \alpha_6^{(1)}$
(0)	9.2575716	8.4206334	7.7566836	7.1377105	6.5396124	5.9535909	5.375448
(1)	9.2547321	8.4150722	7.7484396	7.1267938	6.5260269	5.9373386	5.356530
(2)	9.2445835	8.3947609	7.7181073	7.0864763	6.4757387	5.8770870	5.286320
(3)	9.2308874	8.3674017	7.6772652	7.0321968	6.4080401	5.7959785	5.191807
(4)	9.2177872	8.3414298	7.6385808	6.9808401	6.3440276	5.7193187	5.102504
(5)	9.2083541	8.3229231	7.6111071	6.9444265	6.2986849	5.6650522	5.039317
(6)	9.2043262	8.3151566	7.5996449	6.9292792	6.2798568	5.6425456	5.013133
(7)	9.2063362	8.3192100	7.6057184	6.9373670	6.2899565	5.6546559	5.027252
(8)	9.2140731	8.3344040	7.6282828	6.9672800	6.3272093	5.6992441	5.079175
(9)	9.2261996	8.3581908	7.6635882	7.0140681	6.3854657	5.7689615	5.160349
(10)	9.2401094	8.3856285	7.7043793	7.0681685	6.4528575	5.8496357	5.254300
(11)	9.2519417	8.4092158	7.7395607	7.1149033	6.5111293	5.9194361	5.335624
S	5.3784510	90.1920132	86.0456787	82.1697546	78.4193023	74.7414220	71.110880
S′	5.3784511	90.1920136	86.0456792	82.1697555	78.4193034	74.7414228	71.110879

	$\log \frac{1}{6} \alpha_7^{(1)}$	$\log \frac{1}{6} \alpha_8^{(1)}$	$\log \frac{1}{6} \alpha_9^{(1)}$	$\log \frac{1}{6} \alpha_0^{(3)}$	$\log \frac{1}{6} \alpha_1^{(3)}$	$\log \frac{1}{6} \alpha_2^{(3)}$	$\log \frac{1}{6} \alpha_3^{(3)}$
(0)	4.80286	4.23436	3.66905	9.3851417	9.0071131	8.5587907	8.0827981
(1)	4.78126	4.21011	3.64214	9.3759125	8.9953866	8.5444601	8.0658349
(2)	4.70110	4.11999	3.54207	9.3428932	8.9530189	8.4923559	8.0038910
(3)	4.59318	3.99870	3.40735	9.2985349	8.8960351	8.4222507	7.9205331
(4)	4.49123	3.88410	3.28014	9.2563416	8.8418905	8.3557136	7.8414871
(5)	4.41914	3.80309	3.19020	9.2261067	8.8031994	8.3082698	7.7852125
(6)	4.38928	3.76956	3.15299	9.2132493	8.7868506	8.2883108	7.7616120
(7)	4.40542	3.78770	3.17315	9.2197005	8.7952106	8.2986443	7.7739360
(8)	4.46466	3.85426	3.24704	9.2444834	8.8269489	8.3375791	7.8201308
(9)	4.55729	3.95834	3.36256	9.2834348	8.8767414	8.3986056	7.8924949
(10)	4.66451	4.07885	3.49636	9.3283321	8.9341604	8.4690276	7.9760473
(11)	4.75736	4.18321	3.61224	9.3667520	8.9834097	8.5295479	8.0479558
S	67.51364	63.94112	60.38765	5.7704413	3.3499824	90.5017777	87.4859663
S′	67.51365	63.94115	60.38764	5.7704414	3.3499828	90.5017784	87.4859672

	$\log \frac{1}{6} \alpha_4^{(3)}$	$\log \frac{1}{6} \alpha_5^{(3)}$	$\log \frac{1}{6} \alpha_6^{(3)}$	$\log \frac{1}{6} \alpha_7^{(3)}$	$\log \frac{1}{6} \alpha_8^{(3)}$	$\log \frac{1}{6} \alpha_9^{(3)}$	$\log \frac{1}{6} \alpha_0^{(5)}$
(0)	7.5919423	7.0917965	6.585290	6.07415	5.55949	5.04207	9.57881
(1)	7.5723948	7.0695387	6.560379	6.04659	5.52928	5.00920	9.56246
(2)	7.5005087	6.9878088	6.468733	5.94501	5.41777	4.88776	9.50386
(3)	7.4038440	6.8778100	6.345385	5.80831	5.26770	4.72432	9.42535
(4)	7.3122414	6.7736264	6.228604	5.67892	5.12570	4.56970	9.35096
(5)	7.2471039	6.6996098	6.145700	5.58713	5.02501	4.46010	9.29782
(6)	7.2198491	6.6686942	6.111119	5.54888	4.98309	4.41452	9.27528
(7)	7.2341706	6.6850168	6.129446	5.56922	5.00544	4.43888	9.28664
(8)	7.2876514	6.7457967	6.197532	5.64460	5.08814	4.52889	9.33018
(9)	7.3713956	6.8409426	6.304092	5.76258	5.21754	4.66972	9.39873
(10)	7.4681307	6.9508870	6.427261	5.89899	5.36719	4.83262	9.47800
(11)	7.5514757	7.0456931	6.533543	6.01675	5.49644	4.97336	9.54609
S	84.3803236	81.2186096	78.018539	74.79055	71.54138	68.27556	6.51709
S′	84.3803246	81.2186110	78.018545	74.79058	71.54141	68.27558	6.51709

	$\log \frac{1}{6} \alpha_1^{(5)}$	$\log \frac{1}{6} \alpha_2^{(5)}$	$\log \frac{1}{6} \alpha_3^{(5)}$	$\log \frac{1}{6} \alpha_4^{(5)}$	$\log \frac{1}{6} \alpha_5^{(5)}$	$\log \frac{1}{6} \alpha_6^{(5)}$	$\log \frac{1}{6} \alpha_7^{(5)}$
(0)	9.37475	9.05490	8.67889	8.26947	7.83799	7.38991	6.9308
(1)	9.35641	9.03419	8.65564	8.24383	7.80960	7.36030	6.9000
(2)	9.29036	8.95915	8.57122	8.14963	7.70605	7.24583	6.7746
(3)	9.20166	8.85829	8.45759	8.02292	7.56586	7.09149	6.6061
(4)	9.11752	8.76265	8.34996	7.90314	7.43419	6.94840	6.4516
(5)	9.05741	8.69438	8.27315	7.81777	7.33990	6.84633	6.3417
(6)	9.03198	8.66557	8.24084	7.78184	7.30048	6.80297	6.2945
(7)	9.04491	8.68036	8.25754	7.80047	7.32110	6.82525	6.3184
(8)	9.09419	8.73636	8.32046	7.87062	7.39760	6.90982	6.4110
(9)	9.17164	8.82423	8.41933	7.98041	7.51916	7.04098	6.5518
(10)	9.26104	8.92570	8.53336	8.10735	7.65870	7.19389	6.7181
(11)	9.33781	9.01286	8.63148	8.21656	7.77944	7.32565	6.8608
S	5.16984	3.10433	0.69473	88.08205	85.33501	82.49082	79.5806
S′	5.16984	3.10431	0.69473	88.08196	85.33506	82.49000	79.5788

The values of the coefficients A, precisely as in Chapter III, but relative to $[D-f\cos(\varepsilon-F)]^{-\frac{1}{2}}$, instead of to $\frac{a'}{\Delta}$, are:

	$A_0^{(c)}$	$A_1^{(c)}$	$A_1^{(s)}$	$A_2^{(c)}$	$A_2^{(s)}$	$A_3^{(c)}$	$A_3^{(s)}$	$A_4^{(c)}$
	7	8	8	8	8	8	8	8
(0)	1809554	− 2406800	−1070436	+ 382451	+ 424080	− 42587	−130542	− 3567
(1)	1797762	2432143	920739	419850	371066	62439	118455	+ 4126
(2)	1756238	2354727	783854	418273	313:78	69584	100250	8420
(3)	1701717	2220979	705192	388508	274375	65043	85835	8558
(4)	1651152	2082389	693957	348112	261003	54482	78659	6189
(5)	1615675	1969751	737833	307911	268324	41843	77403	+ 2720
(6)	1600760	1896572	819688	272566	289721	28893	79905	− 1161
(7)	1608186	1869782	923701	245115	320369	16711	84942	5099
(8)	1637092	1896051	1034178	230048	357232	6745	92497	8789
(9)	1683448	1981141	1131199	234252	396909	1484	103282	11741
(10)	1738239	2120579	1186866	264745	431527	4683	116902	12854
(11)	1786248	− 2283311	−1170303	+ 320554	+ 445679	−19456	−128827	−10321
S	1.0193035	−12757118	−5588979	+1916195	+2076741	−206974	−598755	−11762
S′	1.0193036	−12757107	−5588967	+1916190	+2076722	−206976	−598744	−11757

	$A_4^{(s)}$	$A_5^{(c)}$	$A_5^{(s)}$	$A_6^{(c)}$	$A_6^{(s)}$	$A_7^{(c)}$	$A_7^{(s)}$	$A_8^{(c)}$	$A_8^{(s)}$	$A_9^{(c)}$	$A_9^{(s)}$
	8	8	8	8	8	8	8	8	8	8	8
(0)	+ 34459	+ 4478	− 7791	−1917	+1400	+ 621	−134	−168	−35	+ 38	+27
(1)	33321	2047	8411	1284	1875	496	345	157	+40	44	+ 5
(2)	28695	+ 271	7530	676	1811	315	391	111	71	34	− 9
(3)	24115	− 210	6248	421	1497	215	328	77	63	24	9
(4)	21196	+ 197	5236	445	1185	195	241	65	41	18	− 7
(5)	19705	1015	4512	600	916	212	155	61	+17	15	+ 1
(6)	19013	1984	3917	792	659	235	− 69	58	− 7	12	7
(7)	18818	2989	3384	985	403	254	+ 18	53	31	8	12
(8)	19339	3998	3008	1187	174	273	101	47	54	4	17
(9)	21266	5016	3058	1446	42	318	170	48	77	1	23
(10)	25291	5876	3937	1790	144	420	193	71	97	4	31
(11)	+ 30758	+ 5944	− 5803	−2069	+ 640	+ 563	+ 99	−122	−92	+ 18	+37
S	+147993	+16804	−31419	−6807	+5373	+2059	− 541	−520	−81	+110	+66
S′	+147983	+16801	−31416	−6805	+5373	+2058	−541	−518	−80	+110	+69

In the case of $[D-f\cos(\varepsilon-F)]^{-\frac{3}{2}}$ we have

	$A_0^{(c)}$	$A_1^{(c)}$	$A_1^{(s)}$	$A_2^{(c)}$	$A_2^{(s)}$	$A_3^{(c)}$	$A_3^{(s)}$	$A_4^{(c)}$	$A_4^{(s)}$
	7	7	7	7	7	7	7	7	7
(0)	2427402	− 928795	− 413086	+ 242484	+ 268878	− 37529	−115037	− 4023	+ 38871
(1)	2376362	925345	350309	262491	231991	54262	102943	+ 4591	37071
(2)	2202385	851527	283461	248719	186226	57534	82890	8914	30379
(3)	1988543	750201	238200	215966	152521	50297	66374	8475	23883
(4)	1804437	659208	219681	181490	136075	39528	57068	5752	19700
(5)	1683088	595234	222964	153316	133605	29000	53647	+ 2416	17499
(6)	1633989	561906	242853	133088	141464	19640	54316	− 1011	16559
(7)	1658443	559489	276396	120864	157971	11470	58303	4484	16550
(8)	1755834	589379	321470	117792	182914	4807	65914	8024	17656
(9)	1920591	653832	373327	127263	215630	1121	78064	11366	20588
(10)	2129767	749871	419695	153984	250990	3788	94558	13314	26196
(11)	2326762	− 856562	− 439028	+ 197646	+ 274795	− 16677	−110423	−11326	+ 33753
S	1.1953814	−4340686	−1900246	+1077557	+1166547	−162826	−469783	−11706	+149361
S′	1.1953789	−4340663	−1900224	+1077546	+1166513	−162827	−469754	−11694	+149344

	$A_5^{(c)}$	$A_5^{(s)}$	$A_6^{(c)}$	$A_6^{(s)}$	$A_7^{(c)}$	$A_7^{(s)}$	$A_8^{(c)}$	$A_8^{(s)}$	$A_9^{(c)}$	$A_9^{(s)}$
	7	7	7	7	7	7	7	7	7	7
(0)	+ 6156	−10711	−3108	+2269	+1160	−250	− 355	− 74	+ 89	+ 64
(1)	2775	11404	2054	2998	914	636	328	+ 82	101	+ 12
(2)	+ 349	9717	1029	2757	553	686	220	141	75	− 19
(3)	− 253	7542	599	2132	353	537	144	117	49	19
(4)	+ 223	5934	595	1585	301	371	113	72	34	− 9
(5)	1099	4885	766	1170	312	229	102	+ 29	29	+ 2
(6)	2108	4160	993	826	340	−100	95	− 12	22	13
(7)	3205	3629	1247	510	370	+ 26	87	51	15	23
(8)	4451	3348	1559	229	414	153	81	92	7	33
(9)	5920	3609	2013	58	511	272	88	140	2	47
(10)	7419	4971	2666	214	720	330	137	188	8	68
(11)	+ 7949	− 7761	−3264	+1009	+1024	+180	− 250	−189	+ 41	+ 85
S	+20706	−38841	−9950	+7880	+3488	−924	−1001	−152	+235	+150
S′	+20695	−38830	−9943	+7877	+3484	−924	− 999	−153	+237	+150

In the case of $[D-f\cos(\varepsilon-F)]^{-\frac{5}{2}}$ we have

	$A_0^{(c)}$	$A_1^{(c)}$	$A_1^{(s)}$	$A_2^{(c)}$	$A_2^{(s)}$	$A_3^{(c)}$	$A_3^{(s)}$
	5	5	5	5	5	5	5
(0)	37915	—21655	— 9631	+ 7600	+ 8428	—1481	— 4538
(1)	36514	21249	8044	8106	7165	2110	4003
(2)	31905	18516	6164	7286	5455	2125	3061
(3)	26629	15163	4815	5894	4163	1732	2286
(4)	22437	12435	4144	4632	3473	1275	1840
(5)	19853	10688	4003	3730	3250	892	1650
(6)	18849	9881	4270	3172	3372	592	1638
(7)	19348	9942	4912	2911	3805	349	1775
(8)	21388	10905	5948	2951	4581	152	2086
(9)	25046	12893	7362	3391	5745	38	2626
(10)	30061	15917	8909	4407	7183	137	3412
(11)	35163	—19371	— 9929	+ 6014	+ 8360	— 639	— 4233
S	1.62555	—89309	—39066	+30048	+32492	—5762	—16575
S′	1.62553	—89306	—39065	+30046	+32488	—5760	—16573

	$A_4^{(c)}$	$A_4^{(s)}$	$A_5^{(c)}$	$A_5^{(s)}$	$A_6^{(c)}$	$A_6^{(s)}$	$A_7^{(c)}$	$A_7^{(s)}$
	5	5	5	5	5	5	5	5
(0)	—191	+1850	+343	—597	—198	+141	+ 83	—18
(1)	+215	1740	153	627	130	189	65	45
(2)	397	1354	+ 18	508	62	165	37	46
(3)	353	994	— 12	368	33	119	22	34
(4)	224	768	+ 10	272	31	83	18	22
(5)	+ 90	651	48	213	38	59	18	13
(6)	— 37	604	90	178	49	41	19	— 6
(7)	165	610	139	157	62	25	21	+ 1
(8)	307	676	200	150	80	12	24	9
(9)	462	837	282	172	110	3	31	17
(10)	580	1142	379	254	156	13	47	22
(11)	—524	+1561	+ 431	— 420	—202	+ 62	+ 71	+13
S	—494	+6394	+1040	—1959	—576	+455	+228	—61
S′	—493	+6393	+1041	—1957	—575	+457	+228	—61

From these coefficients are derived the developments of the form $[D-f\cos(\varepsilon-F)]^{-\frac{n}{2}}$, where n successively is 1, 3, and 5; which, as they are so nearly those of $\frac{a'}{\Delta}$, etc., we do not give, but pass immediately to the expressions of the latter functions. We have

Arg$=i'g'+i\varepsilon$	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
0 0	1.0193035		1.195379		1.6255	
0 − 1	−0.00229705	−0.00015659	−0.0673943	−0.0205274	−0.2675	−0.0854
0 − 2	+0.0000457	+0.0000144	+0.001493	+0.000834	+0.0114	+0.0070
0 − 3	−0.0000005	−0.0000004	−0.000028	−0.000026	−0.0004	−0.0003
1 + 2	+0.0000040	−0.0000010	+0.000155	−0.000062	+0.0015	−0.0006
1 + 1	−0.0004047	−0.0000231	−0.006846	+0.001093	−0.0356	+0.0068
1 0	+0.0623805	+0.0055532	+0.235847	+0.021001	+0.5626	+0.0502
1 − 1	−0.2551421	−0.1117795	−0.868133	−0.380047	−1.7861	−0.7813
1 − 2	−0.0004761	−0.0007546	+0.012844	+0.009831	+0.0805	+0.0678
1 − 3	−0.0000068	−0.0000085	−0.000259	−0.000301	−0.0027	−0.0034
2 + 1	−0.0000362	−0.0000063	−0.000678	+0.000022	−0.0043	+0.0003
2 0	+0.0037678	+0.0006847	+0.022799	+0.004199	+0.0763	+0.0142
2 − 1	−0.0280461	−0.0147239	−0.149878	−0.079949	−0.4252	−0.2285
2 − 2	+0.0383234	+0.0415345	+0.215505	+0.233305	+0.6009	+0.6498
2 − 3	+0.0001270	+0.0004814	−0.001697	−0.002229	−0.0158	−0.0294
2 − 4	+0.0000001	+0.0000044	+0.000031	+0.000088	+0.0005	+0.0013
3 + 1	−0.0000030	−0.0000008	−0.000065	−0.000005	−0.0005	0.0000
3 0	+0.0002550	+0.0000720	+0.002142	+0.000615	+0.0092	+0.0027
3 − 1	−0.0026134	−0.0016411	−0.019031	−0.012257	−0.0688	−0.0449
3 − 2	+0.0069628	+0.0086560	+0.051549	+0.064745	+0.1787	+0.2259
3 − 3	−0.0041391	−0.0119747	−0.032561	−0.093951	−0.1152	−0.3315
3 − 4	+0.0000239	−0.0001866	+0.000259	+0.000119	+0.0012	+0.0092
3 − 5	+0.0000009	−0.0000017	+0.000003	−0.000024	0.0000	−0.0004
4 0	+0.0000182	+0.0000072	+0.000197	+0.000078	+0.0010	+0.0004
4 − 1	−0.0002277	−0.0001706	−0.002108	−0.001630	−0.0093	−0.0073
4 − 2	+0.0008709	+0.0012602	+0.008001	+0.011812	+0.0332	+0.0498
4 − 3	−0.0009323	−0.0034235	−0.008829	−0.032913	−0.0365	−0.1373
4 − 4	−0.0002354	+0.0029595	−0.002342	+0.029867	−0.0099	+0.1279
4 − 5	−0.0000316	+0.0000542	−0.000155	+0.000121	+0.0004	−0.0022
4 − 6	−0.0000006	+0.0000005	−0.000004	+0.000006		
5 − 1	−0.0000191	−0.0000170	−0.000215	−0.000199	−0.0011	−0.0011
5 − 2	+0.0000915	+0.0001570	+0.001003	+0.001774	+0.0049	+0.0088
5 − 3	−0.0001233	−0.0006229	−0.001353	−0.007117	−0.0064	−0.0345
5 − 4	−0.0001576	+0.0010713	−0.001894	+0.012619	−0.0094	+0.0616
5 − 5	+0.0003361	−0.0006282	+0.004140	−0.007765	+0.0208	−0.0392
5 − 6	+0.0000158	−0.0000117	+0.000103	−0.000051	−0.0001	+0.0004
5 − 7	+0.0000003	0.0000000	+0.000003	−0.000001		
6 − 1	−0.0000015	−0.0000016	−0.000020	−0.000023		
6 − 2	+0.0000086	+0.0000179	+0.000110	+0.000237	+0.0006	+0.0014

Arg $= i'g' + i\varepsilon$	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
6 − 3	−0.0000112	−0.0000919	−0.000136	−0.001218	−0.0007	−0.0067
6 − 4	−0.0000510	+0.0002332	−0.000710	+0.003155	−0.0040	+0.0174
6 − 5	+0.0001702	−0.0002726	+0.002389	−0.003807	+0.0134	−0.0213
6 − 6	−0.0001361	+0.0001074	−0.001989	+0.001575	−0.0115	+0.0091
6 − 7	−0.0000058	+0.0000013	−0.000050	+0.000006		
7 − 1	−0.000000130	−0.000000160				
7 − 2	+0.0000007	+0.0000019	+0.000011	+0.000029		
7 − 3	−0.0000005	−0.0000119	−0.000006	−0.000181	0.0000	−0.0011
7 − 4	−0.0000117	+0.0000396	−0.000185	+0.000607	−0.0012	+0.0037
7 − 5	+0.0000494	−0.0000683	+0.000782	−0.001069	+0.0049	−0.0066
7 − 6	−0.0000772	+0.0000529	−0.001252	+0.000855	−0.0079	+0.0053
7 − 7	+0.0000412	−0.0000108	+0.000697	−0.000184	+0.0046	−0.0012
7 − 8	+0.0000017	+0.0000003	+0.000019	+0.000005		
8 − 3	0.0000000	−0.0000015	+0.000001	−0.000024		
8 − 4	−0.0000022	+0.0000058	−0.000039	+0.000099		
8 − 5	+0.0000109	−0.0000130	+0.000192	−0.000225		
8 − 6	−0.0000245	+0.0000145	−0.000441	+0.000257		
8 − 7	+0.0000264	−0.0000050	+0.000486	−0.000092		
8 − 8	−0.0000104	−0.0000016	−0.000200	−0.000030		
9 − 4	−0.0000004	+0.0000008	−0.000007	+0.000014		
9 − 5	+0.0000020	−0.0000021	+0.000039	−0.000039		
9 − 6	−0.0000058	+0.0000029	−0.000115	+0.000056		
9 − 7	+0.0000093	−0.0000012	+0.000186	−0.000022		
9 − 8	−0.0000075	−0.0000017	−0.000154	−0.000035		
9 − 9	+0.0000022	+0.0000013	+0.000047	+0.000030		

These expressions are now transformed so as to involve arguments of the general form $i'g + ig$. The numerical data and formulæ for this transformation have already been given (pages 52, 53).

Arg $= i'g + ig$	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
0 0	1.0193589		1.197005		1.6320	
0 − 1	−0.00229724	−0.00015724	−0.0674075	−0.0205617	−0.2678	−0.0857
0 − 2	−0.0000097	+0.0000106	−0.000131	+0.000339	+0.0050	+0.0049
0 − 3	−0.0000003	+0.0000002	−0.000015	−0.000004	−0.0001	−0.0001
1 + 2	−0.0000034	−0.0000005	−0.000002	−0.000032	+0.0006	−0.0004
1 + 1	−0.0003305	+0.0000094	−0.006596	+0.001206	−0.0353	+0.0070
1 0	+0.0685447	+0.0082500	+0.256952	+0.030142	+0.6065	+0.0688
1 − 1	−0.2549706	−0.1116781	−0.868245	−0.380300	−1.7890	−0.7841
1 − 2	−0.0066217	−0.0034453	−0.008083	+0.000674	+0.0375	+0.0490
1 − 3	−0.0002520	−0.0001422	−0.000396	−0.000157	−0.0004	−0.0008

Arg$=i'g+ig$	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
1− 4	−0.0000111	−0.0000066	−0.000021	−0.000013		
1− 5	−0.0000005	−0.0000003				
2+ 1	−0.0000282	−0.0000022	−0.000635	+0.000044	−0.0042	+0.0004
2 0	+0.0044452	+0.0010401	+0.026430	+0.006126	+0.0867	+0.0197
2− 1	−0.0298780	−0.0167181	−0.160186	−0.091157	−0.4540	−0.2597
2− 2	+0.0375494	+0.0410484	+0.211516	+0.230997	+0.5904	+0.6449
2− 3	+0.0019459	+0.0024642	+0.008547	+0.008931	+0.0128	+0.0018
2− 4	+0.0000971	+0.0001350	+0.000403	+0.000464	+0.0008	+0.0007
2− 5	+0.0000050	+0.0000073	+0.000021	+0.000025		
3+ 1	−0.0000022	−0.0000003	−0.000060	−0.000001	−0.0005	0.0000
3 0	+0.0003181	+0.0001116	+0.002603	+0.000911	+0.0109	+0.0038
3− 1	−0.0029513	−0.0020680	−0.021534	−0.015454	−0.0775	−0.0561
3− 2	+0.0071828	+0.0094615	+0.053324	+0.071090	+0.1849	+0.2483
3− 3	−0.0037870	−0.0114790	−0.029953	−0.090367	−0.1062	−0.3198
3− 4	−0.0002584	−0.0010272	−0.001969	−0.006495	−0.0067	−0.0143
3− 5	−0.0000140	−0.0000706	−0.000107	−0.000413	−0.0004	−0.0009
3− 6	−0.0000007	−0.0000044	−0.000006	−0.000026		
4 0	+0.0000237	+0.0000113	+0.000248	+0.000117	+0.0012	+0.0006
4− 1	−0.0002704	−0.0002343	−0.002501	−0.002228	−0.0109	−0.0098
4− 2	+0.0009303	+0.0015075	+0.008564	+0.014194	+0.0355	+0.0597
4− 3	−0.0008631	−0.0036297	−0.008176	−0.035048	−0.0337	−0.1465
4− 4	−0.0002945	+0.0026818	−0.002918	+0.027233	−0.0123	+0.1172
4− 5	−0.0000574	+0.0003220	−0.000414	+0.002838	−0.0008	+0.0095
4− 6	−0.0000062	+0.0000266	−0.000040	+0.000219	−0.0001	+0.0006
4− 7	−0.0000005	+0.0000019	−0.000003	+0.000015		
5− 1	−0.0000236	−0.0000251	−0.000264	−0.000291	−0.0013	−0.0015
5− 2	+0.0000993	+0.0002037	+0.001090	+0.002308	+0.0054	+0.0114
5− 3	−0.0001016	−0.0007178	−0.001098	−0.008243	−0.0052	−0.0400
5− 4	−0.0002050	+0.0010922	−0.002467	+0.012925	−0.0123	+0.0632
5− 5	+0.0003133	−0.0005174	+0.003878	−0.006467	+0.0196	−0.0329
5− 6	+0.0000545	−0.0000793	+0.000582	−0.000892	+0.0023	−0.0039
5− 7	+0.0000059	−0.0000075	+0.000059	−0.000079	+0.0002	−0.0004
5− 8	+0.0000005	−0.0000007	+0.000004	−0.000007		
6− 1	−0.0000019	−0.0000026	−0.000025	−0.000035		
6− 2	+0.0000093	+0.0000250	+0.000118	+0.000330	+0.0007	+0.0019
6− 3	−0.0000051	−0.0001141	−0.000051	−0.001522	−0.0002	−0.0084
6− 4	−0.0000727	+0.0002578	−0.001014	+0.003507	−0.0058	+0.0194
6− 5	+0.0001823	−0.0002621	+0.002571	−0.003681	+0.0145	−0.0206
6− 6	−0.0001124	+0.0000741	−0.001660	+0.001109	−0.0097	+0.0065
6− 7	−0.0000235	+0.0000140	−0.000309	+0.000194	−0.0015	+0.0011
6− 8	−0.0000028	+0.0000015	−0.000033	+0.000020		
7− 1	−0.000000165	−0.000000262				
7− 2	+0.0000007	+0.0000029	+0.000011	+0.000044		
7− 3	+0.0000008	−0.0000158	+0.000016	−0.000242	+0.0001	−0.0015
7− 4	−0.0000180	+0.0000469	−0.000286	+0.000722	−0.0019	+0.0044

Arg=$i'g+ig$	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$		$\left(\frac{a'}{\Delta}\right)^5$	
	cos.	sin.	cos.	sin.	cos.	sin.
i' i						
7— 5	+0.0000591	—0.0000713	+0.000940	—0.001122	+0.0058	—0.0069
7— 6	—0.0000767	+0.0000458	—0.001250	+0.000744	—0.0079	+0.0046
7— 7	+0.0000292	—0.0000037	+0.000502	—0.000069	+0.0034	—0.0006
7— 8	+0.0000073	—0.0000009	+0.000117	—0.000014	+0.0007	—0.0001
7— 9	+0.0000009	—0.0000001	+0.000016	—0.000001		
8— 3	+0.0000002	—0.0000022	+0.000006	—0.000035		
8— 4	—0.0000037	+0.0000073	—0.000065	+0.000125		
8— 5	+0.0000143	—0.0000145	+0.000253	—0.000250		
8— 6	—0.0000272	+0.0000134	—0.000493	+0.000241		
8— 7	+0.0000243	—0.0000026	+0.000449	—0.000048		
8— 8	—0.0000059	—0.0000021	—0.000118	—0.000040		
8— 9	—0.0000015	—0.0000004	—0.000029	—0.000008		
9— 4	—0.0000006	+0.0000011	—0.000013	+0.000019		
9— 5	+0.0000029	—0.0000024	+0.000056	—0.000045		
9— 6	—0.0000072	+0.0000027	—0.000141	+0.000054		
9— 7	+0.0000096	—0.0000005	+0.000195	—0.000005		
9— 8	—0.0000063	—0.0000021	—0.000129	—0.000043		
9— 9	+0.0000009	+0.0000009	+0.000019	+0.000022		
9—10	+0.0000003	+0.0000003	+0.000006	+0.000005		

We compute the following Besselian functions, corresponding to the multiples of half the eccentricity of Uranus, by means of the process given, (page 52):

	$l=\frac{1}{2}e'$	$l=e'$	$l=\frac{3}{2}e'$	$l=2e'$	$l=\frac{5}{2}e'$
$\log J_i^{(0)}$	9.9997610	9.9990432	9.9978458	9.9961666	9.99400
$\log J_i^{(1)}$	8.3702419	8.6709131	8.8464064	8.97051	9.06634
$\log J_i^{(2)}$	6.4396130	7.0414338	7.3932179	7.64254	7.83564
$\log J_i^{(3)}$	4.3328729	5.2357835	5.7637586	6.13816	6.42835
$\log J_i^{(4)}$	2.1011862	3.3051637	4.0092888	4.50871	4.89592
$\log J_i^{(5)}$			2.1578731	2.78229	3.26648

Then, for the three multipliers of $\left(\frac{a}{\Delta}\right)^3$, we have:

$$\alpha^2\frac{r^2}{a^2} = [8.8677713] - 2[7.5495636]\cos g - 2[5.63073]\cos 2g - 2[4.0129]\cos 3g$$

$$\frac{r'^2}{a'^2} = [0.0014320] - 2[8.6712719]\cos g' - 2[6.74040]\cos 2g' - 2[5.1105]\cos 3g'$$

$$\frac{r'}{a'}\sin(f'+\Pi') = \begin{array}{ll} & +\;[8.6624013] \\ -2[9.5776690]\sin g' & -2[9.5135301]\cos g' \\ -2[7.94783\;\;]\sin 2g' & -2[7.88361\;\;]\cos 2g' \\ -2[6.4941\;\;]\sin 3g' & -2[6.4298\;\;]\cos 3g' \\ -2[5.1139\;\;]\sin 4g' & -2[5.0500\;\;]\cos 4g' \end{array}$$

The products are:

Arg=$i'g'+ig$	$\alpha^2\frac{r^2}{a^2}\left(\frac{a'}{\Delta}\right)^3$		$\frac{r'^2}{a'^2}\left(\frac{a'}{\Delta}\right)^3$	
	cos.	sin.	cos.	sin.
i' i				
0 0	+0.088520		1.188889	
0 − 1	−0.134538	−0.015185	−0.265023	−0.026828
0 − 2	+0.000127	+0.000098	+0.000131	+0.000178
0 − 3	0.000000	0.000000	−0.000001	−0.000001
1 + 2	+0.000013	−0.000007	+0.000005	−0.000013
1 + 1	−0.001361	−0.000002	−0.002946	+0.000033
1 0	+0.022050	+0.003567	+0.144114	+0.029971
1 − 1	−0.064916	−0.028157	−0.860421	−0.376306
1 − 2	+0.002471	+0.001398	−0.018055	−0.010215
1 − 3	+0.000037	+0.000002	−0.000781	−0.000527
1 − 4	0.000000	0.000000	−0.000039	−0.000031
2 + 2			−0.000015	+0.000004
2 + 1	−0.000134	−0.000015	−0.000277	−0.000029
2 0	+0.002511	+0.000765	+0.013021	+0.004689
2 − 1	−0.012658	−0.007564	−0.118937	−0.072881
2 − 2	+0.016137	+0.017327	+0.210087	+0.228385
2 − 3	−0.000114	−0.000158	+0.010003	+0.013225
2 − 4	−0.000009	−0.000008	+0.000499	+0.000757
2 − 5	+0.000001	0.000000	+0.000026	+0.000042
3 + 1	−0.000012	−0.000002	−0.000025	−0.000004
3 0	+0.000266	+0.000119	+0.001188	+0.000605
3 − 1	−0.001785	−0.001391	−0.013495	−0.010915
3 − 2	+0.004115	+0.005618	+0.043179	+0.059822
3 − 3	−0.002390	−0.006893	−0.030068	−0.089435
3 − 4	−0.000041	−0.000161	−0.001856	−0.007823
3 − 5	0.000000	−0.000003	−0.000091	−0.000544
3 − 6	0.000000	−0.000001	−0.000004	−0.000036
4 0	+0.000027	+0.000016	+0.000109	+0.000071
4 − 1	−0.000215	−0.000213	−0.001388	−0.001441
4 − 2	+0.000670	+0.001178	+0.005924	+0.010671
4 − 3	−0.000623	−0.002732	−0.006751	−0.030542
4 − 4	−0.000185	+0.002121	−0.002719	+0.027020
4 − 5	−0.000021	+0.000112	−0.000593	+0.003171
4 − 6	−0.000002	+0.000005	−0.000066	+0.000262
4 − 7			−0.000006	+0.000019
5 0			−0.000013	−0.000006
5 − 1	−0.000023	−0.000029	−0.000133	−0.000176
5 − 2	+0.000085	+0.000199	+0.000654	+0.001593
5 − 3	−0.000076	−0.000662	−0.000700	−0.006505
5 − 4	−0.000192	+0.001005	−0.002289	+0.011529
5 − 5	+0.000293	−0.000520	+0.003788	−0.006447
5 − 6	+0.000029	−0.000044	+0.000665	−0.000957

$\text{Arg}=i'g'+ig$	$\alpha^2\frac{r^2}{a^2}\left(\frac{a'}{\Delta}\right)^3$		$\frac{r'^2}{a'^2}\left(\frac{a'}{\Delta}\right)^3$	
	cos.	sin.	cos.	sin.
i' i				
5 − 7	+0.000002	−0.000003	+0.000073	−0.000088
5 − 8			+0.000006	−0.000008
6 − 1	−0.000002	−0.000004	−0.000012	−0.000020
6 − 2	+0.000009	+0.000029	+0.000060	+0.000212
6 − 3	0.000000	−0.000125	+0.000002	−0.001109
6 − 4	−0.000084	+0.000277	−0.000886	+0.002864
6 − 5	+0.000200	−0.000287	+0.002354	−0.003339
6 − 6	−0.000130	+0.000094	−0.001633	+0.001120
6 − 7	−0.000017	+0.000010	−0.000337	+0.000202
6 − 8	−0.000001	0.000000	−0.000038	+0.000021
7 − 2	+0.000001	+0.000004	+0.000004	+0.000028
7 − 3	+0.000002	−0.000021	+0.000019	−0.000165
7 − 4	−0.000024	+0.000058	−0.000235	+0.000546
7 − 5	+0.000074	−0.000089	+0.000808	−0.000937
7 − 6	−0.000097	+0.000059	−0.001153	+0.000683
7 − 7	+0.000041	−0.000008	+0.000497	−0.000076
7 − 8	+0.000007	−0.000001	+0.000125	−0.000013
7 − 9	+0.000001	0.000000	+0.000017	−0.000001
8 − 3	0.000000	−0.000003	+0.000005	−0.000023
8 − 4	−0.000006	+0.000010	−0.000050	+0.000088
8 − 5	+0.000021	−0.000019	+0.000206	−0.000194
8 − 6	−0.000039	+0.000019	−0.000428	+0.000203
8 − 7	+0.000035	−0.000005	+0.000417	−0.000045
8 − 8	−0.000011	−0.000003	−0.000117	−0.000037
8 − 9	−0.000002	−0.000001	−0.000031	−0.000009
9 − 4	−0.000001	+0.000001	−0.000010	+0.000013
9 − 5	+0.000004	−0.000003	+0.000043	−0.000032
9 − 6	−0.000011	+0.000004	−0.000117	+0.000043
9 − 7	+0.000014	0.000000	+0.000175	−0.000003
9 − 8	−0.000011	−0.000003	−0.000123	−0.000041
9 − 9	+0.000001	+0.000002	+0.000020	+0.000022
9 − 10			+0.000006	+0.000005

Arg.	$\left(\frac{a'}{\Delta}\right)^3\frac{r'}{a'}\sin(f'+\Pi')$	
	sin.	cos.
i' i		
0 0		−0.040463
0 − 1	−0.2030405	+0.4277100
0 − 2	−0.003162	−0.001314
0 − 3	−0.000076	+0.000079
0 − 4	−0.000003	+0.000008

For the computation of $a'\frac{r}{r'^2}H$ we have (page 63)

$\log h = 9.3949092n$ $\qquad \log l = 9.0364730$
$\log h_1 = 9.3939339n$ $\qquad \log l_1 = 9.0364229$
$\log P_0' = 9.1485126n$
$\log P_1' = 9.9996414$ $\qquad \log Q_1' = 9.9998806$
$\log P_2' = 8.3697236$ $\qquad \log Q_2' = 8.3700426$
$\log P_3' = 6.9159173$ $\qquad \log Q_3' = 6.9162760$
$\log P_4' = 5.53591$ $\qquad \log Q_4' = 5.53629$
$\log P_5' = 4.1967$ $\qquad \log Q_5' = 4.1971$

There is then obtained

Arg.	$-a'\frac{r}{r'^2}H$		Arg.	$-a'\frac{r}{r'^2}H$	
	cos.	sin.		cos.	sin.
1+ 2	+0.0000027	−0.0000004	3+ 1	+0.0000007	0.0000000
1+ 1	+0.0001378	−0.0000080	3 0	−0.0001334	−0.0000585
1 0	−0.0179504	−0.0078673	3− 1	+0.0018387	+0.0008065
1− 1	+0.2477022	+0.1086316	3− 2	+0.0000444	+0.0000195
1− 2	+0.0059715	+0.0026188	3− 3	+0.0000016	+0.0000007
1− 3	+0.0002159	+0.0000947	4 0	−0.0000099	−0.0000043
1− 4	+0.0000093	+0.0000041	4− 1	+0.0001363	+0.0000598
1− 5	+0.0000003	−0.0000002	4− 2	+0.0000032	+0.0000014
2+ 1	+0.0000108	+0.0000002	5− 1	+0.0000097	+0.0000043
2 0	−0.0016835	−0.0007380	5− 2	+0.0000002	+0.0000001
2− 1	+0.0232334	+0.0101892	6− 1	+0.0000007	+0.0000003
2− 2	+0.0005600	+0.0002456	7− 1	+0.000000047	+0.000000021
2− 3	+0.0000202	+0.0000089			
2− 4	+0.0000009	+0.0000004			

The logarithms of the factors proportional to the mass of Uranus are

$\log \mu = 0.4249247$ $\qquad \log (\mu\alpha \sin J) = 7.9456112$

The following are the expressions for the forces :

Arg$=i'g'+ig$	$a\dfrac{d\Omega}{dg}$		$ar\dfrac{d\Omega}{dr}$	
	sin.	cos.	cos.	sin.
i' i	″	″	″	″
0 0			+0.10775	
0−1	−0.0061113	+0.0004183	−0.0143006	−0.0013396
0−2	−0.0000516	−0.0000564	+0.000019	+0.000092
0−3	−0.000002	−0.000002		
1+1	+0.00051	0.00000	−0.00130	+0.00001
1 0			+0.02343	+0.00322
1−1	−0.01934	+0.00810	−0.06003	−0.02555
1−2	−0.00346	+0.00440	−0.00261	−0.00390
1−3	−0.00029	+0.00038	−0.00018	−0.00026
1−4	−0.00003	+0.00003	−0.00001	−0.00002
2+1	+0.00005	−0.00001	−0.00012	−0.00002
2 0			+0.00359	+0.00187
2−1	−0.01768	+0.01737	−0.03982	−0.03754
2−2	+0.20276	−0.21971	+0.20952	+0.22679
2−3	+0.01569	−0.01974	+0.01092	+0.01455
2−4	+0.00104	−0.00144	+0.00055	+0.00084
2−5	+0.00007	−0.00010	+0.00003	+0.00005
3 0			+0.00045	+0.00034
3−1	−0.00296	+0.00336	−0.00676	−0.00777
3−2	+0.03845	−0.05044	+0.04252	+0.05956
3−3	−0.03021	+0.09161	−0.03177	−0.09452
3−4	−0.00275	+0.01093	−0.00207	−0.00882
3−5	−0.00019	+0.00094	−0.00010	−0.00063
3−6	−0.00001	+0.00007	0.00000	−0.00004
4 0			+0.00005	+0.00005
4−1	−0.00036	+0.00046	−0.00084	−0.00116
4−2	+0.00497	−0.00803	+0.00576	+0.01062
4−3	−0.00689	+0.02897	−0.00700	−0.03216
4−4	−0.00313	−0.02854	−0.00298	+0.02955
4−5	−0.00076	−0.00428	−0.00068	+0.00364
4−6	−0.00010	−0.00042	−0.00008	+0.00031
4−7	−0.00001	−0.00004	−0.00001	+0.00002
5−1	−0.00004	+0.00006	−0.00009	−0.00015
5−2	+0.00053	−0.00108	+0.00063	+0.00158
5−3	−0.00081	+0.00573	−0.00069	−0.00682
5−4	−0.00218	−0.01162	−0.00252	+0.01254
5−5	+0.00417	+0.00688	+0.00423	−0.00720
5−6	+0.00087	+0.00127	+0.00077	−0.00111
5−7	+0.00011	+0.00014	+0.00009	−0.00010
5−8	+0.00001	+0.00002	+0.00001	+0.00001

Arg$=i'g'+ig$	$a\frac{d\Omega}{dg}$		$ar\frac{d\Omega}{dr}$	
	sin.	cos.	cos.	sin.
i' i	″	″	″	″
6— 1	0.00000	+0.00001	—0.00001	—0.00002
6— 2	+0.00005	—0.00013	+0.00006	+0.00021
6— 3	—0.00004	+0.00091	+0.00001	—0.00116
6— 4	—0.00077	—0.00274	—0.00097	+0.00310
6— 5	+0.00242	+0.00349	+0.00262	—0.00371
6— 6	—0.00179	—0.00118	—0.00185	+0.00127
6— 7	—0.00044	—0.00026	—0.00040	+0.00024
6— 8	—0.00006	—0.00003	—0.00005	+0.00003
7— 1	—0.00000031	+0.00000064		
7— 2	0.00000	—0.00002	0.00000	+0.00003
7— 3	+0.00001	+0.00013	+0.00002	—0.00017
7— 4	—0.00019	—0.00050	—0.00026	+0.00059
7— 5	+0.00079	+0.00095	+0.00090	—0.00103
7— 6	—0.00122	—0.00073	—0.00130	+0.00077
7— 7	+0.00054	+0.00007	+0.00057	—0.00009
7— 8	+0.00016	+0.00002	+0.00015	—0.00001
7— 9	+0.00002	0.00000	+0.00002	0.00000
8— 3	0.00000	+0.00002	+0.00001	—0.00002
8— 4	—0.00004	—0.00008	—0.00005	+0.00009
8— 5	+0.00019	+0.00019	+0.00023	—0.00021
8— 6	—0.00043	—0.00021	—0.00048	+0.00023
8— 7	+0.00045	+0.00005	+0.00048	—0.00005
8— 8	—0.00013	+0.00004	—0.00013	—0.00004
8— 9	—0.00004	+0.00001	—0.00004	—0.00001
9— 4	—0.00001	—0.00001	—0.00001	+0.00001
9— 5	+0.00004	+0.00003	+0.00005	—0.00004
9— 6	—0.00011	—0.00004	—0.00013	+0.00005
9— 7	+0.00018	+0.00001	+0.00020	0.00000
9— 8	—0.00013	+0.00004	—0.00014	—0.00005
9— 9	+0.00002	—0.00002	+0.00002	+0.00003

Arg.	$a^2\frac{d\Omega}{dZ}$	
	sin.	cos.
i' i	″	″
0 0		—0.000357
0— 1	—0.0017914	+0.0037736
0— 2	—0.000028	—0.000012

The expressions for the three multipliers, A, B, and C, have already been given (page 73). The resulting expressions for T and $\frac{1}{n}\frac{dR}{dt}$ follow: in the latter the terms having the argument γ are alone retained, as the periodic perturbations of the latitude are quite insignificant.

Arg=$\varkappa\gamma+i'g'+ig$			T.	
$\varkappa$	i'	i	sin.	cos.
			″	″
1	0	− 1	−0. 2141836	−0. 0000405
− 1	0	0	−0. 0213460	+0. 0021751
0	0	− 1	+0. 0183339	−0. 0012549
1	0	− 2	−0. 00312	−0. 00049
− 1	0	− 1	+0. 00031	−0. 00022
0	0	− 2	+0. 00015	+0. 00017
1	0	− 3	−0. 00011	−0. 00004
− 1	1	+ 2	+0. 00022	−0. 00006
0	1	+ 1	−0. 00153	0. 00000
1	1	0	+0. 00178	+0. 00010
− 1	1	+ 1	+0. 02152	−0. 00240
1	1	− 1	−0. 01965	+0. 00162
− 1	1	0	−0. 09839	+0. 04193
0	1	− 1	+0. 05802	−0. 02430
1	1	− 2	+0. 02129	−0. 00989
− 1	1	− 1	−0. 00867	+0. 01236
0	1	− 2	+0. 01038	−0. 01320
1	1	− 3	−0. 00333	+0. 00442
− 1	1	− 2	−0. 00040	+0. 00058
0	1	− 3	+0. 00087	−0. 00114
1	1	− 4	−0. 00038	+0. 00050
− 1	1	− 3	−0. 00002	+0. 00003
0	1	− 4	+0. 00006	−0. 00008
1	1	− 5	−0. 00004	+0. 00003
− 1	2	+ 2	+0. 00003	−0. 00001
0	2	+ 1	−0. 00014	+0. 00002
1	2	0	+0. 00012	+0. 00002
− 1	2	+ 1	+0. 00249	−0. 00090
1	2	− 1	−0. 00050	−0. 00111
− 1	2	0	−0. 06519	+0. 06152
0	2	− 1	+0. 05304	−0. 05211
1	2	− 2	− 0. 02517	+0. 02927
− 1	2	− 1	+0. 61758	−0. 66899
0	2	− 2	−0. 60828	+0. 65913
1	2	− 3	+0. 19461	−0. 21063
− 1	2	− 2	+0. 02296	−0. 03307
0	2	− 3	−0. 04707	+0. 05922
1	2	− 4	+0. 02020	−0. 02462
− 1	2	− 3	+0. 00101	−0. 00169
0	2	− 4	−0. 00312	+0. 00432
1	2	− 5	+0. 00159	−0. 00209
− 1	2	− 4	+0. 00004	−0. 00009
0	2	− 5	−0. 00020	+0. 00029
1	2	− 6	+0. 00011	−0. 00014
− 1	3	+ 1	+0. 00028	−0. 00017
1	3	− 1	+0. 00007	−0. 00026
− 1	3	0	−0. 01077	+0. 01196
0	3	− 1	+0. 00888	−0. 01008
1	3	− 2	−0. 00484	+0. 00648
− 1	3	− 1	+0. 11826	−0. 15634
0	3	− 2	−0. 11535	+0. 15132
1	3	− 3	+0. 03893	−0. 05484
− 1	3	− 2	−0. 09603	+0. 28318
0	3	− 3	+0. 09063	− 0. 27483
1	3	− 4	−0. 02840	+0. 08748
− 1	3	− 3	−0. 00470	+0. 02198
0	3	− 4	+0. 00825	−0. 03279
1	3	− 5	−0. 00340	+0. 01289
− 1	3	− 4	−0. 00023	+0. 00140
0	3	− 5	+0. 00057	−0. 00282
1	3	− 6	−0. 00024	+0. 00126
− 1	3	− 5	0. 00000	+0. 00008
0	3	− 6	+0. 00003	−0. 00021
1	3	− 7	−0. 00002	+0. 00010
− 1	4	+ 1	+0. 00003	−0. 00002
1	4	− 1	+0. 00001	−0. 00004
− 1	4	0	−0. 00130	+0. 00169
0	4	− 1	+0. 00108	−0. 00138
1	4	− 2	−0. 00061	+0. 00100
− 1	4	− 1	+0. 01539	−0. 02530
0	4	− 2	−0. 01491	+0. 02409
1	4	− 3	+0. 00518	−0. 00973
− 1	4	− 2	−0. 02141	+0. 08951
0	4	− 3	+0. 02067	−0. 08691
1	4	− 4	−0. 00635	+0. 03003
− 1	4	− 3	−0. 00862	−0. 08964
0	4	− 4	+0. 00939	+0. 08562
1	4	− 5	−0. 00319	−0. 02703
− 1	4	− 4	−0. 00191	−0. 00950
0	4	− 5	+0. 00228	+0. 01284
1	4	− 6	−0. 00083	−0. 00487
− 1	4	− 5	−0. 00021	−0. 00071
0	4	− 6	+0. 00030	+0. 00126
1	4	− 7	−0. 00011	−0. 00054
− 1	4	− 6	−0. 00002	−0. 00005
0	4	− 7	+0. 00003	+0. 00012
1	4	− 8	−0. 00001	−0. 00005
− 1	5	0	−0. 00013	+0. 00020
0	5	− 1	+0. 00011	−0. 00017
1	5	− 2	−0. 00006	+0. 00013
− 1	5	− 1	+0. 00165	−0. 00348
0	5	− 2	−0. 00159	+0. 00324

Arg=$\varkappa\gamma+i'g'+ig$			T		Arg=$\varkappa\gamma+i'g'+ig$			T	
			sin.	cos.				sin.	cos.
$\varkappa$	i'	i	″	″	$\varkappa$	i'	i	″	″
1	5	—3	+0.00055	—0.00144	1	7	—3	+0.00001	—0.00005
—1	5	—2	—0.00247	+0.01781	—1	7	—2	+0.00003	+0.00041
0	5	—3	+0.00243	—0.01719	0	7	—3	—0.00002	—0.00038
1	5	—4	—0.00061	+0.00636	1	7	—4	+0.00001	+0.00014
—1	5	—3	—0.00661	—0.03602	—1	7	—3	—0.00062	—0.00159
0	5	—4	+0.00654	+0.03486	0	7	—4	+0.00057	+0.00150
1	5	—5	—0.00245	—0.01173	1	7	—5	—0.00021	—0.00052
—1	5	—4	+0.01282	+0.02214	—1	7	—4	+0.00246	+0.00301
0	5	—5	—0.01251	—0.02064	0	7	—5	—0.00237	—0.00285
1	5	—6	+0.00399	+0.00641	1	7	—6	+0.00082	+0.00095
—1	5	—5	+0.00211	+0.00298	—1	7	—5	—0.00383	—0.00232
0	5	—6	—0.00261	—0.00381	0	7	—6	+0.00366	+0.00219
1	5	—7	+0.00098	+0.00140	1	7	—7	—0.00120	—0.00066
—1	5	—6	+0.00023	+0.00026	—1	7	—6	+0.00181	+0.00032
0	5	—7	—0.00033	—0.00042	0	7	—7	—0.00162	—0.00021
1	5	—8	+0.00013	+0.00017	1	7	—8	+0.00047	+0.00007
—1	5	—7	+0.00004	+0.00002	—1	7	—7	+0.00039	+0.00004
0	5	—8	—0.00003	—0.00005	0	7	—8	—0.00047	—0.00006
1	5	—9	+0.00001	+0.00002	1	7	—9	+0.00017	+0.00003
—1	6	—0	—0.00002	+0.00002	—1	7	—8	+0.00002	+0.00001
0	6	—1	+0.00001	—0.00002	0	7	—9	—0.00006	—0.00001
1	6	—2	—0.00001	—0.00002	1	7	—10	+0.00004	+0.00001
—1	6	—1	+0.00016	—0.00043	—1	8	—3	—0.00013	—0.00026
0	6	—2	—0.00015	+0.00040	0	8	—4	+0.00012	+0.00023
1	6	—3	+0.00004	—0.00020	1	8	—5	—0.00005	—0.00008
—1	6	—2	—0.00012	+0.00281	—1	8	—4	+0.00060	+0.00060
0	6	—3	+0.00012	—0.00273	0	8	—5	—0.00057	—0.00057
1	6	—4	+0.00002	+0.00106	1	8	—6	+0.00020	+0.00021
—1	6	—3	—0.00240	—0.00849	—1	8	—5	—0.00136	—0.00068
0	6	—4	+0.00231	+0.00822	0	8	—6	+0.00129	+0.00063
1	6	—5	—0.00093	—0.00291	1	8	—7	—0.00044	—0.00021
—1	6	—4	+0.00748	+0.01089	—1	8	—6	+0.00144	+0.00018
0	6	—5	—0.00726	—0.01047	0	8	—7	—0.00135	—0.00015
1	6	—6	+0.00249	+0.00343	1	8	—8	+0.00044	+0.00003
—1	6	—5	—0.00569	—0.00399	—1	8	—7	—0.00043	+0.00012
0	6	—6	+0.00537	+0.00354	0	8	—8	+0.00039	—0.00012
1	6	—7	—0.00168	—0.00106	1	8	—9	—0.00011	+0.00005
—1	6	—6	—0.00110	—0.00065	—1	8	—8	—0.00005	+0.00001
0	6	—7	+0.00132	+0.00078	0	8	—9	+0.00011	—0.00003
1	6	—8	—0.00047	—0.00028	1	8	—10	—0.00007	+0.00002
—1	6	—7	—0.00013	—0.00007	—1	9	—4	+0.00008	+0.00006
0	6	—8	+0.00018	+0.00010	0	9	—5	—0.00012	—0.00009
1	6	—9	—0.00007	—0.00003	1	9	—6	+0.00009	+0.00007
0	7	—1	+0.00000094	—0.00000192	—1	9	—5	—0.00036	—0.00009
—1	7	—1	+0.000012	—0.000053	0	9	—6	+0.00033	+0.00012
0	7	—2	—0.00001	+0.00005	1	9	—7	—0.00012	—0.00009

Arg=$\varkappa\gamma+i'g'+ig$	T.		Arg=$\varkappa\gamma+i'g'+ig$	T.	
	sin.	cos.		sin.	cos.
$\varkappa$ i' i	$''$	$''$	$\varkappa$ i' i	$''$	$''$
−1 9 − 6	+0.00057	+0.00003	−1 9 − 7	−0.00043	+0.00009
0 9 − 7	−0.00054	−0.00003	0 9 − 8	+0.00039	−0.00012
1 9 − 8	+0.00018	+0.00001	1 9 − 9	−0.00013	+0.00009

$$\frac{1}{n}\frac{dR}{dt}=-0''.0008950 \cos(-\gamma)-0''.0019103 \sin(-\gamma)$$

The logarithms of the integrating factors are contained in the following table:

Arg.	$\log \frac{n}{i'n'+in}$	Arg.	$\log \frac{n}{i'n'+in}$	Arg.	$\log \frac{n}{i'n'+in}$	Arg.	$\log \frac{n}{i'n'+in}$
i' i		i' i		i' i		i' i	
0 − 1	0.00000*n*	3 + 1	9.84662	5 − 2	9.88805*n*	7 − 5	9.3967*n*
0 − 2	9.69897*n*	3 0	0.37308	5 − 3	9.63940*n*	7 − 6	9.3000*n*
0 − 3	9.52388*n*	3 − 1	0.23925*n*	5 − 4	9.48227*n*	7 − 7	9.2210*n*
1 + 2	9.66934	3 − 2	9.80233*n*	5 − 5	9.36713*n*	7 − 8	9.1542*n*
1 + 1	9.94264	3 − 3	9.58898*n*	5 − 6	9.27621*n*	8 − 3	9.7280*n*
1 0	0.85020	3 − 4	9.44655*n*	5 − 7	9.20107*n*	8 − 4	9.5421*n*
1 − 1	0.06610*n*	3 − 5	9.33947*n*	6 − 1	0.81568*n*	8 − 5	9.4122*n*
1 − 2	9.73076*n*	3 − 6	9.2537*n*	6 − 2	9.93822*n*	8 − 6	9.3124*n*
1 − 3	9.54381*n*	4 0	0.24814	6 − 3	9.66698*n*	8 − 7	9.2313*n*
1 − 4	9.41355*n*	4 − 1	0.36126*n*	6 − 4	9.50129*n*	8 − 8	9.1630*n*
2 + 1	9.89198	4 − 2	9.84307*n*	6 − 5	9.38165*n*	8 − 9	9.1040*n*
2 0	0.54917	4 − 3	9.61346*n*	6 − 6	9.28795*n*	9 − 4	9.5640*n*
2 − 1	0.14411*n*	4 − 4	9.46404*n*	6 − 7	9.21092*n*	9 − 5	9.4284*n*
2 − 2	9.76507*n*	4 − 5	9.35308*n*	7 − 1	1.9325*n*	9 − 6	9.3252*n*
2 − 3	9.56581*n*	4 − 6	9.26478*n*	7 − 2	9.9950*n*	9 − 7	9.2419*n*
2 − 4	9.42974*n*	5 0	0.15123	7 − 3	9.6964*n*	9 − 8	9.1720*n*
2 − 5	9.3263*n*	5 − 1	0.53156*n*	7 − 4	9.5212*n*	9 − 9	9.1119*n*

In the integration we put

$$k_0 = +0.''2142 \qquad k_1 = -0.''0328 \qquad k_2 = +0.''0037$$

Arg=$i\,g'+ig$	$\frac{d\delta z}{dt}$		$\frac{1}{n}\frac{d\nu}{dt}$	
	cos.	sin.	sin.	cos.
i' i	$''$	$''$	$''$	$''$
0 0				−0.0000263*nt*
0 − 1	−0.0213	−0.0022	+0.0208	−0.0019
	+0.0021751*nt*	−0.0213460*nt*	−0.0010875*nt*	−0.0106730*nt*
0 − 2	−0.0004	0.0000	+0.0009	0.0000
	+0.0000524*nt*	−0.0005148*nt*	−0.0000524*nt*	−0.0005148*nt*
0 − 3	+0.0000018*nt*	−0.0000186*nt*	−0.0000027*nt*	−0.0000279*nt*

Arg=$i'g'+ig$	$\frac{d\delta z}{dt}$		$\frac{1}{n}\frac{d\nu}{dt}$	
	cos.	sin.	sin.	cos.
i' i	$''$	$''$	$''$	$''$
1+ 1	−0.0120	+0.0007	−0.0069	−0.0003
1 0	−0.04146	−0.00386	−0.00175	−0.00024
1− 1	+0.7753	+0.3305	−0.3422	+0.1458
1− 2	+0.0111	−0.0016	−0.0124	+0.0007
1− 3	+0.0004	0.0000	−0.0006	0.0000
2+ 1	−0.0003	+0.0001	−0.0002	0.0000
2 0	−0.00293	+0.00051	+0.00036	−0.00081
2− 1	+0.2918	+0.2754	−0.1210	+0.1155
2− 2	+0.5838	+0.6315	−0.4000	+0.4325
2− 3	+0.0230	+0.0268	−0.0247	+0.0291
2− 4	+0.0010	+0.0011	−0.0016	+0.0018
3 0	−0.0002	+0.0003	0.0000	−0.0001
3− 1	+0.0381	+0.0421	−0.0138	+0.0156
3− 2	+0.1475	+0.1968	−0.0955	+0.1264
3− 3	−0.0287	−0.0910	+0.0216	−0.0710
3− 4	−0.0015	−0.0062	+0.0018	−0.0068
3− 5	−0.0001	−0.0004	+0.0001	−0.0006
4− 1	+0.0045	+0.0056	−0.0012	+0.0017
4− 2	+0.0272	+0.0452	−0.0168	+0.0273
4− 3	−0.0073	−0.0339	+0.0056	−0.0256
4− 4	−0.0019	+0.0164	+0.0018	+0.0140
4− 5	−0.0004	+0.0016	+0.0003	+0.0018
5− 1	+0.0006	+0.0008	−0.0001	+0.0001
5− 2	+0.0046	+0.0099	−0.0027	+0.0056
5− 3	−0.0009	−0.0078	+0.0007	−0.0058
5− 4	−0.0015	+0.0075	+0.0011	+0.0062
5− 5	+0.0017	−0.0027	−0.0014	−0.0023
5− 6	+0.0003	−0.0004	−0.0002	−0.0004
6− 1	+0.00007	+0.00016	−0.00001	0.00000
6− 2	+0.0009	+0.0026	−0.0005	+0.0014
6− 3	0.0000	−0.0013	0.0000	−0.0010
6− 4	−0.0006	+0.0019	+0.0004	+0.0015
6− 5	+0.0011	−0.0016	−0.0010	−0.0013
6− 6	−0.0006	+0.0004	+0.0005	+0.0003
6− 7	−0.0001	+0.0002	+0.0001	0.0000
7− 1	+0.000081	+0.000165		
7− 2	+0.0010	+0.0045	−0.0005	+0.0023
7− 3	0.0000	−0.0002	0.0000	−0.0002
7− 4	−0.0002	+0.0004	+0.0001	+0.0004
7− 5	+0.0004	−0.0005	−0.0003	−0.0004
7− 6	−0.0005	+0.0003	+0.0004	+0.0003
7− 7	+0.0002	−0.0001	−0.0002	0.0000

Arg $= i''g'' + ig$	$n\delta z$		ν		$\frac{u}{\cos i}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i'' i	″	″	″	″	″	″
0 − 0			−0.0353			+0.0000648*nt*
			−0.0000263*nt*			
0 − 1	0.0000	0.0000	+0.0101	+0.0008		
	−0.0021751*nt*	−0.0213460*nt*	−0.0010875*nt*	+0.0106730*nt*	−0.0019103*nt*	−0.0008950*nt*
0 − 2	+0.0002	−0.0001	+0.0001	0.0000		
	−0.0000262*nt*	−0.0002574*nt*	−0.0000262*nt*	+0.0002574*nt*	−0.0000461*nt*	−0.0000216*nt*
0 − 3	−0.0000006*nt*	−0.0000062*nt*	−0.0000009*nt*	+0.0000093*nt*	−0.0000017*nt*	−0.0000008*nt*
1 + 1	−0.0105	−0.0006	+0.0060	−0.0003		
1 0	−0.2936	+0.0273	+0.0124	−0.0017		
1 − 1	−0.9028	+0.3848	−0.3985	−0.1698		
1 − 2	−0.0060	−0.0009	−0.0067	−0.0004		
1 − 3	−0.0001	0.0000	−0.0002	0.0000		
2 + 1	−0.0002	−0.0001	+0.0002	0.0000		
2 0	−0.0104	−0.0018	−0.0013	−0.0029		
2 − 1	−0.4066	+0.3837	−0.1686	−0.1610		
2 − 2	−0.3399	+0.3676	−0.2329	−0.2518		
2 − 3	−0.0085	+0.0099	−0.0091	−0.0107		
2 − 4	−0.0003	+0.0003	−0.0004	−0.0005		
3 0	−0.0005	−0.0007	0.0000	−0.0002		
3 − 1	−0.0661	+0.0730	−0.0239	−0.0271		
3 − 2	−0.0936	+0.1249	−0.0606	−0.0802		
3 − 3	+0.0111	−0.0353	+0.0083	+0.0276		
3 − 4	+0.0004	−0.0017	+0.0005	+0.0019		
4 − 1	−0.0103	+0.0129	−0.0028	−0.0039		
4 − 2	−0.0190	+0.0315	−0.0117	−0.0190		
4 − 3	+0.0030	−0.0139	+0.0023	+0.0105		
4 − 4	+0.0006	+0.0048	+0.0005	−0.0041		
4 − 5	+0.0001	+0.0004	+0.0001	−0.0004		
5 − 1	−0.0020	+0.0027	−0.0003	−0.0003		
5 − 2	−0.0036	+0.0076	−0.0022	−0.0043		
5 − 3	+0.0004	−0.0034	+0.0003	+0.0025		
5 − 4	+0.0005	+0.0023	+0.0003	−0.0019		
5 − 5	−0.0004	−0.0006	−0.0003	+0.0005		
6 − 1	−0.0005	+0.0010	−0.0001	0.0000		
6 − 2	−0.0008	+0.0023	−0.0004	−0.0012		
6 − 3	0.0000	−0.0006	0.0000	+0.0005		
6 − 4	+0.0002	+0.0006	+0.0002	−0.0007		
6 − 5	−0.0003	−0.0004	−0.0003	+0.0004		
6 − 6	+0.0001	+0.0001	+0.0001	−0.0001		
7 − 1	−0.0069	+0.0141				
7 − 2	−0.0010	+0.0045	−0.0005	−0.0022		
7 − 3	0.0000	−0.0001	0.0000	+0.0001		
7 − 4	+0.0001	+0.0001	0.0000	+0.0001		
7 − 5	−0.0001	−0.0001	−0.0001	+0.0001		
7 − 6	+0.0001	+0.0001	+0.0001	−0.0001		

CHAPTER V.

PERTURBATIONS OF SATURN BY NEPTUNE.

In determining the action of Neptune on our two planets it will not be necessary to go beyond terms of the first order with respect to disturbing forces. Following the previous custom, no accents will be given to quantities pertaining to Saturn, and a single one to those belonging to Neptune.

The elements of Neptune employed are the following:*

Epoch 1850, Jan. 0.0, Greenwich M. T.

$$\begin{array}{lrrrcl}
 & ^\circ & ' & '' & & \\
L' = & 335 & 5 & 38.91 & e' & = 0.0084962 \\
\pi' = & 43 & 17 & 30.3 & n' & = 7864''.935 \\
\theta' = & 130 & 7 & 31.83 & \log a' & = 1.4781414 \\
i' = & 1 & 47 & 1.68 & m' & = \frac{1}{19700}
\end{array}$$

These elements include the effect of the 4000-year inequality produced by the action of Uranus. Also, log a′ includes the constant term of the perturbations of the logarithm of the radius vector. As the similarly corrected log a of Saturn is 0.9796819, we have here $\log \alpha = 9.5015405$.

The coefficients of the terms of the developments of the reciprocal of the distance between Saturn and Neptune and its odd powers are then functions of the following six elements:

$$\begin{array}{rlrrrr}
 & & & ^\circ & ' & '' \\
\log \alpha & = 9.5015405 & J = & 0 & 57 & 51.54 \\
e & = 0.05605688 & \Pi = & 192 & 9 & 3.80 \\
e' & = 0.0084962 & \Pi' = & 145 & 19 & 5.10
\end{array}$$

Π and Π' are measured from the ascending node of the orbit of Neptune on that of Saturn. The developments will be made first in terms of the eccentric anomaly of Saturn.

---

* An Investigation of the Orbit of Neptune, with General Tables of its Motion. By Prof. S. Newcomb, p. 76.

The values of the auxiliary constants are

$\log k \quad = 9.9999973$
$\log k_1 \quad = 9.9999413$
$\log p \quad = 0.0030502$
$\log v \quad = 9.8021937$
$\log w \quad = 9.8023874$
$\log w_1 \quad = 9.8019933$
$\log \frac{1}{2}\gamma_2 = 6.1993090$

$K = 313^\circ\ 10'\ 7''.32$
$K_1 = 313^\circ\ 9'\ 55''.30$
$P = 117^\circ\ 19'\ 1''.88$
$V = 313^\circ\ 7'\ 5''.68$
$W = 195^\circ\ 53'\ 27''.92$
$W_1 = 195^\circ\ 48'\ 30''.74$

$$D = 1.1002373 - [8.2302547] \cos \varepsilon' + [5.85845] \cos^2 \varepsilon' + [8.7486289] f \cos F$$

The circumference will be divided into twelve parts with reference to the mean anomaly of Neptune. For five points of the division we have

g'	ε'
30°	30° 14′ 42″.72
60	60 25 24 .11
90	90 29 12 .40
120	120 25 11 .22
150	150 14 29 .83

We get the following table of values of D, $\log f$, and F:

g'	D	$\log f$	$F - g'$		
°			°	′	″
0	1.1080847	9.8038257	313	57	23.41
30	1.1201363	9.8067290	313	58	0.76
60	1.1268758	9.8081619	313	44	49.23
90	1.1265486	9.8077789	313	23	27.53
120	1.1193373	9.8059257	313	0	32.53
150	1.1072172	9.8033118	312	41	4.15
180	1.0933864	9.8006159	312	28	10.54
210	1.0814585	9.7983207	312	24	18.35
240	1.0745845	9.7968135	312	31	35.97
270	1.0746535	9.7965143	312	50	18.03
300	1.0817412	9.7977580	313	16	27.68
330	1.0939957	9.8004464	313	41	50.59
S	6.6040099	8.8131007	1878	58	59.36
S′	6.6040098	8.8131011	1878	58	59.41

Employing the same procedure as in the preceding chapter, we have the following table of values of $\log \frac{1}{6}\alpha_i^{(n)}$:

	$\log \frac{1}{6}\alpha_0^{(1)}$	$\log \frac{1}{6}\alpha_1^{(1)}$	$\log \frac{1}{6}\alpha_2^{(1)}$	$\log \frac{1}{6}\alpha_3^{(1)}$	$\log \frac{1}{6}\alpha_4^{(1)}$	$\log \frac{1}{6}\alpha_5^{(1)}$
(0)	9.2315543	8.4357231	7.8122246	7.2335367	6.6756563	6.1298182
(1)	9.2288883	8.4308073	7.8050982	7.2242096	6.6641326	6.1161000
(2)	9.2273807	8.4278328	7.8006822	7.2183586	6.6568491	6.1073854
(3)	9.2273991	8.4275301	7.8000639	7.2174261	6.6556030	6.1058260
(4)	9.2289570	8.4302584	7.8039425	7.2224499	6.6617699	6.1131350
(5)	9.2316935	8.4356442	7.8119314	7.2330302	6.6749369	6.1288863
(6)	9.2349179	8.4423421	7.8220415	7.2465368	6.6918337	6.1491699
(7)	9.2377498	8.4482880	7.8310455	7.2585846	6.7069196	6.1672911
(8)	9.2393674	8.4515017	7.8358261	7.2649249	6.7148166	6.1767431
(9)	9.2392929	8.4510133	7.8349315	7.2636260	6.7131142	6.1746376
(10)	9.2375690	8.4472532	7.8291720	7.2558765	6.7033784	6.1629176
(11)	9.2347227	8.4416289	7.8208196	7.2448086	6.6896001	6.1464315
S	5.3997463	80.6349113	76.9038889	73.4416834	70.1043040	66.8391692
S′	5.3997463	80.6349118	76.9038901	73.4416851	70.1043064	66.8391725

	$\log \frac{1}{6}\alpha_6^{(1)}$	$\log \frac{1}{6}\alpha_7^{(1)}$	$\log \frac{1}{6}\alpha_8^{(1)}$	$\log \frac{1}{6}\alpha_0^{(3)}$	$\log \frac{1}{6}\alpha_1^{(3)}$	$\log \frac{1}{6}\alpha_2^{(3)}$	$\log \frac{1}{6}\alpha_3^{(3)}$
(0)	5.591837	5.05938	4.5309	9.3190254	8.9783566	8.5691675	8.1327836
(1)	5.575927	5.04127	4.5106	9.3103118	8.9676166	8.5562958	8.1177519
(2)	5.565782	5.02970	4.4975	9.3053264	8.9613088	8.5485976	8.1086449
(3)	5.563909	5.02752	4.4950	9.3052807	8.9609735	8.5479578	8.1076966
(4)	5.572360	5.03713	4.5058	9.3103224	8.9670706	8.5551646	8.1160277
(5)	5.590696	5.05803	4.5294	9.3193733	8.9785081	8.5691124	8.1325192
(6)	5.614363	5.08510	4.5599	9.3301657	8.9924249	8.5863178	8.1530576
(7)	5.635518	5.10926	4.5871	9.3396806	9.0047361	8.6015746	8.1713003
(8)	5.646523	5.12183	4.6012	9.3450617	9.0115486	8.6098959	8.1811513
(9)	5.644014	5.11893	4.5979	9.3447009	9.0108168	8.6087730	8.1796319
(10)	5.630312	5.10324	4.5802	9.3388572	9.0031462	8.5991772	8.1680842
(11)	5.611122	5.08134	4.5556	9.3294120	8.9912057	8.5846085	8.1508514
S	63.621177	60.43638	57.2755	5.9487588	3.9138557	1.4683206	88.8597493
S′	63.621186	60.43635	57.2756	5.9487593	3.9138568	1.4683221	88.8597513

	$\log \frac{1}{6} \alpha_4^{(3)}$	$\log \frac{1}{6} \alpha_5^{(3)}$	$\log \frac{1}{6} \alpha_6^{(3)}$	$\log \frac{1}{6} \alpha_7^{(3)}$	$\log \frac{1}{6} \alpha_8^{(3)}$	$\log \frac{1}{6} \alpha_0^{(5)}$	$\log \frac{1}{6} \alpha_1^{(5)}$
(0)	7.6817331	7.2214931	6.754952	6.28383	5.8093	9.48476	9.30965
(1)	7.6645296	7.2021118	6.733389	6.26008	5.7834	9.46928	9.29264
(2)	7.6540062	7.1901681	6.720023	6.24529	5.7671	9.46037	9.28273
(3)	7.6527477	7.1885985	6.718141	6.24309	5.7646	9.46015	9.28229
(4)	7.6622091	7.1991932	6.729870	6.25595	5.7785	9.46903	9.29196
(5)	7.6812582	7.2208070	6.754053	6.28270	5.8079	9.48524	9.31000
(6)	7.7051480	7.2480577	6.784671	6.31670	5.8452	9.50472	9.33183
(7)	7.7263934	7.2723143	6.811943	6.34698	5.8786	9.52192	9.35114
(8)	7.7377828	7.2852466	6.826422	6.36302	5.8962	9.53158	9.36188
(9)	7.7358646	7.2829285	6.823703	6.35990	5.8926	9.53081	9.36083
(10)	7.7223540	7.2674493	6.806251	6.34047	5.8712	9.52018	9.34882
(11)	7.7024422	7.2448507	6.780961	6.31247	5.8405	9.50324	9.33000
S	86.1632332	83.4116080	80.622189	77.80526	74.9675	6.97064	5.92687
S′	86.1632357	83.4116108	80.622190	77.80522	74.9675	6.97064	5.92690

	$\log \frac{1}{6} \alpha_2$	$\log \frac{1}{6} \alpha_3^{(5)}$	$\log \frac{1}{6} \alpha_4^{(5)}$	$\log \frac{1}{6} \alpha_5^{(5)}$	$\log \frac{1}{6} \alpha_6^{(5)}$	$\log \frac{1}{6} \alpha_7^{(5)}$	$\log \frac{1}{6} \alpha_8^{(5)}$
(0)	9.02547	8.68717	8.31656	7.92414	7.51682	7.0977	6.6697
(1)	9.00651	8.66618	8.29338	7.89886	7.48884	7.0675	6.6373
(2)	8.99537	8.65374	8.27969	7.88403	7.47342	7.0507	6.6191
(3)	8.99465	8.65271	8.27829	7.88227	7.47089	7.0478	6.6158
(4)	9.00531	8.66440	8.29097	7.89581	7.48486	7.0630	6.6322
(5)	9.02559	8.68717	8.31610	7.92390	7.51477	7.0955	6.6673
(6)	9.05044	8.71511	8.34757	7.95831	7.55430	7.1383	6.7134
(7)	9.07237	8.73990	8.37508	7.98894	7.58678	7.1738	6.7519
(8)	9.08445	8.75340	8.39001	8.00521	7.60417	7.1928	6.7725
(9)	9.08307	8.75168	8.38800	8.00303	7.60221	7.1904	6.7697
(10)	9.06933	8.73606	8.37052	7.98343	7.58088	7.1671	6.7444
(11)	9.04815	8.71237	8.34427	7.95462	7.54964	7.1331	6.7077
S	4.23037	92.20988	89.99532	87.65093	85.21445	82.7096	80.1513
S′	4.23034	92.21001	89.99512	87.65162	85.21313	82.7081	80.1499

The values of the coefficients A for the development of $[D-f\cos(\varepsilon-F)]^{-\frac{1}{2}}$ are

	$A_0^{(c)}$	$A_1^{(c)}$	$A_1^{(s)}$	$A_2^{(c)}$	$A_2^{(s)}$	$A_3^{(c)}$	$A_3^{(s)}$	$A_4^{(c)}$
	s	s	s	s	s	s	s	s
(0)	+ 17043325	+ 1893009	+ 1963249	− 23633	+ 648539	−127497	+114274	− 47261
(1)	16939019	1872055	1940813	23018	637993	124727	111914	46026
(2)	16880322	1851866	1934686	27631	631345	124321	108992	45205
(3)	16881035	1838527	1944800	35425	630055	126060	106427	44963
(4)	16941700	1837025	1969346	44214	635174	129654	105093	45453
(5)	17048788	1848624	2004424	52362	646415	134662	105412	46691
(6)	17175838	1869708	2042604	58556	661219	140131	107172	48420
(7)	17288199	1893150	2072893	61303	674934	144444	109700	50090
(8)	17352713	1911637	2084238	59085	682662	145856	112243	51087
(9)	17349737	1920786	2071478	51548	681858	143575	114266	51068
(10)	17281004	1919800	2039070	40623	673571	138437	115436	50144
(11)	+ 17168119	+ 1909907	+ 1998787	− 30088	+ 661257	−−132428	+115492	− 48731
S	+1.02674902	+11283045	+12033193	−253742	+3932510	−805896	+663210	−287570
S′	+1.02674897	+11283049	+12033195	−253744	+3932512	−805896	+663211	−287569

	$A_4^{(s)}$	$A_5^{(c)}$	$A_5^{(s)}$	$A_6^{(c)}$	$A_6^{(s)}$	$A_7^{(c)}$	$A_7^{(s)}$	$A_8^{(c)}$	$A_8^{(s)}$
	s	s	s	s	s	s	s	s	s
(0)	− 3449	− 8628	−10362	+ 426	− 3884	+ 907	− 701	+ 336	+ 49
(1)	3325	8369	10032	407	3744	869	673	321	47
(2)	3964	8012	9989	481	3648	864	633	310	55
(3)	5072	7671	10196	614	3612	886	592	305	70
(4)	6359	7451	10623	773	3655	933	562	308	88
(5)	7615	7411	11230	936	3783	1002	550	321	107
(6)	8644	7544	11911	1078	3971	1076	558	341	126
(7)	9175	7795	12462	1160	4162	1148	581	361	137
(8)	8910	8101	12651	1135	4283	1172	615	376	135
(9)	7766	8401	12366	989	4293	1140	655	378	118
(10)	6070	8630	11717	767	4199	1065	689	369	91
(11)	− 4444	− 8719	−10966	+ 555	− 4046	+ 977	− 707	+ 353	+ 65
S	−37396	−48366	−67253	+4660	−23640	+6017	−3758	+2040	+544
S′	−37397	−48366	−67252	+4661	−23640	+6022	−3758	+2039	+544

For the development of $[D - f\cos(\varepsilon - F)]^{-\frac{3}{2}}$ they are

	$A_0^{(c)}$	$A_1^{(c)}$	$A_1^{(s)}$	$A_2^{(c)}$	$A_2^{(s)}$	$A_3^{(c)}$	$A_3^{(s)}$	$A_4^{(c)}$
	7	8	8	7	7	7	7	7
(0)	+ 2084613	+ 6603683	+ 6848712	− 13504	+ 370578	−101099	+ 90614	− 47927
(1)	2043204	6443588	6680254	12980	359760	97612	87584	46068
(2)	2019884	6325364	6608250	15464	353331	96567	84660	44910
(3)	2019671	6279330	6642298	19824	352592	97915	82665	44669
(4)	2043254	6323061	6778509	24934	358191	101476	82253	45499
(5)	2086283	6452269	6996058	29936	369566	106840	83633	47376
(6)	2138778	6635239	7248813	34029	384257	112994	86418	49927
(7)	2186153	6817625	7464919	36142	397915	118145	89726	52388
(8)	2213409	6941484	7568230	35119	405766	120269	92552	53861
(9)	2211571	6970797	7517678	30624	405075	118327	94173	53815
(10)	2182013	6904765	7333733	23921	396633	113099	94308	52384
(11)	+ 2135069	+ 6770006	+ 7085054	− 17466	+ 383848	−106665	+ 93024	− 50193
S	+1. 2681951	+39733596	+42386247	−146971	+2268756	−645504	+530805	−294508
S′	+1. 2681951	+39733615	+42386261	−146972	+2268756	−645504	+530805	−294509

	$A_4^{(s)}$	$A_5^{(c)}$	$A_5^{(s)}$	$A_6^{(c)}$	$A_6^{(s)}$	$A_7^{(c)}$	$A_7^{(s)}$	$A_8^{(c)}$	$A_8^{(s)}$
	7	8	8	8	8	8	8	8	8
(0)	− 3498	−106561	−127975	+ 6203	− 56540	+ 15211	−11755	+ 6378	+ 936
(1)	3328	102018	122297	5844	53807	14387	11147	6010	873
(2)	3939	96950	120862	6867	52032	14190	10397	5760	1018
(3)	5039	92813	123369	8763	51517	14555	9720	5669	1296
(4)	6365	90839	129514	11113	52525	15443	9302	5774	1648
(5)	7726	91581	138772	13629	55101	16803	9234	6093	2041
(6)	8913	94724	149561	15951	58782	18427	9506	6567	2423
(7)	9596	99273	158714	17407	62475	19838	10036	7070	2680
(8)	9394	104003	162417	17174	64817	20427	10718	7410	2666
(9)	8183	107804	158679	14955	64935	19861	11406	7456	2321
(10)	6341	109779	149055	11504	62968	18384	11903	7219	1774
(11)	− 4577	−109364	−137554	+ 8212	− 59828	+ 16637	−12035	+ 6812	+ 1253
S	−38450	−602856	−839384	+68812	−347664	+102082	−63581	+39108	+10465
S′	−38449	−602853	−839385	+68810	−347663	+102081	−63578	+39110	+10464

For the development of $[D-f\cos(\varepsilon-F)]^{-\frac{5}{2}}$ they are

	$A_0^{(c)}$	$A_1^{(c)}$	$A_1^{(s)}$	$A_2^{(c)}$	$A_2^{(s)}$	$A_3^{(c)}$	$A_3^{(s)}$	$A_4^{(c)}$
	5	5	5	5	5	5	5	5
(0)	+ 30532	+14161	+14686	− 386	+10597	− 3623	+ 3248	− 2067
(1)	29463	13619	14120	366	10144	3451	3096	1960
(2)	28865	13259	13852	433	9884	3388	2970	1897
(3)	28850	13159	13920	554	9862	3434	2900	1886
(4)	29446	13360	14323	703	10099	3587	2908	1936
(5)	30566	13842	15009	856	10572	3832	2999	2044
(6)	31968	14496	15837	991	11188	4122	3152	2192
(7)	33260	15137	16574	1069	11765	4375	3323	2333
(8)	34008	15552	16955	1047	12101	4492	3457	2418
(9)	33948	15606	16831	913	12073	4417	3515	2415
(10)	33127	15304	16256	706	11709	4183	3488	2330
(11)	+ 31859	+14770	+15457	− 508	+11161	− 3886	+ 3389	− 2200
S	+1.87946	+86132	+91909	−4266	+65578	−23395	+19223	−12840
S′	+1.87946	+86133	+91911	−4266	+65577	−23395	+19222	−12838

	$A_4^{(s)}$	$A_5^{(c)}$	$A_5^{(s)}$	$A_6^{(c)}$	$A_6^{(s)}$	$A_7^{(c)}$	$A_7^{(s)}$	$A_8^{(c)}$	$A_8^{(s)}$
	5	5	5	5	5	5	5	5	5
(0)	− 151	− 537	− 645	+ 36	− 327	+ 99	− 77	+ 46	+ 7
(1)	142	507	608	33	306	90	72	43	6
(2)	166	479	597	39	295	91	66	40	7
(3)	213	458	609	50	292	93	62	40	9
(4)	271	452	644	63	299	99	60	41	12
(5)	333	462	700	79	318	109	60	44	15
(6)	391	486	767	94	346	122	63	48	18
(7)	427	517	826	104	372	133	67	53	20
(8)	422	546	852	103	389	138	72	56	20
(9)	367	566	833	90	390	134	77	56	17
(10)	282	571	775	68	375	123	80	54	13
(11)	− 201	− 560	− 705	+ 48	− 351	+110	− 80	+ 50	+ 9
S	−1683	−3071	−4280	+403	−2031	+672	−418	+285	+77
S′	−1683	−3070	−4281	+404	−2029	+669	−418	+286	+76

The corrections to pass from the development of $[D-f\cos(\varepsilon-F)]^{-\frac{n}{2}}$ to that of $\left(\frac{a'}{\Delta}\right)^n$ are so small that it is deemed unnecessary to give the expressions for the former quantities, but we pass immediately to $\left(\frac{a'}{\Delta}\right)$ and $\left(\frac{a'}{\Delta}\right)^3$.

Arg $= i'g'+ig$	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$	
	cos.	sin.	cos.	sin.
i' i				
0 0	1.0267490		1.2681946	
0 − 1	−0.0031008	+0.0000839	−0.0272078	+0.0087021
0 − 2	+0.0000625	+0.0000001	+0.0007300	−0.0001262
0 − 3	−0.0000003	+0.0000001	−0.0000093	+0.0000041
1 + 2	−0.0000004	−0.0000012	−0.0000046	−0.0000241
1 + 1	+0.0002016	+0.0002385	+0.0007387	+0.0013937
1 0	−0.0039807	−0.0140587	−0.0163039	−0.0575499
1 − 1	+0.2256609	+0.2406638	+0.7946716	+0.8477242
1 − 2	−0.0005044	−0.0008631	−0.0095635	−0.0073459
1 − 3	+0.0000110	+0.0000139	+0.0002308	+0.0002017
1 − 4	−0.0000001	−0.0000001	−0.0000034	−0.0000021
2 + 1	+0.0000043	−0.0000046	+0.0000367	−0.0000368
2 0	−0.0001730	+0.0001609	−0.0011676	+0.0012326
2 − 1	+0.0044940	−0.0048541	+0.0252693	−0.0327933
2 − 2	−0.0050749	+0.0786501	−0.0293938	+0.4537504
2 − 3	+0.0001437	−0.0003109	+0.0001228	−0.0048500
2 − 4	−0.0000012	+0.0000053	−0.0000035	+0.0001197
2 − 5			−0.0000002	−0.0000015
3 + 1			−0.0000011	−0.0000006
3 0	+0.0000048	+0.0000019	+0.0000501	+0.0000174
3 − 1	−0.0001257	−0.0000449	−0.0012236	−0.0003071
3 − 2	+0.0026027	+0.0000998	+0.0219110	−0.0009048
3 − 3	−0.0161178	+0.0132642	−0.1291000	+0.1061610
3 − 4	+0.0001148	−0.0000269	+0.0015373	−0.0011111
3 − 5	−0.0000016	+0.0000007	−0.0000359	+0.0000280
3 − 6			+0.0000004	−0.0000005
4 0			+0.0000001	−0.0000015
4 − 1	+0.0000003	+0.0000032	+0.0000074	+0.0000397
4 − 2	−0.0000170	−0.0000645	−0.0002713	−0.0007194
4 − 3	+0.0006156	+0.0007381	+0.0070411	+0.0073737
4 − 4	−0.0057514	−0.0007479	−0.0589015	−0.0076895
4 − 5	+0.0000322	+0.0000244	+0.0006986	+0.0001979
4 − 6	−0.0000005	−0.0000003	−0.0000164	−0.0000036
5 − 1			+0.0000011	−0.0000005
5 − 2	−0.0000014	+0.0000013	−0.0000188	+0.0000179
5 − 3	+0.0000183	−0.0000241	+0.0002167	−0.0003515
5 − 4	−0.0000456	+0.0003394	−0.0003340	+0.0043736
5 − 5	−0.0009673	−0.0013450	−0.0120572	−0.0167875
5 − 6	+0.0000008	+0.0000139	+0.0001083	+0.0002509
5 − 7	−0.0000001	−0.0000001	−0.0000029	−0.0000054

Arg $= i'g' + i\varepsilon$	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$	
	cos.	sin.	cos.	sin.
i' i				
6 − 3	−0.0000008	−0.0000002	−0.0000149	−0.0000029
6 − 4	+0.0000128	−0.0000006	+0.0002016	−0.0000272
6 − 5	−0.0000976	+0.0000686	−0.0014138	+0.0011020
6 − 6	+0.0000932	−0.0004728	+0.0013761	−0.0069533
6 − 7	−0.0000032	+0.0000033	−0.0000456	+0.0000905
6 − 8			+0.0000008	−0.0000021
7 − 4	−0.0000002	−0.0000004	−0.0000035	−0.0000072
7 − 5	+0.0000036	+0.0000039	+0.0000697	+0.0000628
7 − 6	−0.0000401	−0.0000086	−0.0006964	−0.0001170
7 − 7	+0.0001204	−0.0000752	+0.0020415	−0.0012716
7 − 8	−0.0000016	−0.0000001	−0.0000360	+0.0000087
8 − 5	+0.0000001	−0.0000001	+0.0000017	−0.0000036
8 − 6	−0.0000001	+0.0000021	−0.0000007	+0.0000413
8 − 7	−0.0000071	−0.0000119	−0.0001478	−0.0002268
8 − 8	+0.0000408	+0.0000109	+0.0007822	+0.0002093

These expressions are now changed into others in which ε is replaced by g; the formulæ and numerical data for this transformation have already been given (pages 52, 53).

Arg $= i'g' + ig$	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$	
	cos.	sin.	cos.	sin.
i' i				
0 0	1.0268359		1.2689572	
0 − 1	−0.0031007	+0.0000838	−0.0272166	+0.0087058
0 − 2	−0.0000245	+0.0000024	−0.0000325	+0.0001175
0 − 3	−0.0000004	+0.0000002	−0.0000004	+0.0000073
0 − 4			−0.0000001	+0.0000004
1 + 3			+0.0000001	−0.0000003
1 + 2	+0.0000019	+0.0000020	+0.0000044	+0.0000026
1 + 1	+0.0001128	+0.0001439	+0.0004265	+0.0010612
1 0	−0.0103113	−0.0208108	−0.0385980	−0.0813494
1 − 1	+0.2255119	+0.2405231	+0.7945833	+0.8474697
1 − 2	+0.0058113	+0.0058732	+0.0126855	+0.0163834
1 − 3	+0.0002480	+0.0002485	+0.0006296	+0.0007868
1 − 4	+0.0000124	+0.0000124	+0.0000325	+0.0000414
1 − 5	+0.0000007	+0.0000007	+0.0000018	+0.0000024
2 + 2			+0.0000006	−0.0000007
2 + 1	+0.0000025	−0.0000033	+0.0000270	−0.0000272
2 0	−0.0002990	+0.0002971	−0.0018768	+0.0021527
2 − 1	+0.0047751	−0.0092579	+0.0268967	−0.0581990
2 − 2	−0.0049454	+0.0782935	−0.0286047	+0.4518156
2 − 3	−0.0001354	+0.0040783	−0.0014899	+0.0204783

Arg=$i'g'+ig$	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$	
	cos.	sin.	cos.	sin.
i' i				
2− 4	−0.0000048	+0.0002250	−0.0000837	+0.0011308
2− 5	−0.0000002	+0.0000132	−0.0000052	+0.0000661
2− 6	0.0000000	+0.0000008	−0.0000003	+0.0000039
3+ 1			−0.0000008	−0.0000004
3 0	+0.0000083	+0.0000032	+0.0000844	+0.0000260
3− 1	−0.0002904	−0.0000349	−0.0026025	−0.0001311
3− 2	+0.0039446	−0.0010155	+0.0326512	−0.0098267
3− 3	−0.0158714	+0.0131791	−0.1271383	+0.1054850
3− 4	−0.0012251	+0.0010819	−0.0091958	+0.0077667
3− 5	−0.0000827	+0.0000753	−0.0006163	+0.0005255
3− 6	−0.0000054	+0.0000050	−0.0000408	+0.0000348
3− 7	−0.0000003	+0.0000003	−0.0000027	+0.0000023
4 0			−0.0000001	−0.0000026
4− 1	+0.0000021	+0.0000077	+0.0000318	+0.0000888
4− 2	−0.0000867	−0.0001285	−0.0010463	−0.0013591
4− 3	+0.0012530	+0.0008129	+0.0135606	+0.0081418
4− 4	−0.0056325	−0.0006804	−0.0576734	−0.0070069
4− 5	−0.0006032	−0.0000547	−0.0058103	−0.0006159
4− 6	−0.0000496	−0.0000036	−0.0004666	−0.0000452
4− 7	−0.0000038	−0.0000003	−0.0000350	−0.0000032
4− 8			−0.0000026	−0.0000003
5− 1			+0.0000025	−0.0000019
5− 2	−0.0000029	+0.0000045	−0.0000370	+0.0000622
5− 3	+0.0000175	−0.0000694	+0.0001805	−0.0009354
5− 4	+0.0000912	+0.0005205	+0.0013684	+0.0066298
5− 5	−0.0009535	−0.0012834	−0.0118753	−0.0160168
5− 6	−0.0001332	−0.0001691	−0.0015631	−0.0020339
5− 7	−0.0000131	−0.0000159	−0.0001488	−0.0001885
5− 8	−0.0000011	−0.0000013	−0.0000125	−0.0000160
6− 2			+0.0000020	0.0000000
6− 3	−0.0000028	+0.0000004	−0.0000459	+0.0000080
6− 4	+0.0000270	−0.0000146	+0.0004077	−0.0002459
6− 5	−0.0001098	+0.0001459	−0.0015937	+0.0022366
6− 6	+0.0000778	−0.0004506	+0.0011529	−0.0066235
6− 7	+0.0000110	−0.0000739	+0.0001638	−0.0010444
6− 8	+0.0000010	−0.0000081	+0.0000160	−0.0001123
7− 3			+0.0000009	+0.0000012
7− 4	−0.0000011	−0.0000010	−0.0000205	−0.0000164
7− 5	+0.0000118	+0.0000042	+0.0002117	+0.0000629
7− 6	−0.0000618	+0.0000066	−0.0010628	+0.0001411
7− 7	+0.0001095	−0.0000736	+0.0018577	−0.0012434
7− 8	+0.0000208	−0.0000147	+0.0003435	−0.0002371
8− 4			−0.0000001	+0.0000010
8− 5	0.0000000	−0.0000007	−0.0000008	−0.0000137
8− 6	+0.0000021	+0.0000045	+0.0000427	+0.0000874
8− 7	−0.0000158	−0.0000136	−0.0003143	−0.0002574
8− 8	+0.0000374	+0.0000081	+0.0007151	+0.0001563

The expressions for the two factors by which $\left(\frac{a'}{\Delta}\right)^3$ must be multiplied are

$$\begin{aligned}
\frac{1}{2}\left(\frac{r'^2}{a'^2}-\alpha^2\frac{r^2}{a^2}\right) = {} & [9.6526918] \\
& -2[7.6281908]\cos g' + 2[7.4505092]\cos(-g) \\
& -2[4.9553\quad]\cos 2g' + 2[5.5968\quad]\cos(-2g) \\
& \qquad\qquad\qquad\qquad\quad + 2[4.0441\quad]\cos(-3g) \\
\frac{r'}{a'}\sin(f'+\Pi') = {} & \qquad\qquad\qquad\qquad\quad - [7.8604435] \\
& -2[9.6139931]\sin g' + 2[9.4540858]\cos g' \\
& -2[7.24218\quad]\sin 2g' + 2[7.08227\quad]\cos 2g' \\
& -2[5.0465\quad]\sin 3g' + 2[4.8866\quad]\cos 3g'
\end{aligned}$$

The products are

Arg=$i'g'+ig$	$\frac{1}{2}\left(\frac{r'^2}{a'^2}-\alpha^2\frac{r^2}{a^2}\right)\left(\frac{a'}{\Delta}\right)^3$		$\frac{r'}{a'}\sin(f'+\Pi')\left(\frac{a'}{\Delta}\right)^3$	
	cos.	sin.	sin.	cos.
i' i				
0 0	+0.0570160		+0.013258	
0 − 1	−0.0068999	+0.0002754	−0.122353	+0.567232
0 − 2	−0.0000325	+0.0000025	−0.003952	+0.010369
0 − 3	−0.0000011	+0.0000003	−0.000185	+0.000505
1 + 2	−0.0000005	−0.0000004	−0.00008	−0.00006
1 + 1	+0.0000581	+0.0001562	−0.01182	+0.00629
1 0	−0.0024662	−0.0043117	+0.15198	−0.22056
1 − 1	+0.0275617	+0.0295937	+0.02166	+0.00201
1 − 2	−0.0009115	−0.0015739	−0.19389	+0.11675
1 − 3	−0.0000213	−0.0000506	−0.00917	+0.00512
1 − 4	−0.0000006	−0.0000023	−0.00050	+0.00028
2 + 2	+0.0000003	−0.0000002		
2 + 1	+0.0000033	−0.0000124	+0.00051	+0.00012
2 0	−0.0001682	+0.0013487	−0.05101	+0.00229
2 − 1	+0.0025682	−0.0277736	+0.57359	−0.08621
2 − 2	−0.0106082	+0.1636826	+0.02389	+0.00678
2 − 3	−0.0001487	+0.0079848	−0.07902	−0.02243
2 − 4	−0.0000012	+0.0004384	−0.00585	−0.00170
2 − 5	+0.0000002	+0.0000254	−0.00041	−0.00011
2 − 6	+0.0000001	+0.0000010		
3 + 1	−0.0000004	0.0000000		
3 0	+0.0000404	+0.0000057	+0.00005	+0.00152
3 − 1	−0.0011111	+0.0001180	−0.01385	−0.02796
3 − 2	+0.0124610	−0.0055267	+0.17770	+0.13955
3 − 3	−0.0491933	+0.0406944	+0.00943	+0.01358
3 − 4	−0.0036343	+0.0032734	−0.01303	−0.02541
3 − 5	−0.0002429	+0.0002272	−0.00135	−0.00259
3 − 6	−0.0000158	+0.0000152	−0.00011	−0.00020
3 − 7	−0.0000011	+0.0000010		

$\mathrm{Arg} = i'g' + ig$	$\frac{1}{2}\left(\frac{r'^2}{a'^2} - \alpha^2\frac{r^2}{a^2}\right)\left(\frac{a'}{\Delta}\right)^3$		$\frac{r'}{a'}\sin(f' + \Pi')\left(\frac{a'}{\Delta}\right)^3$	
	cos.	sin.	sin.	cos.
i' i				
4 0	−0.0000003	−0.0000011		
4— 1	+0.0000209	+0.0000329	−0.00084	+0.00091
4— 2	−0.0005291	−0.0004864	+0.00598	−0.01561
4— 3	+0.0058429	+0.0027851	+0.00757	+0.08205
4— 4	−0.0230505	−0.0028491	−0.00135	+0.00849
4— 5	−0.0024204	−0.0002033	+0.00329	−0.00904
4— 6	−0.0001969	−0.0000120	+0.00040	−0.00119
4— 7	−0.0000148	−0.0000006	+0.00004	−0.00011
4— 8	−0.0000012	0.0000000		
5— 1	+0.0000009	−0.0000011		
5— 2	−0.0000105	+0.0000294		
5— 3	+0.0000194	−0.0004030		
5— 4	+0.0007797	+0.0027024		
5— 5	−0.0048298	−0.0065511		
5— 6	−0.0006722	−0.0008468		
5— 7	−0.0000659	−0.0000787		
5— 8	−0.0000057	−0.0000067		
6— 2	+0.0000010	−0.0000003		
6— 3	−0.0000186	+0.0000067		
6— 4	+0.0001600	−0.0001252		
6— 5	−0.0006073	+0.0009807		
6— 6	+0.0004864	−0.0027402		
6— 7	+0.0000640	−0.0004447		
6— 8	+0.0000058	−0.0000486		
7— 3	+0.0000005	+0.0000005		
7— 4	−0.0000098	−0.0000058		
7— 5	+0.0000931	+0.0000173		
7— 6	−0.0004461	+0.0000845		
7— 7	+0.0007789	−0.0005169		
7— 8	+0.0001461	−0.0001030		
8— 4	+0.0000001	+0.0000005		
8— 5	−0.0000012	−0.0000060		
8— 6	+0.0000218	+0.0000359		
8— 7	−0.0001392	−0.0001030		
8— 8	+0.0003003	+0.0000665		

For the computation of $a'\frac{r}{r'^2}H$ we have (page 63)

$\log h = 9.3366883$ $\log l = 9.3637535n$

$\log h_1 = 9.3359062$ $\log l_1 = 9.3644536n$

We have then

Arg=$i'g'+ig$	$-a'\frac{r}{r'^2}H$		Arg=$i'g'+ig$	$-a'\frac{r}{r'^2}H$	
	cos.	sin.		cos.	sin.
i' i			i' i		
1 + 2	−0.0000022	−0.0000019	2 − 1	−0.0036830	−0.0039265
1 + 1	−0.0001080	−0.0000975	2 − 2	−0.0001032	−0.0001099
1 0	+0.0182556	+0.0194612	2 − 3	−0.0000043	−0.0000046
1 − 1	−0.2167446	−0.2310759	2 − 4	−0.0000002	−0.0000002
1 − 2	−0.0060702	−0.0064716	3 0	+0.0000044	+0.0000047
1 − 3	−0.0002550	−0.0002718	3 − 1	−0.0000528	−0.0000563
1 − 4	−0.0000127	−0.0000135	3 − 2	−0.0000015	−0.0000016
1 − 5	−0.0000007	−0.0000007	3 − 3	−0.0000001	−0.0000001
2 + 1	−0.0000018	−0.0000017	4 − 1	−0.0000007	−0.0000008
2 0	+0.0003102	+0.0003307			

In addition

$$-\frac{a'^2}{r'^2}\sin(f'+\Pi') = +0.82229 \sin g' - 0.56900 \cos g'$$
$$+0.01397 \sin 2g' - 0.00967 \cos 2g'$$
$$+0.00020 \sin 3g' - 0.00014 \cos 3g'$$

The logarithms of the factors which depend on the mass of Neptune are

$\log \mu = 0.5211900$ $\log(\mu\alpha \sin J) = 8.2488067$

The expressions for the forces are then

Arg=$i'g'+ig$	$a\frac{d\Omega}{dg}$		$ar\frac{d\Omega}{dr}$		$a^2\frac{d\Omega}{dZ}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	″	″	″	″	″	″
0 0			+0.189316			+0.000235
0 − 1	−0.010295	−0.000278	−0.022910	+0.000914	+0.0100593	−0.0021698
0 − 2	−0.000163	−0.000016	−0.000108	+0.000008	+0.000184	−0.000070
0 − 3	−0.000004	−0.000002	−0.000004	+0.000001	−0.000009	−0.000003
1 + 2	+0.000002	+0.000001	−0.000002	−0.000001		
1 + 1	−0.000016	+0.000154	+0.000193	+0.000519	+0.000112	−0.000210
1 0			−0.008189	−0.014317	−0.003911	+0.002695
1 − 1	+0.029111	−0.031368	+0.091516	+0.098263	+0.000035	+0.000380
1 − 2	−0.001719	+0.003974	−0.003027	−0.005226	+0.002070	−0.003438
1 − 3	−0.000070	+0.000232	−0.000071	−0.000168	+0.000089	−0.000163
1 − 4	−0.000004	+0.000015	−0.000002	−0.000008	0.000000	−0.000009
2 + 1	−0.000002	−0.000017	+0.000011	−0.000041	0.000000	+0.000009
2 0			−0.000558	+0.004478	+0.000041	−0.000905
2 − 1	+0.003626	+0.043777	+0.008527	−0.092219	−0.001529	+0.010172

Arg=$i'g'+ig$	$a\frac{d\Omega}{dg}$		$ar\frac{d\Omega}{dr}$		$a^2\frac{d\Omega}{dZ}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	″	″	″	″	″	″
2— 2	−0.033527	−0.519201	−0.035223	+0.543491	+0.000120	+0.000424
2— 3	−0.001392	−0.040579	−0.000494	+0.026512	−0.000398	−0.001401
2— 4	--0.000066	−0.002986	−0.000004	+0.001456	−0.000030	−0.000104
2— 5	−0.000003	−0.000219	+0.000001	+0.000084	0.000000	−0.000007
2— 6	0.000000	−0.000016	0.000000	+0.000003		
3 0			+0.000134	+0.000019	+0.000027	0.000000
3— 1	−0.001140	+0.000303	−0.003689	+0.000392	−0.000496	−0.000246
3— 2	+0.026185	+0.006754	+0.041375	−0.018351	+0.002474	+0.003151
3— 3	−0.158099	−0.131279	−0.163341	+0.135122	+0.000241	+0.000167
3— 4	−0.016271	−0.014369	−0.012067	+0.010869	−0.000451	−0.000231
3— 5	−0.001373	−0.001250	−0.000810	+0.000754	−0.000046	−0.000024
3— 6	−0.000108	−0.000100	−0.000052	+0.000050		
3— 7	−0.000007	−0.000007	−0.000004	+0.000003		
4 0			−0.000001	−0.000004		
4— 1	+0.000005	−0.000023	+0.000069	+0.000109	+0.000015	−0.000015
4— 2	−0.000576	+0.000853	−0.001757	−0.001615	−0.000277	+0.000106
4— 3	+0.012481	−0.008097	+0.019401	+0.009248	+0.001455	+0.000134
4— 4	−0.074808	+0.009034	−0.076537	−0.009460	+0.000151	−0.000024
4— 5	−0.010014	+0.000908	−0.008037	−0.000675	−0.000160	+0.000059
4— 6	−0.000988	+0.000072	−0.000654	−0.000040	−0.000021	+0.000007
4— 7	−0.000088	+0.000007	−0.000049	−0.000002		
4— 8			−0.000004	0.000000		
5— 2	−0.000019	−0.000030	−0.000035	+0.000098		
5— 3	+0.000174	+0.000691	+0.000064	−0.001338		
5— 4	+0.001211	−0.006912	+0.002589	+0.008973		
5— 5	−0.015831	+0.021307	−0.016037	−0.021752		
5— 6	−0.002654	+0.003368	−0.002232	−0.002812		
5— 7	−0.000304	+0.000370	−0.000219	−0.000261		
5— 8	−0.000029	+0.000035	−0.000019	−0.000022		
6— 3	−0.000028	−0.000004	−0.000062	+0.000022		
6— 4	+0.000359	+0.000194	+0.000531	−0.000415		
6— 5	−0.001823	−0.002423	−0.002016	+0.003255		
6— 6	+0.001550	+0.008976	+0.001615	−0.009098		
6— 7	+0.000256	+0.001718	+0.000213	−0.001477		
6— 8	+0.000026	+0.000215	+0.000019	−0.000161		
7— 4	−0.000015	+0.000013	−0.000033	−0.000019		
7— 5	+0.000196	−0.000070	+0.000309	+0.000057		
7— 6	−0.001231	−0.000131	−0.001481	+0.000281		
7— 7	+0.002546	+0.001711	+0.002586	−0.001716		
7— 8	+0.000553	+0.000390	+0.000485	−0.000342		
8— 5	0.000000	+0.000012	−0.000004	−0.000020		
8— 6	+0.000042	−0.000090	+0.000072	+0.000118		
8— 7	−0.000367	+0.000316	−0.000462	−0.000342		
8— 8	+0.000994	−0.000215	+0.000997	+0.000221		

The expressions by which these forces must be multiplied in order that we may obtain T and $\frac{1}{n}\frac{dR}{dt}$ have already been given (page 74), and the resulting developments are

Arg=$\varkappa\gamma+i'g'+ig$			T	
			sin.	cos.
$\varkappa$	i'	i	"	"
1	0—	1	—0.37621	+0.00003
—1	0	0	—0.0329355	—0.0014737
0	0—	1	+0.03088	+0.00083
1	0—	2	—0.00823	+0.00036
—1	0—	1	+0.00037	—0.00003
0	0—	2	+0.00049	+0.00005
1	0—	3	—0.00032	0.00000
—1	0—	2	+0.00001	0.00000
0	0—	3	+0.00001	+0.00001
1	0—	4	—0.00001	0.00000
—1	1+	2	+0.00006	+0.00005
0	1+	1	+0.00005	—0.00046
1	1	0	+0.00001	+0.00043
—1	1+	1	—0.00480	+0.01068
1	1—	1	+0.00153	—0.00717
—1	1	0	+0.14961	—0.16060
0	1—	1	—0.08733	+0.09410
1	1—	2	—0.03279	+0.03447
—1	1—	1	—0.00799	+0.01484
0	1—	2	+0.00516	—0.01192
1	1—	3	—0.00214	+0.00456
—1	1—	2	—0.00006	+0.00022
0	1—	3	+0.00021	—0.00070
1	1—	4	—0.00013	+0.00035
—1	2+	2	0.00000	—0.00003
0	2+	1	+0.00001	+0.00005
1	2	0	+0.00001	+0.00006
—1	2+	1	—0.00029	—0.00172
1	2—	1	—0.00019	—0.00425
—1	2	0	+0.01386	+0.15007
0	2—	1	—0.01088	—0.13133
1	2—	2	+0.00445	+0.08363
—1	2—	1	—0.10276	—1.58983
0	2—	2	+0.10058	+1.55760
1	2—	3	—0.03181	—0.49060
—1	2—	2	+0.00044	—0.05033
0	2—	3	+0.00418	+0.12174
1	2—	4	—0.00225	—0.05367
—1	2—	3	+0.00006	—0.00249
0	2—	4	+0.00020	+0.00896
1	2—	5	—0.00015	—0.00466
—1	2—	4	0.00000	—0.00019
0	2—	5	+0.00001	+0.00066
1	2—	6	—0.00001	—0.00042
—1	3+	1	+0.00004	0.00000
1	3—	1	+0.00013	—0.00001
—1	3	0	—0.00439	+0.00066
0	3—	1	+0.00342	—0.00091
1	3—	2	—0.00343	—0.00047
—1	3—	1	+0.08492	+0.02437
0	3—	2	—0.07855	—0.02026
1	3—	3	+0.03787	+0.01740
—1	3—	2	—0.48361	—0.39943
0	3—	3	+0.47430	+0.39384
1	3—	4	—0.15090	—0.12563
—1	3—	3	—0.02713	—0.02509
0	3—	4	+0.04881	+0.04311
1	3—	5	—0.02017	—0.01760
—1	3—	4	—0.00162	—0.00155
0	3—	5	+0.00412	+0.00375
1	3—	6	—0.00197	—0.00177
—1	3—	5	—0.00010	—0.00009
0	3—	6	+0.00032	+0.00030
1	3—	7	—0.00018	—0.00015
—1	4	0	+0.00004	—0.00011
0	4—	1	—0.00001	+0.00007
1	4—	2	+0.00007	—0.00011
—1	4—	1	—0.00217	+0.00286
0	4—	2	+0.00173	—0.00256
1	4—	3	—0.00169	+0.00149
—1	4—	2	+0.04020	—0.02503
0	4—	3	—0.03744	+0.02429
1	4—	4	+0.01826	—0.00853
—1	4—	3	—0.22822	+0.02851
0	4—	4	+0.22442	—0.02710
1	4—	5	—0.07174	+0.00850
—1	4—	4	—0.01981	+0.00150
0	4—	5	+0.03004	—0.00272
1	4—	6	—0.01181	+0.00111
—1	4—	5	—0.00145	+0.00007
0	4—	6	+0.00296	—0.00022
1	4—	7	—0.00134	+0.00011

Arg=$\varkappa\gamma+i'g'+ig$			T	
			sin.	cos.
$\varkappa$	i'	i	″	″
−1	4	−6	−0.00011	0.00000
0	4	−7	+0.00026	−0.00002
1	4	−8	−0.00014	+0.00001
−1	5	−1	−0.00007	−0.00015
0	5	−2	+0.00006	+0.00009
1	5	−3	−0.00003	−0.00010
−1	5	−2	+0.00048	+0.00233
0	5	−3	−0.00052	−0.00207
1	5	−4	+0.00005	+0.00126
−1	5	−3	+0.00411	−0.02168
0	5	−4	−0.00363	+0.02074
1	5	−5	+0.00251	−0.00847
−1	5	−4	−0.04800	+0.06536
0	5	−5	+0.04749	−0.06392
1	5	−6	−0.01526	+0.02041
−1	5	−5	−0.00580	+0.00721
0	5	−6	+0.00796	−0.01010
1	5	−7	−0.00301	+0.00387
−1	5	−6	−0.00052	+0.00062
0	5	−7	+0.00091	−0.00111
1	5	−8	−0.00039	+0.00048
−1	5	−7	−0.00005	+0.00005
0	5	−8	+0.00009	−0.00010
1	5	−9	−0.00004	+0.00005
−1	6	−2	−0.00010	0.00000
0	6	−3	+0.00008	+0.00001
1	6	−4	−0.00006	−0.00003
−1	6	−3	+0.00114	+0.00066
0	6	−4	−0.00108	−0.00058
1	6	−5	+0.00051	+0.00040
−1	6	−4	−0.00563	−0.00762
0	6	−5	+0.00547	+0.00727
1	6	−6	−0.00189	−0.00311
−1	6	−5	+0.00494	+0.02743
0	6	−6	−0.00465	−0.02693
1	6	−7	+0.00144	+0.00861
−1	6	−6	+0.00054	+0.00393
0	6	−7	−0.00077	−0.00515
1	6	−8	+0.00030	+0.00191
−1	6	−7	+0.00004	+0.00039
0	6	−8	−0.00008	−0.00064
1	6	−9	+0.00003	+0.00029
−1	7	−3	−0.00004	+0.00005
0	7	−4	+0.00004	−0.00004
1	7	−5	−0.00004	+0.00002
−1	7	−4	+0.00063	−0.00021
0	7	−5	−0.00059	+0.00021
1	7	−6	+0.00029	−0.00005
−1	7	−5	−0.00383	−0.00043
0	7	−6	+0.00369	+0.00039
1	7	−7	−0.00142	−0.00027
−1	7	−6	+0.00785	+0.00518
0	7	−7	−0.00764	−0.00513
1	7	−8	+0.00243	+0.00166
−1	7	−7	+0.00131	+0.00093
0	7	−8	−0.00166	−0.00117
1	7	−9	+0.00062	+0.00044
−1	8	−4	0.00000	+0.00004
0	8	−5	0.00000	−0.00004
1	8	−6	−0.00001	+0.00001
−1	8	−5	+0.00013	−0.00028
0	8	−6	−0.00013	+0.00027
1	8	−7	+0.00007	−0.00011
−1	8	−6	−0.00115	+0.00096
0	8	−7	+0.00110	−0.00095
1	8	−8	−0.00045	+0.00033
−1	8	−7	+0.00303	−0.00068
0	8	−8	−0.00298	+0.00064
1	8	−9	+0.00099	−0.00021

Arg=$\varkappa\gamma+i'g'+ig$			$\frac{1}{n}\frac{dR}{dt}$		Arg=$\varkappa\gamma+i'g'+ig$			$\frac{1}{n}\frac{dR}{dt}$	
			cos.	sin.				cos.	sin.
$\varkappa$	i'	i	″	″	$\varkappa$	i'	i	″	″
1	0	−1	+0.00056	+0.00029	−1	2	−2	−0.00021	+0.00072
−1	0	0	+0.0050243	+0.0011042	1	2	−4	+0.00020	−0.00068
1	0	−2	−0.00500	−0.00109	−1	2	−3	0.00000	−0.00001
−1	0	−1	−0.00033	−0.00006	1	2	−5	+0.00003	−0.00007
1	0	−3	−0.00023	−0.00006	−1	3	+1	0.00000	0.00000
−1	1	+2	0.00000	+0.00006	1	3	−1	−0.00003	+0.00001
1	1	0	−0.00022	−0.00021	−1	3	0	−0.00022	+0.00008
−1	1	+1	−0.00195	−0.00137	1	3	−2	+0.00035	−0.00025
1	1	−1	+0.00195	+0.00133	−1	3	−1	+0.00126	−0.00158
−1	1	0	+0.00021	−0.00003	1	3	−3	−0.00122	+0.00156
1	1	−2	+0.00013	+0.00037	−1	3	−2	+0.00001	+0.00005
−1	1	−1	+0.00103	+0.00174	1	3	−4	−0.00017	+0.00013
1	1	−3	−0.00103	−0.00170	−1	3	−3	−0.00023	+0.00013
−1	1	−2	−0.00005	−0.00006	1	3	−5	+0.00022	−0.00012
−1	1	−4	−0.00007	−0.00013	−1	4	−1	−0.00012	−0.00005
1	2	0	0.00000	+0.00004	1	4	−3	+0.00020	+0.00004
−1	2	+1	0.00000	+0.00031	−1	4	−2	+0.00074	−0.00007
1	2	−1	−0.00008	−0.00088	1	4	−4	−0.00072	+0.00007
−1	2	0	−0.00076	−0.00513	−1	4	−3	+0.00002	+0.00002
1	2	−2	+0.00077	+0.00505	1	4	−5	−0.00011	−0.00001
−1	2	−1	+0.00011	+0.00024	−1	4	−4	−0.00009	−0.00003
1	2	−3	−0.00006	+0.00041	1	4	−6	+0.00008	+0.00003

The table of logarithms of the integrating factors follows:

Arg.	log $\frac{n}{i'n'+in}$	Arg.	log $\frac{n}{i'n'+in}$	Arg.	log $\frac{n}{i'n'+in}$	Arg.	log $\frac{n}{i'n'+in}$
i' i		i' i		i' i		i' i	
0 −1	0.00000n	2 −4	9.43860n	4 −5	9.36806n	6 −6	9.3074n
0 −2	9.69897n	2 −5	9.3332n	4 −6	9.2770n	6 −7	9.2271n
0 −3	9.5229n	2 −6	9.2485n	4 −7	9.2017n	6 −8	9.1594n
1 +2	9.6618	3 +1	9.8135	4 −8	9.1376n	6 −9	9.1009n
1 +1	9.9286	3 0	0.2706	5 −1	0.9740n	7 −3	9.7574n
1 0	0.74772	3 −1	0.33375n	5 −2	9.9562n	7 −4	9.5609n
1 −1	0.08553n	3 −2	9.83455n	5 −3	9.6765n	7 −5	9.4261n
1 −2	9.73963n	3 −3	9.60841n	5 −4	9.5078n	7 −6	9.3234n
1 −3	9.54956n	3 −4	9.46046n	5 −5	9.3866n	7 −7	9.2404n
1 −4	9.4178n	3 −5	9.35031n	5 −6	9.2919n	7 −8	9.1708n
1 −5	9.3168n	3 −6	9.2625n	5 −7	9.2142n	7 −9	9.1108n
2 +2	9.6275	3 −7	9.1895n	5 −8	9.1484n	8 −4	9.5901n
2 +1	9.8673	4 0	0.1457	5 −9	9.0912n	8 −5	9.4473n
2 0	0.44669	4 −1	0.54525n	6 −2	0.0327n	8 −6	9.3401n
2 −1	0.19215n	4 −2	9.89112n	6 −3	9.7150n	8 −7	9.2541n
2 −2	9.78450n	4 −3	9.64112n	6 −4	9.5335n	8 −8	9.1924n
2 −3	9.57799n	4 −4	9.48347n	6 −5	9.4059n	8 −9	9.1209n

In integrating we put

$k_0 = +0''.3763 \qquad k_1 = -0''.0492 \qquad k_2 = -0''.0025 \qquad k_3 = +0''.0025 \qquad k_4 = +0''.0005$

And we have

Arg$=i'g'+ig$	$\frac{d\,.\,\delta z}{dt}$		$\frac{1}{n}\frac{dv}{dt}$	
	cos.	sin.	sin.	cos.
i' i	$''$	$''$	$''$	$''$
0 0				+0.0000207
0 − 1	−0.0329	+0.0015	−0.0162	+0.0014
	−0.0014737nt	−0.0329355nt	−0.0007368nt	+0.0164677nt
0 − 2	−0.0012	+0.0001	+0.0018	+0.0001
	−0.0000412nt	−0.0009224nt	−0.0000412nt	+0.0009224nt
1 + 1	0.0000	+0.0024	+0.0001	−0.0014
1 0	+0.00543	+0.01726	−0.00161	+0.00070
1 − 1	−0.9611	−1.0316	+0.4094	−0.4394
1 − 2	−0.0311	−0.0384	+0.0279	−0.0334
1 − 3	−0.0012	−0.0016	+0.0018	−0.0022
2 + 1	0.0000	+0.0002	0.0000	−0.0001
2 0	−0.00002	+0.00413	−0.00020	−0.00284
2 − 1	−0.0533	+0.5784	+0.0205	+0.2302
2 − 2	−0.1118	+1.7242	+0.0751	+1.1559
2 − 3	−0.0032	+0.0690	+0.0042	+0.0779
2 − 4	−0.0001	+0.0032	+0.0003	+0.0053
3 0	+0.0002	0.0000	0.0000	0.0000
3 − 1	+0.01360	+0.00335	−0.00491	+0.00060
3 − 2	+0.1438	−0.0448	−0.0852	−0.0238
3 − 3	−0.1765	+0.1479	+0.1382	+0.1167
3 − 4	−0.0105	+0.0092	+0.0122	+0.0106
3 − 5	−0.0007	+0.0006	+0.0009	+0.0008
4 − 1	−0.00008	−0.00033	+0.00003	−0.00010
4 − 2	−0.0068	−0.0086	+0.0036	−0.0048
4 − 3	+0.0198	+0.0111	−0.0131	+0.0082
4 − 4	−0.0475	−0.0058	+0.0405	−0.0047
4 − 5	−0.0040	−0.0005	+0.0047	−0.0005
4 − 6	+0.0002	0.0000	+0.0007	0.0000
5 − 2	−0.0006	+0.0013	+0.0003	+0.0007
5 − 3	+0.0002	−0.0014	−0.0002	−0.0010
5 − 4	+0.0013	+0.0055	−0.0008	+0.0041
5 − 5	−0.0068	−0.0091	+0.0061	−0.0082
5 − 6	−0.0006	−0.0010	+0.0008	−0.0012
6 − 4	+0.0003	−0.0002	−0.0002	−0.0001
6 − 5	−0.0009	+0.0013	+0.0008	+0.0010
6 − 6	+0.0005	−0.0029	−0.0004	−0.0027
6 − 7	0.0000	−0.0004	−0.0001	−0.0005
7 − 5	+0.0001	0.0000	−0.0001	0.0000
7 − 6	−0.0004	0.0000	+0.0004	+0.0001
7 − 7	+0.0008	−0.0004	−0.0006	−0.0004
7 − 8	+0.0001	−0.0001	−0.0001	−0.0001
8 − 7	−0.0002	−0.0001	+0.0001	−0.0001
8 − 8	+0.0001	0.0000	−0.0002	+0.0001

In fine we have the perturbations of Saturn by Neptune. Here we give a single accent to the symbols belonging to Saturn and three accents to the mean anomaly of Neptune.

$\text{Arg}=i'g'''+ig'$	$n'\delta z'$		ν'		$\frac{u'}{\cos i'}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	"	"	"	"	"	"
0 0			-0.0620			+0.0003
			+0.0000207$n't$			-0.0004230$n't$
0 - 1	0.0000	0.0000	-0.0327	-0.0007	0.0000	0.0000
	+0.0014737$n't$	-0.0329355$n't$	+0.0007363$n't$	+0.0164677$n't$	+0.0011042$n't$	+0.0050243$n't$
0 - 2	+0.0004	0.0000	+0.0007	0.0000	+0.0001	-0.0001
	+0.0000206$n't$	-0.0004612$n't$	+0.0000206$n't$	+0.0004612$n't$	+0.0000309$n't$	+0.0001407$n't$
1 + 1	0.0000	-0.0020	-0.0001	-0.0012	-0.0013	+0.0012
1 0	+0.0304	-0.0965	+0.0090	+0.0039	-0.0041	+0.0028
1 - 1	+1.1703	-1.2561	+0.4984	+0.5350	+0.0013	+0.0001
1 - 2	+0.0171	-0.0211	+0.0153	+0.0183	-0.0009	+0.0015
1 - 3	+0.0004	-0.0006	+0.0006	+0.0008		
2 0	-0.0001	-0.0116	+0.0006	-0.0079	+0.0003	-0.0027
2 - 1	+0.0829	+0.9001	+0.0319	-0.3582	-0.0026	+0.0175
2 - 2	+0.0681	+1.0498	+0.0457	-0.7037	-0.0003	+0.0008
2 - 3	+0.0012	+0.0261	+0.0016	-0.0295		
2 - 4	0.0000	+0.0009	+0.0001	-0.0015		
3 0	+0.0004	0.0000	0.0000	0.0000		
3 - 1	-0.0293	+0.0072	-0.0106	-0.0013	-0.0004	0.0000
3 - 2	-0.0982	-0.0306	-0.0582	+0.0163	-0.0022	-0.0028
3 - 3	+0.0716	+0.0600	+0.0561	-0.0474	-0.0001	-0.0001
3 - 4	+0.0030	+0.0027	+0.0035	-0.0031		
3 - 5	+0.0002	+0.0001	+0.0002	-0.0002		
4 - 1	+0.0003	-0.0012	+0.0001	+0.0004		
4 - 2	+0.0053	-0.0067	+0.0028	+0.0037	+0.0004	-0.0002
4 - 3	-0.0087	+0.0049	-0.0057	-0.0036	-0.0004	-0.0001
4 - 4	+0.0145	-0.0018	+0.0123	+0.0015		
4 - 5	+0.0009	-0.0001	+0.0011	+0.0001		
5 - 2	+0.0005	+0.0012	+0.0003	-0.0006		
5 - 3	-0.0001	-0.0007	-0.0001	+0.0005		
5 - 4	-0.0004	+0.0018	-0.0003	-0.0013		
5 - 5	+0.0017	-0.0022	+0.0015	+0.0020		
5 - 6	+0.0001	-0.0002	+0.0002	+0.0002		
6 - 4	-0.0001	-0.0001	-0.0001	0.0000		
6 - 5	+0.0002	+0.0003	+0.0002	-0.0003		
6 - 6	-0.0001	-0.0006	-0.0001	+0.0005		
6 - 7	0.0000	-0.0001	0.0000	+0.0001		
7 - 6	+0.0001	0.0000	+0.0001	0.0000		
7 - 7	-0.0001	-0.0001	-0.0001	+0.0001		

CHAPTER VI.

PERTURBATIONS OF JUPITER BY NEPTUNE.

In this case also first-order terms suffice. The elements of the two planets have already been given (pages 19, 161). The coefficients of the terms of the developments of the reciprocal of the distance between Jupiter and Neptune, and its odd powers, are functions of the six elements

$$\begin{array}{lll} \log \alpha = 9.2380919 & \qquad & \qquad\qquad\qquad {}^{\circ} \quad\ {}' \quad\ {}'' \\ \quad e = 0.04824277 & & J = \ \ 0 \quad 56 \quad 53.38 \\ \quad e' = 0.0084962 & & \Pi = 196 \quad 4 \quad 4.53 \\ & & \Pi' = 227 \quad 24 \quad 47.47 \end{array}$$

Π and Π' are measured from the ascending node of the orbit of Neptune on that of Jupiter. The developments will be made first in terms of the eccentric anomaly of the latter planet.

The values of the auxiliary constants are

$$\begin{array}{lll} & & \qquad\qquad {}^{\circ} \quad\ {}' \quad\ {}'' \\ \log k \ \ = 9.9999955 & & K \ = 31 \quad 20 \quad 50.47 \\ \log k_1 \ = 9.9999450 & & K_1 = 31 \quad 20 \quad 35.43 \\ \log p \ \ = 9.2675773 & & P \ = 13 \quad 53 \quad 1.47 \\ \log v \ \ = 9.5387100 & & V \ = 31 \quad 22 \quad 43.86 \\ \log w \ \ = 9.5385860 & & W \ = 17 \quad 26 \quad 35.35 \\ \log w_1 \ = 9.5390766 & & W_1 = 17 \quad 28 \quad 47.54 \\ \log \tfrac{1}{2}\gamma_2 = 5.5420182 & & \end{array}$$

$$D = 1.0298308 - [8.2302547] \cos \varepsilon' + [5.8584494] \cos^2 \varepsilon' + [8.6834322] f \cos F$$

The circumference will be divided into eight parts with reference to g', the mean anomaly of Neptune. For three points of division we have

g'	ε'
45°	45° 20′ 46″.64
90	90 29 12 .40
135	135 20 31 .76

We get the following table of values of D, log f, and F:

g'	D	log f	F—g'		
°			°	′	″
0	1.0271856	9.5383650	31	3	45.80
45	1.0217845	9.5368144	31	33	22.27
90	1.0211885	9.5368308	31	56	48.30
135	1.0257276	9.5381879	31	56	6.93
180	1.0326568	9.5395481	31	33	35.96
225	1.0379352	9.5403230	31	6	29.59
270	1.0385560	9.5405918	30	48	57.11
315	1.0341396	9.5400099	30	47	8.07
S	4.1195869	8.1553357	125	23	7.17
S′	4.1195869	8.1553352	125	23	6.86

Employing the same method as in the preceding chapter, we have the following values of log $\alpha_i^{(n)}$:

	log $\alpha_0^{(1)}$	log $\alpha_1^{(1)}$	log $\alpha_2^{(1)}$	log $\alpha_3^{(1)}$	log $\alpha_4^{(1)}$	log $\alpha_5^{(1)}$	log $\alpha_6^{(1)}$
(0)	0.0038932	8.9430342	8.0571682	7.2167845	6.397480	5.59035	4.79117
(1)	0.0050731	8.9450050	8.0599259	7.2203281	6.401808	5.59546	4.79706
(2)	0.0052126	8.9454332	8.0606412	7.2213302	6.403097	5.59705	4.79894
(3)	0.0042225	8.9438341	8.0584363	7.2185203	6.399684	5.59303	4.79431
(4)	0.0026867	8.9406253	8.0535630	7.2119846	6.391486	5.58317	4.78279
(5)	0.0015122	8.9379116	8.0493179	7.2062100	6.384182	5.57433	4.77243
(6)	0.0013828	8.9377920	8.0492080	7.2061098	6.384092	5.57425	4.77235
(7)	0.0023676	8.9401337	8.0528997	7.2111498	6.390480	5.58199	4.78144
S	0.0131753	5.7668847	2.2205804	88.8562091	85.576155	82.34482	79.14525
S′	0.0131754	5.7668844	2.2205798	88.8562082	85.576154	82.34481	79.14524

	log $\alpha_0^{(3)}$	log $\alpha_1^{(3)}$	log $\alpha_2^{(3)}$	log $\alpha_3^{(3)}$	log $\alpha_4^{(3)}$	log $\alpha_5^{(3)}$	log $\alpha_6^{(3)}$
(0)	0.0314812	9.4412053	8.7749893	8.0796296	7.368805	6.64838	5.92143
(1)	0.0350934	9.4455847	8.7801476	8.0855698	7.375529	6.65590	5.92973
(2)	0.0355384	9.4463096	8.7811567	8.0868642	7.377108	6.65776	5.93188
(3)	0.0325122	9.4426928	8.7769403	8.0820459	7.371687	6.65172	5.92523
(4)	0.0277517	9.4363094	8.7689093	8.0723610	7.360346	6.63873	5.91058
(5)	0.0240883	9.4311526	8.7622367	8.0641668	7.350628	6.62750	5.89782
(6)	0.0237010	9.4307747	8.7618684	8.0638081	7.350278	6.62715	5.89748
(7)	0.0267786	9.4351689	8.7675989	8.0708801	7.358694	6.63690	5.90858
S	0.1184723	7.7545990	5.0869237	92.3026629	89.456537	86.57202	83.66137
S′	0.1184725	7.7545990	5.0869235	92.3026626	89.456538	86.57202	83.66136

The values of the coefficients A for the development of $[D-f\cos(\varepsilon-F)]^{-\frac{1}{2}}$ and $[D-f\cos(\varepsilon-F)]^{-\frac{3}{2}}$, no division by an integer being made, are

	$A_0^{(c)}$	$A_1^{(c)}$	$A_1^{(s)}$	$A_2^{(c)}$	$A_2^{(s)}$	$A_3^{(c)}$
		7	7	7	7	7
(0)	1.0090047	+ 751301	− 452547	+ 53332	−100834	− 916
(1)	1.0117498	750774	461089	51915	102386	1352
(2)	1.0120748	748351	466655	50598	103254	1694
(3)	1.0097700	745694	464791	50383	102711	1673
(4)	1.0062056	743211	456511	51147	100903	1329
(5)	1.0034881	742136	447830	52219	99111	932
(6)	1.0031891	744206	443915	53214	98548	686
(7)	1.0054665	+ 748465	− 445919	+ 53773	− 99332	− 669
S	4.0304742	+2987069	−1819628	+208291	−403539	−4625
S′	4.0304744	+2987069	−1819629	+208290	−403540	−4626

	$A_3^{(s)}$	$A_4^{(c)}$	$A_4^{(s)}$	$A_5^{(c)}$	$A_5^{(s)}$	$A_6^{(c)}$	$A_6^{(s)}$
	7	7	7	7	7	7	7
(0)	−16448	−1406	−2064	− 354	−163	− 61	+ 7
(1)	16553	1491	2035	365	149	62	10
(2)	16560	1550	1999	371	137	62	13
(3)	10455	1536	1985	367	136	61	13
(4)	16238	1456	1987	355	145	60	10
(5)	16050	1369	1998	342	155	59	7
(6)	16059	1328	2025	337	164	59	5
(7)	−16247	−1343	−2058	− 343	−168	− 60	+ 5
S	−65305	−5740	−8075	−1417	−609	−242	+35
S′	−65305	−5739	−8076	−1417	−608	−242	+35

	$A_0^{(c)}$	$A_1^{(c)}$	$A_1^{(s)}$	$A_2^{(c)}$	$A_2^{(s)}$	$A_3^{(c)}$
		7	7	7	7	7
(0)	1.0751800	+2365838	−1425066	+ 278487	− 526537	− 6681
(1)	1.0841600	2377329	1460037	272595	537603	9912
(2)	1.0852715	2371275	1478675	265858	542528	12429
(3)	1.0777355	2351904	1465940	263505	537181	12219
(4)	1.0659865	2326999	1429340	265562	523905	9638
(5)	1.0570324	2310597	1394294	269616	511729	6723
(6)	1.0560902	2315666	1381283	274592	508519	4946
(7)	1.0636007	+2339952	−1394093	+ 278783	− 514979	− 4841
S	4.2825282	+9379778	−5714364	+1084499	−2101489	−33694
S′	4.2825286	+9379782	−5714364	+1084499	−2101492	−33695

	$A_3^{(s)}$	$A_4^{(c)}$	$A_4^{(s)}$	$A_5^{(c)}$	$A_5^{(s)}$	$A_6^{(c)}$	$A_6^{(s)}$
	7	7	7	7	7	7	7
(0)	−119938	−13157	−19324	− 4044	−1859	− 829	+ 93
(1)	121374	14031	19153	4192	1712	839	138
(2)	121508	14601	18832	4266	1575	837	173
(3)	120175	14405	18610	4205	1558	825	169
(4)	117736	13554	18492	4030	1645	803	132
(5)	115727	12677	18491	3861	1756	785	92
(6)	115721	12287	18731	3812	1852	787	67
(7)	−117629	−12487	−19124	− 3893	−1905	− 807	+ 67
S	−474903	−53599	−75379	−16152	−6931	−3256	+465
S′	−474905	−53600	−75378	−16151	−6931	−3256	+466

The following are the formulæ for mechanical quadratures in the case of the division of the circumference into eight parts:

If

$$(0.4) = Y_0 + Y_4 \qquad \left(\tfrac{0}{4}\right) = Y_0 - Y_4$$
$$(1.5) = Y_1 + Y_5 \qquad \left(\tfrac{1}{5}\right) = Y_1 - Y_5$$
$$(2.6) = Y_2 + Y_6 \qquad \left(\tfrac{2}{6}\right) = Y_2 - Y_6$$
$$(3.7) = Y_3 + Y_7 \qquad \left(\tfrac{3}{7}\right) = Y_3 - Y_7$$

$$(0.2) = (0.4) + (2.6)$$
$$(1.3) = (1.5) + (3.7)$$

Then

$$2(c_0 + c_4) = (0.2) \qquad 2(c_1 + c_3) = \left(\tfrac{0}{4}\right)$$
$$2(c_0 - c_4) = (1.3) \qquad 2(c_1 - c_3) = \left[\left(\tfrac{1}{5}\right) - \left(\tfrac{3}{7}\right)\right] \cos 45^\circ$$
$$4\, c_2 = (0.4) - (2.6) \qquad 2(s_1 + s_3) = \left[\left(\tfrac{1}{5}\right) + \left(\tfrac{3}{7}\right)\right] \cos 45^\circ$$
$$4\, s_2 = (1.5) - (3.7) \qquad 2(s_1 - s_3) = \left(\tfrac{2}{6}\right)$$

The quantity $\frac{1}{2}\gamma_2$ is so small that we may neglect the terms multiplied by it, and thus may take $[D - f \cos(\varepsilon - F)]^{-\frac{1}{2}}$ as the equivalent of $\frac{a'}{\Delta}$.

Arg=$i'g'+i\varepsilon$	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$	
	cos.	sin.	cos.	sin.
i' i				
0 0	1.0076186		1.0706321	
0 − 1	−0.0007326	−0.0000090	−0.0029278	−0.0025664
0 − 2	+0.0000303	+0.0000021	+0.0001607	+0.0000170
0 − 3	0.0000000	0.0000000	−0.0000002	−0.0000005
1 + 2	+0.0000001	+0.0000002	+0.0000007	+0.0000018
1 + 1	+0.0000937	−0.0000690	+0.0002899	−0.0002370
1 0	+0.0013995	+0.0044427	+0.0045952	+0.0145895
1 − 1	+0.1493534	−0.0909814	+0.4689890	−0.2857182
1 − 2	−0.0001261	+0.0001343	−0.0010542	+0.0003051
1 − 3	+0.0000059	−0.0000031	+0.0000440	−0.0000217
1 − 4	−0.0000002	−0.0000001	−0.0000001	0.0000000

$\text{Arg}=i'g'+ie$	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$	
	cos.	sin.	cos.	sin.
i' i				
2+ 1	+0.0000011	+0.0000002	+0.0000051	+0.0000017
2 0	—0.0000134	+0.0000003	—0.0000488	—0.0000359
2— 1	+0.0015404	+0.0004056	+0.0068064	+0.0029958
2— 2	+0.0104145	—0.0201770	+0.0542249	—0.1050745
2— 3	—0.0000045	+0.0000399	—0.0001415	+0.0002640
2— 4	+0.0000006	—0.0000012	+0.0000066	—0.0000109
3 0	+0.0000001	—0.0000002	+0.0000015	—0.0000011
3— 1	+0.0000041	+0.0000066	+0.0000049	+0.0000406
3— 2	+0.0003445	—0.0001271	+0.0023460	—0.0005665
3— 3	—0.0002313	—0.0032652	—0.0016847	—0.0237452
3— 4	+0.0000036	+0.0000074	+0.0000148	+0.0000741
3— 5	0.0000000	—0.0000003	0.0000000	—0.0000030
4— 2	+0.0000031	+0.0000011	+0.0000193	+0.0000132
4— 3	+0.0000457	—0.0000609	+0.0004375	—0.0004838
4— 4	—0.0002870	—0.0004038	—0.0026800	—0.0037689
4— 5	+0.0000013	+0.0000008	+0.0000131	+0.0000120
4— 6			—0.0000003	—0.0000006
5— 3	+0.0000009	—0.0000003	+0.0000088	—0.0000005
5— 4	+0.0000014	—0.0000148	+0.0000250	—0.0001573
5— 5	—0.0000708	—0.0000304	—0.0008076	—0.0003465
5— 6	+0.0000004	—0.0000001	+0.0000040	+0.0000005
6— 4	+0.0000002	—0.0000002	+0.0000022	—0.0000017
6— 5	—0.0000013	—0.0000026	—0.0000145	—0.0000334
6— 6	—0.0000121	+0.0000017	—0.0001628	+0.0000233
7— 5	0.0000000	—0.0000001	+0.0000002	—0.0000008
7— 6	—0.0000004	—0.0000003	—0.0000066	—0.0000045

These expressions are now changed into others dependent on the argument $i'g'+ig$. The formulæ and data for this transformation have already been given (pages 52, 53).

$\text{Arg}=i'g'+ig$	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$	
	cos.	sin.	cos.	sin.
i' i				
0 0	1.0076363		1.0707027	
0— 1	—0.0007335	—0.0000091	—0.0029329	—0.0025664
0— 2	+0.0000125	+0.0000019	+0.0000898	—0.0000448
0— 3	+0.0000009	+0.0000001	+0.0000049	—0.0000019
1+ 2	+0.0000010	—0.0000006	+0.0000033	—0.0000013
1+ 1	+0.0000502	—0.0000431	+0.0001533	—0.0001539
1 0	—0.0022054	+0.0066390	—0.0067243	+0.0214871

Arg=$i'g'+ig$	$\frac{a'}{\Delta}$		$\left(\frac{a'}{\Delta}\right)^3$	
	cos.	sin.	cos.	sin.
i' i				
1— 1	+0.1492726	—0.0909350	+0.4687669	—0.2855666
1— 2	+0.0034722	—0.0020578	+0.0102446	—0.0065780
1— 3	+0.0001299	—0.0000728	+0.0004016	—0.0002558
1— 4	+0.0000055	—0.0000034	+0.0000183	—0.0000115
2+ 1	+0.0000007	+0.0000001	+0.0000028	+0.0000013
2 0	—0.0000506	—0.0000095	—0.0002131	—0.0001082
2— 1	+0.0010373	+0.0013786	+0.0041871	+0.0080620
2— 2	+0.0104277	—0.0201231	+0.0542730	—0.1047770
2— 3	+0.0004978	—0.0009306	+0.0024736	—0.0047896
2— 4	+0.0000246	—0.0000451	+0.0001224	—0.0002355
2— 5	+0.0000013	—0.0000022	+0.0000063	—0.0000121
3— 0	0.0000000	—0.0000004	+0.0000014	—0.0000021
3— 1	—0.0000127	+0.0000099	—0.0001097	+0.0000472
3— 2	+0.0003605	+0.0001094	+0.0024622	+0.0011523
3— 3	—0.0002138	—0.0032549	—0.0015644	—0.0236554
3— 4	—0.0000123	—0.0002282	—0.0001012	—0.0016378
3— 5	—0.0000007	—0.0000138	—0.0000057	—0.0000991
3— 6	0.0000000	—0.0000007	—0.0000003	—0.0000059
4— 1	0.0000000	—0.0000002	—0.0000004	—0.0000013
4— 2	—0.0000009	+0.0000046	—0.0000186	+0.0000393
4— 3	+0.0000732	—0.0000216	+0.0006941	—0.0001179
4— 4	—0.0002812	—0.0004045	—0.0026252	—0.0037701
4— 5	—0.0000257	—0.0000382	—0.0002419	—0.0003512
4— 6	—0.0000018	—0.0000027	—0.0000172	—0.0000252
4— 7			—0.0000011	—0.0000017
5— 3	+0.0000006	+0.0000010	+0.0000028	+0.0000131
5— 4	+0.0000100	—0.0000112	+0.0001223	—0.0001142
5— 5	—0.0000697	—0.0000314	—0.0007941	—0.0003567
5— 6	—0.0000080	—0.0000038	—0.0000923	—0.0000420
5— 7	—0.0000007	—0.0000003	—0.0000076	—0.0000034
6— 4	+0.0000004	+0.0000001	+0.0000028	+0.0000025
6— 5	+0.0000004	—0.0000029	+0.0000093	—0.0000365
6— 6	—0.0000121	+0.0000014	—0.0001611	+0.0000188
6— 7	—0.0000017	+0.0000003	—0.0000233	+0.0000029

There are now computed certain values of the Besselian function $J_l^{(i)}$ corresponding to the multiples of half of the eccentricity of Neptune's orbit, by the method of page 52.

Values of log $J_l^{(i)}$

i	$l=\frac{1}{2}e'$	$l=e'$	$l=\frac{3}{2}e'$
0	9.9999922	9.9999686	9.9999295
1	7.6281908	7.9292090	8.1052808
2	4.9554	5.5574	5.9096
3	2.1064	3.0095	3.5378

With these we obtain the expressions for the two multipliers of $\left(\frac{a'}{\Delta}\right)^3$

$$\frac{1}{2}\left[\frac{r'^2}{a'^2}-\alpha^2\frac{r^2}{a^2}\right]= \quad [9.6857724]$$
$$+2[6.85846\ ]\cos(-\ g)-2[7.62819]\cos\ g'$$
$$+2[4.9396\ ]\cos(-2g)-2[4.9553\ ]\cos 2g'$$

$$\frac{r'}{a'}\sin(f'+\Pi')= \quad [7.97234\ ]$$
$$-2[9.5293508]\sin\ g'-2[9.5659853]\cos\ g'$$
$$-2[7.15754\ ]\sin 2g'-2[7.19417\ ]\cos 2g'$$

The expressions for the products are

Arg=$i'g+ig$	$\frac{1}{2}\left[\frac{r'^2}{a'^2}-\alpha^2\frac{r^2}{a^2}\right]\left(\frac{a'}{\Delta}\right)^3$		Arg=$i'g'+ig$	$\frac{1}{2}\left[\frac{r'^2}{a'^2}-\alpha^2\frac{r^2}{a^2}\right]\left(\frac{a'}{\Delta}\right)^3$	
	cos.	sin.		cos.	sin.
i' i			i' i		
0 0	+0.5193540		3− 3	−0.0007705	−0.0114533
0− 1	−0.0018686	−0.0000325	3− 4	−0.0000395	−0.0007946
0− 2	+0.0000161	+0.0000053	3− 5	−0.0000019	−0.0000479
0− 3	+0.0000012	+0.0000002	3− 6	0.0000000	−0.0000029
1+ 2	+0.0000011	−0.0000011	4− 1	+0.0000003	−0.0000009
1+ 1	+0.0000818	−0.0000751	4− 2	−0.0000200	+0.0000151
1 0	−0.0120187	+0.0102154	4− 3	+0.0003415	+0.0000406
1− 1	+0.2273652	−0.1385221	4− 4	−0.0012736	−0.0018216
1− 2	+0.0050765	−0.0029533	4− 5	−0.0001158	−0.0001711
1− 3	+0.0001958	−0.0001108	4− 6	−0.0000081	−0.0000123
1− 4	+0.0000088	−0.0000049	4− 7	−0.0000005	−0.0000008
2+ 1	+0.0000006	+0.0000013	5− 3	−0.0000014	+0.0000070
2 0	−0.0000906	−0.0001389	5− 4	+0.0000699	−0.0000397
2− 1	+0.0000790	+0.0050475	5− 5	−0.0003842	−0.0001714
2− 2	+0.0262751	−0.0507951	5− 6	−0.0000446	−0.0000207
2− 3	+0.0012440	−0.0022972	5− 7	−0.0000037	−0.0000016
2− 4	+0.0000620	−0.0001116	6− 4	+0.0000009	+0.0000017
2− 5	+0.0000032	−0.0000057	6− 5	+0.0000078	−0.0000162
3 0	+0.0000016	−0.0000017	6− 6	−0.0000777	+0.0000093
3− 1	−0.0000734	−0.0000081	6− 7	−0.0000114	+0.0000014
3− 2	+0.0009624	+0.0009868			

Arg.	$\frac{r'}{a'}\sin(f'+\Pi')\left(\frac{a'}{\Delta}\right)^3$	
	sin.	cos.
i' i		
0 0		+0.0052523
0− 1	+0.2635842	−0.0759927
0− 2	+0.0061272	−0.0014797
0− 3	+0.0002412	−0.0000582
0− 4	+0.0000110	−0.0000027

For the computation of $a'\frac{r}{r'^2}H$ we have

$\log h = 9.1695601$ $\log l = 8.9536702$
$\log h_1 = 9.1690072$ $\log l_1 = 8.9542630$

And the expression for $a'\frac{r}{r'^2}H$ follows

Arg$=i'g'+ig$	$-a'\frac{r}{r'^2}H$		Arg$=i'g'+ig$	$-a'\frac{r}{r'^2}H$	
	cos.	sin.		cos.	sin.
i' i			i' i		
1+ 2	−0.0000008	+0.0000007	2 0	+0.0001817	−0.0001107
1+ 1	−0.0000497	+0.0000360	2− 1	−0.0025076	+0.0015274
1 0	+0.0106923	−0.0065130	2− 2	−0.0000604	+0.0000368
1− 1	−0.1475785	+0.0898888	2− 3	−0.0000022	+0.0000013
1− 2	−0.0035576	+0.0021669	3 0	+0.0000026	−0.0000016
1− 3	−0.0001286	+0.0000784	3− 1	−0.0000360	+0.0000219
1− 4	−0.0000055	+0.0000034	3− 2	−0.0000009	+0.0000005
1− 5	−0.0000003	+0.0000002	4− 1	−0.0000005	+0.0000003
2+ 1	−0.0000008	+0.0000006			

The logarithms of the factors which depend on the mass of Neptune are

$\log \mu = 0.2576408$ $\log (\mu a \sin J) = 7.7144717$

The expressions for the forces are

Arg$=i'g'+ig$	$a\frac{d\Omega}{dg}$		$ar\frac{d\Omega}{dr}$	
	sin.	cos.	cos.	sin.
i' i	″	′	″	″
0 0			+0.028117	
0− 1	−0.0013275	+0.0000165	−0.0027182	+0.0000507
0− 2	+0.0000452	−0.0000069	+0.0000179	+0.0000079
1+ 1	−0.000001	−0.000013	+0.000013	−0.000032
1 0			−0.000405	+0.000693
1− 1	+0.003066	+0.001893	+0.009321	−0.005730
1− 2	−0.000309	−0.000395	−0.000393	+0.000439
1− 3	+0.000007	−0.000014	+0.000004	+0.000010
2 0			+0.000211	−0.000443
2− 1	−0.002661	−0.005259	−0.005334	+0.010652
2− 2	+0.037527	+0.072707	+0.038008	−0.073655
2− 3	+0.002691	+0.005046	+0.001797	−0.003313
2− 4	+0.000177	+0.000326	+0.000089	−0.000161
2− 5	+0.000012	+0.000020	0.000000	−0.000008

Arg=$i'g'+ig$	$a\frac{d\Omega}{dg}$		$ar\frac{d\Omega}{dr}$	
	sin.	cos.	cos.	sin.
i' i	″	″	″	″
3 0			+0.000008	—0.000006
3— 1	—0.000088	—0.000058	—0.000187	+0.000016
3— 2	+0.001302	—0.000398	+0.001414	+0.001688
3— 3	—0.001161	+0.017672	—0 001201	—0.017783
3— 4	—0.000089	+0.001652	—0.000060	—0.001231
3— 5	—0.000006	+0.000125	0.000000	—0.000074
3— 6	0.000000	+0.000007		
4— 2	—0.000003	—0.000017	—0.000035	+0.000023
4— 3	+0.000397	+0.000117	+0.000552	+0.000093
4— 4	—0.002036	+0.002928	—0.002051	—0.002931
4— 5	—0.000233	+0.000346	—0.000186	—0.000275
4— 6	—0.000019	+0.000030	—0.000013	—0.000020
5— 3	+0.000003	—0.000005	0.000000	+0.000012
5— 4	+0.000072	+0.000081	+0.000117	—0.000062
5— 5	—0.000631	+0.000284	—0.000632	—0.000282
5— 6	—0.000087	+0.000042	—0.000074	—0.000034
5— 7	—0.000009	+0.000004	—0.000006	0.000000
6— 4	+0.000003	—0.000001		
6— 5	+0.000004	+0.000026	+0.000014	—0.000027
6— 6	—0.000131	—0.000015	—0.000130	+0.000015
6— 7	—0 000021	—0.000002	—0.000019	0.000000

Arg.	$a^2\frac{d\Omega}{dZ}$	
	sin.	cos.
i' i	″	″
0 0		+0.000034
0— 1	+0.0013659	—0.0003937
0— 2	+0.000032	—0.000008
0— 3	+0.000001	0.000000

The expressions for the multipliers A, B, and C have already been given (page 73), and we obtain the following expressions for T and $\frac{1}{n}\frac{dR}{dt}$:

Arg=$\varkappa\gamma+i'g'+ig$			T		Arg=$\varkappa\gamma+i'g'+ig$			T	
			sin.	cos.				sin.	cos.
$\varkappa$	i'	i	″	″	$\varkappa$	i'	i	″	″
1	0−	1	−0.05597	−0.00002	0	3−	2	−0.00391	+0.00119
−1	0	0	−0.0040184	+0.0000833	1	3−	3	+0.00136	−0.00167
0	0−	1	+0.00398	−0.00005	−1	3−	2	−0.00365	+0.05327
1	0−	2	−0.00131	−0.00002	0	3−	3	+0.00348	−0.05302
−1	0−	1	+0.00020	−0.00004	1	3−	4	−0.00111	+0.01739
0	0−	2	−0.00014	+0.00002	−1	3−	3	−0.00013	+0.00284
1	0−	3	+0.00006	0.00000	0	3−	4	+0.00027	−0.00496
−1	1+	2	+0.00001	−0.00002	1	3−	5	−0.00012	+0.00206
0	1+	1	0.00000	+0.00004	−1	3−	4	0.00000	+0.00015
1	1	0	0.00000	−0.00004	0	3−	5	+0.00002	−0.00037
−1	1+	1	−0.00012	−0.00050	1	3−	6	−0.00001	+0.00018
1	1−	1	−0.00019	+0.00032	−1	4−	1	−0.00002	−0.00005
−1	1	0	+0.01544	+0.00949	0	4−	2	+0.00001	+0.00005
0	1−	1	−0.00920	−0.00568	1	4−	3	−0.00001	−0.00002
1	1−	2	−0.00313	−0.00187	−1	4−	2	+0.00125	+0.00028
−1	1−	1	−0.00115	−0.00132	0	4−	3	−0.00119	−0.00035
0	1−	2	+0.00093	+0.00118	1	4−	4	+0.00055	−0.00010
1	1−	3	−0.00038	−0.00044	−1	4−	3	−0.00617	+0.00880
−1	1−	2	+0.00004	0.00000	0	4−	4	+0.00611	−0.00878
0	1−	3	−0.00002	+0.00004	1	4−	5	−0.00200	+0.00288
1	1−	4	0.00000	−0.00002	−1	4−	4	−0.00046	+0.00069
−1	2+	1	+0.00007	+0.00015	0	4−	5	+0.00070	−0.00104
1	2−	1	+0.00024	+0.00046	1	4−	6	−0.00028	+0.00041
−1	2	0	−0.00883	−0.01766	−1	4−	5	−0.00002	+0.00005
0	2−	1	+0.00798	+0.01578	0	4−	6	+0.00006	−0.00009
1	2−	2	−0.00545	−0.01045	1	4−	7	−0.00004	+0.00004
−1	2−	1	+0.11348	+0.21990	−1	5−	3	+0.00024	+0.00023
0	2−	2	−0.11258	−0.21812	0	5−	4	−0.00022	−0.00024
1	2−	3	+0.03680	+0.07128	1	5−	5	+0.00012	+0.00006
−1	2−	2	+0.00358	+0.00643	−1	5−	4	−0.00192	+0.00085
0	2−	3	−0.00807	−0.01514	0	5−	5	+0.00189	−0.00085
1	2−	4	+0.00356	+0.00672	1	5−	6	−0.00062	+0.00028
−1	2−	3	+0.00017	+0.00028	−1	5−	5	−0.00018	+0.00009
0	2−	4	−0.00053	−0.00098	0	5−	6	+0.00026	−0.00013
1	2−	5	+0.00028	+0.00051	1	5−	7	−0.00012	+0.00006
−1	2−	4	0.00000	+0.00001	−1	6−	4	+0.00001	+0.00008
0	2−	5	−0.00004	−0.00006	0	6−	5	−0.00001	−0.00008
1	2−	6	+0.00002	+0.00004	1	6−	6	+0.00001	+0.00002
−1	3	0	−0.00031	−0.00016	−1	6−	5	−0.00039	−0.00005
0	3−	1	+0.00026	+0.00017	0	6−	6	+0.00039	+0.00004
1	3−	2	−0.00018	−0.00001	1	6−	7	−0.00013	−0.00001
−1	3−	1	+0.00398	−0.00162					

$$\frac{1}{n}\frac{dR}{dt} = +0''.0006825 \cos(-\gamma) + 0''.0001987 \sin(-\gamma)$$

The logarithms of the integrating factors follow:

Arg.	$\log \frac{n}{i'n'+in}$	Arg.	$\log \frac{n}{i'n'+in}$	Arg.	$\log \frac{n}{i'n'+in}$	Arg.	$\log \frac{n}{i'n'+in}$
i' i		i' i		i' i		i' i	
0 − 1	0.00000n	2 0	0.84172	3 − 3	9.55532n	4 − 6	9.24321n
0 − 2	9.69897n	2 − 1	0.06751n	3 − 4	9.42204n	5 − 3	9.5784n
0 − 3	9.52288n	2 − 2	9.73142n	3 − 5	9.32021n	5 − 4	9.4389n
1 + 1	9.96981	2 − 3	9.54424n	3 − 6	9.23777n	5 − 5	9.3335n
1 0	1.14275	2 − 4	9.41386n	4 − 1	0.14749n	5 − 6	9.2487n
1 − 1	0.03245n	2 − 5	9.31372n	4 − 2	9.76649n	5 − 7	9.1778n
1 − 2	9.71489n	3 0	0.66563	4 − 3	9.56670n	6 − 4	9.4476n
1 − 3	9.53343n	3 − 1	0.10566n	4 − 4	9.43039n	6 − 5	9.3403n
2 + 1	9.94158	3 − 2	9.74860n	4 − 5	9.32679n	6 − 6	9.2542n
						6 − 7	9.1825n

In integrating we put

$k_0 = +0''.0560$ $k_1 = -0''.0060$ $k_2 = +0''.0001$

Arg $= i'g' + ig$	$\frac{d\delta z}{dt}$		$\frac{1}{n}\frac{d\nu}{dt}$	
	cos.	sin.	sin.	cos.
i' i	$''$	$''$	$''$	$''$
0 0				−0.0000010
0 − 1	−0.0040	−0.0001	+0.0040	0.0000
	+0.0000833nt	−0.0040184nt	−0.0000416nt	−0.0020092nt
0 − 2	0.0000	0.0000	0.0000	0.0000
	+0.0000020nt	−0.0000968nt	−0.0000020nt	−0.0000968nt
1 + 1	0.0000	−0.0005	0.0000	+0.0003
1 0	−0.00013	−0.00079	−0.00020	−0.00008
1 − 1	−0.2259	+0.1389	+0.1064	+0.0654
1 − 2	−0.0060	+0.0042	+0.0057	+0.0038
1 − 3	−0.0002	+0.0001	+0.0003	+0.0002
2 − 0	+0.00016	−0.00029	+0.00012	+0.00023
2 − 1	+0.0680	−0.1361	−0.0319	−0.0635
2 − 2	+0.0863	−0.1674	−0.0614	−0.1189
2 − 3	+0.0033	−0.0062	−0.0038	−0.0072
2 − 4	+0.0002	−0.0002	−0.0002	−0.0003
3 − 1	+0.0016	−0.0009	−0.0008	−0.0004
3 − 2	+0.0034	+0.0019	−0.0023	+0.0008
3 − 3	−0.0009	−0.0155	+0.0008	−0.0126
3 − 4	+0.0001	−0.0008	0.0000	−0.0010
4 − 3	+0.0004	−0.0001	−0.0003	−0.0001
4 − 4	−0.0011	−0.0014	+0.0009	−0.0013
4 − 5	−0.0001	−0.0002	+0.0002	−0.0002

In fine, for the perturbations of Jupiter by Neptune we have

$\text{Arg}=i'g'''+ig$	$n\delta z$		ν		$\frac{u}{\cos i}$	
	sin.	cos.	cos.	sin.	sin.	cos.
i' i	″	″	″	″	″	″
0 0			−0.0093			
			−0.0000010*nt*			−0.0000494*nt*
0 − 1	0.0000	0.0000	+0.0020	0.0000		
	−0.0000833*nt*	−0.0040184*nt*	−0.0000416*nt*	+0.0020092*nt*	+0.0001987*nt*	+0.0006825*nt*
0 − 2	−0.0000010*nt*	−0.0000484*nt*	−0.0000010*nt*	+0.0000484*nt*	+0.0000048*nt*	+0.0000165*nt*
1 + 1	0.0000	+0.0005	0.0000	+0.0003		
1 0	−0.0018	+0.0110	+0.0028	−0.0011		
1 − 1	+0.2434	+0.1497	+0.1147	−0.0705		
1 − 2	+0.0031	+0.0022	+0.0030	−0.0020		
2 0	+0.0011	+0.0020	−0.0008	+0.0016		
2 − 1	−0.0794	−0.1590	−0.0373	+0.0742		
2 − 2	−0.0465	−0.0902	−0.0331	+0.0641		
2 − 3	−0.0012	−0.0022	−0.0013	+0.0025		
3 − 1	−0.0020	−0.0011	−0.0010	+0.0005		
3 − 2	−0.0019	+0.0011	−0.0013	−0.0004		
3 − 3	+0.0003	−0.0056	+0.0003	+0.0045		
3 − 4	0.0000	−0.0002	0.0000	+0.0003		
4 − 3	−0.0001	0.0000	−0.0001	0.0000		
4 − 4	+0.0003	−0.0004	+0.0002	+0.0004		

CHAPTER VII.

PERTURBATIONS OF JUPITER AND SATURN BY THE FOUR INTERIOR PLANETS.

Here we need take account only of the secular terms, the constant terms of ν, and, in the action of Venus and the Earth, of the terms of $n\delta z$ and ν, which depend on the single multiple of the elongation of the disturbing planet from the disturbed. The terms to be derived are so few and small that they may most readily be got by an algebraical development of $\frac{\mathrm{a}'}{\Delta}$.

The elements of the interior planets for the epoch 1850.0 needed for the computation are

	Mercury.	Venus.	Earth.	Mars.
π	75° 7′ 13″.62	129° 27′ 42″.83	100° 21′ 39″.73	333° 17′ 51″.74
i	7° 0′ 7″.71	3° 23′ 35″.01	0 0 0	1° 51′ 2″.24
☊	46° 33′ 8″.63	75° 19′ 53″.08		48° 23′ 54″.59
e	0.20560476	0.00684311	0.01677114	0.09326803
n	5381016″.260	2106641″.357	1295977″.416	689050″.784
log a	9.5878217	9.8593378	0.0000000	0.1828971
m	$\frac{1}{5000000}$	$\frac{1}{425000}$	$\frac{1}{322800}$	$\frac{1}{3093500}$

Action of Mercury on Jupiter.

In this case

$$\log \alpha = 8.8715885 \quad \log b_{\frac{1}{2}}^{(0)} = 0.30163 \quad \log b_{\frac{1}{2}}^{(1)} = 8.87249 \quad \log \alpha b_{\frac{1}{2}}^{(1)} = 8.22481$$

$$\log \alpha\frac{db_{\frac{1}{2}}^{(0)}}{d\alpha} = 7.74588 \quad \log \alpha\frac{db_{\frac{1}{2}}^{(1)}}{d\alpha} = 8.87430 \quad \log \alpha^2\frac{d^2b_{\frac{1}{2}}^{(0)}}{d\alpha^2} = 7.75129 \quad \log \alpha^2\frac{d^2b_{\frac{1}{2}}^{(1)}}{d\alpha^2} = 6.9720$$

The constant term of ν is given by the formula

$$\nu = \frac{1}{6}\frac{m}{\mu'}\left[b_{\frac{1}{2}}^{(0)} + \alpha\frac{db_{\frac{1}{2}}^{(0)}}{d\alpha}\right]$$

and in this case is

$$\nu = +0''.0138$$

We have also the following terms in $\frac{\mathrm{a}'}{\Delta}$

$$\frac{\mathrm{a}'}{\Delta} = [7.32172]\,(e^2 + e'^2) - [7.92378]\sin^2\tfrac{1}{2}J - [6.59106]ee'\cos(\Pi' - \Pi)$$

In which we have

$$\log \sin \tfrac{1}{2}J = 8.7391 \qquad \Pi' - \Pi = 296^\circ\ 43'$$

From this expression it is easy to find that the secular terms of $n\delta z$, ν, and $\frac{u}{\cos i}$ are

$$n\delta z = -\overset{''}{0.0000059}nt \sin(-g) - \overset{''}{0.0000137}nt \cos(-g)$$

$$\nu = -0.0000030nt \cos(-g) + 0.0000069nt \sin(-g)$$

$$\frac{u}{\cos i} = +0.0000080nt \sin(-g) - 0.0000171nt \cos(-g)$$

Action of Venus on Jupiter.

In this case

$\log \alpha = 9.1431046$

$\log b_{\frac{1}{2}}^{(0)} = 0.30315$

$\log \alpha \frac{db_{\frac{1}{2}}^{(0)}}{d\alpha} = 8.29574$

$\log \alpha^2 \frac{d^2b_{\frac{1}{2}}^{(0)}}{d\alpha^2} = 8.31457$

$\log \alpha^3 \frac{d^3b_{\frac{1}{2}}^{(0)}}{d\alpha^3} = 7.43109$

$\log \alpha^4 \frac{d^4b_{\frac{1}{2}}^{(0)}}{d\alpha^4} = 7.48662$

$\log \alpha^2 \frac{db_{\frac{3}{2}}^{(0)}}{d\alpha} = 8.41003$

$\log \alpha^3 \frac{d^2b_{\frac{3}{2}}^{(0)}}{d\alpha^2} = 8.46044$

$\log b_{\frac{1}{2}}^{(1)} = 9.14628$

$\log \alpha \frac{db_{\frac{1}{2}}^{(1)}}{d\alpha} = 9.15264$

$\log \alpha^2 \frac{d^2b_{\frac{1}{2}}^{(1)}}{d\alpha^2} = 7.79914$

$\log \alpha^3 \frac{d^3b_{\frac{1}{2}}^{(1)}}{d\alpha^3} = 7.83341$

$\log \alpha^4 \frac{d^4b_{\frac{1}{2}}^{(1)}}{d\alpha^4} = 7.20768$

$\log \alpha\, b_{\frac{3}{2}}^{(1)} = 8.77923$

$\log \alpha^2 \frac{db_{\frac{3}{2}}^{(1)}}{d\alpha} = 8.81022$

$\log \alpha^3 \frac{d^2b_{\frac{3}{2}}^{(1)}}{d\alpha^2} = 8.14175$

$\log b_{\frac{1}{2}}^{(2)} = 8.16479$

$\log \alpha \frac{db_{\frac{1}{2}}^{(2)}}{d\alpha} = 8.46937$

$\log \alpha^2 \frac{d^2b_{\frac{1}{2}}^{(2)}}{d\alpha^2} = 8.48344$

$\log \alpha^3 \frac{d^3b_{\frac{1}{2}}^{(2)}}{d\alpha^3} = 7.47526$

$\log \alpha^4 \frac{d^4b_{\frac{1}{2}}^{(2)}}{d\alpha^4} = 7.52801$

$\log \alpha^2 b_{\frac{3}{2}}^{(0)} = 8.63965$

$\log \alpha^2 b_{\frac{3}{2}}^{(2)} = 7.54622$

$\log \alpha^2 \frac{db_{\frac{3}{2}}^{(2)}}{d\alpha} = 8.33395$

$\log \alpha^3 \frac{d^2b_{\frac{3}{2}}^{(2)}}{d\alpha^2} = 8.38980$

The constant term of ν is

$$\nu = +0''.1640$$

The non-periodic portion of $\frac{\mathrm{a}'}{\triangle}$ contains the following terms:

$$\begin{aligned}\frac{\mathrm{a}'}{\triangle} = {} & [7.87614](e^2 + e'^2) \quad - [8.47820] \sin^2 \tfrac{1}{2}J \quad - [7.41622]\, ee' \cos(\Pi' - \Pi) \\ & + [8.53188] \sin^4 \tfrac{1}{2}J \quad + [6.03454]\, e^4 \quad + [8.08876]\, e^2e'^2 \\ & + [7.99054]\, e'^4 \quad - [8.69081](e^2 + e'^2) \sin^2 \tfrac{1}{2}J \quad - [7.32535]\, e^3e' \cos(\Pi' - \Pi) \\ & - [7.83041]\, ee'^3 \cos(\Pi' - \Pi) + [8.48167]\, ee' \sin^2 \tfrac{1}{2}J \cos(\Pi' - \Pi) + [6.69417]\, e^2e'^2 \cos 2(\Pi' - \Pi) \\ & + [8.89363]\, e^2 \sin^2 \tfrac{1}{2}J \cos 2\Pi - [8.21647]\, ee' \sin^2 \tfrac{1}{2}J \cos(\Pi' + \Pi) + [6.93763]\, e'^2 \sin^2 \tfrac{1}{2}J \cos 2\Pi'\end{aligned}$$

In which we have

$$\mathrm{J} = 2^\circ\ 15'\ 11''.34 \qquad \Pi' = 310^\circ\ 4'\ 23''.11 \qquad \Pi = 67^\circ\ 36'\ 52''.64$$

From this expression we find

$$-\frac{1}{e'}\frac{d\left(\frac{\mathrm{a}'}{\Delta}\right)}{d\Pi'} = +\,0.0000158 \qquad -\frac{d\left(\frac{\mathrm{a}'}{\Delta}\right)}{de'} = -\,0.0007369$$

$$\frac{d\left(\frac{\mathrm{a}'}{\Delta}\right)}{d\mathrm{J}} = -\,0.0005933 \qquad \frac{1}{\sin \mathrm{J}}\left[\frac{d\left(\frac{\mathrm{a}'}{\Delta}\right)}{d\Pi} + \cos \mathrm{J}\,\frac{d\left(\frac{\mathrm{a}'}{\Delta}\right)}{d\Pi'}\right] = +\,0.0000001$$

From these four quantities it is easy to conclude that the secular terms are

$$\begin{aligned} n\delta z &= -\,\overset{''}{0}.0000153nt \sin(-g) - \overset{''}{0}.0007144nt \cos(-g) \\ \nu &= -\,0.0000077nt \cos(-g) + 0.0003572nt \sin(-g) \\ \frac{u}{\cos i} &= +\,0.0002201nt \sin(-g) - 0.0001852nt \cos(-g) \end{aligned}$$

The terms of $n\delta z$ and ν, which depend on the elongation of Venus from Jupiter, are*

$$n\delta z = +\,0''.0675 \sin(\text{♀} - \text{♃}) \qquad \nu = +\,0''.0675 \cos(\text{♀} - \text{♃})$$

Action of the Earth on Jupiter.

In this case

$$\log \alpha = 9.2837668$$

$$\log b_{\frac{1}{2}}^{(0)} = 0.30511 \qquad \log b_{\frac{1}{2}}^{(1)} = 9.28988 \qquad \log b_{\frac{1}{2}}^{(2)} = 8.44939$$

$$\log \alpha\frac{db_{\frac{1}{2}}^{(0)}}{d\alpha} = 8.58591 \qquad \log \alpha\frac{db_{\frac{1}{2}}^{(1)}}{d\alpha} = 9.30214 \qquad \log \alpha\frac{db_{\frac{1}{2}}^{(2)}}{d\alpha} = 8.75728$$

$$\log \alpha^2\frac{d^2b_{\frac{1}{2}}^{(0)}}{d\alpha^2} = 8.62181 \qquad \log \alpha^2\frac{d^2b_{\frac{1}{2}}^{(1)}}{d\alpha^2} = 8.23752 \qquad \log \alpha^2\frac{d^2b_{\frac{1}{2}}^{(2)}}{d\alpha^2} = 8.78433$$

$$\log \alpha^3\frac{d^3b_{\frac{1}{2}}^{(0)}}{d\alpha^3} = 8.02093 \qquad \log \alpha^3\frac{d^3b_{\frac{1}{2}}^{(1)}}{d\alpha^3} = 8.30187 \qquad \log \alpha^3\frac{d^3b_{\frac{1}{2}}^{(2)}}{d\alpha^3} = 8.06368$$

$$\log \alpha^4\frac{d^4b_{\frac{1}{2}}^{(0)}}{d\alpha^4} = 8.12288 \qquad \log \alpha^4\frac{d^4b_{\frac{1}{2}}^{(1)}}{d\alpha^4} = 7.95070 \qquad \log \alpha^4\frac{d^4b_{\frac{1}{2}}^{(2)}}{d\alpha^4} = 8.16093$$

$$\log \alpha b_{\frac{1}{2}}^{(1)} = 9.07533 \qquad \log \alpha^2 b_{\frac{1}{2}}^{(0)} = 8.96871 \qquad \log \alpha^2 b_{\frac{1}{2}}^{(2)} = 8.13844$$

$$\log \alpha^2\frac{db_{\frac{1}{2}}^{(0)}}{d\alpha} = 8.85658 \qquad \log \alpha^2\frac{db_{\frac{1}{2}}^{(1)}}{d\alpha} = 9.13378 \qquad \log \alpha^2\frac{db_{\frac{1}{2}}^{(2)}}{d\alpha} = 8.78328$$

$$\log \alpha^3\frac{d^2b_{\frac{1}{2}}^{(0)}}{d\alpha^2} = 8.94979 \qquad \log \alpha^3\frac{d^2b_{\frac{1}{2}}^{(1)}}{d\alpha^2} = 8.74229 \qquad \log \alpha^3\frac{d^2b_{\frac{1}{2}}^{(2)}}{d\alpha^2} = 8.88564$$

The constant term of ν is

$$\nu = +\,0''.2189$$

---

* For the formulæ of computation see Mécanique Céleste, Tome I, pp. 279, 280.

The non-periodic portion of $\frac{a'}{\triangle}$ contains the following terms:

$$
\begin{aligned}
\frac{a'}{\triangle} = \; & [8.17224](e^2 + e'^2) - [8.77430]\sin^2\tfrac{1}{2}\text{J} + [7.53817]e'^2\sin^2\tfrac{1}{2}\text{J}\cos 2\Pi' \\
& + [8.87474]\sin^4\tfrac{1}{2}\text{J} + [6.63508]e^4 - [7.85190]ee'\cos(\Pi' - \Pi) \\
& + [8.30292]e'^4 - [9.01985](e^2 + e'^2)\sin^2\tfrac{1}{2}\text{J} + [8.41780]e^2e'^2 \\
& - [8.28145]ee'^3\cos(\Pi' - \Pi) + [8.93970]ee'\sin^2\tfrac{1}{2}\text{J}\cos(\Pi' - \Pi) - [7.79205]e^3e'\cos(\Pi' - \Pi) \\
& + [9.20601]e^2\sin^2\tfrac{1}{2}\text{J}\cos 2\Pi - [8.67277]ee'\sin^2\tfrac{1}{2}\text{J}\cos(\Pi' + \Pi) + [7.28650]e^2e'^2\cos 2(\Pi' - \Pi)
\end{aligned}
$$

In which we have

$$\text{J} = 1^\circ\ 18'\ 42''.10 \qquad \Pi' = 92^\circ\ 59'\ 49'' \qquad \Pi = 181^\circ\ 25'\ 20''$$

From this expression we find

$$-\frac{1}{e'}\frac{d\left(\frac{a'}{\triangle}\right)}{d\Pi'} = +0.0001198 \qquad -\frac{d\left(\frac{a'}{\triangle}\right)}{de'} = -0.0014394$$

$$\frac{d\left(\frac{a'}{\triangle}\right)}{d\text{J}} = -0.0006832 \qquad \frac{1}{\sin \text{J}}\left[\frac{d\left(\frac{a'}{\triangle}\right)}{d\Pi} + \cos \text{J}\,\frac{d\left(\frac{a'}{\triangle}\right)}{d\Pi'}\right] = -0.0000003$$

From these four quantities it is easy to conclude that the secular terms are

$$
\begin{aligned}
n\delta z &= -\overset{''}{0.0001530}nt\sin(-g) - \overset{''}{0.0018372}nt\cos(-g) \\
\nu &= -0.0000765nt\cos(-g) + 0.0009186nt\sin(-g) \\
\frac{u}{\cos i} &= -0.0004354nt\sin(-g) + 0.0000230nt\cos(-g)
\end{aligned}
$$

The terms of $n\delta z$ and ν, which depend on the elongation of the Earth from Jupiter, are

$$n\delta z = +0''.1225\sin(\text{♁} - \text{♃}) \qquad \nu = +0''.1225\cos(\text{♁} - \text{♃})$$

Action of Mars on Jupiter.

In this case

$$\log\alpha = 9.4666639 \qquad \log b_{\frac{1}{2}}^{(0)} = 0.31071 \qquad \log b_{\frac{1}{2}}^{(1)} = 9.48119 \qquad \log \alpha b_{\frac{1}{2}}^{(1)} = 9.48360$$

$$\log\alpha\frac{db_{\frac{1}{2}}^{(0)}}{d\alpha} = 8.97705 \qquad \log\alpha\frac{db_{\frac{1}{2}}^{(1)}}{d\alpha} = 9.51039$$

$$\log\alpha^2\frac{d^2b_{\frac{1}{2}}^{(0)}}{d\alpha^2} = 9.05994 \qquad \log\alpha^2\frac{d^2b_{\frac{1}{2}}^{(1)}}{d\alpha^2} = 8.83322$$

The constant term of ν is

$$\nu = +0''.0238$$

The non-periodic portion of $\frac{\mathrm{a}'}{\triangle}$ contains the terms

$$\frac{\mathrm{a}'}{\triangle} = [8.58050]\,(e^2 + e'^2) - [9.18256]\sin^2 \tfrac{1}{2}\mathrm{J} - [8.44029]\,ee' \cos(\Pi' - \Pi)$$

In which we have

$$\log \sin \mathrm{J} = 8.3988 \qquad \Pi' = 8^\circ\ 24' \qquad \Pi = 329^\circ\ 46'$$

Whence we conclude that the secular terms are

$$\begin{aligned} n\delta z &= +\overset{''}{0}.0002138nt \sin(-g) - \overset{''}{0}.0002217nt \cos(-g) \\ \nu &= +0.0001069nt \cos(-g) + 0.0001108nt \sin(-g) \\ \frac{u}{\cos i} &= -0.0000186nt \sin(-g) - 0.0001257nt \cos(-g) \end{aligned}$$

Action of Mercury on Saturn.

In this case

$$\log \alpha = 8.6081398 \qquad \log b_{\frac{1}{2}}^{(0)} = 0.3012 \qquad \log \alpha\frac{db_{\frac{1}{2}}^{(0)}}{d\alpha} = 7.2171$$

The constant term of ν' is

$$\nu' = +0''.0138$$

The non-periodic portion of $\frac{\mathrm{a}'}{\triangle}$ contains the terms

$$\frac{\mathrm{a}'}{\triangle} = [6.7918]\,(e^2 + e'^2) - [7.3939]\sin^2 \tfrac{1}{2}\mathrm{J} - [5.7977]\,ee' \cos(\Pi' - \Pi)$$

In which we have

$$\log \sin \mathrm{J} = 9.0469 \qquad \Pi' = 64^\circ\ 18' \qquad \Pi = 49^\circ\ 27'$$

Whence we conclude that the secular terms are

$$\begin{aligned} n'\delta z' &= \overset{''}{0}.000000n't \sin g' - \overset{''}{0}.000005n't \cos g' \\ \nu' &= 0.000000n't \cos g' - 0.000002n't \sin g' \\ \frac{u'}{\cos i'} &= +0.000005n't \sin g' + 0.000002n't \cos g' \end{aligned}$$

Action of Venus on Saturn.

In this case

$$\log \alpha = 8.8796559 \qquad \log b_{\frac{1}{2}}^{(0)} = 0.3017 \qquad \log \alpha\frac{db_{\frac{1}{2}}^{(0)}}{d\alpha} = 7.7621$$

The constant term of ν' is

$$\nu' = +0''.1624$$

The non-periodic portion of $\frac{a'}{\Delta}$ contains the terms

$$\frac{a'}{\Delta} = [7.3380](e^2 + e'^2) - [7.9401] \sin^2 \tfrac{1}{2}J - [6.6153] ee' \cos(\Pi' - \Pi)$$

In which we have

$$\log \sin J = 8.5544 \qquad \Pi' = 61^\circ\ 44' \qquad \Pi = 101^\circ\ 4'$$

Whence we conclude that the secular terms are

$$\begin{aligned} n'\delta z' &= +0''.000002 n't \sin g' - 0''.000235 n't \cos g' \\ \nu' &= -0.000001 n't \cos g' - 0.000117 n't \sin g' \\ \frac{u'}{\cos i'} &= +0.000067 n't \sin g' - 0.000036 n't \cos g' \end{aligned}$$

The terms of $n'\delta z'$ and ν', which depend on the elongation of Venus from Saturn, are

$$n'\delta z' = +0''.0369 \sin(\text{♀} - \text{♄}) \qquad \nu' = +0''.0369 \cos(\text{♀} - \text{♄})$$

Action of the Earth on Saturn.

In this case

$$\log \alpha = 9.0203181 \qquad \log b_{\frac{1}{2}}^{(0)} = 0.3022 \qquad \log \alpha \frac{db_{\frac{1}{2}}^{(0)}}{d\alpha} = 8.0460$$

The constant term of ν' is

$$\nu' = +0''.2147$$

The non-periodic portion of $\frac{a'}{\Delta}$ contains the terms

$$\frac{a'}{\Delta} = [7.6236](e^2 + e'^2) - [8.2257] \sin^2 \tfrac{1}{2}J - [7.0413] ee' \cos(\Pi' - \Pi)$$

In which we have

$$\log \sin J = 8.6387 \qquad \Pi' = 157^\circ\ 46' \qquad \Pi = 168^\circ\ 1'$$

Whence we conclude that the secular terms are

$$\begin{aligned} n'\delta z' &= +0''.000004 n't \sin g' - 0''.000579 n't \cos g' \\ \nu' &= -0.000002 n't \cos g' - 0.000289 n't \sin g' \\ \frac{u'}{\cos i'} &= +0.000086 n't \sin g' + 0.000212 n't \cos g' \end{aligned}$$

The terms of $n'\delta z'$ and ν', which depend on the elongation of the Earth from Saturn, are

$$n'\delta z' = +0''.0672 \sin(\text{♁} - \text{♄}) \qquad \nu' = +0''.0672 \cos(\text{♁} - \text{♄})$$

Action of Mars on Saturn.

In this case

$$\log \alpha = 9.2032152 \qquad \log b_{\frac{1}{2}}^{(0)} = 0.3038 \qquad \log \alpha \frac{db_{\frac{1}{2}}^{(0)}}{d\alpha} = 8.4190$$

The constant term of ν' is

$$\nu' = +\,0''.0226$$

The non-periodic portion of $\frac{\mathrm{a}'}{\triangle}$ contains the terms

$$\frac{\mathrm{a}'}{\triangle} = [8.0014]\,(e^2 + e'^2) - [8.6035]\,\sin^2 \tfrac{1}{2}\mathrm{J} - [7.6012]\,ee' \cos(\Pi' - \Pi)$$

In which we have

$$\log \sin \mathrm{J} = 8.6153 \qquad \Pi' = 113^\circ\ 5' \qquad \Pi = 356^\circ\ 19'$$

Whence we conclude that the secular terms are

$$\begin{aligned} n'\delta z' &= -\,\overset{\prime\prime}{0}.000044 n't \sin g' - \overset{\prime\prime}{0}.000172 n't \cos g' \\ \nu' &= +\,0.000022 n't \cos g' - 0.000086 n't \sin g' \\ \frac{u'}{\cos i'} &= +\,0.000051 n't \sin g' + 0.000022 n't \cos g' \end{aligned}$$

CHAPTER VIII.

PERTURBATIONS OF THE SECOND ORDER WITH RESPECT TO DISTURBING FORCES IN THE LONGITUDES AND RADII-VECTORES, ARISING FROM THE MUTUAL ACTION OF JUPITER AND SATURN—DERIVATION OF THE FACTORS OF δT AND δT'.

Having completed the determination of all the perturbations of the first order with respect to the disturbing forces, we now arrive at the consideration of those of the second order. And, in the first place, we confine our attention to those of the fundamental arguments and radii-vectores which arise from the mutual action of Jupiter and Saturn. The more important part of these perturbations arises from attributing to the variables involved in T and T' no longer elliptical values but elliptical values augmented by the perturbations of the first order.

In a similar manner to HANSEN'S, we can put for Jupiter

$$\delta T = A n \delta z + B \nu + C \delta \frac{h}{h_0} + D \frac{u}{\cos i} + E \frac{u_1}{\cos i} + F n' \delta z' + G \nu' + H \frac{u'}{\cos i'}$$

and for Saturn

$$\delta T' = A' n' \delta z' + B' \nu' + C' \delta \frac{h'}{h_0'} + D' \frac{u'}{\cos i'} + E' \frac{u_1'}{\cos i'} + F' n \delta z + G' \nu + H' \frac{u}{\cos i}$$

Here u_1 and u_1' denote the differential coefficients of u and u' with respect to the time. In the present chapter we shall be engaged in determining the factors entering into the right members of these two equations.

Of these factors, two are very readily found, viz, A and F. For, evidently,

$$A = \frac{dT}{dg} \qquad\qquad F = \frac{dT}{dg'}$$

As to B, we have

$$B = r\frac{dT}{dr}$$

Then, supposing T to have the form

$$T = Aa\frac{d\Omega}{dg} + Bar\frac{d\Omega}{dr}$$

(where the reader is asked not to confound this A and B with the same symbols denoting two factors of δT), we can suppose that B$=$V$+$X, where V denotes the portion of B which arises from making the forces in T variable with respect to r, and X the portion which arises from making the multipliers A and B so variable. Then it is plain that V has the expression

$$V = A\frac{d\,.\,ar\frac{d\Omega}{dr}}{dg} + Bar\frac{d\,.\,r\frac{d\Omega}{dr}}{dr}$$

In order to find X we take the expression

$$\mathrm{T}=\frac{a}{\cos\varphi}\left\{\left[2\frac{\rho}{r}\cos(f-\omega)-1+2\frac{h^2\rho}{h_0^2 a\cos^2\varphi}[\cos(f-\omega)-1]\right]\frac{d\Omega}{df}+2\frac{\rho}{r}\sin(f-\omega)\,r\frac{d\Omega}{dr}\right\}$$

whence

$$\mathrm{X}=-\frac{a}{\cos\varphi}\left[2\frac{\rho}{r}\cos(f-\omega)\frac{d\Omega}{df}+2\frac{\rho}{r}\sin(f-\omega)\,r\frac{d\Omega}{dr}\right]$$

But we have

$$\frac{d\Omega}{df}=\frac{r^2}{a^2\cos\varphi}\frac{d\Omega}{dg}-\frac{er\sin f}{a\cos^2\varphi}r\frac{d\Omega}{dr}$$

Thus

$$\mathrm{X}=-\frac{2r\rho}{a^2\cos^2\varphi}\cos(f-\omega)\,a\frac{d\Omega}{dg}-\frac{2\rho}{a\cos^3\varphi}\Big[\sin(f-\omega)-e\sin\omega\Big]ar\frac{d\Omega}{dr}$$

Since, in computing the forces we have supposed $\log r$ to be augmented by the constant term of its perturbations, but in deriving the multipliers A and B have given to r its elliptic value, it follows that, calling the constant term of ν, c, we ought to write $\mathrm{B}(\nu-c)+c\mathrm{X}$ for $\mathrm{B}\nu$.

Differentiating, partially with reference to $\frac{h}{h_0}$, the expression for T, we get

$$\mathrm{O}=-\frac{4\rho}{\cos^3\varphi}\Big[\cos(f-\omega)-1\Big]\frac{d\Omega}{df}$$

We note that

$$\mathrm{T}=\frac{a}{\cos\varphi}\frac{d\Omega}{df}$$

Whence, it follows that*

$$\mathrm{O}=2[\mathrm{T}+\mathrm{X}+\bar{\mathrm{T}}]$$

As to the other factor it can be got from either of the two equations†

$$\delta\frac{h}{h_0}=-\left(\frac{d\delta z}{dt}+2\nu\right)\qquad\qquad \delta\frac{h}{h_0}=-\int\bar{\mathrm{T}}n\,dt$$

We get D, E, and H from the following equations:‡

$$\mathrm{D}=\mathrm{A}\frac{d\cdot a^2\frac{d\Omega}{d\mathrm{Z}}}{dg}+\mathrm{B}\left[a^2r\frac{d\cdot\frac{d\Omega}{d\mathrm{Z}}}{dr}+a^2\frac{d\Omega}{d\mathrm{Z}}\right]$$

$$\mathrm{E}=\mathrm{A}a^2\frac{d\Omega}{d\mathrm{Z}}$$

$$\mathrm{H}=\mathrm{A}aa'\frac{d\cdot\frac{d\Omega}{d\mathrm{Z}'}}{dg}+\mathrm{B}aa'r\frac{d\cdot\frac{d\Omega}{d\mathrm{Z}'}}{dr}$$

---

*Auseinandersetzung, Abth. I, s. 128, gl. (69).

†Auseinandersetzung, Abth. I, ss. 128, 129, gl. (70), (71).

‡Auseinandersetzung, Abth. I, s. 130, gl. (72).

Lastly, we have*

$$G = -V - T$$

In all these equations A and B denote the same multipliers as appear in T.

In these expressions four functions of the co-ordinates are present which did not appear in the determination of the first-order terms.

For Jupiter we have †

$$\frac{a}{\mu}r\frac{d\left(r\frac{d\Omega}{dr}\right)}{dr} = \frac{3}{4}\left(\alpha^2\frac{r^2}{a^2} - \frac{r'^2}{a'^2}\right)^2\left(\frac{a'}{\Delta}\right)^5 - \frac{r'^2}{a'^2}\left(\frac{a'}{\Delta}\right)^3 + \frac{1}{4}\frac{a'}{\Delta} - a'\frac{r}{r'^3}H$$

$$r\frac{d\left(a^2\frac{d\Omega}{dZ}\right)}{dr} = -\frac{3}{2}\mu\alpha\sin J\left\{\left(\alpha^2\frac{r^2}{a^2} - \frac{r'^2}{a'^2}\right)\left(\frac{a'}{\Delta}\right)^5 + \left(\frac{a'}{\Delta}\right)^3\right\}\frac{r'}{a'}\sin(f' + \Pi')$$

$$aa'\frac{d\Omega}{dZ'} = \mu\alpha\sin J\left\{-\left(\frac{a'}{\Delta}\right)^3 + \left(\frac{a'}{r'}\right)^3\right\}\frac{r}{a}\sin(f + \Pi)$$

$$r\frac{d\left(aa'\frac{d\Omega}{dZ'}\right)}{dr} = \mu\alpha\sin J\left\{\frac{3}{2}\left(\alpha^2\frac{r^2}{a^2} - \frac{r'^2}{a'^2}\right)\left(\frac{a'}{\Delta}\right)^5 + \frac{1}{2}\left(\frac{a'}{\Delta}\right)^3 + \left(\frac{a'}{r'}\right)^3\right\}\frac{r}{a}\sin(f + \Pi)$$

For Saturn the similar quantities are

$$\frac{a'}{\mu'}r'\frac{d\left(r'\frac{d\Omega'}{dr'}\right)}{dr'} = \frac{3}{4}\left(\alpha^2\frac{r^2}{a^2} - \frac{r'^2}{a'^2}\right)^2\left(\frac{a'}{\Delta}\right)^5 - \alpha^2\frac{r^2}{a^2}\left(\frac{a'}{\Delta}\right)^3 + \frac{1}{4}\frac{a'}{\Delta} - a'\frac{r'}{r^3}H$$

$$r'\frac{d\left(a'^2\frac{d\Omega'}{dZ'}\right)}{dr'} = -\frac{3}{2}\mu'\alpha\sin J\left\{\left(\alpha^2\frac{r^2}{a^2} - \frac{r'^2}{a'^2}\right)\left(\frac{a'}{\Delta}\right)^5 - \left(\frac{a'}{\Delta}\right)^3\right\}\frac{r}{a}\sin(f + \Pi)$$

$$aa'\frac{d\Omega'}{dZ} = \mu'\alpha\sin J\left\{\left(\frac{a'}{\Delta}\right) - \frac{1}{\alpha^3}\left(\frac{a}{r}\right)^3\right\}\frac{r'}{a'}\sin(f' + \Pi')$$

$$r'\frac{d\left(aa'\frac{d\Omega'}{dZ}\right)}{dr'} = \mu'\alpha\sin J\left\{\frac{3}{2}\left(\alpha^2\frac{r^2}{a^2} - \frac{r'^2}{a'^2}\right)\left(\frac{a'}{\Delta}\right)^5 - \frac{1}{2}\left(\frac{a'}{\Delta}\right)^3 - \frac{1}{a^3}\left(\frac{a}{r}\right)^3\right\}\frac{r'}{a'}\sin(f' + \Pi')$$

For the principal multiplier involved in these expressions, from the data at page 59, we have

$$\begin{aligned}\alpha^2\frac{r^2}{a^2} - \frac{r'^2}{a'^2} = &-\ [9.8490745] \\ &+ 2[8.7484582]\cos g' - 2[8.1564087]\cos g \\ &+ 2[6.8947428]\cos 2g' - 2[6.2375704]\cos 2g \\ &+ 2[5.3420289]\cos 3g' - 2[4.6197410]\cos 3g \\ &+ 2[3.9142411]\cos 4g' - 2[3.1268416]\cos 4g\end{aligned}$$

---

*Auseinandersetzung, Abth. I, s. 130, gl. (74).

† Auseinandersetzung, Abth. I, s. 120.

We then multiply the expression for $\left(\frac{a'}{\Delta}\right)^5$, given at pages 53–56, by this factor, and the product again by three-fourths of this factor, and thus obtain

Arg $= i'g' + ig$	$\left(\alpha^2\frac{r^2}{a^2}-\frac{r'^2}{a'^2}\right)\left(\frac{a'}{\Delta}\right)^5$		$\frac{3}{4}\left(\alpha^2\frac{r^2}{a^2}-\frac{r'^2}{a'^2}\right)^2\left(\frac{a'}{\Delta}\right)^5$		$\left(\alpha^2\frac{r^2}{a^2}-\frac{r'^2}{a'^2}\right)\times\left(\frac{a'}{\Delta}\right)^5\frac{r'}{a'}\sin(f'+\Pi')$		$-\left(\alpha^2\frac{r^2}{a^2}-\frac{r'^2}{a'^2}\right)\times\left(\frac{a'}{\Delta}\right)^5\frac{r}{a}\sin(f+\Pi)$	
	cos.	sin.	cos.	sin.	sin.	cos.	sin.	cos.
i' i								
0 0	−5.02721		+2.59302			−0.29		+0.35
0 − 1	+0.567068	+1.175051	−0.264241	−0.244635	−1.78	+4.07	+1.98	−4.57
0 − 2	+0.07576	−0.06441	−0.01099	+0.01861	−0.35	−0.27	+0.46	+0.39
0 − 3	−0.00476	−0.00399	+0.00103	+0.00051	+0.02	−0.01	−0.04	+0.04
0 − 4	−0.00018	+0.00031	+0.00002	−0.00005				
1 + 3	−0.00080	+0.00015	+0.00014	0.00000				
1 + 2	+0.00431	+0.01246	−0.00008	−0.00258	+0.02	−0.06	−0.03	+0.08
1 + 1	+0.16021	−0.09824	−0.04441	+0.00915	+0.63	+0.43	−0.76	−0.52
1 0	−1.531692	−1.210440	+0.401926	+0.535226	−4.01	+2.80	+3.66	−2.50
1 − 1	−1.825288	+8.852622	+0.925569	−4.560693	+0.58	+0.42	−0.70	−0.49
1 − 2	+0.709941	−0.097781	−0.075858	+0.075590	−3.47	−0.73	+4.40	+0.92
1 − 3	−0.01743	−0.04487	+0.00869	+0.00836	+0.05	−0.24	−0.08	+0.32
1 − 4	−0.00255	+0.00146	+0.00053	−0.00026	+0.01	+0.01	−0.02	−0.01
1 − 5	+0.00013	+0.00014	−0.00003	−0.00002				
2 + 3	−0.00010	−0.00001	+0.00002	0.00000				
2 + 2	−0.00033	+0.00168	+0.00013	−0.00028			0.00	+0.01
2 + 1	+0.02574	−0.00012	−0.00526	−0.00182	+0.10	−0.01	−0.15	−0.01
2 0	−0.094530	−0.301683	−0.001913	+0.089814	−0.38	+1.00	+0.44	−1.04
2 − 1	−1.760858	+1.573472	+0.768184	−0.486025	−3.10	−2.97	+2.44	+2.42
2 − 2	+6.575664	+2.851119	−3.383346	−1.442006	−0.65	+0.62	+0.78	−0.79
2 − 3	+0.123630	−0.299932	−0.007497	−0.053802	+0.02	−2.61	−0.01	+3.55
2 − 4	−0.02072	+0.00326	+0.00634	−0.00731	+0.11	−0.07	−0.17	+0.08
2 − 5	+0.00024	+0.00146	+0.00013	−0.00046				
2 − 6	+0.00010	−0.00002	−0.00001	0.00000				
3 + 2	−0.00014	+0.00016	+0.00003	−0.00001				
3 + 1	+0.00294	+0.00170	−0.00044	−0.00051	+0.01	−0.01	−0.02	+0.02
3 0	+0.01214	−0.04496	−0.00670	+0.00934	+0.04	+0.17	−0.04	+0.21
3 − 1	−0.443932	+0.054753	+0.143260	+0.019989	−1.22	−0.20	+1.14	+0.24
3 − 2	+1.264647	+2.048382	−0.422232	−0.897606	+1.77	−2.81	−1.33	+1.99
3 − 3	+3.04058	−4.27059	−1.53345	+2.19691	−0.67	−0.69	+0.88	+0.82
3 − 4	−0.02192	−0.16314	−0.12479	+0.01842	+1.76	−0.37	−2.55	+0.52
3 − 5	+0.00413	+0.00813	−0.00839	−0.00637	+0.10	+0.01	−0.14	−0.03
3 − 6	+0.00099	+0.00016	−0.00046	−0.00045				
3 − 7	+0.00019	−0.00008	−0.00011	+0.00001				
4 + 1	+0.00021	+0.00037	−0.00001	−0.00008				
4 0	+0.00469	−0.00445	−0.00133	+0.00054	+0.01	+0.02	−0.01	−0.02
4 − 1	−0.065543	−0.032528	+0.014718	+0.014596	−0.23	+0.10	+0.24	−0.10
4 − 2	−0.023878	+0.535892	+0.046419	−0.183692	−0.01	−1.22	−0.03	+1.02
4 − 3	+2.030304	−0.783534	−0.898122	+0.263698	+2.18	+0.81	−1.43	−0.59
4 − 4	−2.43782	−2.68073	+1.25694	+1.34814	+0.56	−0.68	−0.67	+0.90

Arg $=i'g'+ig$	$\left(\alpha^2\frac{r^2}{a^2}-\frac{r'^2}{a'^2}\right)\left(\frac{a'}{\Delta}\right)^5$		$\frac{3}{4}\left(\alpha^2\frac{r^2}{a^2}-\frac{r'^2}{a'^2}\right)^2\left(\frac{a'}{\Delta}\right)^5$		$\left(\alpha^2\frac{r^2}{a^2}-\frac{r'^2}{a'^2}\right)\times\left(\frac{a'}{\Delta}\right)^5\frac{r'}{a'}\sin(f'+\Pi')$		$-\left(\alpha^2\frac{r^2}{a^2}-\frac{r'^2}{a'^2}\right)\times\left(\frac{a'}{\Delta}\right)^5\frac{r}{a}\sin(f+\Pi)$	
	cos.	sin.	cos.	sin.	sin.	cos.	sin.	cos.
i' i								
4−5	−0.1141	−0.1218	−0.0020	+0.1438	+0.50	+1.10	−0.73	−1.63
4−6	+0.0048	−0.0082	−0.0077	+0.0086	+0.04	+0.09	−0.06	−0.12
4−7	+0.0003	−0.0007	−0.0006	+0.0006				
5 0	+0.0009	−0.0003	−0.0002	+0.0001				
5−1	−0.00569	−0.00939	+0.00050	+0.00273	−0.03	+0.03	+0.03	−0.04
5−2	−0.058167	+0.079796	+0.024204	−0.019233	−0.17	−0.24	+0.14	+0.24
5−3	+0.551950	+0.120960	−0.197581	−0.077911	+1.08	−0.22	−0.83	+0.15
5−4	−0.315926	−1.777410	+0.090186	+0.793893	−0.21	+1.52	+0.18	−0.91
5−5	−2.0842	+1.1902	+1.0454	−0.6186	+0.63	+0.38	−0.84	−0.44
5−6	−0.1657	+0.0437	+0.1291	+0.0264	−0.62	+0.46	+0.95	−0.70
5−7	−0.0105	−0.0056	+0.0078	+0.0087	−0.06	+0.05	+0.07	−0.07
5−8	−0.0006	−0.0009	+0.0002	+0.0010				
6−1	0.0000	−0.0017	0.0000	+0.0004				
6−2	−0.01517	+0.00584	+0.00460	−0.00023	−0.04	−0.02	+0.05	+0.02
6−3	+0.08196	+0.08361	−0.02069	−0.03359	+0.20	−0.21	−0.21	+0.15
6−4	+0.2070	−0.4977	−0.1060	+0.1836	+0.34	+0.81	−0.23	−0.59
6−5	−1.4045	−0.0319	+0.6327	+0.0452	−0.95	+0.08	+0.49	−0.02
6−6	+0.4461	+1.4712	−0.2381	−0.7361	−0.22	+0.53	+0.24	−0.72
6−7	−0.0121	+0.1528	+0.0421	−0.0994	−0.36	−0.31	+0.58	+0.48
6−8	−0.0079	+0.0097	+0.0092	−0.0053	−0.06	−0.02	+0.09	+0.03
6−9	−0.0009	+0.0005	+0.0009	0.0000				
7−2	−0.0023	−0.0005	+0.0006	+0.0003				
7−3	+0.0042	+0.0206	+0.0004	−0.0064	+0.02	−0.05	−0.01	+0.06
7−4	+0.1028	−0.0712	−0.0407	+0.0183	+0.23	+0.16	−0.18	−0.15
7−5	−0.3985	−0.2611	+0.1500	+0.1229	−0.55	+0.39	+0.38	−0.24
7−6	−0.2339	+1.0139	+0.1248	−0.4598	−0.17	−0.52	+0.06	+0.23
7−7	+0.9557	−0.0613	−0.4770	+0.0408	−0.41	−0.08	+0.55	+0.08
7−8	+0.1159	+0.0451	−0.0677	−0.0473	+0.12	−0.24	−0.21	+0.42
7−9	+0.0074	+0.0095	−0.0031	−0.0088	+0.03	−0.09	0.00	+0.07
7−10	+0.0003	−0.0002	+0.0001	−0.0002				
8−3	−0.0012	+0.0033	+0.0005	−0.0008				
8−4	+0.0248	−0.0008	−0.0079	−0.0016	+0.06	+0.01	−0.06	0.00
8−5	−0.0513	−0.1116	+0.0124	+0.0439	−0.10	+0.23	+0.07	−0.17
8−6	−0.2767	+0.2846	+0.1260	−0.1082	−0.35	−0.33	+0.21	+0.22
8−7	+0.6718	+0.3124	−0.3064	−0.1539	+0.27	−0.18	−0.06	+0.05
8−8	+0.1004	−0.5741	−0.0430	+0.2862	+0.02	−0.30	−0.03	+0.41
8−9	+0.0544	−0.0758	−0.0433	+0.0407	+0.13	+0.07	−0.27	−0.04
8−10	+0.0091	−0.0039	−0.0073	+0.0009	+0.01	+0.02	−0.10	−0.02
8−11	+0.0009	+0.0001	−0.0007	−0.0004				
9−4	+0.0039	+0.0022	−0.0010	−0.0009				
9−5	+0.0036	−0.0267	−0.0030	+0.0087	+0.01	+0.06	−0.01	−0.06
9−6	−0.1095	+0.0279	+0.0433	−0.0050	−0.19	−0.04	+0.15	+0.03
9−7	+0.1783	+0.2592	−0.0675	−0.1160	+0.17	−0.29	−0.12	+0.18

Arg=$i'g'+ig$	$\left(\alpha^2\frac{r^2}{a^2}-\frac{r'^2}{a'^2}\right)\left(\frac{a'}{\Delta}\right)^5$		$\frac{3}{4}\left(\alpha^2\frac{r^2}{a^2}-\frac{r'^2}{a'^2}\right)^2\left(\frac{a'}{\Delta}\right)^5$		$\left(\alpha^2\frac{r^2}{a^2}-\frac{r'^2}{a'^2}\right)\times$ $\left(\frac{a'}{\Delta}\right)^5\frac{r'}{a'}\sin(f'+\Pi')$		$-\left(\alpha^2\frac{r^2}{a^2}-\frac{r'^2}{a'^2}\right)\times$ $\left(\frac{a'}{\Delta}\right)^5\frac{r}{a}\sin(f+\Pi)$	
	cos.	sin.	cos.	sin.	sin.	cos.	sin.	cos.
i i								
9— 8	+0.3082	—0.4072	—0.1482	+0.1864	+0.14	+0.08	+0.02	—0.02
9— 9	—0.3177	—0.1409	+0.1585	+0.0652	+0.10	+0.18	—0.25	+0.04
9—10	—0.0430	—0.0512	+0.0212	+0.0354	+0.11	—0.02	+0.05	—0.15
9—11	—0.0015	—0.0080	—0.0004	+0.0056	+0.03	0.00	+0.02	—0.01
9—12	+0.0002	—0.0007	—0.0004	+0.0005				
10— 4	+0.0004	+0.0007	0.0000	—0.0003				
10— 5	+0.0032	—0.0039	—0.0013	+0.0010				
10— 6	—0.0256	—0.0081	+0.0085	+0.0044	—0.05	+0.02	+0.05	—0.02
10— 7	+0.0059	+0.0980	+0.0024	—0.0388	0.00	—0.15	—0.03	+0.13
10— 8	+0.2205	—0.0938	—0.0980	+0.0345	+0.22	+0.06	+0.10	—0.13
10— 9	—0.2225	—0.2608	+0.1020	+0.1243	—0.15	+0.18	—0.25	+0.04
10—10	—0.1265	+0.1594	+0.0595	—0.0796	+0.07	+0.01	+0.13	—0.07
10—11	—0.0415	+0.0202	+0.0263	—0.0086	0.00	—0.06	—0.05	—0.09
10—12	—0.0063	—0.0005	+0.0040	+0.0014	+0.01	—0.01	0.00	—0.02
11— 5	+0.0009	—0.0004	—0.0004	0.0000				
11— 6	—0.0038	—0.0042	+0.0009	+0.0016				
11— 7	—0.0118	+0.0223	+0.0055	—0.0075	—0.03	—0.04	0.00	+0.05
11— 8	+0.0807	+0.0108	—0.0320	—0.0075	+0.13	0.00	+0.07	—0.06
11— 9	—0.0347	—0.1730	+0.0110	+0.0766	—0.17	+0.01	—0.14	—0.03
11—10	—0.1989	+0.1047	+0.0943	—0.0480	+0.04	+0.14	+0.06	+0.07
11—11	+0.0697	+0.0945	—0.0351	—0.0444	+0.02	—0.05	—0.01	—0.09
11—12	+0.0065	+0.0298	—0.0015	—0.0177	—0.04	—0.01	—0.05	+0.01
12— 6	—0.0002	—0.0010	0.0000	+0.0003				
12— 7	—0.0051	+0.0028	+0.0020	—0.0007				
12— 8	+0.0181	+0.0148	—0.0062	—0.0067	+0.04	0.00	+0.03	0.00
12— 9	+0.0225	—0.0634	—0.0122	+0.0259	—0.06	—0.06	—0.03	—0.05
12—10	—0.1308	—0.0035	+0.0602	+0.0055	—0.05	+0.09	—0.03	+0.08
12—11	+0.0358	+0.1474	—0.0154	—0.0739	+0.05	+0.02	+0.06	—0.03
12—12	+0.0701	—0.0207	—0.0372	+0.0107	+0.01	+0.01	—0.08	—0.01

In order to form the expressions for the forces in the second-order terms we still need developments for the two quantities

$$\left(\frac{a'}{r'}\right)^3\frac{r}{a}\sin(f+\Pi) \qquad\qquad -\frac{1}{\alpha^3}\left(\frac{a}{r}\right)^3\frac{r'}{a'}\sin(f'+\Pi')$$

The expression for $\left(\frac{a}{r}\right)^3$ is*

$$\left(\frac{a}{r}\right)^3=\frac{1}{\cos^3\varphi}+\left(\frac{3}{2}e+\frac{27}{16}e^3\right)\cos g+\left(\frac{9}{4}e^2+\frac{7}{4}e^4\right)\cos 2g+\frac{53}{16}e^3\cos 3g+\frac{231}{48}e^4\cos 4g+\ \cdot\ \cdot$$

with a similar formula for $\left(\frac{a'}{r'}\right)^3$

---

*Auseinandersetzung, Abth. I, s. 179.

When the numerical values of e and e' are substituted we obtain

$$\left(\frac{\mathrm{a}'}{r'}\right)^3 = [0.00205] + 2[8.92625]\cos g' + 2[7.85050]\cos 2g' + 2[6.76604]\cos 3g'$$

$$\left(\frac{\mathrm{a}}{r}\right)^3 = [0.00152] + 2[8.86066]\cos g + 2[7.71983]\cos 2g + 2[6.57045]\cos 3g$$

The expressions for the factors $\frac{r}{\mathrm{a}}\sin(f+\Pi)$ and $\frac{r'}{\mathrm{a}'}\sin(f'+\Pi')$ have already been given (page 60), and we obtain the products

Arg $=i'g'+ig$	$\left(\frac{\mathrm{a}'}{r'}\right)^3\frac{r}{\mathrm{a}}\sin(f+\Pi)$		Arg $=i'g'+ig$	$-\frac{1}{\alpha^3}\left(\frac{\mathrm{a}}{r}\right)^3\frac{r'}{\mathrm{a}'}\sin(f+\Pi)$	
	sin.	cos.		sin.	cos.
i' i			i' i		
0 0		+0.066	0 0		−0.308
0 − 1	+0.415	−0.914	1 0	−4.984	+3.657
0 − 2	+0.010	−0.022	2 0	−0.140	+0.102
0 − 3	0.000	−0.001	3 0	−0.006	+0.004
1 + 2	−0.001	−0.002	−2 − 1	+0.010	+0.007
1 + 1	−0.035	−0.077	−1 − 1	+0.360	+0.264
1 0	0.000	+0.011	0 − 1	0.000	−0.045
1 − 1	+0.035	−0.077	1 − 1	−0.360	+0.264
1 − 2	+0.001	−0.002	2 − 1	−0.010	+0.007
2 + 1	−0.003	−0.006	−2 − 2	+0.001	+0.001
2 0	0.000	+0.001	−1 − 2	+0.026	+0.019
2 − 1	+0.003	−0.006	0 − 2	0.000	−0.003
3 + 1	0.000	−0.001	1 − 2	−0.026	+0.019
3 − 1	0.000	−0.001	2 − 2	−0.001	+0.001
			−1 − 3	+0.002	+0.001
			1 − 3	−0.002	+0.001

We are now able to get the four functions severally for Jupiter and Saturn, the remaining data required being found in Chapters I and II.

Arg $=i'g'+ig$	$ar\frac{d\left(r\frac{d\Omega}{dr}\right)}{dr}$		$a^2r\frac{d^2\Omega}{dr\,dZ}+a^2\frac{d\Omega}{dZ}$		$aa'\frac{d\Omega}{dZ'}$		$aa'r\frac{d^2\Omega}{dr\,dZ'}$	
	cos.	sin.	sin.	cos.	sin.	cos.	sin.	cos.
i' i	″	″	″	″	″	″	″	′
0 0	+21.2696			+0.146		−0.0279		−0.150
0 − 1	− 3.22710	− 3.26268	+0.900	−2.060	−0.1825	+0.4243	−0.809	+1.890
0 − 2	− 0.1675	+ 0.2695	+0.185	+0.149	−0.0507	−0.0376	−0.234	−0.219
0 − 3	+ 0.0170	+ 0.0087			+0.0015	−0.0028	+0.023	−0.023
0 − 4	+ 0.0003	− 0.0010			+0.0001	+0.0001		
1 + 3	+ 0.0026	0.0000			−0.0003	−0.0002		
1 + 2	− 0.0016	− 0.0401	−0.008	+0.032	+0.0036	−0.0066	+0.015	−0.043
1 + 1	− 0.5939	+ 0.1297	−0.333	−0.224	+0.0767	+0.0442	+0.380	+0.234
1 0	+ 4.69009	+ 5.11622	+1.655	−1.139	−0.5053	+0.3533	−1.852	+1.269

Arg=i′g′+ig	$ar\frac{d\left(r\frac{d\Omega}{dr}\right)}{dr}$		$a^2r\frac{d^2\Omega}{drdZ}+a^2\frac{d\Omega}{dZ}$		$aa'\frac{d\Omega}{dZ'}$		$aa'r\frac{d^2\Omega}{drdZ'}$	
	cos.	sin.	sin.	cos.	sin.	cos.	sin.	cos.
i′ i	″	″	″	″	″	″	″	″
1— 1	+ 6.54843	—32.97927	—0.303	—0.215	+0.0995	+0.0488	+0.372	+0.215
1— 2	— 1.19864	+ 1.65391	+1.798	+0.379	—0.6050	—0.1315	—2.228	—0.464
1— 3	+ 0.1290	+ 0.1624	—0.030	+0.128	—0.0052	—0.0301	+0.050	—0.169
1— 4	+ 0.0096	— 0.0032			+0.0008	—0.0001	+0.011	0.000
1— 5	— 0.0010	— 0.0003						
2+ 3	+ 0.0003	0.0000						
2+ 2	+ 0.0019	— 0.0045			0.0000	—0.0009		
2+ 1	— 0.0748	— 0.0253	—0.050	+0.010	+0.0133	+0.0005	+0.076	0.000
2 0	— 0.08518	+ 0.98315	+0.153	—0.486	—0.0611	+0.1189	—0.222	+0.541
2— 1	+10.24927	— 3.07938	+1.566	+1.503	—0.2762	—0.2742	—1.265	—1.258
2— 2	—48.84956	—20.69473	+0.332	—0.332	—0.1062	+0.0811	—0.395	+0.414
2— 3	+ 0.05380	— 1.03378	—0.012	+1.377	+0.0018	—0.4024	0.000	—1.840
2— 4	+ 0.1300	— 0.1239	—0.058	+0.033	+0.0105	—0.0162	+0.091	—0.038
2— 5	+ 0.0032	— 0.0077			0.0000	—0.0003		
3+ 1	— 0.0064	— 0.0077			+0.0016	—0.0007	+0.011	—0.011
3 0	— 0.0995	+ 0.1011	—0.027	—0.085	+0.0005	+0.0212	+0.023	+0.111
3— 1	+ 1.91765	+ 0.79348	+0.627	+0.095	—0.1140	—0.0327	—0.598	—0.123
3— 2	— 5.14840	—14.27037	—0.916	+1.452	+0.1231	—0.1900	+0.702	—1.051
3— 3	—26.1007	+37.5759	+0.359	+0.364	—0.0806	—0.0941	—0.464	—0.426
3— 4	— 2.4779	+ 0.1749	—0.939	+0.198	+0.2420	—0.0515	+1.346	—0.272
3— 5	— 0.1595	— 0.1425	—0.055	—0.005	+0.0155	—0.0011	+0.073	+0.019
3— 6	— 0.0083	— 0.0103			+0.0006	—0.0003		
3— 7	— 0.0032	+ 0.0006						
4+ 1	0.0000	— 0.0013			+0.0001	—0.0002		
4 0	— 0.0193	+ 0.0042			+0.0015	+0.0026	0.000	+0.011
4— 1	+ 0.17361	+ 0.27159	+0.123	—0.052	—0.0229	+0.0052	—0.127	+0.054
4— 2	+ 1.16692	— 2.82988	+0.010	+0.640	+0.0072	—0.0918	+0.015	—0.541
4— 3	—15.95049	+ 3.61154	—1.145	—0.430	+0.1154	+0.0437	+0.763	+0.318
4— 4	+24.0627	+25.6448	—0.298	+0.367	+0.0668	—0.0746	+0.353	—0.479
4— 5	— 0.180	+ 3.016	—0.267	—0.596	+0.0612	+0.1335	+0.387	+0.870
4— 6	— 0.180	+ 0.176	—0.021	—0.047	+0.0061	+0.0108	+0.030	+0.065
4— 7	— 0.013	+ 0.013			+0.0006	+0.0005		
5 0	— 0.003	+ 0.002			+0.0003	+0.0002		
5— 1	— 0.0013	+ 0.0446	+0.014	—0.014	—0.0029	+0.0026	—0.015	+0.023
5— 2	+ 0.45743	— 0.24968	+0.093	+0.128	—0.0091	—0.0202	—0.076	—0.127
5— 3	— 3 38332	— 1.77554	—0.576	+0.118	+0.0649	—0.0082	+0.445	—0.081
5— 4	+ 1.07637	+15.35642	+0.112	—0.812	—0.0083	+0.0623	—0.100	+0.491
5— 5	+21.509	—12.841	—0.341	—0.204	+0.0627	+0.0397	+0.452	+0.234
5— 6	+ 2.839	+ 0.674	+0.338	—0.249	—0.0674	+0.0519	—0.514	+0.380
5— 7	+ 0.170	+ 0.205	+0.032	—0.027	—0.0060	+0.0071	—0.038	+0.038
5— 8	+ 0.003	+ 0.026			—0.0002	+0.0007		
6— 1	0.000	+ 0.006			—0.0002	+0.0005		
6— 2	+ 0.0796	+ 0.0109			—0.0034	—0.0025	—0.027	—0.011

Arg$=i'g'+ig$	$ar\frac{d\left(r\frac{d\Omega}{dr}\right)}{dr}$		$a^2r\frac{d^2\Omega}{drdZ}+a^2\frac{d\Omega}{dZ}$		$aa'\frac{d\Omega}{dZ'}$		$aa'r\frac{d^2\Omega}{drdZ'}$	
	cos.	sin.	sin.	cos.	sin.	cos.	sin.	cos.
i' i	″	″	″	″	″	″	″	″
6—3	−0.2936	−0.6683	−0.106	+0.112	+0.0153	−0.0109	+0.111	−0.081
6—4	−2.355	+3.417	−0.185	−0.438	+0.0143	+0.0412	+0.123	+0.318
6—5	+13.074	+1.335	+0.513	−0.044	−0.0291	+0.0037	−0.268	+0.008
6—6	−5.267	−16.052	+0.119	−0.287	−0.0193	+0.0482	−0.127	+0.391
6—7	+1.023	−2.268	+0.195	+0.172	−0.0376	−0.0305	−0.314	−0.261
6—8	+0.221	−0.115	+0.032	+0.013	−0.0060	−0.0026	−0.050	−0.015
6—9	+0.023	+0.003			−0.0006	0.0000		
7—2	+0.010	+0.006			−0.0007	−0.0001		
7—3	+0.026	−0.115	−0.008	+0.028	+0.0018	−0.0037	0.000	−0.030
7—4	−0.847	+0.279	−0.125	−0.083	+0.0108	+0.0100	+0.100	+0.081
7—5	+2.977	+2.750	+0.298	−0.209	−0.0234	+0.0149	−0.207	+0.130
7—6	+2.878	−10.005	+0.087	+0.282	−0.0053	−0.0109	−0.030	−0.127
7—7	−10.879	+0.985	+0.225	+0.041	−0.0344	−0.0060	−0.299	−0.043
7—8	−1.595	−1.158	−0.063	+0.129	+0.0115	−0.0245	+0.115	−0.230
7—9	−0.067	−0.218	−0.017	+0.046	+0.0006	−0.0044	0.000	−0.038
7—10	+0.003	0.000			−0.0001	−0.0005		
8—3	+0.010	−0.013			0.0000	−0.0008		
8—4	−0.154	−0.051	−0.033	−0.007	+0.0035	+0.0009	+0.030	0.000
8—5	+0.186	+0.943	+0.054	−0.123	−0.0056	+0.0093	−0.038	+0.091
8—6	+2.887	−2.256	+0.181	+0.182	−0.0117	−0.0121	−0.115	−0.119
8—7	−6.947	−3.635	−0.148	+0.098	+0.0021	−0.0034	+0.034	−0.027
8—8	−0.979	+6.767	−0.011	+0.179	−0.0004	−0.0226	+0.019	−0.226
8—9	−1.075	+0.982	−0.072	−0.041	+0.0147	+0.0028	+0.150	+0.023
8—10	−0.183	+0.016			+0.0029	−0.0004	+0.058	+0.011
8—11	−0.016	−0.013			+0.0003	−0.0002		
9—4	−0.019	−0.019			+0.0007	−0.0002		
9—5	−0.083	+0.176	−0.006	−0.029	−0.0001	+0.0030	0.000	+0.034
9—6	+0.966	−0.051	+0.102	+0.021	−0.0072	−0.0025	−0.081	−0.015
9—7	−1.450	−2.714	−0.089	+0.160	+0.0053	−0.0083	+0.065	−0.100
9—8	−3.562	+4.366	−0.078	−0.046	+0.0012	−0.0012	−0.011	+0.011
9—9	+3.863	+1.562	−0.051	−0.104	+0.0139	−0.0032	+0.138	−0.023
9—10	+0.523	+0.892	−0.063	+0.013	+0.0005	+0.0081	−0.030	+0.081
9—11	−0.013	+0.141	−0.017	0.000	+0.0007	+0.0017	−0.011	0.000
9—12	−0.010	+0.013			+0.0002	+0.0002		
10—4	0.000	−0.006			+0.0001	−0.0001		
10—5	−0.029	+0.019			+0.0003	+0.0006		
10—6	+0.180	+0.112	+0.032	−0.011	−0.0024	+0.0004	−0.027	+0.011
10—7	+0.103	−0.889	0.000	+0.083	+0.0006	−0.0052	+0.015	−0.072
10—8	−2.345	+0.754	−0.120	−0.032	+0.0052	+0.0018	−0.062	+0.072
10—9	+2.451	+3.048	+0.087	−0.098	+0.0019	−0.0003	+0.142	−0.023
10—10	+1.467	−1.989	−0.035	+0.006	+0.0038	+0.0080	−0.076	+0.034
10—11	+0.670	−0.212	−0.006	+0.035	−0.0041	+0.0015	+0.030	+0.050
10—12	+0.103	+0.038	−0.006	+0.006	−0.0009	+0.0007	0.000	+0.011

Arg=$i'g'+ig$	$ar\frac{d\left(r\frac{d\Omega}{dr}\right)}{dr}$		$a^2r\frac{d^2\Omega}{drdZ}+a^2\frac{d\Omega}{dZ}$		$aa'\frac{d\Omega}{dZ'}$		$aa'r\frac{d^2\Omega}{drdZ'}$	
	cos.	sin.	sin.	cos.	sin.	cos.	sin.	cos.
i' i	″	″	″	″	″	″	″	″
11— 5	−0.010	0.000			+0.0001	+0.0001		
11— 6	+0.013	+0.038			−0.0005	+0.0004		
11— 7	+0.138	−0.164	+0.017	+0.017	−0.0006	−0.0017	0.000	−0.027
11— 8	−0.751	−0.209	−0.074	0.000	+0.0034	−0.0003	−0.042	+0.034
11— 9	+0.225	+1.867	+0.098	+0.006	−0.0002	+0.0029	+0.081	+0.015
11—10	+2.358	+1.178	−0.029	−0.079	+0.0011	+0.0016	−0.034	−0.043
11—11	−0.895	−1.120	−0.017	+0.035	−0.0042	+0.0033	+0.008	+0.050
11—12	−0.035	−0.459	+0.023	+0.006	−0.0013	−0.0020	+0.030	0.000
12— 6	0.000	+0.006			−0.0001	+0.0001		
12— 7	+0.048	−0.013			−0.0004	−0.0003		
12— 8	−0.141	−0.170	−0.023	0.000	+0.0011	−0.0007	−0.019	0.000
12— 9	−0.324	+0.629	+0.035	+0.040	+0.0007	+0.0020	+0.015	+0.027
12—10	+1.521	+0.173	+0.023	−0.050	−0.0014	+0.0003	+0.019	−0.046
12—11	−0.375	−1.919	−0.029	−0.017	−0.0010	+0.0012	−0.034	+0.015
12—12	−0.985	+0.282	−0.006	−0.006	−0.0023	−0.0017	+0.046	+0.008

Arg=$i'g'+ig$	$a'r'\frac{d\left(r'\frac{d\Omega'}{dr'}\right)}{dr'}$		$a'^2r'\frac{d^2\Omega'}{dr'dZ'}+a'^2\frac{d\Omega'}{dZ'}$		$aa'\frac{d\Omega'}{dZ}$		$aa'r'\frac{d^2\Omega'}{dr'dZ}$	
	cos.	sin.	sin.	cos.	sin.	cos.	sin.	cos.
i' i	″	″	″	″	″	″	″	″
0 0	+432.962			+ 1.40		−0.497		− 1.86
1 0	+ 52.9395	+78.1156	+14.46	− 9.88	−7.536	+5.621	−27.94	+19.96
2 0	− 1.0354	+11.9696	+ 1.74	− 4.03	+0.049	−0.530	− 1.86	+ 4.15
3 0	− 0.956	+ 1.143	− 0.14	− 0.82	−0.020	−0.108	+ 0.13	+ 0.67
4 0	− 0.175	+ 0.057	− 0.03	− 0.09	−0.009	−0.012	+ 0.04	+ 0.08
5 0	− 0.02	+ 0.02			−0.002	−0.001		
−4— 1	− 0.002	+ 0.010			−0.001	−0.001		
−3— 1	− 0.055	+ 0.061	+ 0.06	+ 0.07	−0.006	−0.001	− 0.04	− 0.04
−2— 1	− 0.637	+ 0.205	+ 0.58	− 0.05	−0.036	+0.029	− 0.36	− 0.01
−1— 1	− 5.362	− 1.084	+ 2.97	− 2.05	+0.463	+0.935	− 1.57	+ 2.29
0— 1	− 20.9473	−83.7676	+14.01	−31.68	+1.534	−3.566	− 7.04	+15.98
1— 1	+ 15.3859	−93.2674	− 2.72	− 1.95	−1.221	+0.258	+ 1.39	+ 2.28
2— 1	+115.7133	−51.8406	+ 9.37	+ 9.37	+2.666	+2.534	−12.30	−11.71
3— 1	+ 20.7068	+ 5.0202	+ 4.38	+ 0.93	+0.883	+0.234	− 4.74	− 0.82
4— 1	+ 1.9290	+ 2.3042	+ 0.92	− 0.36	+0.153	−0.039	− 0.89	+ 0.37
5— 1	+ 0.031	+ 0.393	+ 0.12	− 0.16	+0.017	−0.016	− 0.11	+ 0.11
6— 1	0.00	+ 0.06			+0.001	−0.003		
−3— 2	+ 0.004	0.000						
−2— 2	+ 0.014	+ 0.037	0.00	+ 0.03	+0.001	−0.002		
−1— 2	− 0.030	+ 0.336	− 0.12	+ 0.32	+0.046	+0.023	− 0.02	− 0.18
0— 2	− 0.372	− 3.003	+ 2.36	+ 0.21	+0.217	+0.096	− 1.34	− 1.01

Arg=$i'g'+ig$	$a'r'\frac{d\left(r'\frac{d\Omega'}{dr'}\right)}{dr'}$		$a'^2r'\frac{d^2\Omega'}{dr'dZ'}+a'^2\frac{d\Omega'}{dZ'}$		$aa'\frac{d\Omega'}{dZ}$		$aa'r'\frac{d^2\Omega'}{dr'dZ}$	
	cos.	sin.	sin.	cos.	sin.	cos.	sin.	cos.
i' i	″	″	″	″	″	″	″	″
1— 2	− 17.0443	+ 68.5236	+17.38	+ 3.64	+2.359	+0.569	−13.51	− 2.79
2— 2	−562.3187	−238.6124	+ 3.09	− 3.03	+0.485	−0.339	− 2.54	+ 2.36
3— 2	− 64.3581	−145.4312	− 5.06	+ 7.61	−1.211	+1.994	+ 6.85	−10.90
4— 2	+ 8.5789	− 28.4054	− 0.13	+ 3.88	−0.045	+0.784	− 0.01	− 4.69
5— 2	+ 3.9135	− 2.7124	+ 0.51	+ 0.90	+0.072	+0.150	− 0.63	− 0.92
6— 2	+ 0.700	+ 0.022	+ 0.19	+ 0.09	+0.023	+0.016	− 0.15	− 0.08
7— 2	+ 0.08	+ 0.06			+0.004	0.000		
—2— 3	+ 0.004	0.000						
—1— 3	+ 0.017	+ 0.004			+0.005	+0.003		
0— 3	+ 0.216	− 0.350	− 0.09	+ 0.06	−0.005	+0.009	+ 0.07	− 0.04
1— 3	+ 0.207	+ 6.174	− 0.28	+ 1.23	+0.022	+0.117	+ 0.16	− 0.90
2— 3	− 4.1224	− 15.3762	− 0.02	+13.75	−0.005	+1.531	+ 0.07	− 9.97
3— 3	−261.3672	+373.3284	+ 3.35	+ 3.21	+0.335	+0.434	− 2.53	− 2.65
4— 3	−149.9928	+ 41.4181	− 5.39	− 2.20	−1.310	−0.452	+ 8.34	+ 3.06
5— 3	− 31.8426	− 13.7741	− 3.13	+ 0.55	−0.594	+0.088	+ 4.10	− 0.82
6— 3	− 3.068	− 5.571	− 0.79	+ 0.57	−0.122	+0.092	+ 0.77	− 0.79
7— 3	+ 0.14	− 1.00	− 0.02	+ 0.23	−0.012	+0.028	+ 0.08	− 0.19
8— 3	+ 0.08	− 0.12			+0.001	+0.005		
0— 4	+ 0.010	− 0.039						
1— 4	+ 0.006	+ 0.356	− 0.07	− 0.04	−0.003	+0.001	+ 0.04	+ 0.04
2— 4	+ 0.932	− 1.349	− 0.64	+ 0.32	−0.038	+0.063	+ 0.41	− 0.28
3— 4	− 25.880	+ 4.025	− 9.73	+ 1.98	−0.893	+0.194	+ 6.65	− 1.40
4— 4	+217.452	+233.907	− 2.55	+ 3.39	−0.312	+0.312	+ 2.13	− 2.55
5— 4	+ 13.7922	+135.5802	+ 0.65	− 3.39	+0.086	−0.783	− 0.78	+ 5.75
6— 4	− 18.57	+ 30.46	− 0.85	− 2.21	−0.149	−0.399	+ 1.27	+ 3.06
7— 4	− 6.88	+ 2.81	− 0.67	− 0.57	−0.095	−0.084	+ 0.86	+ 0.61
8— 4	− 1.28	− 0.33	− 0.15	0.00	−0.028	−0.006	+ 0.22	+ 0.04
9— 4	− 0.15	− 0.15			−0.005	+0.002		
10— 4	0.00	− 0.06			−0.001	+0.001		
1— 5	− 0.010	+ 0.024						
2— 5	+ 0.030	− 0.083	0.00	− 0.04	0.000	+0.001		
3— 5	− 1.652	− 1.090	− 0.54	− 0.11	−0.058	+0.005	+ 0.38	+ 0.03
4— 5	− 0.43	+ 27.91	− 2.76	− 6.16	−0.225	−0.482	+ 1.88	+ 4.12
5— 5	+183.95	−108.48	− 3.14	− 1.66	−0.266	−0.191	+ 2.35	+ 1.44
6— 5	+109.92	+ 8.83	+ 1.83	− 0.09	+0.428	−0.054	− 3.56	+ 0.31
7- 5	+ 25.44	+ 21.61	+ 1.41	− 0.90	+0.241	−0.155	− 2.06	+ 1.45
8— 5	+ 1.91	+ 7.53	+ 0.25	− 0.61	+0.049	−0.086	− 0.38	+ 0.85
9— 5	− 0.57	+ 1.46	− 0.04	− 0.23	+0.001	−0.025	+ 0.04	+ 0.22
10— 5	− 0.22	+ 0.15			−0.003	−0.005		
11— 5	− 0.08	0.00			−0.001	−0.001		
3— 6	− 0.089	− 0.087			−0.002	+0.001		
4— 6	− 1.42	+ 1.69	− 0.25	− 0.46	−0.023	−0.039	+ 0.15	+ 0.34
5— 6	+ 24.48	+ 5.07	+ 3.57	− 2.64	+0.238	−0.188	− 2.31	+ 1.72
6— 6	− 42.15	−130.96	+ 0.90	− 2.70	+0.095	−0.208	− 0.82	+ 1.97
7— 6	+ 22.44	− 80.96	+ 0.22	+ 0.85	+0.083	+0.213	− 0.64	− 1.94

$\text{Arg}=i'g'+ig$	$a'r'\frac{d\left(r'\frac{d\Omega'}{dr'}\right)}{dr'}$		$a'^2r'\frac{d^2\Omega'}{dr'dZ'}+a'^2\frac{d\Omega'}{dZ'}$		$aa'\frac{d\Omega'}{dZ}$		$aa'r'\frac{d^2\Omega'}{dr'dZ}$	
	cos.	sin.	sin.	cos.	sin.	cos.	sin.	cos.
i' i	″	″	″	″	″	″	″	″
8—6	+22.33	−18.63	+0.77	+0.81	+0.133	+0.129	−1.30	−1.23
9—6	+ 7.53	− 0.73	+0.55	+0.13	+0.070	+0.022	−0.71	−0.15
10—6	+ 1.44	+ 0.81	+0.20	−0.07	+0.021	−0.004	−0.19	+0.07
11—6	+ 0.14	+ 0.28			+0.004	−0.003		
12—6	0.00	+ 0.06			0.000	−0.001		
4—7	− 0.12	+ 0.12			−0.002	−0.002		
5—7	+ 1.51	+ 1.63	+0.28	−0.28	+0.021	−0.025	−0.22	+0.19
6—7	+ 7.99	−18.65	+2.17	+1.79	+0.134	+0.104	−1.34	−1.14
7—7	−85.62	+ 7.18	+2.05	+0.32	+0.148	+0.037	−1.52	−0.30
8—7	−54.52	−27.77	−0.24	+0.19	−0.094	+0.070	+1.00	−0.67
9—7	−11.72	−20.69	−0.43	+0.65	−0.062	+0.098	+0.63	−1.07
10—7	+ 0.55	− 6.82	−0.10	+0.46	−0.005	+0.052	0.00	−0.55
11—7	+ 0.98	− 1.28	0.00	+0.17	+0.006	−0.015	−0.11	−0.15
12—7	+ 0.35	− 0.12			+0.003	+0.003		
5—8	+ 0.04	+ 0.19			+0.001	−0.002		
6—8	+ 1.75	− 1.00	+0.35	+0.13	+0.021	+0.009	−0.22	−0.08
7—8	−12.65	− 8.91	−0.78	+1.54	−0.038	+0.086	+0.44	−0.89
8—8	− 7.87	+51.76	−0.11	+1.52	−0.004	−0.099	+0.07	−1.01
9—8	−26.85	+33.46	+0.08	−0.08	−0.047	−0.034	+0.52	+0.30
10—8	−17.60	+ 6.02	+0.28	−0.46	−0.067	−0.023	+0.81	+0.22
11—8	− 5.68	− 1.42	+0.23	−0.21	−0.035	+0.004	+0.47	0.00
12—8	− 1.08	− 1.22	+0.11	0.00	−0.011	+0.007	+0.14	0.00
6—9	+ 0.17	0.00			+0.002	0.000		
7—9	− 0.59	− 1.67	0.00	+0.26	−0.002	+0.015	+0.10	−0.32
8—9	− 8.14	+ 7.59	−1.00	−0.16	−0.051	−0.008	+0.48	+0.25
9—9	+28.83	+11.92	−0.94	+0.16	−0.062	+0.010	+0.38	+0.63
10—9	+18.41	+22.62	−0.91	+0.14	+0.008	−0.026	−0.53	+0.65
11—9	+ 1.87	+13.85	−0.49	−0.13	+0.004	−0.042	−0.60	+0.06
12—9	− 2.26	+ 4.64	−0.11	−0.21	−0.008	−0.025	−0.20	−0.20
7—10	+ 0.02	− 0.04			0.000	+0.002		
8—10	− 1.37	+ 0.17	−0.37	−0.07	−0.010	+0.002	+0.04	+0.07
9—10	+ 3.97	+ 6.65	+0.18	−0.55	−0.003	−0.028	+0.39	−0.06
10—10	+10.92	−14.53	+0.45	−0.28	−0.014	−0.035	+0.25	+0.05
11—10	+17.27	− 8.71	+0.21	+0.25	+0.013	−0.002	+0.13	+0.49
12—10	+11.02	+ 1.10	−0.10	+0.28	+0.031	−0.002	−0.19	+0.32
8—11	− 0.14	− 0.08			−0.001	+0.001		
9—11	− 0.08	+ 1.06	+0.07	−0.04	−0.003	−0.006	+0.11	0.00
10—11	+ 4.94	− 1.59	−0.14	−0.32	+0.014	−0.006	−0.01	−0.21
11—11	− 6.43	− 8.18	0.00	−0.35	+0.019	−0.013	+0.06	−0.17
12—11	− 2.77	−13.69	+0.21	−0.11	−0.001	+0.014	+0.18	+0.06
9—12	− 0.08	+ 0.10			−0.001	−0.001		
10—12	+ 0.75	+ 0.28	0.00	−0.07	+0.003	−0.002	+0.03	−0.03
11—12	− 0.28	− 3.32	−0.18	+0.07	+0.005	+0.006	−0.14	−0.04
12—12	− 6.94	+ 1.97	−0.25	−0.04	+0.002	+0.003	+0.04	+0.03

Before we can write the expressions for the first eight factors of δT we have to pass through the intermediate stage of deriving V, X, and $\bar{T}$. It will be seen that in all cases, except that of X, the factors involving γ are the factors A and B, whose expressions have been given at pages 73, 74. In the case of X, putting

$$X = Ma\frac{d\Omega}{dg} + Nar\frac{d\Omega}{dr}$$

we have

$$M = -\frac{2r\rho}{a^2\cos^2\varphi}\cos(f-\omega) \qquad N = -\frac{2\rho}{a\cos^3\varphi}[\sin f\cos\omega - (\cos f + e)\sin\omega]$$

For a similar reason, as in deriving the expression for T, we can dispense with any direct computation of the terms of δT involving higher multiples of γ than the first, as all such terms in W are computed very readily by the formulæ of page 74. This abbreviation in determining T is applicable, no matter how far the approximation may be pushed; it is also true for each portion of T under restrictions which are readily seen. In consequence, availing ourselves of the quantities P and Q of page 63, in M and N, it suffices to put

$$\frac{\rho}{a}\cos\omega = \frac{1}{2}P_0 + P_1\cos\gamma \qquad \frac{\rho}{a}\sin\omega = Q_1\cos\varphi\sin\gamma$$

and we also have

$$\begin{aligned}
\frac{r}{a}\cos f &= \frac{1}{2}P_0 + P_1\cos g + P_2\cos 2g + P_3\cos 3g + \ . \ . \ . \\
\frac{r}{a}\sin f &= \cos\varphi\,[Q_1\sin g + Q_2\sin 2g + Q_3\sin 3g + \ . \ . \ .] \\
\frac{\cos f + e}{\cos\varphi} &= \cos\varphi\,[Q_1\cos g + 2Q_2\cos 2g + 3Q_3\cos 3g + \ . \ . \ .] \\
\frac{\sin f}{\cos\varphi} &= \qquad P_1\sin g + 2P_2\sin 2g + 3P_3\sin 3g + \ . \ . \ .
\end{aligned}$$

From these equations we derive

$$\begin{aligned}
M = &-\frac{P_0}{\cos^2\varphi}\left[\frac{1}{2}P_0 + P_1\cos g + P_2\cos 2g + P_3\cos 3g + \ . \ . \ .\right] \\
&-\sum_{i=0}^{i=+\infty}{}_i\left[\frac{P_1P_i}{\cos^2\varphi} \pm Q_1Q_i\right]\cos(\gamma \mp ig) \\
N = &-\frac{P_0}{\cos^2\varphi}\left[P_1\sin g + 2P_2\sin 2g + 3P_3\sin 3g + \ . \ . \ .\right] \\
&+\sum_{i=0}^{i=\infty}{}_i\ i\left[Q_1Q_i \pm \frac{P_1P_i}{\cos^2\varphi}\right]\sin(\gamma \mp ig)
\end{aligned}$$

It is evident that M and N are connected by the equation

$$N = -\frac{dM}{dg}$$

Applying these formulæ to Jupiter we get

$$\begin{array}{lll}
M = -\ [8.0210889] & +\ 2[8.8601562]\cos\gamma & -\ 2[6.76559]\cos(\gamma + g) \\
\quad +\ 2[8.8601562]\cos g & -\ 2[0.0000005]\cos(\gamma - g) & -\ 2[5.06865]\cos(\gamma + 2g) \\
\quad +\ 2[7.2422636]\cos 2g & -\ 2[8.3821499]\cos(\gamma - 2g) & \\
\quad +\ 2[5.8004832]\cos 3g & -\ 2[6.9403906]\cos(\gamma - 3g) & \\
\quad +\ 2[4.4324972]\cos 4g & -\ 2[5.5724173]\cos(\gamma - 4g) &
\end{array}$$

$$
\begin{aligned}
\mathrm{N} = &+ 2[8.8601562] \sin g \\
&+ 2[7.5432936] \sin 2g \\
&+ 2[6.2776045] \sin 3g \\
&+ 2[5.0345572] \sin 4g \\
&+ 2[0.0000005] \sin (\gamma - g) \\
&+ 2[8.6831799] \sin (\gamma - 2g) \\
&+ 2[7.4175119] \sin (\gamma - 3g) \\
&+ 2[6.1744773] \sin (\gamma - 4g) \\
&- 2[6.76559\ \ ] \sin (\gamma + g) \\
&- 2[5.36968\ \ ] \sin (\gamma + 2g)
\end{aligned}
$$

And, similarly for Saturn, we get

$$
\begin{aligned}
\mathrm{M}' = &- \ [8.1518373] \\
&+ 2[8.9255751] \cos g' \\
&+ 2[7.3727757] \cos 2g' \\
&+ 2[5.9960961] \cos 3g' \\
&+ 2[4.6932130] \cos 4g' \\
&+ 2[8.9255751] \cos \gamma' \\
&- 2[0.0000009] \cos (\gamma' - g') \\
&- 2[8.4472585] \cos (\gamma' - 2g') \\
&- 2[7.0706074] \cos (\gamma' - 3g') \\
&- 2[5.7677413] \cos (\gamma' - 4g') \\
&- 2[4.5057301] \cos (\gamma' - 5g') \\
&- 2[6.89630\ \ ] \cos (\gamma' + g') \\
&- 2[5.26447\ \ ] \cos (\gamma' + 2g')
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{N}' = &+ 2[8.9255751] \sin g' \\
&+ 2[7.6738057] \sin 2g' \\
&+ 2[6.4732174] \sin 3g' \\
&+ 2[5.2952730] \sin 4g' \\
&+ 2[0.0000009] \sin (\gamma' - g') \\
&+ 2[8.7482885] \sin (\gamma' - 2g') \\
&+ 2[7.5477287] \sin (\gamma' - 3g') \\
&+ 2[6.3698013] \sin (\gamma' - 4g') \\
&+ 2[5.2047001] \sin (\gamma' - 5g') \\
&- 2[6.89630\ \ ] \sin (\gamma' + g') \\
&- 2[5.56550\ \ ] \sin (\gamma' + 2g')
\end{aligned}
$$

We can now give the values of V and X. It is thought unnecessary to give the partial derivatives with respect to g or g' of any expression which has already been given, for the reason that they can so easily be formed. The expressions for B and G are appended:

Arg= $\varkappa\gamma+i'g'+ig$			V		X		B		G	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0				+0.06138		+0.06138		
1	0	−1	−42.307	−0.066	+14.278	−0.025	−28.029	−0.091	+56.586	+0.082
−1	0	0	−3.951	+4.621	+1.17913	−0.84751	−2.772	+3.773	+5.093	−5.638
0	0	−1	+2.60940	−2.04419	−1.03659	−0.00354	+1.57281	−2.04773	−3.53363	+2.54858
1	0	−2	+0.478	−1.882	+0.127	+0.510	+0.605	−1.372	−0.389	+2.224
−1	0	−1	−0.269	−0.447	+0.035	+0.073	−0.234	−0.374	+0.318	+0.530
0	0	−2	+0.19153	+0.26064	−0.00889	−0.03706	+0.18264	+0.22358	−0.24191	−0.31404
1	0	−3	+0.059	+0.034	−0.012	+0.004	+0.047	+0.038	−0.058	−0.030
−1	0	−2	+0.032	−0.015	−0.004	+0.002	+0.028	−0.013	−0.036	+0.018
0	0	−3	−0.01704	+0.01396	+0.00088	−0.00020	−0.01616	+0.01376	+0.01850	−0.01740
1	0	−4	−0.004	+0.002	0.000	−0.001	−0.004	+0.001	+0.005	−0.001
−1	0	−3	+0.001	0.000			+0.001	0.000	−0.001	0.000
0	0	−4	−0.00100	−0.00102	−0.00004	+0.00003	−0.00104	−0.00099	+0.00111	+0.00102
1	0	−5	+0.001	+0.001			+0.001	+0.001	−0.001	−0.001
−1	1	+4	0.000	−0.007			0.000	−0.007	0.000	+0.007

Arg= $\varkappa\gamma+i'g'+ig$			V		X		B		G	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	1	+ 3	+ 0.00236	+ 0.01264	− 0.00011	+ 0.00008	+ 0.00225	+ 0.01272	− 0.00259	− 0.01285
1	1	+ 2	− 0.005	− 0.008	0.000	+ 0.001	− 0.005	− 0.007	+ 0.005	+ 0.009
−1	1	+ 3	− 0.007	+ 0.012	+ 0.002	− 0.001	− 0.005	+ 0.011	+ 0.007	− 0.012
0	1	+ 2	− 0.00704	+ 0.03441	− 0.00138	− 0.00381	− 0.00842	+ 0.03060	+ 0.00996	− 0.04073
1	1	+ 1	+ 0.006	− 0.063	0.000	+ 0.009	+ 0.006	− 0.054	− 0.008	+ 0.073
−1	1	+ 2	− 0.230	− 0.170	+ 0.018	+ 0.052	− 0.212	− 0.118	+ 0.249	+ 0.203
0	1	+ 1	− 0.36721	− 0.07128	+ 0.10334	− 0.05272	− 0.26387	− 0.12400	+ 0.47094	+ 0.08718
1	1	0	+ 0.726	+ 0.301	− 0.158	− 0.035	+ 0.568	+ 0.266	− 0.885	− 0.364
−1	1	+ 1	+ 4.870	− 4.130	− 1.426	+ 0.719	+ 3.444	− 3.411	− 6.254	+ 5.054
0	1	0	0.00000	0.00000	+ 0.16312	+ 0.78650	+ 0.16312	+ 0.78650	0.00000	0.00000
1	1	− 1	− 5.0412	+ 3.3090	+ 1.365	− 1.009	− 3.676	+ 2.300	+ 6.4715	− 3.9986
−1	1	0	+ 9.922	+49.739	− 2.095	−10.824	+ 7.827	+38.915	− 12.499	−62.969
0	1	− 1	− 4.91261	−25.32954	− 0.08079	+ 0.00778	− 4.99340	−25.32176	+ 6.25857	+32.55616
1	1	− 2	− 3.370	−15.844	+ 1.271	+ 5.956	− 2.099	− 9.888	+ 4.173	+19.399
−1	1	− 1	− 1.0709	− 3.6163	− 0.248	+ 0.902	− 1.319	− 2.714	+ 0.5641	+ 4.7236
0	1	− 2	− 0.24520	+ 2.59490	− 0.08917	− 0.43467	− 0.33437	+ 2.16023	+ 0.96309	− 3.47275
1	1	− 3	+ 1.229	− 0.649	− 0.125	+ 0.205	+ 1.104	− 0.444	− 1.633	+ 0.942
−1	1	− 2	+ 0.257	− 0.302	− 0.040	+ 0.046	+ 0.217	− 0.256	− 0.316	+ 0.370
0	1	− 3	− 0.25009	+ 0.30458	+ 0.00917	− 0.01476	− 0.24092	+ 0.28982	+ 0.33884	− 0.39241
1	1	− 4	+ 0.063	− 0.045	− 0.002	+ 0.001	+ 0.061	− 0.044	− 0.098	+ 0.072
−1	1	− 3	+ 0.017	+ 0.008	0.000	+ 0.001	+ 0.017	+ 0.009	− 0.020	− 0.008
0	1	− 4	− 0.02090	+ 0.00453	+ 0.00011	− 0.00011	− 0.02079	+ 0.00442	+ 0.02744	− 0.00854
1	1	− 5	+ 0.004	− 0.005	0.000	0.000	+ 0.004	− 0.005	− 0.007	+ 0.007
−1	1	− 4	− 0.002	0.000			− 0.002	0.000	+ 0.002	0.000
0	1	− 5	− 0.00038	+ 0.00005	− 0.00003	− 0.00002	− 0.00041	+ 0.00002	+ 0.00071	− 0.00028
1	1	− 6	+ 0.001	0.000			+ 0.001	0.000	− 0.001	0.000
0	2	+ 3	+ 0.00026	− 0.00098			+ 0.00026	− 0.00098	− 0.00026	+ 0.00098
−1	2	+ 3	− 0.001	0.000	+ 0.001	− 0.001	0.000	− 0.001	+ 0.001	0.000
0	2	+ 2	+ 0.00104	+ 0.00381	− 0.00036	− 0.00027	+ 0.00068	+ 0.00354	− 0.00102	− 0.00446
1	2	+ 1	− 0.003	− 0.007	0.000	+ 0.001	− 0.003	− 0.006	+ 0.003	+ 0.007
−1	2	+ 2	− 0.042	− 0.003	+ 0.005	+ 0.004	− 0.037	+ 0.001	+ 0.047	+ 0.004
0	2	+ 1	− 0.03928	+ 0.01129	+ 0.00691	− 0.01064	− 0.03237	+ 0.00065	+ 0.04882	− 0.01335
1	2	0	+ 0.103	− 0.009	− 0.018	+ 0.005	+ 0.085	− 0.004	− 0.123	+ 0.010
−1	2	+ 1	+ 0.171	− 0.874	− 0.096	+ 0.146	+ 0.075	− 0.728	− 0.224	+ 1.044
0	2	0	0.00000	0.00000	+ 0.23489	+ 0.05967	+ 0.23489	+ 0.05967	0.00000	0.00000
1	2	− 1	− 0.5469	+ 0.8643	+ 0.079	− 0.201	− 0.468	+ 0.663	+ 0.7447	− 1.0511
−1	2	0	+ 14.636	+ 4.668	− 3.224	− 0.828	+ 11.412	+ 3.840	− 19.532	− 5.243
0	2	− 1	− 9.60269	− 1.11394	− 2.38232	+ 1.03341	− 11.98501	− 0.08053	+ 13.32044	+ 0.91722
1	2	− 2	+ 1.5995	− 4.6390	+ 0.8718	+ 0.8956	+ 2.4713	− 3.7434	− 3.119	+ 6.101
−1	2	− 1	−119.8230	+51.0393	+32.814	−14.067	− 87.009	+36.972	+167.6953	−71.5954
0	2	− 2	+106.01668	−45.27941	− 0.04681	+ 0.00420	+105.96987	−45.27521	−151.09782	+64.68104
1	2	− 3	− 21.930	+ 9.193	− 2.562	+ 1.159	− 24.492	+10.352	+ 34.297	−14.464
−1	2	− 2	+ 1.5773	+ 3.3876	− 0.2253	− 0.9535	+ 1.3520	+ 2.4341	− 1.499	− 5.021
0	2	− 3	+ 2.35012	− 5.49634	+ 0.18871	− 0.08052	+ 2.53883	− 5.57686	− 4.15523	+ 7.92094
1	2	− 4	− 1.313	+ 2.471	− 0.143	+ 0.017	− 1.456	+ 2.488	+ 2.193	− 3.443
−1	2	− 3	+ 0.329	+ 0.256	− 0.044	− 0.045	+ 0.285	+ 0.211	− 0.380	− 0.340

Arg= $\varkappa\gamma+i'g'+ig$			V		X		B		G	
$\varkappa$	i'	i	sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
			″	″	″	″	″	″	″	″
0	2	−4	−0.15465	−0.49077	+0.01045	−0.00100	−0.14420	−0.49177	+0.11231	+0.67980
1	2	−5	−0.025	+0.213	−0.004	+0.002	−0.029	+0.215	+0.068	−0.300
−1	2	−4	+0.007	+0.015	−0.002	−0.001	+0.005	+0.014	−0.009	−0.019
0	2	−5	−0.01114	−0.03543	+0.00028	−0.00014	−0.01086	−0.03557	+0.01030	+0.04823
1	2	−6	+0.004	+0.016	−0.001	0.000	+0.003	+0.016	−0.002	−0.022
0	2	−6	−0.00063	−0.00167	+0.00003	−0.00002	−0.00060	−0.00169	+0.00064	+0.00244
−1	3	+2	−0.004	+0.003	+0.001	+0.001	−0.003	+0.004	+0.005	−0.003
0	3	+1	−0.00325	+0.00302	+0.00007	−0.00140	−0.00318	+0.00162	+0.00398	−0.00353
1	3	0	+0.010	−0.008	−0.001	+0.001	+0.009	−0.007	−0.012	+0.009
−1	3	+1	−0.048	−0.110	−0.002	+0.017	−0.050	−0.093	+0.055	+0.130
0	3	0	0.00000	0.00000	+0.04330	−0.01001	+0.04330	−0.01001	0.00000	0.00000
1	3	−1	−0.014	+0.151	−0.004	−0.026	−0.018	+0.125	+0.031	−0.185
−1	3	0	+2.774	−0.827	−0.596	+0.138	+2.178	−0.689	−3.652	+1.145
0	3	−1	−1.67802	+0.77926	−0.34634	+0.44085	−2.02436	+1.22011	+2.32527	−1.08947
1	3	−2	−0.156	−1.046	+0.196	+0.050	+0.040	−0.996	−0.024	+1.418
−1	3	−1	−14.922	+28.851	+4.787	−6.057	−10.135	+22.794	+21.657	−37.882
0	3	−2	+12.87302	−24.26454	−0.99807	−1.40617	+11.87495	−25.67073	−19.23030	+32.41300
1	3	−3	−0.237	+6.511	−0.474	+0.666	−0.711	+7.177	+1.382	−9.236
−1	3	−2	−69.022	−99.317	+13.584	+19.364	−55.438	−79.953	+89.055	+127.729
0	3	−3	+64.65315	+91.45689	−0.05234	−0.04272	+64.60081	+91.41417	−84.13717	−118.41682
1	3	−4	−16.577	−23.474	−0.806	−1.074	−17.383	−24.548	+22.261	+31.276
−1	3	−3	−6.595	+1.056	+1.195	−0.078	−5.400	+0.978.	+8.559	−0.940
0	3	−4	+9.10247	+2.31940	+0.05439	+0.08122	+9.15686	+2.40062	−11.81818	−3.46238
1	3	−5	−3.450	−1.223	−0.052	−0.076	−3.502	−1.299	+4.476	+1.767
−1	3	−4	−0.332	+0.407	+0.054	−0.048	−0.278	+0.359	+0.434	−0.469
0	3	−5	+0.71640	−0.26172	+0.00363	+0.00582	+0.72003	−0.25590	−0.93326	+0.26690
1	3	−6	−0.319	+0.031	−0.004	−0.004	−0.323	+0.027	+0.416	−0.016
−1	3	−5	−0.014	+0.023	+0.004	−0.003	−0.010	+0.020	+0.019	−0.028
0	3	−6	+0.04652	−0.03167	+0.00027	+0.00021	+0.04679	−0.03146	−0.06108	+0.04687
1	3	−7	−0.024	+0.011	0.000	−0.001	−0.024	+0.010	+0.031	−0.112
−1	3	−6	−0.004	+0.003			−0.004	+0.003	+0.004	−0.003
0	3	−7	+0.00216	−0.00285	+0.00002	+0.00002	+0.00218	−0.00283	−0.00306	+0.00326
−1	4	+1	−0.014	−0.009	0.000	+0.002	−0.014	−0.007	+0.016	+0.011
0	4	0	0.00000	0.00000	+0.00501	−0.00354	+0.00501	−0.00354	0.00000	0.00000
1	4	−1	+0.009	+0.018	−0.002	−0.002	+0.007	+0.016	−0.009	−0.023
−1	4	0	+0.294	−0.312	−0.068	+0.049	+0.226	−0.263	−0.390	+0.399
0	4	−1	−0.15912	+0.20917	−0.01243	+0.08450	−0.17155	+0.29367	+0.22774	−0.27809
1	4	−2	−0.1275	−0.1234	+0.029	−0.008	−0.098	−0.131	+0.1295	+0.1795
−1	4	−1	+1.076	+5.719	+0.173	−1.165	+1.249	+4.554	−0.913	−7.434
0	4	−2	−0.80131	−4.64775	−0.46277	−0.23345	−1.26408	−4.88120	+0.59118	+6.16744
1	4	−3	+1.113	+0.841	−0.014	+0.165	+1.099	+1.006	−1.287	−1.291
−1	4	−2	−37.5113	−12.8716	+6.362	+3.232	−31.149	−9.640	+46.9598	+17.4477
0	4	−3	+33.88113	+11.94483	+0.72197	−0.79007	+34.60310	+11.15476	−42.76022	−16.35448
1	4	−4	−9.727	−1.061	−0.463	−0.208	−10.190	−1.269	+12.570	+1.919
−1	4	−3	+66.142	−70.334	−9.953	+10.743	+56.189	−59.591	−80.770	+86.232
0	4	−4	−61.53333	+67.26806	+0.02210	−0.06522	−61.51123	+67.20284	+75.35198	−82.84294
1	4	−5	+17.070	−18.743	+0.422	−0.498	+17.492	−19.241	−21.161	+23.418

Arg= $\varkappa\gamma+i'g'+ig$			V		X		B		G	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	4	−4	− 1.666	− 7.835	+0.157	+1.109	− 1.509	− 6.726	+ 1.786	+ 9.621
0	4	−5	− 0.62573	+10.33047	−0.03440	+0.03220	− 0.66013	+10.36267	+ 1.16777	−13.49758
1	4	−6	+ 0.541	− 3.759	+0.034	−0.049	+ 0.575	− 3.808	− 0.762	+ 4.636
−1	4	−5	− 0.507	− 0.373	+0.053	+0.054	− 0.454	− 0.319	+ 0.584	+ 0.471
0	4	−6	+ 0.44699	+ 0.80753	−0.00274	+0.00340	+ 0.44425	+ 0.81093	− 0.50166	− 1.00475
1	4	−7	− 0.115	− 0.361	0.000	−0.003	− 0.115	− 0.364	+ 0.126	+ 0.448
−1	4	−6	− 0.035	− 0.016	+0.005	+0.001	− 0.030	− 0.015	+ 0.042	+ 0.020
0	4	−7	+ 0.05370	+ 0.04836	−0.00009	+0.00028	+ 0.05361	+ 0.04864	− 0.06209	− 0.06136
1	4	−8	− 0.024	− 0.019	0.000	0.000	− 0.024	− 0.019	+ 0.027	+ 0.026
−1	4	−7	− 0.003	0.000			− 0.003	0.000	+ 0.003	0.000
0	4	−8	+ 0.00746	+ 0.00266	+0.00002	+0.00004	+ 0.00748	+ 0.00270	− 0.00823	− 0.00341
0	5	0	0.00000	0.00000	+0.00039	−0.00062	+ 0.00039	− 0.00062	0.00000	0.00000
−1	5	0	+ 0.014	− 0.054	−0.006	+0.008	+ 0.008	− 0.046	− 0.021	+ 0.068
0	5	−1	− 0.00532	+ 0.03191	+0.00424	+0.01045	− 0.00108	+ 0.04236	+ 0.01005	− 0.04195
1	5	−2	− 0.0265	− 0.0022	+0.002	−0.004	− 0.024	− 0.006	+ 0.0302	+ 0.0075
−1	5	−1	+ 0.658	+ 0.615	−0.059	−0.143	+ 0.599	+ 0.472	− 0.760	− 0.820
0	5	−2	− 0.50428	− 0.48746	−0.09999	+0.00414	− 0.60427	− 0.48332	+ 0.58105	+ 0.66878
1	5	−3	+ 0.2505	− 0.0502	+0.0121	+0.0258	+ 0.2626	− 0.0244	− 0.312	+ 0.019
−1	5	−2	− 8.1002	+ 2.5012	+1.378	−0.055	− 6.722	+ 2.446	+10.1287	− 2.6538
0	5	−3	+ 7.20416	− 2.07513	+0.10788	−0.40042	+ 7.31204	− 2.47555	+ 9.09206	+ 2.16585
1	5	−4	− 1.698	+ 1.452	−0.123	+0.013	− 1.821	+ 1.465	+ 2.252	− 1.669
−1	5	−3	+ 6.2989	−39.2994	−1.5016	+5.5027	+ 4.7973	−33.7967	− 8.396	+47.462
0	5	−4	− 6.02469	+36.65055	+0.55353	+0.31829	− 5.47116	+36.96884	+ 8.05541	−44.45554
1	5	−5	+ 0.102	−10.769	+0.065	−0.297	+ 0.167	−11.066	− 0.425	+13.220
−1	5	−4	+60.330	+36.232	−7.518	−4.406	+52.812	+31.826	−71.476	−42.695
0	5	−5	−58.23577	−33.40774	+0.05776	+0.00303	−58.17801	−33.40471	+69.17473	+39.40340
1	5	−6	+16.908	+ 9.586	+0.285	+0.147	+17.193	+ 9.733	−20.242	−11.368
−1	5	−5	+ 7.371	− 2.620	−0.863	+0.256	+ 6.508	− 2.364	− 8.749	+ 2.952
0	5	−6	− 9.50437	+ 1.35024	−0.01759	−0.01457	− 9.52196	+ 1.33567	+11.29325	− 1.45191
1	5	−7	+ 3.415	− 0.232	+0.035	+0.010	+ 3.450	− 0.222	− 4.068	+ 0.225
−1	5	−6	+ 0.348	− 0.581	−0.043	+0.055	+ 0.305	− 0.526	− 0.425	+ 0.665
0	5	−7	− 0.73758	+ 0.61744	−0.00249	−0.00106	− 0.74007	+ 0.61638	+ 0.88915	− 0.70284
1	5	−8	+ 0.320	− 0.203	+0.002	0.000	+ 0.322	− 0.203	− 0.386	+ 0.230
−1	5	−7	− 0.003	− 0.048	0.000	+0.005	− 0.003	− 0.043	0.000	+ 0.055
0	5	−8	− 0.03542	+ 0.07007	−0.00023	+0.00002	− 0.03565	+ 0.07009	+ 0.04486	− 0.08092
1	5	−9	+ 0.026	− 0.025	0.000	+0.001	+ 0.026	− 0.024	− 0.031	+ 0.029
−1	6	0	+ 0.002	− 0.007	−0.001	+0.001	+ 0.001	− 0.006	− 0.002	+ 0.009
0	6	−1	+ 0.00121	+ 0.00347	+0.00111	+0.00082	+ 0.00232	+ 0.00429	− 0.00112	− 0.00460
1	6	−2	− 0.007	+ 0.003	0.000	−0.001	− 0.007	+ 0.002	+ 0.008	− 0.003
−1	6	−1	+ 0.127	+ 0.018	−0.015	−0.012	+ 0.112	+ 0.006	− 0.151	− 0.033
0	6	−2	− 0.09454	− 0.01527	−0.01336	+0.00844	− 0.10790	− 0.00683	+ 0.11348	+ 0.02895
1	6	−3	+ 0.023	− 0.035	+0.004	+0.003	+ 0.027	− 0.032	− 0.032	+ 0.037
−1	6	−2	− 0.906	+ 1.187	+0.183	−0.115	− 0.723	+ 1.072	+ 1.170	− 1.373
0	6	−3	+ 0.80735	− 1.00270	−0.02283	−0.09477	+ 0.78452	− 1.09747	− 1.05474	+ 1.16180
1	6	−4	− 0.035	+ 0.421	−0.020	+0.014	− 0.055	+ 0.435	+ 0.085	− 0.506

Arg= $\varkappa\gamma+i'g'+ig$			V		X		B		G	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	6	−3	−4.312	−9.070	+0.313	+1.306	−3.999	−7.764	+4.826	+10.991
0	6	−4	+3.83036	+8.40510	+0.30416	+0.01677	+4.13452	+8.42187	−4.28505	−10.23334
1	6	−5	−1.860	−2.128	−0.025	−0.083	−1.885	−2.211	+2.138	+2.657
−1	6	−4	+35.265	−0.951	−4.179	−0.245	+31.086	−1.196	−41.461	+0.666
0	6	−5	−33.42872	+0.92270	−0.10883	+0.35440	−33.53755	+1.27710	+39.40245	−0.66207
1	6	−6	+9.919	−1.438	+0.180	+0.001	+10.099	−1.437	−11.775	+1.541
−1	6	−5	−15.257	+45.748	+1.527	−4.810	−13.730	+40.938	+17.475	−52.881
0	6	−6	+13.48590	−44.28526	+0.00850	+0.04282	+13.49440	−44.24244	−15.41530	+51.27238
1	6	−7	−3.910	+13.158	−0.040	+0.152	−3.950	+13.310	+4.470	−15.305
−1	6	−6	+3.285	+5.909	−0.297	−0.593	+2.988	+5.316	−3.710	−6.850
0	6	−7	−2.74770	−7.52924	+0.00664	−0.00916	−2.74106	−7.53840	+3.09747	+8.73198
1	6	−8	+0.790	+2.699	0.000	+0.023	+0.790	+2.722	−0.890	−3.135
−1	6	−7	+0.609	+0.242	−0.052	−0.026	+0.557	+0.216	−0.690	−0.293
0	6	−8	−0.70733	−0.55988	+0.00034	−0.00159	−0.70699	−0.56147	+0.80134	+0.66034
1	6	−9	+0.246	+0.258	0.000	+0.001	+0.246	+0.259	−0.279	−0.302
−1	6	−8	+0.054	−0.009	−0.005	−0.001	+0.049	−0.010	−0.061	+0.008
0	6	−9	−0.08084	−0.02053	−0.00004	−0.00015	−0.08088	−0.02068	+0.09182	+0.02593
1	6	−10	+0.033	+0.016	−0.001	0.000	+0.032	+0.016	−0.037	−0.019
−1	6	−9	+0.003	−0.002			+0.003	−0.002	−0.003	+0.002
0	6	−10	−0.00895	+0.00154	0.00000	+0.00001	−0.00895	+0.00155	+0.00968	−0.00141
1	6	−11	+0.007	−0.001			+0.007	−0.001	−0.007	+0.001
0	7	−1	+0.00028	+0.00024	+0.00017	+0.00001	+0.00045	+0.00025	−0.00031	−0.00031
−1	7	−1	+0.016	−0.004	−0.003	0.000	+0.013	−0.004	−0.019	+0.004
0	7	−2	−0.01121	+0.00514	−0.00103	+0.00196	−0.01224	+0.00710	+0.01384	−0.00493
1	7	−3	−0.002	−0.008	+0.001	0.000	−0.001	−0.008	+0.001	+0.009
−1	7	−2	−0.011	+0.229	+0.015	−0.027	+0.004	+0.202	+0.029	−0.271
0	7	−3	+0.01481	−0.19650	−0.01232	−0.01321	+0.00249	−0.20971	−0.03314	+0.23289
1	7	−4	+0.046	+0.060	−0.002	+0.004	+0.044	+0.064	−0.048	−0.074
−1	7	−3	−1.741	−1.015	+0.169	+0.183	−1.572	−0.832	+2.003	+1.276
0	7	−4	+1.55014	+0.95182	+0.07687	−0.03586	+1.62701	+0.91596	−1.78767	−1.20325
1	7	−5	−0.583	−0.072	−0.014	−0.013	−0.597	−0.085	+0.685	+0.123
−1	7	−4	+8.502	−5.904	−1.060	+0.490	+7.442	−5.414	−10.056	+6.668
0	7	−5	−8.04741	+5.45171	+0.03494	+0.20882	−8.01247	+5.66053	+9.54307	−6.16627
1	7	−6	+2.059	−2.206	+0.052	−0.028	+2.111	−2.234	−2.484	+2.521
−1	7	−5	+6.582	+27.992	−0.468	−2.869	+6.114	+25.123	−7.312	−32.241
0	7	−6	−6.68419	−26.75840	−0.21033	−0.01411	−6.89452	−26.77251	+7.39450	+30.86979
1	7	−7	+3.059	+7.966	+0.048	+0.102	+3.107	+8.068	−3.361	−9.232
−1	7	−6	−31.422	−3.029	+2.852	+0.222	−28.570	−2.807	+35.649	+3.328
0	7	−7	+30.34215	+1.89095	−0.02985	+0.01298	+30.31230	+1.90393	−34.46121	−2.02011
1	7	−8	−9.126	−0.464	−0.077	+0.001	−9.203	−0.463	+10.398	+0.482
−1	7	−7	−4.157	+3.443	+0.362	−0.280	−3.795	+3.163	+4.732	−3.859
0	7	−8	+5.26380	−3.32770	+0.00452	+0.00320	+5.26832	−3.32450	−5.99072	+3.73716
1	7	−9	−1.883	+1.034	−0.014	+0.005	−1.897	+1.039	+2.145	−1.165
−1	7	−8	−0.123	+0.581	+0.013	−0.045	−0.110	+0.536	+0.149	−0.651
0	7	−9	+0.34515	−0.70113	+0.00093	+0.00003	+0.34608	−0.70110	−0.40059	+0.78708
1	7	−10	−0.163	+0.247	−0.003	+0.001	−0.166	+0.248	+0.187	−0.278

Arg= $\varkappa\gamma+i'g'+ig$			V		X		B		G	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
—1	7	—9	+0.018	+ 0.031	+0.002	—0.004	+ 0.020	+ 0.027	— 0.015	— 0.038
0	7	—10	— 0.00414	— 0.08076	+0.00015	—0.00004	— 0.00399	— 0.08080	— 0.00327	+ 0.09105
1	7	—11	— 0.001	+ 0.055	+0.001	0.000	0.000	+ 0.055	+ 0.006	— 0.059
—1	7	—10	+ 0.002	+ 0.002			+ 0.002	+ 0.002	— 0.002	— 0.002
0	7	—11	— 0.00339	— 0.00688	—0.00014	—0.00001	— 0.00353	— 0.00689	+ 0.00285	+ 0.00793
1	7	—12	+ 0.002	+ 0.006			+ 0.002	+ 0.006	— 0.002	— 0.006
0	8	— 2	- 0.00081	+ 0.00125	+0.00001	+0.00028	— 0.00080	+ 0.00153	+ 0.00105	— 0.00137
—1	8	2	+ 0.012	+ 0.029	—0.001	—0.005	+ 0.011	+ 0.024	— 0.013	— 0.035
0	8	— 3	— 0.01377	— 0.02428	—0.00276	—0.00086	— 0.01653	— 0.02514	+ 0.01755	+ 0.02950
1	8	— 4	+ 0.016	+ 0.002	+0.001	+0.001	+ 0.017	+ 0.003	— 0.017	— 0.003
—1	8	— 3	— 0.347	+ 0.022	+0.038	+0.011	— 0.309	+ 0.033	+ 0.405	— 0.007
0	8	— 4	+ 0.31115	— 0.01228	+0.01058	—0.01460	+ 0.32173	— 0.02688	— 0.36383	— 0.00347
1	8	— 5	— 0.092	+ 0.061	—0.003	—0.001	— 0.095	+ 0.060	+ 0.111	— 0.064
—1	8	— 4	+ 0.870	— 2.166	—0.146	+0.201	+ 0.724	— 1.965	— 1.078	+ 2.472
0	8	— 5	— 0.84496	+ 1.99985	+0.04055	+0.05451	— 0.80441	+ 2.05436	+ 1.04920	— 2.28587
1	8	— 6	+ 0.052	— 0.710	+0.009	—0.012	+ 0.061	— 0.722	— 0.091	+ 0.819
—1	8	— 5	+ 6.801	+ 6.823	—0.556	—0.752	+ 6.245	+ 6.071	— 7.650	— 7.924
0	8	— 6	— 6.38814	— 6.52929	—0.13118	+0.05109	— 6.51932	— 6.47820	+ 7.20228	+ 7.59594
1	8	— 7	+ 2.315	+ 1.662	+0.023	+0.030	+ 2.338	+ 1.692	— 2.629	— 1.959
—1	8	— 6	—19.983	+ 9.035	+1.803	—0.694	—18.180	+ 8.341	+22.650	—10.093
0	8	— 7	+19.16027	— 8.87404	—0.01983	—0.11611	+19.14044	— 8.99015	—21.73985	+ 9.93193
1	8	— 8	— 5.693	+ 3.208	—0.054	+0.024	— 5.747	+ 3.232	+ 6.480	— 3.601
—1	8	— 7	— 2.700	—19.744	+0.252	+1.574	— 2.448	—18.170	+ 3.093	+22.074
0	8	— 8	+ 3.40236	+18.95337	—0.01287	—0.01628	+ 3.38949	+18.93709	— 3.89187	—21.20181
1	8	— 9	— 1.123	— 5.738	—0.010	—0.039	— 1.133	— 5.777	+ 1.289	+ 6.431
—1	8	— 8	— 3.127	— 2.554	+0.231	+0.200	— 2.896	— 2.354	+ 3.476	+ 2.871
0	8	— 9	+ 3.23043	+ 3.25042	—0.00182	+0.00243	+ 3.22861	+ 3.25285	— 3.59601	— 3.65134
1	8	—10	— 1.053	— 1.169	—0.004	—0.006	— 1.057	— 1.175	+ 1.174	+ 1.314
—1	8	— 9	— 0.494	— 0.007	+0.035	+0.003	— 0.459	— 0.004	+ 0.549	+ 0.015
0	8	—10	+ 0.62087	+ 0.14754	+0.00006	+0.00058	+ 0.62093	+ 0.14812	— 0.69044	— 0.17265
1	8	—11	— 0.228	— 0.083	—0.001	—0.002	— 0.229	— 0.085	+ 0.253	+ 0.095
—1	8	—10	— 0.041	+ 0.030	+0.003	—0.001	— 0.038	+ 0.029	+ 0.046	— 0.032
0	8	—11	+ 0.06913	— 0.01641	+0.00006	+0.00007	+ 0.06919	— 0.01634	— 0.07708	+ 0.01704
1	8	—12	— 0.031	— 0.003	+0.002	0.000	— 0.029	— 0.003	+ 0.034	+ 0.003
—1	8	—11	0.000	+ 0.001			0.000	+ 0.001	0.000	— 0.001
0	8	—12	+ 0.00508	— 0.00312	0.00000	+0.00001	+ 0.00508	— 0.00312	— 0.00577	+ 0.00369
—1	9	— 2	+ 0.002	+ 0.001			+ 0.002	+ 0.001	— 0.002	— 0.001
0	9	— 3	— 0.00332	— 0.00169	—0.00042	+0.00008	— 0.00374	— 0.00161	+ 0.00371	+ 0.00220
1	9	— 4	+ 0.004	0.000			+ 0.004	0.000	— 0.004	0.000
—1	9	— 3	— 0.047	+ 0.031	+0.006	—0.001	— 0.041	+ 0.030	+ 0.055	— 0.033
0	9	— 4	+ 0.03943	— 0.02830	+0.00037	—0.00324	+ 0.03980	— 0.03154	— 0.04714	+ 0.03019
1	9	— 5	— 0.002	+ 0.020	0.000	—0.001	— 0.002	+ 0.019	+ 0.004	— 0.022
—1	9	— 4	— 0.092	— 0.442	—0.004	+0.045	— 0.096	— 0.397	+ 0.087	+ 0.509
0	9	— 5	+ 0.07310	+ 0.40730	+0.01493	+0.00684	+ 0.08803	+ 0.41414	— 0.06650	— 0.47027
1	9	— 6	— 0.078	— 0.117	0.000	—0.003	— 0.078	— 0.120	+ 0.084	+ 0.138
—1	9	— 5	+ 2.385	+ 0.530	—0.206	—0.095	+ 2.179	+ 0.435	— 2.695	— 0.662
0	9	— 6	— 2.23738	— 0.53064	—0.03406	+0.03874	— 2.27144	— 0.49190	+ 2.53222	+ 0.66261

Arg= $\varkappa\gamma+i'g'+ig$			V		X		B		G	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
1	9	−7	+ 0.744	− 0.027	+0.010	+0.004	+ 0.754	− 0.023	− 0.849	+ 0.007
−1	9	−6	− 4.694	+ 6.878	+0.470	−0.532	− 4.224	+ 6.346	+ 5.380	− 7.680
0	9	−7	+ 4.52390	− 6.57520	−0.05051	−0.07551	+ 4.47339	− 6.65071	− 5.18999	+ 7.35127
1	9	−8	− 1.112	+ 2.276	−0.014	+0.018	− 1.126	+ 2.294	+ 1.291	− 2.553
−1	9	−7	− 9.439	−12.881	+0.688	+1.039	− 8.751	−11.842	+10.476	+14.414
0	9	−8	+ 9.29110	+12.33229	+0.05939	−0.02619	+ 9.35049	+12.30610	−10.32496	−13.80967
1	9	−9	− 3.165	− 3.639	−0.020	−0.027	− 3.185	− 3.666	+ 3.523	+ 4.085
−1	9	−8	+11.377	− 4.479	−0.805	+0.342	+10.572	− 4.137	−12.566	+ 4.999
0	9	−9	−10.77926	+ 4.88540	+0.00831	−0.01058	−10.77095	+ 4.87482	+11.90855	− 5.45723
1	9	−10	+ 3.256	− 1.561	+0.016	−0.009	+ 3.272	− 1.570	− 3.602	+ 1.748
−1	9	−9	+ 1.340	− 2.566	−0.095	+0.172	+ 1.245	− 2.394	− 1.491	+ 2.828
0	9	−10	− 1.73410	+ 2.73406	−0.00139	−0.00115	− 1.73549	+ 2.73291	+ 1.92700	− 3.01675
1	9	−11	+ 0.628	− 0.908	+0.003	−0.003	+ 0.631	− 0.911	− 0.698	+ 1.003
−1	9	−10	− 0.064	− 0.384	+0.003	+0.025	− 0.061	− 0.359	+ 0.066	+ 0.424
0	9	−11	− 0.00942	+ 0.49580	−0.00032	+0.00008	− 0.00974	+ 0.49588	+ 0.01515	− 0.54704
1	9	−12	+ 0.020	− 0.188	0.000	0.000	+ 0.020	− 0.188	− 0.024	+ 0.207
−1	9	−11	− 0.026	− 0.032	+0.002	+0.002	− 0.024	− 0.030	+ 0.028	+ 0.035
0	9	−12	+ 0.02472	+ 0.05371	−0.00002	+0.00005	+ 0.02470	+ 0.05376	− 0.02646	− 0.05914
1	9	−13	− 0.006	− 0.024	0.000	0.000	− 0.006	− 0.024	+ 0.006	+ 0.026
0	10	−3	− 0.00047	+ 0.00003	−0.00005	+0.00004	− 0.00052	+ 0.00007	+ 0.00054	0.00000
−1	10	−3	− 0.002	+ 0.010			− 0.002	+ 0.010	+ 0.003	− 0.011
0	10	−4	+ 0.00266	− 0.00662	−0.00019	−0.00047	+ 0.00247	− 0.00709	− 0.00342	+ 0.00737
1	10	−5	0.000	0.000			0.000	0.000	0.000	0.000
−1	10	−4	− 0.053	− 0.056	+0.002	+0.006	− 0.051	− 0.050	+ 0.058	+ 0.066
0	10	−5	+ 0.04837	+ 0.05150	+0.00330	−0.00027	+ 0.05167	+ 0.05123	− 0.05251	− 0.06080
1	10	−6	− 0.025	− 0.006	0.000	0.000	− 0.025	− 0.006	+ 0.028	+ 0.008
−1	10	−5	+ 0.484	− 0.178	−0.046	+0.004	+ 0.438	− 0.174	− 0.552	+ 0.186
0	10	−6	− 0.45740	+ 0.15826	−0.00316	+0.01357	− 0.46056	+ 0.17183	+ 0.52214	− 0.16399
1	10	−7	+ 0.129	− 0.105	+0.002	0.000	+ 0.131	− 0.105	− 0.149	+ 0.113
−1	10	−6	− 0.095	+ 2.334	+0.044	−0.187	− 0.051	+ 2.147	+ 0.154	− 2.614
0	10	−7	+ 0.12263	− 2.22550	−0.03264	−0.01831	+ 0.08999	− 2.24381	− 0.18224	+ 2.49493
1	10	−8	+ 0.114	+ 0.719	−0.002	+0.007	+ 0.112	+ 0.726	− 0.112	− 0.810
−1	10	−7	− 6.236	− 2.681	+0.448	+0.254	− 5.788	− 2.427	+ 6.909	+ 3.048
0	10	−8	+ 6.01140	+ 2.57700	+0.03940	−0.04143	+ 6.05080	+ 2.53557	− 6.66624	− 2.93199
1	10	−9	− 2.012	− 0.562	−0.014	−0.006	− 2.026	− 0.568	+ 2.236	+ 0.651
−1	10	−8	+ 7.428	− 8.374	−0.543	+0.564	+ 6.885	− 7.810	− 8.226	+ 9.222
0	10	−9	− 7.067	+ 8.243	+0.02192	+0.02769	− 7.045	+ 8.271	+ 7.829	− 9.086
1	10	−10	+ 2.057	− 2.723	+0.012	−0.014	+ 2.069	− 2.737	− 2.282	+ 3.005
−1	10	−9	+ 4.264	+ 5.929	−0.289	−0.375	+ 3.975	+ 5.554	− 4.699	− 6.481
0	10	−10	− 4.479	− 5.497	+0.00774	+0.00358	− 4.471	− 5.493	+ 4.939	+ 6.008
1	10	−11	+ 1.420	+ 1.652	+0.007	+0.007	+ 1.427	+ 1.659	− 1.568	− 1.807
−1	10	−10	+ 1.922	+ 0.528	−0.118	−0.035	+ 1.804	+ 0.493	− 2.103	− 0.584
0	10	−11	− 2.084	− 0.736	+0.00074	−0.00085	− 2.083	− 0.737	+ 2.282	+ 0.811
1	10	−12	+ 0.703	+ 0.280	+0.002	+0.001	+ 0.705	+ 0.281	− 0.770	− 0.308
−1	10	−11	+ 0.276	− 0.113	−0.017	+0.005	+ 0.259	− 0.108	− 0.303	+ 0.120
0	10	−12	− 0.36150	+ 0.07393	−0.00021	−0.00009	− 0.36171	+ 0.07384	+ 0.39615	− 0.07774
1	10	−13	+ 0.139	− 0.010	−0.001	−0.001	+ 0.138	− 0.011	− 0.152	+ 0.010

Arg= $\varkappa\gamma+i'g'+ig$			V		X		B		G	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i		″	″	″	″	″	″	″
0	11	−4	0.00000	+0.00097			0.00000	+0.00097	+0.00004	−0.00110
−1	11	−4	−0.017	−0.004	+0.001	0.000	−0.016	−0.004	+0.018	+0.005
0	11	−5	+0.01059	+0.00241	+0.00049	−0.00033	+0.01108	+0.00208	−0.01184	−0.00362
1	11	−6	+0.001	+0.002	−0.001	0.000	0.000	+0.002	0.000	−0.002
−1	11	−5	+0.057	−0.078	−0.007	+0.005	+0.050	−0.073	−0.067	+0.085
0	11	−6	−0.05602	+0.07047	+0.00084	+0.00298	−0.05518	+0.07345	+0.06550	−0.07710
1	11	−7	+0.008	−0.030	0.000	−0.001	+0.008	−0.031	−0.010	+0.033
−1	11	−6	+0.266	+0.472	−0.012	−0.042	+0.254	+0.430	−0.285	−0.533
0	11	−7	−0.2459	−0.4521	−0.01122	−0.00026	−0.2571	−0.4524	+0.2631	+0.5110
1	11	−8	+0.124	+0.122	+0.001	+0.001	+0.125	+0.123	−0.135	−0.140
−1	11	−7	−2.076	+0.286	+0.153	+0.004	−1.923	+0.290	+2.306	−0.285
0	11	−8	+1.998	−0.2633	+0.00774	−0.02506	+2.006	−0.2884	−2.221	+0.2606
1	11	−9	−0.628	+0.214	−0.004	+0.001	−0.632	+0.215	+0.700	−0.226
−1	11	−8	+1.081	−5.217	−0.108	+0.344	+0.973	−4.871	−1.233	+5.732
0	11	−9	−1.027	+5.007	+0.03033	+0.01796	−0.997	+5.025	+1.173	−5.510
1	11	−10	+0.142	−1.587	+0.002	−0.008	+0.144	−1.595	−0.171	+1.754
−1	11	−9	+6.645	+1.343	−0.412	−0.248	+6.233	+1.095	−7.264	−1.710
0	11	−10	−6.534	−3.462	−0.01132	+0.01530	−6.545	−3.447	+7.147	+3.802
1	11	−11	+2.123	+3.331	+0.010	+0.005	+2.133	+3.336	−2.324	−3.428
−1	11	−10	−2.714	+3.243	+0.154	−0.202	−2.560	+3.041	+2.938	−3.546
0	11	−11	+2.412	−3.395	−0.00101	+0.00503	+2.411	−3.390	−2.609	+3.707
1	11	−12	−0.711	+1.153	−0.001	+0.005	−0.712	+1.158	+0.769	−1.252
−1	11	−11	−0.067	+1.303	+0.004	−0.074	−0.063	+1.229	+0.076	−1.416
0	11	−12	+0.1606	−1.430	+0.00019	−0.00052	+0.1608	−1.431	−0.1760	+1.555
1	11	−13	−0.070	+0.497	0.000	+0.002	−0.070	+0.499	+0.076	−0.541
−1	12	−5	−0.001	−0.013	0.000	+0.002	−0.001	−0.011	0.000	+0.015
0	12	−6	−0.00115	+0.01502	+0.00044	+0.00044	−0.00071	+0.01546	+0.00207	−0.01669
1	12	−7	−0.003	−0.006	0.000	0.000	−0.003	−0.006	+0.003	+0.007
−1	12	−6	+0.107	+0.052	−0.006	−0.006	+0.101	+0.046	−0.116	−0.061
0	12	−7	−0.09298	−0.05255	−0.00244	+0.00129	−0.09542	−0.05126	+0.10147	+0.06083
1	12	−8	+0.033	+0.005	0.000	0.000	+0.033	+0.005	−0.037	−0.007
−1	12	−7	−0.424	+0.361	+0.034	−0.019	−0.390	+0.342	+0.474	−0.389
0	12	−8	+0.4020	−0.3307	−0.00166	−0.00862	+0.4003	−0.3393	−0.4502	+0.3563
1	12	−9	−0.095	+0.133	−0.002	+0.001	−0.097	+0.134	+0.109	−0.145
−1	12	−8	−0.632	−1.759	+0.023	+0.119	−0.609	−1.640	+0.669	+1.935
0	12	−9	+0.5847	+1.676	+0.01815	+0.00133	+0.6028	+1.689	−0.6185	−1.845
1	12	−10	−0.263	−0.482	−0.003	−0.004	−0.266	−0.486	+0.281	+0.534
−1	12	−9	+4.171	−0.089	−0.249	−0.019	+3.922	−0.108	−4.540	+0.064
0	12	−10	−3.974	+0.081	−0.00626	+0.02100	−3.980	+0.102	+4.330	−0.058
1	12	−11	+1.190	−0.126	+0.009	−0.003	+1.199	−0.129	−1.301	+0.128
−1	12	−10	−1.357	+5.235	+0.090	−0.285	−1.267	+4.950	+1.485	−5.656
0	12	−11	+1.218	−4.984	−0.01046	−0.00288	+1.208	−4.987	−1.332	+5.391
1	12	−12	−0.314	+1.444	−0.001	+0.014	−0.315	+1.458	+0.344	−1.567
−1	12	−11	−2.673	−0.881	+0.134	+0.042	−2.539	−0.839	+2.872	+0.940
0	12	−12	+2.587	+0.664	+0.00034	−0.00093	+2.587	+0.663	−2.782	−0.706
1	12	−13	−0.742	−0.166	−0.008	0.000	−0.750	−0.166	+0.801	+0.176

In order to get the value of C we need that of $\overline{\mathrm{T}}$. This can be got by making γ equal to g in the expression for T, taking care to derive the terms involving $\pm 2\gamma$, $\pm 3\gamma$, etc., by means of the process noted at page 74, or more readily by the equation

$$\mathrm{T}=\frac{1}{n}\frac{d}{dt}\left(\frac{d\delta z}{dt}+2\nu\right)$$

For Jupiter we have the following expression:

Arg=$i'g'+ig$	$\overline{\mathrm{T}}$		Arg=$i'g'+ig$	$\overline{\mathrm{T}}$	
	sin.	cos.		sin.	cos.
i' i	″	″	i' i	″	″
0 0		−0.0246	5− 1	−0.0008	−0.0085
0− 1	+ 0.0380	+0.1732	5− 2	+0.0747	+0.0569
0− 2	− 0.0148	−0.0094	5− 3	−0.6815	+0.2266
0− 3	+ 0.0005	−0.0010	5− 4	+0.4213	−2.7627
1+ 3	− 0.0003	−0.0014	5− 5	+3.6051	+2.0622
1+ 2	+ 0.0019	−0.0017	5− 6	+0.5123	−0.0737
1+ 1	+ 0.0011	+0.0281	5− 7	+0.0349	−0.0284
1 0	− 0.0660	−0.3208	5− 8	+0.0018	−0.0036
1− 1	+ 0.4711	+2.4190	6− 1	−0.0007	−0.0004
1− 2	+ 0.2573	−0.2053	6− 2	+0.0127	+0.0002
1− 3	+ 0.0197	−0.0221	6− 3	−0.0702	+0.0997
1− 4	+ 0.0014	−0.0004	6− 4	−0.2997	−0.6185
2+ 1	+ 0.0020	+0.0034	6− 5	+2.0516	−0.0729
2 0	− 0.1181	−0.0140	6− 6	−0.6981	+2.3083
2− 1	+ 2.3998	−0.5665	6− 7	+0.1288	+0.3483
2− 2	−15.0620	+6.4637	6− 8	+0.0296	+0.0223
2− 3	− 0.3040	+0.6788	6− 9	+0.0033	+0.0013
2− 4	+ 0.0073	+0.0396	7− 2	+0.0012	−0.0007
2− 5	+ 0.0004	+0.0017	7− 3	+0.0001	+0.0183
3+ 1	+ 0.0004	+0.0002	7− 4	−0.1169	−0.0658
3 0	− 0.0215	+0.0077	7− 5	+0.4829	−0.3396
3− 1	+ 0.3784	−0.3219	7− 6	+0.3286	+1.3851
3− 2	− 1.6395	+3.4109	7− 7	−1.3668	−0.0836
3− 3	− 6.4194	−9.0498	7− 8	−0.2099	+0.1347
3− 4	− 0.7658	−0.1943	7− 9	−0.0111	+0.0253
3− 5	− 0.0466	+0.0133	7−10	−0.0022	+0.0029
3− 6	− 0.0019	+0.0014	8− 2	0.0000	−0.0004
4 0	− 0.0024	+0.0019	8− 3	+0.0012	+0.0020
4− 1	+ 0.0272	−0.0643	8− 4	−0.0228	+0.0016
4− 2	+ 0.1581	+0.6183	8− 5	+0.0484	−0.1220
4− 3	− 3.3166	−1.0972	8− 6	+0.3348	+0.3331
4− 4	+ 4.6773	−5.1280	8− 7	−0.8581	+0.4017
4− 5	+ 0.0386	−0.6765	8− 8	−0.1352	−0.7512
4− 6	− 0.0233	−0.0439	8− 9	−0.1156	−0.1161
4− 7	− 0.0016	−0.0016	8−10	−0.0201	−0.0041
5 0	− 0.0002	+0.0004	8−11	−0.0016	+0.0016

$\mathrm{Arg}=i'g'+ig$	T		$\mathrm{Arg}=i'g'+ig$	T	
	sin.	cos.		sin	cos.
i' i	″	″	i' i	″	″
9− 3	+0.0004	+0.0001	10−12	+0.0104	−0.0024
9− 4	−0.0029	+0.0021	11− 5	−0.0008	0.0000
9− 5	−0.0052	−0.0249	11− 6	+0.0030	−0.0035
9− 6	+0.1152	+0.0254	11− 7	+0.0111	+0.0206
9− 7	−0.1995	+0.2957	11− 8	−0.0786	+0.0111
9− 8	−0.3698	−0.4862	11− 9	+0.0347	−0.1764
9− 9	+0.3812	−0.1737	11−10	+0.2095	+0.1142
9−10	+0.0555	−0.0880	11−11	−0.0703	+0.0986
9−11	−0.0007	−0.0147	11−12	−0.0045	+0.0386
9−12	−0.0008	−0.0017	12− 6	+0.0002	−0.0008
10− 4	−0.0001	+0.0005	12− 7	+0.0043	+0.0020
10− 5	−0.0028	−0.0031	12− 8	−0.0158	+0.0130
10− 6	+0.0231	−0.0087	12− 9	−0.0200	−0.0567
10− 7	−0.0042	+0.0996	12−10	+0.1224	−0.0016
10− 8	−0.2384	−0.1005	12−11	−0.0345	+0.1382
10− 9	+0.2481	−0.2919	12−12	−0.0652	−0.0172
10−10	+0.1428	+0.1774			
10−11	+0.0614	+0.0209			

The four factors C, D, E, and H for Jupiter have then the expressions

$\mathrm{Arg}=\varkappa\gamma+i'g'+ig$			C		D		E		H	
			sin.	cos.	cos.	sin.	sin.	cos.	cos.	sin.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		+0.0736				−0.111		
1	0−	1	−0.002	−0.082	+0.02	−0.11	−0.04	+0.21	−0.02	+0.14
−1	0	0	+0.074	+0.340	−1.40	−3.19	+0.50	−1.13	+1.18	+2.73
0	0−	1	−0.1488	−0.6692	+0.750	+1.694	−0.720	+1.694	−0.547	−1.273
1	0−	2	+0.076	+0.336	+0.41	+0.93	+0.50	−1.13	−0.45	−1.02
−1	0−	1	−0.028	−0.020	−0.32	+0.24	+0.04	+0.10	+0.43	−0.37
0	0−	2	+0.0534	+0.0138	+0.212	−0.101	−0.106	−0.051	−0.304	+0.225
1	0−	3	−0.026	0.000	+0.06	−0.04	+0.08	+0.02	−0.05	+0.03
−1	0−	2	0.000	−0.002	+0.01	+0.01			−0.04	−0.04
0	0−	3	−0.0012	+0.0044	−0.007	−0.013	+0.002	−0.004	+0.014	+0.025
1	0−	4	−0.002	−0.004						
0	1+	3	−0.0004	−0.0022	−0.002	+0.001				
1	1+	2	0.000	0.000						
−1	1+	3	+0.004	−0.002			0.00	+0.01	−0.02	0.00
0	1+	2	−0.0048	+0.0016	+0.015	+0.022	+0.008	−0.011	−0.022	−0.040
1	1+	1	+0.004	−0.002	−0.02	−0.04	+0.01	+0.02	+0.03	+0.07
−1	1+	2	−0.002	+0.038	+0.16	−0.15	−0.12	−0.11	−0.19	+0.17
0	1+	1	+0.0014	−0.0810	+0.188	−0.153	+0.188	+0.153	−0.230	+0.133
1	1	0	+0.002	+0.056	−0.42	+0.35	−0.18	−0.07	+0.49	−0.35
−1	1+	1	−0.084	−0.410	−1.63	−1.17	+0.76	−0.51	+1.82	+1.30

Arg=$\varkappa\gamma+i'g'+ig$			C		D		E		H	
			sin.	cos.	cos.	sin.	sin.	cos.	cos.	sin.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	1	0	+ 0.1942	+ 0.9314	0.000	0.000	−1.119	+0.777	0.000	0.000
1	1 −	1	− 0.130	− 0.638	+1.63	+1.17	+0.76	−0.51	−1.82	−1.30
−1	1	0	+ 0.964	+ 4.812	+0.32	−0.35	−0.16	−0.09	−0.45	+0.32
0	1 −	1	− 1.9112	− 9.5996	−0.183	+0.178	+0.183	+0.178	+0.299	−0.147
1	1 −	2	+ 0.936	+ 4.802	0.00	+0.09	−0.16	−0.14	−0.07	−0.10
−1	1 −	1	+ 0.518	− 0.410	−3.38	+0.72	+0.80	+0.18	+4.65	−0.99
0	1 −	2	− 1.0996	+ 0.4758	+2.379	−0.513	−1.184	−0.256	−3.630	+0.789
1	1 −	3	+ 0.558	− 0.176	+0.21	−0.04	+0.79	+0.17	+0.20	−0.07
−1	1 −	2	+ 0.038	− 0.044	+0.06	+0.23	−0.04	+0.03	−0.12	−0.33
0	1 −	3	− 0.1196	+ 0.1018	+0.039	−0.167	−0.012	−0.056	−0.047	+0.271
1	1 −	4	+ 0.066	− 0.052	−0.04	−0.03	+0.02	+0.04	+0.06	−0.01
−1	1 −	3	+ 0.006	+ 0.002	0.00	−0.01			−0.02	+0.01
0	1 −	4	− 0.0100	+ 0.0070	−0.005	−0.001	+0.001	0.000	+0.009	+0.001
1	1 −	5	+ 0.006	− 0.004					0.00	0.00
−1	2 +	2	0.000	+ 0.006	+0.03	0.00	−0.02	0.00	−0.04	+0.01
0	2 +	1	− 0.0012	− 0.0102	+0.029	−0.006	+0.029	+0.006	−0.040	+0.002
1	2	0	+ 0.004	+ 0.008	−0.07	0.00	−0.02	+0.01	+0.10	−0.01
−1	2 +	1	− 0.086	− 0.048	−0.20	−0.44	+0.06	−0.19	+0.26	+0.50
0	2	0	+ 0.2336	+ 0.0914	0.000	0.000	−0.081	+0.301	0.000	0.000
1	2 −	1	− 0.238	− 0.028	+0.24	+0.40	0.00	−0.25	−0.28	−0.48
−1	2	0	+ 3.344	− 0.506	−2.44	+2.32	+0.88	+0.83	+1.85	−1.78
0	2 −	1	− 7.4004	+ 1.3272	+1.316	−1.231	−1.316	−1.231	−0.828	+0.823
1	2 −	2	+ 4.784	− 1.130	+0.71	−0.65	+0.87	+0.83	−0.77	+0.68
−1	2 −	1	−30.116	+12.978	−0.64	−0.51	+0.11	−0.15	+0.83	+0.66
0	2 −	2	+59.9446	−25.8674	+0.476	+0.333	−0.238	+0.166	−0.637	−0.487
1	2 −	3	−29.858	+12.860	+0.04	−0.04	+0.17	−0.13	+0.01	+0.12
−1	2 −	2	− 0.608	+ 1.358	+0.01	+2.88	−0.01	+0.51	−0.03	−4.27
0	2 −	3	+ 3.3796	− 3.6526	−0.007	−2.248	+0.002	−0.749	+0.017	+3.622
1	2 −	4	− 2.046	+ 1.978	0.00	+0.13	0.00	+0.50	0.00	−0.58
−1	2 −	3	+ 0.014	+ 0.078	+0.11	+0.05	−0.01	−0.01	−0.18	−0.06
0	2 −	4	+ 0.1200	− 0.3008	−0.074	−0.124	+0.019	−0.031	+0.126	+0.195
1	2 −	5	− 0.094	+ 0.178	−0.01	+0.04	−0.01	+0.03	0.00	−0.08
−1	2 −	4	0.000	+ 0.006						
0	2 −	5	+ 0.0030	− 0.0224	0.000	−0.003			+0.001	+0.004
1	2 −	6	− 0.006	+ 0.012						
−1	3 +	2	0.000	+ 0.002					−0.01	−0.01
0	3 +	1	− 0.0004	− 0.0014	+0.003	+0.001	+0.003	−0.001	−0.005	−0.002
1	3	0	+ 0.002	0.000					+0.01	+0.01
−1	3 +	1	− 0.018	− 0.006	+0.01	−0.09	−0.01	−0.03	−0.01	+0.11
0	3	0	+ 0.0436	− 0.0046	0.000	0.000	+0.013	+0.049	0.000	0.000
1	3 −	1	− 0.042	+ 0.016	+0.01	+0.09	−0.03	−0.03	0.00	−0.11
−1	3	0	+ 0.564	− 0.360	−0.89	+0.23	+0.28	+0.09	+0.80	−0.23
0	3 −	1	− 1.2302	+ 0.8582	+0.432	−0.114	−0.432	−0.114	−0.342	+0.098
1	3 −	2	+ 0.752	− 0.644	+0.27	−0.13	+0.30	+0.04	−0.32	+0.13
−1	3 −	1	− 3.896	+ 5.948	+1.70	+2.76	−0.42	+0.65	−1.17	−1.83
0	3 −	2	+ 7.4394	−12.2874	−1.185	−1.951	+0.592	−0.976	+0.738	+1.140

Arg=$\kappa\gamma+i'g'+ig$			C		D		E		H	
			sin.	cos.	cos.	sin.	sin.	cos.	cos.	sin.
κ	i'	i	″	″	″	″	″	″	″	″
1	3	−3	−3.238	+6.782	−0.08	−0.19	−0.41	+0.64	+0.16	+0.33
−1	3	−2	−12.898	−18.096	−0.69	+0.73	+0.13	+0.10	+0.90	−1.00
0	3	−3	+26.0246	+35.7348	+0.492	−0.636	−0.164	−0.212	−0.725	+0.847
1	3	−4	−12.980	−17.752	−0.07	+0.05	+0.13	+0.15	+0.18	−0.08
−1	3	−3	−1.538	−0.388	+2.12	+0.43	−0.30	+0.05	−3.31	−0.66
0	3	−4	+4.0086	+2.0598	−1.748	−0.379	+0.437	−0.095	+2.903	+0.618
1	3	−5	−2.156	−1.240	+0.23	+0.04	−0.29	+0.06	−0.58	−0.14
−1	3	−4	−0.096	+0.028	+0.09	0.00	0.00	0.00	−0.13	+0.03
0	3	−5	+0.3478	+0.0278	−0.141	−0.012	+0.028	−.0002	+0.232	+0.017
1	3	−6	−0.202	−0.038	+0.03	+0.02	−0.02	0.00	−0.07	−0.03
−1	3	−5	−0.002	+0.004	−0.01	0.00			0.00	0.00
0	3	−6	+0.0258	−0.0048	−0.007	−0.004			+0.011	+0.006
1	3	−7	−0.014	0.000					−0.01	0.00
−1	4	+1	−0.004	0.000					0.00	+0.01
0	4	0	+0.0052	−0.0032	0.000	0.000	+0.005	+0.006	0.000	0.000
1	4	−1	−0.004	+0.006					0.00	−0.01
−1	4	0	+0.056	−0.076	−0.17	−0.03	+0.05	−0.01	+0.18	+0.05
0	4	−1	−0.1076	+0.1782	+0.075	+0.019	−0.075	+0.019	−0.069	−0.016
1	4	−2	+0.054	−0.128	+0.07	−0.01	+0.05	−0.03	−0.08	−0.01
−1	4	−1	+0.020	+1.100	+0.07	+1.13	−0.02	+0.26	−0.08	−0.90
0	4	−2	−0.1892	−2.2696	−0.044	−0.767	+0.022	−0.384	+0.04300	+0.55060
1	4	−3	+0.320	+1.230	−0.07	−0.09	+0.02	+0.27	+0.05	+0.14
−1	4	−2	−6.174	−2.688	+2.44	−0.87	−0.43	−0.17	−1.47	+0.57
0	4	−3	+12.5690	+5.0446	−1.924	+0.663	+0.641	+0.221	+1.039	−0.394
1	4	−4	−6.612	−2.132	+0.11	−0.05	−0.42	−0.16	+0.11	0.00
−1	4	−3	+9.350	−10.310	+0.67	+0.78	−0.07	+0.11	−0.88	−1.05
0	4	−4	−18.2386	+20.7634	−0.611	−0.611	+0.153	−0.153	+0.801	+0.895
1	4	−5	+9.026	−10.346	+0.07	+0.10	−0.11	+0.11	−0.12	−0.23
−1	4	−4	+0.074	−1.354	+0.63	−1.40	−0.06	−0.17	−0.98	+2.24
0	4	−5	−0.8048	+3.4620	−0.551	+1.178	+0.110	+0.236	+0.918	−2.003
1	4	−6	+0.510	−1.852	+0.09	−0.20	−0.07	−0.16	−0.22	+0.46
−1	4	−5	−0.048	−0.088	+0.06	−0.09	−0.01	0.00	−0.07	+0.13
0	4	−6	+0.0574	+0.3134	−0.066	+0.115	+0.011	+0.019	+0.111	−0.194
1	4	−7	−0.022	−0.180	+0.02	−0.03	−0.01	−0.01	−0.04	+0.07
−1	4	−6	−0.004	−0.006	+0.01	0.00			−0.01	0.00
0	4	−7	+0.0134	+0.0234	−0.008	+0.006			+0.013	−0.010
1	4	−8	−0.006	−0.014					−0.01	+0.01
0	5	0	+0.0004	−0.0004	0.000	0.000			0.000	0.000
−1	5	0	+0.002	−0.012	−0.02	−0.02	+0.01	−0.01	+0.03	+0.03
0	5	−1	−0.0026	+0.0239	+0.008	+0.008	−0.008	+0.008	−0.009	−0.008
1	5	−2	−0.004	−0.018	0.00	0.00	+0.01	−0.01	−0.01	−0.01
−1	5	−1	+0.086	+0.124	−0.12	+0.23	+0.02	+0.05	+0.11	−0.21
0	5	−2	−0.2042	−0.2405	+0.070	−0.147	−0.035	−0.074	−0.054	+0.121
1	5	−3	+0.148	+0.114	−0.01	−0.04	+0.03	+0.05	−0.01	+0.05
−1	5	−2	−1.302	+0.196	+1.16	+0.18	−0.19	+0.03	−0.84	−0.11
0	5	−3	+2.6286	−0.5290	−0.872	−0.130	+0.291	−0.043	+0.584	+0.074

Arg=$\varkappa\gamma+i'g'+ig$			C		D		E		H	
			sin.	cos.	cos.	sin.	sin.	cos.	cos.	sin.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
1	5	—4	—1.354	+0.460	+0.01	+0.05	—0.19	+0.05	+0.06	—0.01
—1	5	—3	+1.190	—5.320	—0.23	—1.84	+0.04	—0.26	+0.16	+1.00
0	5	—4	—2.1118	+10.7212	+0.168	+1.533	—0.042	+0.383	—0.100	—0.747
1	5	—5	+0.776	—5.496	—0.04	—0.19	+0.04	—0.26	+0.03	—0.02
—1	5	—4	+7.256	+4.114	+0.76	—0.48	—0.09	—0.04	—1.06	+0.63
0	5	—5	—14.5522	—7.8410	—0.650	+0.467	+0.130	+0.093	+0.940	—0.596
1	5	—6	+7.238	+3.858	+0.13	—0.07	—0.09	—0.06	—0.23	+0.12
—1	5	—5	+1.030	—0.152	—0.83	—0.60	+0.09	—0.06	+1.35	+0.99
0	5	—6	—2.5884	+0.0268	+0.698	+0.553	—0.116	+0.092	—1.214	—0.935
1	5	—7	+1.376	+0.034	—0.13	—0.12	+0.08	—0.06	+0.29	+0.22
—1	5	—6	+0.068	—0.058	—0.06	—0.08	+0.01	—0.01	+0.09	+0.11
0	5	—7	—0.2384	+0.1118	+0.074	+0.087	—0.011	+0.012	—0.127	—0.149
1	5	—8	+0.136	—0.054	—0.02	—0.02	+0.01	—0.01	+0.05	+0.06
—1	5	—7	+0.006	—0.004					—0.01	0.00
0	5	—8	—0.0156	+0.0146	+0.003	+0.010			—0.006	—0.017
1	5	—9	+0.010	—0.006					0.00	+0.01
—1	6	0	—0.002	—0.002						
0	6	—1	+0.0006	+0.0030						
1	6	—2	—0.002	—0.002						
—1	6	—1	+0.018	+0.006			+0.01	+0.01	+0.04	—0.02
0	6	—2	—0.0394	—0.0102	+0.023	—0.016	—0.011	—0.008	—0.020	+0.015
1	6	—3	+0.026	+0.002	—0.02	0.00	+0.01	+0.01	—0.02	0.00
—1	6	—2	—0.162	+0.142	+0.24	+0.18	—0.04	+0.03	—0.20	—0.14
0	6	—3	+0.3088	—0.3084	—0.179	—0.135	+0.060	—0.045	+0.137	+0.098
1	6	—4	—0.140	+0.198	—0.01	+0.02	—0.04	+0.04	+0.02	—0.02
—1	6	—3	—0.402	—1.230	+0.35	—0.97	—0.05	—0.13	—0.22	+0.65
0	6	—4	+0.9184	+2.4530	—0.291	+0.781	+0.073	+0.195	+0.172	—0.495
1	6	—5	—0.606	—1.224	+0.05	—0.07	—0.06	—0.13	—0.02	+0.01
—1	6	—4	+4.034	+0.080	—1.21	—0.12	+0.14	—0.01	+0.56	+0.06
0	6	—5	—8.0618	+0.0418	+1.048	+0.131	—0.210	+0.026	—0.437	—0.056
1	6	—6	+4.072	—0.204	—0.19	—0.02	+0.14	—0.02	+0.01	—0.02
—1	6	—5	—1.382	+4.646	—0.28	—0.69	+0.02	—0.07	+0.37	+0.95
0	6	—6	+2.4796	—9.2720	+0.280	+0.611	—0.047	+0.102	—0.347	—0.867
1	6	—7	—1.200	+4.598	—0.05	—0.13	+0.03	—0.07	+0.05	+0.23
—1	6	—6	+0.256	+0.696	—0.49	+0.42	+0.04	+0.03	+0.83	—0.71
0	6	—7	—0.4288	—1.7272	+0.460	—0.358	—0.066	—0.051	—0.789	+0.640
1	6	—8	+0.200	+0.918	—0.12	+0.08	+0.04	+0.03	+0.21	—0.17
—1	6	—7	+0.058	+0.050	—0.07	+0.02	+0.01	0.00	+0.13	—0.04
0	6	—8	—0.1282	—0.1596	+0.084	—0.035	—0.011	—0.004	—0.145	+0.062
1	6	—9	+0.066	+0.090	—0.03	+0.01	+0.01	0.00	+0.05	—0.02
—1	6	—8	+0.004	0.000	—0.01	0.00			0.00	0.00
0	6	—9	—0.0154	—0.0084	+0.010	0.000			—0.016	+0.001
1	6	—10	+0.006	+0.006					+0.01	0.00
—1	7	—1	0.000	0.000						
0	7	—2	—0.0048	+0.0022	+0.004	0.000			—0.004	+0.001
1	7	—3	+0.004	—0.002						

Arg=$\varkappa\gamma+i'g'+ig$			C		D		E		H	
			sin.	cos.	cos.	sin.	sin.	cos.	cos.	sin.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	7	−2	−0.006	+0.030	+0.02	+0.06	0.00	+0.01	−0.01	−0.05
0	7	−3	+0.0122	−0.0626	−0.018	−0.041	+0.006	−0.014	+0.016	+0.033
1	7	−4	0.000	+0.036	−0.01	+0.01	0.00	+0.01	−0.01	+0.01
−1	7	−3	−0.186	−0.156	+0.23	−0.20	−0.03	−0.03	−0.18	+0.16
0	7	−4	+0.3950	+0.2994	−0.187	+0.165	+0.047	+0.041	+0.130	−0.120
1	7	−5	−0.232	−0.128	+0.03	0.00	−0.03	−0.03	−0.01	−0.01
−1	7	−4	+0.988	−0.548	−0.70	−0.43	+0.08	−0.05	+0.44	+0.28
0	7	−5	−1.9558	+1.1676	+0.590	+0.378	−0.118	+0.076	−0.351	−0.223
1	7	−6	+0.954	−0.686	−0.08	−0.08	+0.08	−0.05	+0.02	+0.03
−1	7	−5	+0.524	+2.760	−0.25	+0.71	+0.03	+0.07	+0.11	−0.27
0	7	−6	−1.1840	−5.4808	+0.244	−0.626	−0.041	−0.104	−0.096	+0.196
1	7	−7	+0.700	+2.736	−0.03	+0.14	+0.03	+0.07	−0.02	+0.01
−1	7	−6	−2.750	−0.154	−0.56	+0.11	+0.05	+0.01	+0.78	−0.12
0	7	−7	+5.4450	+0.1170	+0.508	−0.128	−0.073	−0.018	−0.722	+0.126
1	7	−8	−2.698	−0.034	−0.12	+0.04	+0.05	+0.01	+0.19	−0.01
−1	7	−7	−0.426	+0.272	+0.17	+0.35	−0.01	+0.03	−0.32	−0.61
0	7	−8	+1.0430	−0.5432	−0.147	−0.338	+0.018	−0.042	+0.277	+0.587
1	7	−9	−0.552	+0.272	+0.05	+0.10	−0.01	+0.03	−0.06	−0.16
−1	7	−8	−0.026	+0.050	+0.02	+0.09			0.00	−0.11
0	7	−9	+0.0904	−0.1212	−0.008	−0.068	+0.001	−0.008	+0.015	+0.119
1	7	−10	−0.054	+0.064	−0.01	0.00			−0.01	−0.03
−1	7	−9	−0.002	+0.006	0.00	+0.01			0.00	−0.01
0	7	−10	+0.0107	−0.0149	+0.002	−0.008			−0.003	+0.014
1	7	−11	−0.008	+0.008					0.00	−0.01
−1	8	−2	0.000	+0.002						
0	8	−3	−0.0040	−0.0082	+0.001	−0.008				
1	8	−4	+0.004	+0.004	0.00	+0.01				
−1	8	−3	−0.040	−0.008	+0.07	−0.02	−0.01	0.00	−0.06	+0.01
0	8	−4	+0.0810	+0.0054	−0.055	+0.012	+0.014	+0.003	+0.042	−0.011
1	8	−5	−0.044	+0.004	+0.01	+0.01	−0.01	0.00	0.00	0.00
−1	8	−4	+0.124	−0.210	−0.13	−0.26	+0.02	−0.03	+0.10	+0.18
0	8	−5	−0.2305	+0.4640	+0.120	+0.211	−0.024	+0.042	−0.084	−0.140
1	8	−6	+0.096	−0.242	−0.01	−0.04	+0.02	−0.03	+0.01	+0.01
−1	8	−5	+0.586	+0.698	−0.44	+0.44	+0.04	+0.04	+0.26	−0.27
0	8	−6	−1.2210	−1.3648	+0.392	−0.379	−0.065	−0.063	−0.211	+0.217
1	8	−7	+0.674	+0.654	−0.09	+0.06	+0.04	+0.04	+0.02	−0.02
−1	8	−6	−1.728	+0.728	+0.38	+0.24	−0.03	+0.02	−0.07	−0.08
0	8	−7	+3.4034	−1.5446	−0.322	−0.239	+0.046	−0.034	+0.044	+0.072
1	8	−8	−1.682	+0.834	+0.06	+0.08	−0.03	+0.02	0.00	+0.02
−1	8	−7	−0.282	−1.512	+0.01	−0.09	0.00	−0.03	−0.01	−0.59
0	8	−8	+0.6828	+2.9618	−0.017	+0.388	+0.002	+0.049	−0.010	+0.541
1	8	−9	−0.352	−1.468	−0.01	−0.43	0.00	−0.03	+0.05	−0.13
−1	8	−8	−0.236	−0.234	+0.22	−0.04	−0.02	0.00	−0.41	+0.09
0	8	−9	+0.4964	+0.5744	−0.224	+0.034	+0.025	+0.004	+0.396	−0.077
1	8	−10	−0.250	−0.302	+0.08	+0.02	−0.02	0.00	−0.11	+0.04
−1	8	−9	−0.040	−0.010	+0.02	+0.01			−0.10	0.00

Arg=$\varkappa\gamma+i'g'+ig$			C		D		E		H	
			sin.	cos.	cos.	sin.	sin.	cos.	cos.	sin.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	8	−10	+0.0992	+0.0432	−0.049	−0.008	+0.005	−0.001	+0.086	+0.012
1	8	−11	−0.052	−0.028					0.00	−0.02
−1	8	−10	−0.004	+0.002					−0.01	0.00
0	8	−11	+0.0128	+0.0022	−0.006	−0.003				
1	8	−12	−0.002	0.000					−0.01	0.00
−1	9	−3	−0.004	+0.002						
0	9	−4	+0.0104	−0.0060	−0.010	−0.003			+0.009	+0.002
1	9	−5	−0.004	+0.002	+0.01	0.00				
−1	9	−4	+0.002	−0.044	+0.01	−0.07	0.00	−0.01	0.00	+0.06
0	9	−5	+0.0062	+0.0898	+0.001	+0.062	0.000	+0.012	−0.002	−0.045
1	9	−6	−0.012	−0.048	0.00	−0.01	0.00	−0.01	−0.01	0.00
−1	9	−5	+0.208	+0.074	−0.24	+0.06	+0.02	+0.01	+0.17	−0.05
0	9	−6	−0.4274	−0.1358	+0.205	−0.065	−0.034	−0.011	−0.130	+0.045
1	9	−7	+0.230	+0.048	−0.05	+0.01	+0.02	+0.01	+0.01	0.00
−1	9	−6	−0.432	+0.540	+0.24	+0.38	−0.02	+0.03	−0.14	−0.22
0	9	−7	+0.8322	−1.1118	−0.211	−0.336	+0.030	−0.048	+0.111	+0.173
1	9	−8	−0.386	+0.590	+0.04	+0.07	−0.02	+0.03	−0.01	−0.02
−1	9	−7	−0.698	−0.988	+0.19	−0.15	−0.02	−0.01	−0.01	0.00
0	9	−8	+1.4468	+1.9300	−0.182	+0.134	+0.023	+0.017	+0.029	+0.030
1	9	−9	−0.756	−0.946	+0.03	−0.04	−0.02	−0.01	−0.01	−0.03
−1	9	−8	+0.768	−0.356	+0.22	−0.06	−0.02	0.00	−0.39	−0.08
0	9	−9	−1.4796	+0.7750	−0.273	−0.045	+0.030	−0.005	+0.375	+0.087
1	9	−10	+0.724	−0.392	+0.13	+0.14	−0.02	0.00	−0.11	−0.05
−1	9	−9	+0.112	−0.180	+0.06	−0.08	0.00	−0.01	+0.04	+0.24
0	9	−10	−0.2776	+0.3871	−0.017	+0.135	+0.002	+0.014	+0.016	−0.244
1	9	−11	+0.146	−0.196	−0.05	−0.10	0.00	−0.01	−0.04	+0.08
−1	9	−10	+0.002	−0.030	+0.03	−0.01			−0.01	+0.03
0	9	−11	−0.0134	+0.0732	−0.014	+0.031	+0.001	+0.003	+0.024	−0.056
1	9	−12	+0.008	−0.038	−0.01	−0.02			−0.03	+0.04
−1	9	−11	0.000	−0.002						
0	9	−12	+0.0018	+0.0075	−0.004	+0.004				
1	9	−13	0.000	−0.004						
−1	10	−3	−0.002	+0.002						
0	10	−4	+0.0010	−0.0016						
1	10	−5	0.000	0.000						
−1	10	−4	−0.006	−0.008					0.00	+0.01
0	10	−5	+0.0092	+0.0118	−0.006	+0.012	+0.001	+0.002		
1	10	−6	−0.006	−0.004	0.00	−0.01			0.00	+0.01
−1	10	−5	+0.044	−0.008	−0.07	−0.02	+0.01	0.00	+0.06	+0.01
0	10	−6	−0.0896	+0.0212	+0.060	+0.011	−0.010	+0.002	−0.042	−0.007
1	10	−7	+0.044	−0.016	−0.01	−0.01	+0.01	0.00	0.00	−0.01
−1	10	−6	−0.030	+0.186	+0.01	+0.20	0.00	+0.02	−0.03	−0.14
0	10	−7	+0.0456	−0.3762	−0.017	−0.177	+0.002	−0.025	+0.013	+0.108
1	10	−8	−0.008	+0.196	0.00	+0.04	0.00	+0.02	+0.02	0.00
−1	10	−7	−0.450	−0.226	+0.29	−0.10	−0.02	−0.01	−0.02	+0.10
0	10	−8	+0.9116	+0.4262	−0.261	+0.090	+0.033	+0.011	+0.125	−0.042

Arg$=\kappa\gamma+i'g'+ig$			C		D		E		H	
			sin.	cos.	cos.	sin.	sin.	cos.	cos.	sin.
κ	i'	i	″	″	″	″	″	″	″	″
1	10	−9	−0.476	−0.190	+0.05	−0.03	−0.02	−0.01	−0.14	−0.04
−1	10	−8	+0.510	−0.568	−0.12	−0.18	0.00	−0.01	−0.16	−0.03
0	10	−9	−0.9846	+1.1566	+0.034	+0.116	−0.004	+0.013	+0.052	+0.009
1	10	−10	+0.474	−0.592	+0.07	+0.03	0.00	−0.01	+0.11	0.00
−1	10	−9	+0.292	+0.354	+0.08	−0.11	0.00	−0.01	+0.01	+0.19
0	10	−10	−0.6190	−0.6598	−0.071	+0.176	+0.007	+0.018	+0.114	−0.239
1	10	−11	+0.310	+0.324	+0.02	−0.13	0.00	−0.01	−0.18	+0.13
−1	10	−10	+0.126	+0.042	−0.04	+0.02			+0.06	+0.07
0	10	−11	−0.2710	−0.1106	+0.074	+0.032	−0.007	+0.003	−0.137	−0.048
1	10	−12	+0.138	+0.058	−0.06	−0.05			+0.12	−0.02
−1	10	−11	+0.020	−0.004	0.00	0.00			+0.01	+0.03
0	10	−12	−0.0488	+0.0026	+0.018	+0.014			−0.033	−0.025
1	10	−13	+0.024	−0.002	−0.02	−0.02			+0.02	+0.01
−1	11	−4	0.000	−0.002						
0	11	−5	+0.0020	+0.0012						
1	11	−6	−0.004	0.000						
−1	11	−5	+0.006	−0.004					+0.01	0.00
0	11	−6	−0.0114	+0.0122	+0.011	+0.009				
1	11	−7	+0.004	−0.008	−0.01	−0.01			+0.01	0.00
−1	11	−6	+0.014	+0.038	−0.03	−0.01			+0.01	−0.05
0	11	−7	−0.0346	−0.0772	+0.021	+0.052	−0.003	+0.007	−0.014	+0.035
1	11	−8	+0.024	+0.038	0.00	−0.05			+0.01	+0.01
−1	11	−7	−0.154	+0.006	+0.16	+0.01	−0.01	0.00	−0.01	+0.02
0	11	−8	+0.3044	−0.0226	−0.138	−0.016	+0.017	−0.002	+0.081	+0.008
1	11	−9	−0.152	+0.026	+0.02	+0.02	−0.01	0.00	−0.09	−0.04
−1	11	−8	+0.088	−0.342	−0.12	−0.11	0.00	−0.01	−0.08	+0.07
0	11	−9	−0.1614	+0.6888	+0.019	+0.184	−0.002	+0.020	−0.005	−0.079
1	11	−10	+0.062	−0.350	+0.09	−0.13	0.00	−0.01	+0.08	+0.03
−1	11	−9	+0.414	+0.238	−0.01	−0.08			+0.01	−0.01
0	11	−10	−0.8298	−0.4210	+0.062	+0.009	−0.006	+0.001	+0.032	−0.047
1	11	−11	+0.422	+0.204	−0.07	+0.07			−0.05	+0.07
−1	11	−10	−0.140	+0.202	−0.05	−0.02			+0.08	+0.12
0	11	−11	+0.2514	−0.4176	+0.104	+0.070	−0.009	+0.006	−0.139	−0.110
1	11	−12	−0.118	+0.208	−0.09	−0.08			+0.10	+0.02
−1	11	−11	−0.010	+0.078	−0.04	+0.03			0.00	−0.05
0	11	−12	+0.0222	−0.1746	+0.031	−0.033			−0.046	+0.071
1	11	−13	−0.012	+0.092	0.00	+0.01			+0.06	−0.05
−1	12	−5	+0.002	0.000						
0	12	−6	−0.0006	+0.0026						
1	12	−7	0.000	−0.002						
−1	12	−6	+0.006	+0.006					+0.01	0.00
0	12	−7	−0.0132	−0.0100	+0.011	−0.009				
1	12	−8	+0.008	+0.004	−0.01	+0.01			+0.01	0.00
−1	12	−7	−0.032	+0.018	+0.05	+0.02			0.00	−0.01
0	12	−8	+0.0614	−0.0424	−0.043	−0.028	+0.005	−0.004	+0.026	+0.017
1	12	−9	−0.032	+0.026	+0.01	+0.02			−0.04	−0.01
−1	12	−8	−0.028	−0.114	−0.01	−0.03	0.00	−0.01	−0.03	+0.07

$\mathrm{Arg}=\varkappa\gamma+i'g'+ig$			C		D		E		H	
			sin.	cos.	cos.	sin.	sin.	cos.	cos.	sin.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	12	−9	+0.0639	+0.2280	−0.035	+0.110	+0.004	+0.012	+0.019	−0.055
1	12	−10	−0.042	−0.112	+0.06	−0.11	0.00	−0.01	+0.01	+0.01
−1	12	−9	+0.240	+0.012	−0.12	−0.06	+0.01	0.00	+0.01	−0.04
0	12	−10	−0.4802	−0.0074	+0.150	+0.010	−0.015	+0.001	−0.043	−0.010
1	12	−11	+0.240	−0.010	−0.08	+0.04	+0.01	0.00	+0.05	+0.06
−1	12	−10	−0.076	+0.272	+0.04	+0.03			+0.05	+0.05
0	12	−11	+0.1388	−0.5438	−0.005	−0.077	0.000	−0.007	−0.034	−0.041
1	12	−12	−0.062	+0.274	−0.03	+0.07			−0.01	+0.01
−1	12	−11	−0.130	−0.034	0.00	0.00			+0.01	−0.03
0	12	−12	+0.2610	+0.0470	+0.011	−0.017			−0.083	+0.060
1	12	−13	−0.134	−0.020	−0.02	+0.02			+0.11	−0.05

All the factors belonging to Jupiter having now been given, the similar quantities for Saturn are tabulated:

$\mathrm{Arg}=\varkappa\gamma'+i'g'+ig$			V′		X′		B′		G′	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0				−1.2588		−1.2588		
−1	1	0	+867.04	+2.74	+516.84	+1.90	+1383.88	+4.64	−350.09	−1.67
−1	2	0	+115.60	−126.41	+60.57	−33.59	+176.17	−160.00	−57.02	+83.22
0	1	0	−57.3392	+73.0254	−43.5173	−0.3053	−100.8565	+72.7201	+19.8580	−44.4869
1	0	0	−39.05	+29.64	−6.5936	+14.9534	−45.64	+44.59	+30.42	−24.29
−1	3	0	+2.12	−27.17	+4.03	−6.87	+6.15	−34.04	+0.90	+18.78
0	2	0	−0.1210	+18.5562	−5.1065	+2.8158	−5.2275	+21.3720	−2.1289	−12.0027
1	1	0	−3.03	+5.23	+0.10	+1.72	−2.93	+6.95	+2.22	−4.49
−1	4	0	−1.92	−3.36	−0.02	−0.88	−1.94	−4.24	+1.70	+2.35
0	3	0	+1.5755	+2.4063	−0.3401	+0.5783	+1.2354	+2.9846	−1.3851	−1.5718
1	2	0	−0.07	+0.74	+0.11	+0.14	+0.04	+0.88	+0.01	−0.64
−1	5	0	−0.46	−0.23	−0.05	−0.08	−0.51	−0.31	+0.38	+0.15
0	4	0	+0.3615	+0.1704	+0.0007	+0.0736	+0.3622	+0.2440	−0.2937	−0.0991
1	3	0	+0.04	+0.09	+0.02	+0.01	+0.06	+0.10	−0.04	−0.08
−1	6	0	−0.03	0.00			−0.03	0.00	+0.02	0.00
0	5	0	+0.0520	−0.0045	+0.0048	+0.0068	+0.0568	+0.0023	−0.0416	+0.0076
1	4	0	−0.01	+0.02			−0.01	+0.02	+0.01	−0.02
0	6	0	+0.0053	−0.0032	+0.0009	+0.0005	+0.0062	−0.0027	−0.0042	+0.0030
0	−4	−1	−0.0048	−0.0132	−0.0024	−0.0019	−0.0072	−0.0151	+0.0038	+0.0114
−1	−2	−1	−0.06	−0.03	−0.01	−0.01	−0.07	−0.04	+0.05	+0.03
0	−3	−1	−0.0732	−0.0659	−0.0177	+0.0048	−0.0909	−0.0611	+0.0598	+0.0564
1	−4	−1	+0.15	+0.10	+0.02	+0.01	+0.17	+0.11	−0.13	−0.09
−1	−1	−1	−0.51	−0.03	−0.08	−0.01	−0.59	−0.04	+0.43	+0.02
0	−2	−1	−0.5963	−0.1559	−0.1161	+0.1176	−0.7124	−0.0383	+0.4790	+0.1294
1	−3	−1	+1.25	+0.18	+0.22	−0.06	+1.47	+0.12	−1.03	−0.16

Arg= $\varkappa\gamma'+i'g'+ig$	V′		X′		B′		G′	
	sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$ i' i	″	″	″	″	″	″	″	″
—1 0— 1	— 4.11	+ 2.80	— 0.95	— 0.76	— 5.06	+ 2.04	+ 3.12	— 3.75
0—1— 1	— 2.7934	+ 0.9512	— 0.3836	+ 1.4648	— 3.1770	+ 2.4160	+ 2.0982	— 1.1175
1—2— 1	+ 7.94	— 3.48	+ 1.38	— 1.39	+ 9.32	— 4.87	— 6.34	+ 3.15
—1 1— 1	— 47.620	— 42.884	— 9.356	— 4.845	— 56.976	— 47.729	+ 49.88	+ 93.47
0 0— 1	0.0000	0.0000	+ 25.6875	+ 122.2131	+ 25.6875	+ 122.2131	0.0000	0.0000
1—1— 1	+ 25.87	— 60.65	+ 4.62	— 17.37	+ 30.49	— 78.02	— 15.75	+ 69.20
—1 2— 1	+ 392.406	+1931.364	+ 79.204	+ 393.671	+ 471.610	+2325.035	—423.03	—2060.97
0 1— 1	— 575.7289	—2765.1410	+ 1.9892	+ 1.4766	— 573.7397	—2763.6644	+907.5409	+4344.4247
1 0— 1	+ 368.35	+1751.57	—304.19	—1450.95	+ 64.16	+ 300.62	—782.26	—3729.47
—1 3— 1	+ 263.90	+ 253.95	+ 55.69	+ 66.67	+ 319.59	+ 320.62	—201.00	— 226.40
0 2— 1	— 215.5265	— 260.4742	— 6.4830	— 33.1287	— 222.0095	— 293.6029	+177.9390	+ 301.0658
1 1— 1	+ 0.507	— 9.840	— 14.273	— 12.695	— 13.766	— 22.535	— 1.29	— 12.77
—1 4— 1	+ 58.23	+ 4.33	+ 12.42	+ 4.56	+ 70.65	+ 8.89	— 43.03	— 3.75
0 3— 1	— 48.3766	— 6.7501	— 4.6698	— 5.6234	— 53.0464	— 12.3735	+ 35.8158	+ 9.3200
1 2— 1	— 1.773	— 4.097	— 2.193	— 0.148	— 3.966	— 4.245	+ 2.01	+ 2.31
—1 5— 1	+ 7.27	— 4.86	+ 1.72	— 0.27	+ 8.99	— 5.13	— 5.21	+ 3.96
0 4— 1	— 6.1730	+ 4.0111	— 1.0443	— 0.3889	— 7.2173	+ 3.6222	+ 4.3387	— 3.0443
1 3— 1	— 0.65	— 0.56	— 0.20	+ 0.13	— 0.85	— 0.43	+ 0.62	+ 0.35
—1 6— 1	+ 0.43	— 1.16	+ 0.15	— 0.15	+ 0.58	— 1.31	— 0.25	+ 0.94
0 5— 1	— 0.3672	+ 1.0038	— 0.1449	+ 0.0258	— 0.5121	+ 1.0296	+ 0.2024	— 0.7907
1 4— 1	— 0.12	— 0.03	— 0.02	+ 0.03	— 0.14	0.00	+ 0.11	+ 0.01
—1 7— 1	— 0.02	— 0.17	0.00	— 0.02	— 0.02	— 0.19	+ 0.03	+ 0.14
0 6— 1	+ 0.0358	+ 0.1420	— 0.0134	+ 0.0130	+ 0.0224	+ 0.1550	— 0.0413	— 0.1107
1 5— 1	— 0.04	+ 0.02	0.00	— 0.01	— 0.04	+ 0.01	+ 0.04	— 0.02
0 7— 1	+ 0.0132	+ 0.0136	— 0.0006	+ 0.0022	+ 0.0126	+ 0.0158	— 0.0120	— 0.0102
—1—1— 2	+ 0.01	— 0.03			+ 0.01	— 0.03	— 0.01	+ 0.02
0—2— 2	+ 0.0061	— 0.0256	— 0.0042	— 0.0041	+ 0.0019	— 0.0297	— 0.0061	+ 0.0205
1—3— 2	— 0.01	+ 0.07			— 0.01	+ 0.07	+ 0.01	— 0.06
—1—0— 2	— 0.02	— 0.21	— 0.02	— 0.10	— 0.04	— 0.31	— 0.02	+ 0.07
0—1— 2	— 0.0272	— 0.0724	— 0.0512	— 0.0074	— 0.0784	— 0.0798	+ 0.0153	+ 0.0065
1—2— 2	+ 0.11	+ 0.43	+ 0.06	+ 0.03	+ 0.17	+ 0.46	— 0.08	— 0.33
—1 1— 2	— 2.97	— 7.89	— 0.31	— 0.48	— 3.28	— 8.37	+ 3.70	+ 12.62
0 0— 2	0.0000	0.0000	+ 1.7529	+ 10.7271	+ 1.7529	+ 10.7271	0.0000	0.0000
1—1— 2	+ 1.69	+ 0.53	+ 0.61	+ 0.09	+ 2.30	+ 0.62	— 0.99	+ 1.50
—1 2— 2	+ 40.556	+ 46.705	+ 11.04	— 0.28	+ 51.60	+ 46.42	— 40.390	— 107.716
0 1— 2	— 45.2569	— 191.3331	— 5.2535	+ 2.3768	— 50.5104	— 188.9563	+ 80.6048	+ 373.2938
1 0— 2	+ 54.46	+ 193.37	— 20.80	— 127.32	+ 33.66	+ 66.05	— 91.41	— 379.50
—1 3— 2	—1173.40	+ 502.97	—244.49	+ 104.88	—1417.89	+ 607.85	+835.77	— 363.37
0 2— 2	+ 923.4211	— 410.3923	— 1.5913	+ 1.0079	+ 921.8298	— 409.3844	—645.2088	+ 300.2177
1 1— 2	— 54.88	+ 26.72	+ 62.70	— 27.74	+ 7.82	— 1.02	+ 24.49	— 15.18
—1 4— 2	— 206.893	+ 353.200	— 50.245	+ 62.573	— 257.138	+ 415.773	+141.37	— 267.87
0 3— 2	+ 176.6016	— 299.2082	+ 20.5852	— 8.6506	+ 197.1868	— 307.8588	—118.0527	+ 224.6945
1 2— 2	+ 5.389	+ 28.123	+ 7.86	— 11.68	+ 13.25	+ 16.44	— 7.524	— 18.749
—1 5— 2	+ 8.777	+ 85.869	— 2.580	+ 15.294	+ 6.197	+ 101.163	— 11.29	— 65.49
0 4— 2	— 7.2696	— 75.7210	+ 4.2442	— 5.2484	— 3.0254	— 80.9694	+ 9.8493	+ 57.1032
1 3— 2	+ 6.31	+ 4.03	— 0.04	— 2.12	+ 6.27	+ 1.91	— 5.35	— 2.33
—1 6— 2	+ 9.93	+ 11.31	+ 0.84	+ 2.29	+ 10.77	+ 13.60	— 8.62	— 8.37

Arg= $\varkappa\gamma'+i'g'+ig$	V′		X′		B′		G′	
	sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$ i' i	″	″	″	″	″	″	″	″
0 5— 2	− 8.90733	− 10.25910	+ 0.2205	− 1.2863	− 8.6868	− 11.5454	+ 7.73073	+ 7.48003
1 4— 2	+ 1.445	− 0.202	− 0.171	− 0.231	+ 1.274	− 0.433	− 1.15	+ 0.32
—1 7— 2	+ 2.28	+ 0.58	+ 0.28	+ 0.22	+ 2.56	+ 0.80	− 1.91	− 0.32
0 6— 2	− 2.08643	− 0.53294	− 0.0715	− 0.1932	− 2.1579	− 0.7261	+ 1.73800	+ 0.28121
1 5— 2	+ 0.166	− 0.177	− 0.035	− 0.013	+ 0.131	− 0.190	− 0.13	+ 0.17
—1 8— 2	+ 0.30	− 0.13	+ 0.04	− 0.01	+ 0.34	− 0.14	− 0.24	+ 0.13
0 7— 2	− 0.2969	+ 0.1057	− 0.0227	− 0.0183	− 0.3196	+ 0.0874	+ 0.2404	− 0.1101
1 6— 2	0.00	− 0.04	0.00	0.00	0.00	− 0.04	0.00	+ 0.04
—1 9— 2	+ 0.02	− 0.02			+ 0.02	− 0.02	− 0.02	+ 0.02
0 8— 2	− 0.0260	+ 0.0340	− 0.0038	− 0.0004	− 0.0298	+ 0.0336	+ 0.0198	− 0.0307
1 7— 2	+ 0.01	− 0.02			+ 0.01	− 0.02	− 0.01	+ 0.02
—1 0— 3	+ 0.03	+ 0.02			+ 0.03	+ 0.02	− 0.03	− 0.04
0 1— 3	+ 0.0054	− 0.0213	+ 0.0007	− 0.0014	+ 0.0061	− 0.0227	− 0.0050	+ 0.0163
1—2— 3	− 0.03	+ 0.01			− 0.03	+ 0.01	+ 0.03	0.00
—1—1— 3	+ 0.04	− 0.52	0.00	− 0.02	+ 0.04	− 0.54	+ 0.06	+ 0.93
0 0— 3	0.0000	0.0000	+ 0.1737	+ 0.8449	+ 0.1737	+ 0.8449	0.0000	0.0000
1—1— 3	− 0.18	− 0.06	0.00	+ 0.02	− 0.18	− 0.04	+ 0.20	+ 0.22
—1 2— 3	+ 3.55	+ 2.68	+ 0.57	− 0.32	+ 4.12	+ 2.36	− 3.91	− 8.06
0 1— 3	− 3.9911	− 14.9472	− 0.2141	− 0.0749	− 4.2052	− 15.0221	+ 6.9364	+ 31.0546
1 0— 3	+ 2.58	+ 16.28	− 2.06	− 10.02	+ 0.52	+ 6.26	− 5.83	− 31.36
—1 3— 3	+ 1.51	+ 61.14	− 1.92	+ 13.38	− 0.41	+ 74.52	− 5.36	− 44.01
0 2— 3	+ 16.7787	− 33.3477	− 1.6199	− 2.1267	+ 15.1588	− 35.4744	− 9.2355	+ 24.2283
1 1— 3	− 4.69	+ 10.04	+ 2.53	+ 0.91	− 2.16	+ 10.95	+ 3.06	− 8.23
—1 4— 3	−594.32	−852.79	−95.33	−134.35	−689.65	−987.14	+459.59	+663.23
0 3— 3	+515.7983	+725.8970	− 0.5513	− 1.4375	+515.2470	+724.4595	−396.3397	−560.5710
1 2— 3	− 80.76	−112.87	+18.66	+ 25.56	− 62.10	− 87.31	+ 58.97	+ 82.65
—1 5— 3	−392.325	−148.624	− 57.12	− 29.23	−449.44	−177.85	+312.688	+109.596
0 4— 3	+349.5288	+133.6786	+ 7.8778	+ 11.3333	+357.4066	+145.0119	−276.9491	− 97.6308
1 3— 3	− 51.65	− 3.50	+ 8.46	+ 3.69	− 43.19	+ 0.19	+ 39.56	+ 0.56
—1 6— 3	−101.33	+ 25.03	−15.09	+ 0.44	−116.42	+ 25.47	+ 80.77	− 23.79
0 5— 3	+ 92.8999	− 22.1306	+ 4.7914	+ 2.4759	+ 97.6913	− 19.6547	− 73.6099	+ 21.2037
1 4— 3	− 9.60	+ 8.37	+ 1.74	− 0.28	− 7.86	+ 8.09	+ 7.13	− 7.23
—1 7— 3	− 13.75	+ 15.44	− 2.39	+ 1.45	− 16.14	+ 16.89	+ 10.60	− 13.34
0 6— 3	+ 12.9321	− 14.2449	+ 1.2702	− 0.0320	+ 14.2023	− 14.2769	− 9.8987	+ 12.2941
1 5— 3	− 0.328	+ 2.509	+ 0.21	− 0.20	− 0.12	+ 2.31	+ 0.097	− 2.084
—1 8— 3	− 0.47	+ 3.52	− 0.22	+ 0.38	− 0.69	+ 3.90	+ 0.21	− 2.98
0 7— 3	+ 0.4738	− 3.3320	+ 0.2019	− 0.1210	+ 0.6757	− 3.4530	− 0.2118	+ 2.8115
1 6— 3	+ 0.24	+ 0.29	+ 0.01	− 0.04	+ 0.25	+ 0.25	− 0.23	− 0.21
—1 9— 3	+ 0.23	+ 0.51	+ 0.01	+ 0.06	+ 0.24	+ 0.57	− 0.22	− 0.42
0 8— 3	− 0.2313	− 0.4824	+ 0.0184	− 0.0331	− 0.2129	− 0.5155	+ 0.2251	+ 0.3971
1 7— 3	+ 0.09	+ 0.01	0.00	0.00	+ 0.09	+ 0.01	− 0.08	0.00
—1 10— 3	+ 0.05	+ 0.03			+ 0.05	+ 0.03	− 0.05	− 0.03
0 9— 3	− 0.0680	− 0.0404	− 0.0003	− 0.0055	− 0.0683	− 0.0459	+ 0.0611	+ 0.0308
1 8— 3	+ 0.04	+ 0.01			+ 0.04	+ 0.01	− 0.04	− 0.01
0 10— 3	− 0.0110	0.0000	− 0.0004	− 0.0006	− 0.0114	− 0.0006	+ 0.0124	− 0.0007

Arg= $\varkappa\gamma'+i'g'+ig$			V′		X′		B′		G′	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	1	−4	0.00	− 0.02	0.00	0.00	0.00	− 0.02	+ 0.01	+ 0.05
0	0	−4	0.0000	0.0000	+ 0.0143	+ 0.0656	+ 0.0143	+ 0.0656	0.0000	0.0000
1	−1	−4	− 0.01	− 0.02	+ 0.01	0.00	0.00	− 0.02	+ 0.01	+ 0.03
−1	2	−4	+ 0.21	+ 0.36	+ 0.02	0.00	+ 0.23	+ 0.36	− 0.26	− 0.75
0	1	−4	− 0.2895	− 1.1686	− 0.0016	− 0.0009	− 0.2911	− 1.1695	+ 0.5196	+ 2.3213
1	0	−4	+ 0.16	+ 1.12	− 0.17	− 0.77	− 0.01	+ 0.35	− 0.41	− 2.28
−1	3	−4	+ 2.98	+ 3.42	+ 0.35	+ 0.69	+ 3.33	+ 4.11	− 2.60	− 2.52
0	2	−4	− 0.3590	− 2.1539	− 0.0445	− 0.1286	− 0.4035	− 2.2825	+ 0.5018	+ 1.6387
1	1	−4	− 0.62	+ 0.17	+ 0.01	+ 0.01	− 0.61	+ 0.18	+ 0.50	− 0.12
−1	4	−4	− 77.68	+ 2.60	−12.58	− 0.82	− 90.26	+ 1.78	+ 60.41	− 4.59
0	3	−4	+ 54.3386	+ 15.9151	+ 0.8105	− 1.0164	+ 55.1491	+ 14.8987	− 41.8511	− 10.6560
1	2	−4	− 12.19	− 4.86	+ 0.51	+ 1.55	− 11.68	− 3.31	+ 9.62	+ 3.39
−1	5	−4	+522.50	−561.53	+65.84	−72.61	+588.34	−634.14	−428.27	+457.45
0	4	−4	−461.9541	+507.8785	+ 1.1622	− 0.4024	−460.7919	+507.4761	+377.2372	−414.3948
1	3	−4	+ 94.13	−103.36	− 9.98	+11.37	+ 84.15	− 91.99	− 75.33	+ 82.37
−1	6	−4	+ 66.73	−371.03	+12.36	−45.45	+ 79.09	−416.48	− 50.22	+306.90
0	5	−4	− 61.7309	+340.6751	− 5.5801	+ 6.0077	− 67.3110	+346.6828	+ 46.1689	−280.8633
1	4	−4	− 0.17	− 61.37	− 1.23	+ 5.58	− 1.40	− 55.79	+ 1.41	+ 49.83
−1	7	−4	− 42.85	−100.22	− 2.96	−12.77	− 45.81	−112.99	+ 38.29	+ 82.60
0	6	−4	+ 39.4048	+ 94.1053	− 1.0552	+ 3.8132	+ 38.3496	+ 97.9185	− 35.2235	− 77.2929
1	5	−4	− 10.98	− 12.61	+ 0.44	+ 1.25	− 10.54	− 11.36	+ 9.54	+ 10.05
−1	8	−4	− 20.38	− 13.43	− 1.87	− 2.06	− 22.25	− 15.49	+ 17.70	+ 10.67
0	7	−4	+ 19.1792	+ 12.9092	+ 0.2466	+ 1.0752	+ 19.4258	+ 13.9844	− 16.6308	− 10.2117
1	6	−4	− 3.55	− 0.59	+ 0.18	+ 0.14	− 3.37	− 0.45	+ 3.02	+ 0.34
−1	9	−4	− 4.63	+ 0.01	− 0.49	− 0.16	− 5.12	− 0.15	+ 3.96	− 0.20
0	8	−4	+ 4.4607	+ 0.0425	+ 0.1574	+ 0.1749	+ 4.6181	+ 0.2174	− 3.8149	+ 0.1506
1	7	−4	− 0.62	+ 0.32	+ 0.04	+ 0.02	− 0.58	+ 0.34	+ 0.52	− 0.30
−1	10	−4	− 0.68	+ 0.43	− 0.08	+ 0.03	− 0.76	+ 0.46	+ 0.57	− 0.40
0	9	−4	+ 0.6501	− 0.4164	+ 0.0404	+ 0.0136	+ 0.6905	− 0.4028	− 0.5437	+ 0.3903
1	8	−4	− 0.03	+ 0.12	0.00	0.00	− 0.03	+ 0.12	+ 0.02	− 0.11
−1	11	−4	− 0.05	+ 0.14			− 0.05	+ 0.14	+ 0.05	− 0.14
0	10	−4	+ 0.0525	− 0.1127	+ 0.0067	− 0.0013	+ 0.0592	− 0.1140	− 0.0408	+ 0.1012
1	9	−4	− 0.01	+ 0.04			− 0.01	+ 0.04	+ 0.01	− 0.04
0	11	−4	+ 0.0006	+ 0.0142	+ 0.0007	− 0.0007	+ 0.0013	+ 0.0135	0.0000	− 0.0148
0	0	−5	0.0000	0.0000	+ 0.0009	+ 0.0048	+ 0.0009	+ 0.0048	0.0000	0.0000
−1	2	−5	0.00	+ 0.03	0.00	0.00	0.00	+ 0.03	0.00	− 0.06
0	1	−5	− 0.0190	− 0.0860	− 0.0003	+ 0.0003	− 0.0197	− 0.0857	+ 0.0363	+ 0.1709
1	0	−5	+ 0.02	+ 0.08	− 0.02	− 0.06	0.00	+ 0.02	− 0.04	− 0.17
−1	3	−5	+ 0.16	+ 0.14	+ 0.02	+ 0.03	+ 0.18	+ 0.17	− 0.13	− 0.10
0	2	−5	− 0.0263	− 0.1235	− 0.0034	− 0.0029	− 0.0297	− 0.1264	+ 0.0293	+ 0.0968
1	1	−5	0.00	− 0.02	0.00	0.00	0.00	− 0.02	0.00	+ 0.02
−1	4	−5	− 4.19	+ 3.89	− 0.73	+ 0.46	− 4.92	+ 4.35	+ 3.19	− 3.37
0	3	−5	+ 3.4329	− 0.9817	+ 0.0596	− 0.0658	+ 3.4925	− 1.0475	− 2.6351	+ 0.9629
1	2	−5	− 0.70	− 0.31	+ 0.04	+ 0.03	− 0.66	− 0.28	+ 0.54	+ 0.24
−1	5	−5	− 11.81	− 77.78	− 0.81	−10.10	− 12.62	− 87.88	+ 11.20	+ 63.54
0	4	−5	− 5.0628	+ 62.3156	+ 0.6043	+ 0.2578	− 4.4585	+ 62.5734	+ 3.0690	− 50.6658

Arg= $\varkappa\gamma'+i'g'+ig$			V′		X′		B′		G′	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
1	3	−5	+ 2.78	− 14.50	− 0.73	+ 0.75	+ 2.05	− 13.75	− 2.00	+ 11.81
−1	6	−5	+456.97	+268.52	+49.43	+27.88	+506.40	+296.40	−385.45	−228.24
0	5	−5	−424.0840	−241.5061	+ 0.3585	+ 0.8537	−423.7255	−240.6524	+357.0214	+204.8103
1	4	−5	+ 98.32	+ 56.54	− 6.47	− 3.49	+ 91.85	+ 53.05	− 82.01	− 47.45
−1	7	−5	+309.64	− 4.97	+32.60	+ 1.76	+342.24	− 3.21	−263.24	+ 6.98
0	6	−5	−289.8739	+ 4.8711	− 4.0926	− 2.3865	−293.9665	+ 2.4846	+245.9267	− 6.7882
1	5	−5	+ 58.51	− 8.62	− 3.43	− 0.04	+ 55.08	− 8.66	− 49.22	+ 8.01
−1	8	−5	+ 85.86	− 56.25	+ 9.56	− 4.41	+ 95.42	− 60.66	− 72.55	+ 49.77
0	7	−5	− 81.9068	+ 52.9242	− 2.7370	− 0.1633	− 84.6438	+ 52.7609	+ 69.0697	− 46.7912
1	6	−5	+ 12.60	− 13.20	− 0.81	+ 0.47	+ 11.79	− 12.73	− 10.44	+ 11.55
−1	9	−5	+ 10.49	− 23.42	+ 1.53	− 2.05	+ 12.02	− 25.47	− 8.46	+ 20.51
0	8	−5	− 10.2918	+ 22.4218	− 0.8052	+ 0.3684	− 11.0970	+ 22.7902	+ 8.2981	− 19.6162
1	7	−5	+ 0.43	− 4.34	− 0.10	+ 0.16	+ 0.33	− 4.18	− 0.23	+ 3.76
−1	10	−5	− 0.85	− 5.34	+ 0.04	− 0.51	− 0.81	− 5.85	+ 0.91	+ 4.63
0	9	−5	+ 0.7335	+ 5.1892	− 0.1279	+ 0.1716	+ 0.6056	+ 5.3608	− 0.8062	− 4.4917
1	8	−5	− 0.45	− 0.75	− 0.01	+ 0.04	− 0.46	− 0.71	+ 0.42	+ 0.64
−1	11	−5	− 0.67	− 0.75	− 0.04	− 0.08	− 0.71	− 0.83	+ 0.62	+ 0.64
0	10	−5	+ 0.6437	+ 0.7452	− 0.0054	+ 0.0429	+ 0.6383	+ 0.7881	− 0.5929	− 0.6312
1	9	−5	− 0.16	− 0.07	0.00	0.00	− 0.16	− 0.07	+ 0.14	+ 0.06
−1	12	−5	− 0.20	− 0.04			− 0.20	− 0.04	+ 0.18	+ 0.03
0	11	−5	+ 0.1623	+ 0.0454	+ 0.0029	+ 0.0070	+ 0.1652	+ 0.0524	− 0.1454	− 0.0331
1	10	−5	+ 0.02	− 0.01			+ 0.02	− 0.01	− 0.02	+ 0.01
0	12	−5	+ 0.0212	− 0.0071	+ 0.0010	+ 0.0007	+ 0.0222	− 0.0064	− 0.0179	+ 0.0075
0	2	−6	− 0.0013	− 0.0049	− 0.0004	− 0.0002	− 0.0017	− 0.0051	+ 0.0013	+ 0.0033
−1	4	−6	− 0.16	+ 0.30	0.00	+ 0.02	− 0.16	+ 0.32	+ 0.12	− 0.25
0	3	−6	+ 0.1872	− 0.1092	+ 0.0013	− 0.0041	+ 0.1885	− 0.1133	− 0.1426	+ 0.0969
1	2	−6	− 0.03	− 0.01	+ 0.01	0.00	− 0.02	− 0.01	+ 0.02	+ 0.01
−1	5	−6	− 4.80	− 4.22	− 0.50	− 0.60	− 5.30	− 4.82	+ 4.16	+ 3.35
0	4	−6	+ 2.0503	+ 4.1065	+ 0.0544	+ 0.0187	+ 2.1047	+ 4.1252	− 1.8269	− 3.3004
1	3	−6	− 0.06	− 1.06	− 0.01	+ 0.06	− 0.07	− 1.00	+ 0.06	+ 0.86
−1	6	−6	+ 66.21	− 21.15	+ 7.22	− 1.89	+ 73.43	− 23.04	− 55.86	+ 18.78
0	5	−6	− 57.6956	+ 7.4175	− 0.0377	+ 0.3441	− 57.7333	+ 7.7616	+ 48.5565	− 6.8981
1	4	−6	+ 14.41	− 0.40	− 0.64	− 0.26	+ 13.77	− 0.66	− 12.11	+ 0.50
−1	7	−6	−105.94	+332.79	− 9.02	+30.88	−114.96	+363.67	+ 92.81	−287.80
0	6	−6	+ 94.5054	−314.3314	− 0.5596	+ 0.3265	+ 93.9458	−314.0049	− 82.6770	+271.4960
1	5	−6	− 24.42	+ 78.99	+ 0.95	− 3.50	− 23.47	+ 75.49	+ 21.22	− 67.86
−1	8	−6	+ 53.65	+232.74	+ 3.63	+21.45	+ 57.28	+254.19	− 48.23	−202.02
0	7	−6	− 52.8883	−220.7926	+ 0.7951	− 2.5592	− 52.0932	−223.3518	+ 47.8079	+191.3864
1	6	−6	+ 18.36	+ 48.00	− 0.58	− 1.97	+ 17.78	+ 46.03	− 16.80	− 41.38
−1	9	−6	+ 61.84	+ 64.45	+ 4.76	+ 6.41	+ 66.60	+ 70.82	− 54.93	− 55.45
0	8	−6	− 58.8726	− 62.0903	− 0.2938	− 1.8018	− 59.1664	− 63.8921	+ 52.2177	+ 56.3714
1	7	−6	+ 13.85	+ 10.23	− 0.39	− 0.49	+ 13.46	+ 9.74	− 12.17	− 8.67
−1	10	−6	+ 24.13	+ 6.12	+ 1.95	+ 0.90	+ 26.08	+ 7.02	− 21.35	− 4.91
0	9	−6	− 23.2857	− 6.0930	− 0.3986	− 0.5393	− 23.6843	− 6.6323	+ 20.5747	+ 4.8798
1	8	−6	+ 4.64	− 0.09	− 0.15	− 0.05	+ 4.49	− 0.14	− 4.07	+ 0.19
−1	11	−6	+ 5.45	− 1.78	+ 0.49	− 0.05	+ 5.94	− 1.83	− 4.78	+ 1.71
0	10	−6	− 5.3346	+ 1.6757	− 0.1644	− 0.0771	− 5.4990	+ 1.5986	+ 4.6731	− 1.6173

Arg= $\varkappa\gamma'+i'g'+ig$	V′		X′		B′		G′	
	sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$ i' i	″	″	″	″	″	″	″	″
1 9 − 6	+ 0.83	− 0.62	− 0.04	+0.01	+ 0.79	− 0.61	− 0.72	+ 0.57
−1 12 − 6	+ 0.74	− 0.90	+ 0.08	−0.05	+ 0.82	− 0.95	− 0.63	+ 0.82
0 11 − 6	− 0.7335	+ 0.8634	− 0.0403	+0.0025	− 0.7738	+ 0.8659	+ 0.6271	− 0.7888
1 10 − 6	+ 0.06	− 0.20	0.00	+0.01	+ 0.06	− 0.19	− 0.05	+ 0.18
−1 13 − 6	+ 0.03	− 0.22			+ 0.03	− 0.22	− 0.03	+ 0.22
0 12 − 6	− 0.0283	+ 0.2053	− 0.0065	+0.0043	− 0.0348	+ 0.2096	+ 0.0170	− 0.1848
1 11 − 6	+ 0.02	− 0.07			+ 0.02	− 0.07	− 0.02	+ 0.07
0 3 − 7	+ 0.0080	− 0.0086			+ 0.0080	− 0.0086	− 0.0056	+ 0.0074
−1 5 − 7	− 0.41	− 0.17	− 0.05	−0.03	− 0.46	− 0.20	+ 0.35	+ 0.13
0 4 − 7	+ 0.2175	+ 0.2149	+ 0.0040	−0.0002	+ 0.2215	+ 0.2147	− 0.1881	− 0.1693
1 3 − 7	− 0.03	− 0.02	+ 0.01	0.00	− 0.02	− 0.02	+ 0.03	+ 0.01
−1 6 − 7	+ 3.58	− 5.32	+ 0.43	−0.51	+ 4.01	− 5.83	− 2.94	+ 4.64
0 5 − 7	− 3.8937	+ 3.0776	+ 0.0011	+0.0368	− 3.8926	+ 3.1144	+ 3.2301	− 2.7035
1 4 − 7	+ 1.07	− 0.48	− 0.04	+0.01	+ 1.03	− 0.47	− 0.89	+ 0.43
−1 7 − 7	+ 26.43	+ 49.63	+ 2.24	+4.67	+ 28.67	+ 54.30	− 23.38	− 42.85
0 6 − 7	− 16.2761	− 45.8851	− 0.1860	+0.0386	− 16.4621	− 45.8465	+ 14.4380	+ 39.5650
1 5 − 7	+ 3.09	+ 12.22	+ 0.02	−0.42	+ 3.11	+ 11.80	− 2.76	− 10.52
−1 8 − 7	−220.85	− 16.99	−17.90	−0.86	−238.75	− 17.85	+194.62	+ 15.74
0 7 − 7	+211.2685	+ 12.3845	− 0.2789	−0.3532	+210.9896	+ 12.0313	−186.0160	− 11.5928
1 6 − 7	− 56.18	− 3.88	+ 1.79	+0.05	− 54.39	− 3.83	+ 49.29	+ 3.56
−1 9 − 7	−159.11	+ 71.86	−13.02	+5.17	−172.13	+ 77.03	+140.37	− 64.25
0 8 − 7	+152.3186	− 69.5874	+ 1.4855	+0.1005	+153.8041	− 69.4869	−134.2451	+ 62.1753
1 7 − 7	− 34.85	+ 19.38	+ 1.08	−0.47	− 33.77	+ 18.91	+ 30.60	− 17.28
−1 10 − 7	− 42.07	+ 60.01	− 3.84	+4.35	− 45.91	+ 64.36	+ 36.71	− 53.72
0 9 − 7	+ 40.9072	− 57.9441	+ 1.0950	−0.4257	+ 42.0022	− 58.3698	− 35.6578	+ 51.8271
1 8 − 7	− 6.92	+ 13.93	+ 0.26	−0.34	− 6.66	+ 13.59	+ 5.95	− 12.42
−1 11 − 7	− 1.42	+ 22.40	− 0.39	+1.69	− 1.81	+ 24.09	+ 0.91	− 19.99
0 10 − 7	+ 1.5933	− 21.8425	+ 0.3222	−0.3622	+ 1.9155	− 22.2047	− 1.0710	+ 19.4824
1 9 − 7	+ 0.62	+ 4.46	+ 0.03	−0.12	+ 0.65	+ 4.34	− 0.63	− 3.96
−1 12 − 7	+ 2.60	+ 4.96	+ 0.13	+0.41	+ 2.73	+ 5.37	− 2.43	− 4.39
0 11 − 7	− 2.5381	− 4.9204	+ 0.0339	−0.1419	− 2.5042	− 5.0623	+ 2.3719	+ 4.3525
1 10 − 7	+ 0.76	+ 0.80	− 0.01	−0.03	+ 0.75	+ 0.77	− 0.70	− 0.70
−1 13 − 7	+ 1.11	+ 0.66	+ 0.06	+0.06	+ 1.17	+ 0.72	− 1.02	− 0.57
0 12 − 7	− 1.0693	− 0.6373	− 0.0094	−0.0344	− 1.0787	− 0.6717	+ 0.9802	+ 0.5503
1 11 − 7	+ 0.21	+ 0.04	0.00	−0.01	+ 0.21	+ 0.03	− 0.19	− 0.03
0 4 − 8	+ 0.0258	+ 0.0105			+ 0.0258	+ 0.0105	− 0.0234	− 0.0081
−1 6 − 8	− 0.02	− 0.54	0.00	−0.04	− 0.02	− 0.58	+ 0.04	+ 0.47
0 5 − 8	− 0.1711	+ 0.3098	+ 0.0012	+0.0028	− 0.1699	+ 0.3126	+ 0.1348	− 0.2681
1 4 − 8	+ 0.05	− 0.02	0.00	0.00	+ 0.05	− 0.02	− 0.04	+ 0.01
−1 7 − 8	+ 5.36	+ 2.33	+ 0.45	+0.26	+ 5.81	+ 2.59	− 4.73	− 1.93
0 6 − 8	− 3.6846	− 3.0367	− 0.0223	+0.0083	− 3.7069	− 3.0284	+ 3.2523	+ 2.5747
1 5 − 8	+ 0.75	+ 1.02	0.00	−0.04	+ 0.75	+ 0.98	− 0.66	− 0.88
−1 8 − 8	− 33.15	+ 26.99	− 2.74	+2.09	− 35.89	+ 29.08	+ 29.14	− 24.07
0 7 − 8	+ 32.1359	− 20.0470	− 0.0544	−0.0945	+ 32.0815	− 20.1415	− 28.2365	+ 17.8507
1 6 − 8	− 9.00	+ 4.53	+ 0.26	−0.05	− 8.74	+ 4.48	+ 7.90	− 4.02
−1 9 − 8	− 21.89	−134.67	− 1.92	−9.65	− 23.81	−144.32	+ 19.07	+120.47

Arg= $\varkappa\gamma'+i'g'+ig$	V′		X′		B′		G′	
	sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$ i' i	″	″	″	″	″	″	″	″
0 8—8	+23.8594	+129.9802	+0.1968	−0.2210	+24.0562	+129.7592	−20.8585	−116.1964
1 7—8	− 6.11	− 36.09	+0.18	+0.87	− 5.93	− 35.22	+ 5.34	+ 32.19
−1 10—8	−72.99	− 98.88	−4.98	−7.28	−77.97	−106.16	+65.72	+ 88.37
0 9—8	+71.2107	+ 95.2406	+0.1408	+0.8024	+71.3515	+ 96.0430	−64.0803	− 85.0511
1 8—8	−19.23	− 22.60	+0.39	+0.54	−18.84	− 22.06	+17.28	+ 20.12
−1 11—8	−52.45	− 23.01	−3.55	−1.99	−56.00	− 25.00	+47.32	+ 20.24
0 10—8	+51.0787	+ 22.4700	+0.4113	+0.6128	+51.4900	+ 23.0828	−46.0609	− 19.7496
1 9—8	−12.40	− 3.59	+0.24	+0.13	−12.16	− 3.46	+11.16	+ 3.10
−1 12—8	−19.05	+ 2.38	−1.35	−0.03	−20.40	+ 2.35	+17.14	− 2.39
0 11—8	+18.7200	− 2.2004	+0.2967	+0.1672	+19.0167	− 2.0332	−16.8393	+ 2.2242
1 10—8	− 3.94	+ 1.27	+0.09	0.00	− 3.85	+ 1.27	+ 3.53	− 1.20
−1 13—8	− 4.22	+ 3.44	−0.32	+0.17	− 4.54	+ 3.61	+ 3.77	− 3.19
0 12—8	+ 4.1429	− 3.2712	+0.1114	+0.0031	+ 4.2543	− 3.2681	− 3.6995	+ 3.0360
1 11—8	− 0.65	+ 0.82	+0.02	−0.01	− 0.63	+ 0.81	+ 0.57	− 0.76
0 5—9	− 0.0030	+ 0.0207			− 0.0030	+ 0.0207	+ 0.0015	− 0.0177
−1 7—9	+ 0.49	− 0.06	+0.04	+0.01	+ 0.53	− 0.05	− 0.43	+ 0.07
0 6—9	− 0.3753	− 0.1059	−0.0018	+0.0014	− 0.3771	− 0.1045	+ 0.3303	+ 0.0840
1 5—9	+ 0.08	+ 0.06	0.00	−0.01	+ 0.08	+ 0.05	− 0.07	− 0.05
−1 8—9	− 1.18	+ 4.90	−0.12	+0.38	− 1.30	+ 5.28	+ 0.98	− 4.37
0 7—9	+ 1.9100	− 3.7525	−0.0095	−0.0110	+ 1.9005	− 3.7625	− 1.6457	+ 3.3427
1 6—9	− 0.67	+ 0.86	+0.02	−0.01	− 0.65	+ 0.85	+ 0.59	− 0.76
−1 9—9	−23.90	− 19.56	−1.68	−1.45	−25.58	− 21.01	+21.50	+ 17.41
0 8—9	+19.5961	+ 19.8982	+0.0408	−0.0484	+19.6369	+ 19.8498	−17.6041	− 17.7133
1 7—9	− 4.96	− 5.88	+0.07	+0.14	− 4.89	− 5.74	+ 4.45	+ 5.23
−1 10—9	+75.25	− 32.42	+4.79	−2.32	+80.04	− 34.74	−68.15	+ 29.01
0 9—9	−73.0070	+ 33.4570	+0.1624	+0.0978	−72.8446	+ 33.5548	+66.0839	− 29.9518
1 8—9	+21.01	− 9.20	−0.38	+0.20	+20.63	− 9.00	−18.98	+ 8.23
−1 11—9	+55.16	− 63.10	+3.69	−3.96	+58.85	− 67.06	−49.82	+ 57.27
0 10—9	−53.3262	+ 61.8900	−0.4010	+0.1807	−53.7272	+ 62.0707	+47.1332	− 56.1510
1 9—9	+12.97	− 16.74	−0.24	+0.28	+12.73	− 16.46	−11.68	+ 15.17
−1 12—9	+ 8.98	− 42.09	+0.80	−2.64	+ 9.78	− 44.73	− 7.87	+ 38.26
0 11—9	− 8.7822	+ 41.2860	−0.3123	+0.3297	− 9.0945	+ 41.6157	+ 7.6905	− 37.5189
1 10—9	+ 0.98	− 10.17	−0.05	+0.16	+ 0.93	− 10.01	− 0.81	+ 9.23
−1 13—9	− 5.49	− 15.55	−0.20	−1.00	− 5.69	− 16.55	+ 5.19	+ 14.13
0 12—9	+ 5.0559	+ 15.0834	−0.0679	+0.2194	+ 4.9880	+ 15.3028	− 4.7796	− 13.6980
1 11—9	− 1.57	− 3.10	+0.02	+0.07	− 1.55	− 3.03	+ 1.47	+ 2.81
0 6—10	− 0.0358	+ 0.0053			− 0.0358	+ 0.0053	+ 0.0331	− 0.0059
−1 8—10	+ 0.13	+ 0.34	0.00	+0.03	+ 0.13	+ 0.37	− 0.14	− 0.29
0 7—10	+ 0.0140	− 0.3907	−0.0015	−0.0007	+ 0.0125	− 0.3914	+ 0.0178	+ 0.3466
1 6—10	− 0.02	+ 0.22	0.00	−0.01	− 0.02	+ 0.21	0.00	− 0.21
−1 9—10	− 4.03	− 0.14	−0.28	−0.04	− 4.31	− 0.18	+ 3.62	+ 0.08
0 8—10	+ 3.3860	+ 0.8470	+0.0056	−0.0081	+ 3.3916	+ 0.8389	− 3.0446	− 0.7237
1 7—10	− 0.89	− 0.42	+0.02	+0.02	− 0.87	− 0.40	+ 0.80	+ 0.37
−1 10—10	+ 9.90	− 19.10	+0.65	−1.23	+10.55	− 20.33	− 8.91	+ 17.33
0 9—10	−10.6321	+ 16.6454	+0.0361	+0.0164	−10.5960	+ 16.6618	+ 9.5680	− 15.0857
1 8—10	+ 3.28	− 4.49	−0.07	+0.06	+ 3.21	− 4.43	− 2.95	+ 4.07

Arg= $\varkappa\gamma'+i'g'+ig$	V′		X′		B′		G′	
	sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$ i' i	″	″	″	″	″	″	″	″
−1 11−10	+29.57	+37.85	+1.86	+2.17	+31.43	+40.02	−26.81	−34.65
0 10−10	−30.2760	−36.8380	−0.0397	+0.1117	−30.3157	−36.7263	+27.4554	+33.7060
1 9−10	+8.64	+11.01	−0.14	−0.17	+8.50	+10.84	−7.83	−10.06
−1 12−10	+48.89	+26.58	+2.86	+1.66	+51.75	+28.24	−44.71	−24.22
0 11−10	−48.2060	−25.6390	−0.1465	−0.1824	−48.3525	−25.8214	+44.0714	+23.3455
1 10−10	+13.14	+6.29	−0.19	−0.11	+12.95	+6.18	−12.00	−5.72
−1 13−10	+33.25	−0.89	+1.87	+0.11	+35.12	−0.78	−30.53	+1.04
0 12−10	−31.8490	+0.4249	−0.2339	−0.1371	−32.0829	+0.2878	+29.2282	−0.5947
1 11−10	+7.50	−0.62	−0.13	+0.01	+7.37	−0.61	−6.87	+0.62
−1 8−11	+0.03	+0.02			+0.03	+0.02	−0.03	−0.02
0 7−11	−0.0112	−0.0310			−0.0112	−0.0310	+0.0133	+0.0268
−1 9−11	−0.37	+0.21	−0.02	+0.01	−0.39	+0.22	+0.33	−0.20
0 8−11	+0.3436	−0.0760	+0.0003	−0.0011	+0.3439	−0.0771	−0.3082	+0.0733
1 7−11	−0.10	−0.03	0.00	+0.01	−0.10	−0.02	+0.09	+0.03
−1 10−11	−0.49	−3.06	+0.01	−0.20	−0.48	−3.26	+0.47	+2.77
0 9−11	−0.0759	+2.7440	+0.0057	+0.0012	−0.0702	+2.7452	+0.0471	−2.4869
1 8−11	+0.14	−0.77	0.00	0.00	+0.14	−0.77	−0.12	+0.70
−1 11−11	+14.01	+3.73	+0.82	+0.23	+14.83	+3.96	−12.81	−3.37
0 10−11	−12.7140	−4.5200	−0.0032	+0.0246	−12.7172	−4.4954	+11.6130	+4.0997
1 9−11	+3.57	+1.53	−0.03	−0.02	+3.54	+1.51	−3.26	−1.39
−1 12−11	−16.52	+22.11	−0.83	+1.26	−17.35	+23.37	+15.28	−20.23
0 11−11	+15.9920	−22.7300	−0.0716	−0.0068	+15.9204	−22.7768	−16.7842	+20.8142
1 10−11	−4.98	+6.69	+0.08	−0.09	−4.90	+6.60	+4.60	−6.13
−1 13−11	−9.05	+38.23	−0.55	+1.97	−9.60	+40.20	+8.27	−35.37
0 12−11	+8.9080	−36.0750	+0.0682	−0.0999	+8.9762	−36.1749	−8.1424	+33.3516
1 11−11	−2.24	+9.08	+0.02	−0.14	−2.22	+8.94	+2.05	−8.37
0 8−12	+0.0236	−0.0151			+0.0236	−0.0151	−0.0209	+0.0127
−1 10−12	−0.24	−0.27	0.00	−0.02	−0.24	−0.29	+0.22	+0.24
0 9−12	+0.1216	+0.2719	+0.0008	−0.0002	+0.1224	+0.2717	−0.1135	−0.2470
1 8−12	0.00	−0.07	0.00	0.00	0.00	−0.07	0.00	+0.06
−1 11−12	+2.14	−0.86	+0.13	−0.04	+2.27	−0.90	−1.95	+0.81
0 10−12	−2.0240	+0.3954	0.0000	+0.0036	−2.0240	+0.3990	+1.8470	−0.3759
1 9−12	+0.62	+0.02	−0.01	0.00	+0.61	+0.02	−0.57	−0.02
−1 12−12	−0.30	+9.30	−0.03	+0.50	−0.33	+9.80	+0.26	−8.56
0 11−12	+0.9865	−8.7420	−0.0173	+0.0020	+0.9692	−8.7400	−0.9001	+8.0373
1 10−12	−0.47	+2.56	+0.02	−0.02	−0.45	+2.54	+0.43	−2.35
−1 13−12	−18.36	−4.69	−0.87	−0.18	−19.23	−4.87	+17.08	+4.41
0 12−12	+17.0600	+4.3260	−0.0012	−0.0432	+17.0588	+4.2828	−15.8618	−4.0710
1 11−12	−4.38	−1.42	+0.07	+0.02	−4.31	−1.40	+3.96	+1.33

Next in order follows the expression for T':

Arg=$i'g'+ig$	T'	
	sin.	cos.
i' i	″	″
0 0		+ 0.1478
1 0	+ 1.9899	+ 9.5637
2 0	+ 0.8926	+ 0.9746
3 0	+ 0.1482	+ 0.0426
4 0	+ 0.0172	— 0.0044
—2— 1	+ 0.0058	— 0.0188
—1— 1	+ 0.0502	+ 0.1843
0— 1	— 11.6426	— 55.5290
1— 1	+110.6520	+527.2337
2— 1	— 13.3822	+ 9.8429
3— 1	— 2.4227	+ 1.6236
4— 1	— 0.1852	+ 0.3373
5— 1	+ 0.0022	+ 0.0458
—1— 2	+ 0.0161	+ 0.0358
0— 2	— 1.0375	— 5.2269
1— 2	+ 10.9496	+ 61.1107
2— 2	+ 92.6790	— 38.1746
3— 2	+ 10.0689	— 20.8725
4— 2	— 0.9708	— 3.8075
5— 2	— 0.4612	— 0.3546
6— 2	— 0.0792	— 0.0013
7— 2	— 0.0084	+ 0.0057
0— 3	— 0.0909	— 0.4232
1— 3	+ 0.9394	+ 6.0984
2— 3	+ 1.8926	— 4.0378
3— 3	+ 39.3716	+ 55.5085
4— 3	+ 20.3422	+ 6.7285
5— 3	+ 4.1889	— 1.3921
6— 3	+ 0.4345	— 0.6126
7— 3	— 0.0004	— 0.1146
8— 3	— 0.0096	— 0.0133
9— 3	— 0.0020	— 0.0007
1— 4	+ 0.0732	+ 0.3879
2— 4	— 0.0262	— 0.2236
3— 4	+ 4.6982	+ 1.1890
4— 4	— 28.6894	+ 29.4477
5— 4	— 2.5835	+ 16.9438
6— 4	+ 1.8419	+ 3.7988
7— 4	+ 0.7184	+ 0.4055
8— 4	+ 0.1398	— 0.0133
9— 4	+ 0.0171	— 0.0140
10— 4	+ 0.0008	— 0.0031
3— 5	+ 0.2732	— 0.0770
4— 5	— 0.2338	+ 4.1548
5— 5	— 22.1066	— 12.6478
6— 5	— 12.5826	+ 0.4445
7— 5	— 2.9721	+ 2.0845
8— 5	—0.2953	+ 0.7502
9— 5	+0.0321	+ 0.1493
10— 5	+0.0194	+ 0.0174
11— 5	+0.0041	+ 0.0001
4— 6	+0.1482	+ 0.2746
5— 6	—3.1481	+ 0.4643
6— 6	+4.2774	—14.1586
7— 6	—2.0135	— 8.4942
8— 6	—2.0513	— 2.0309
9— 6	—0.7069	— 0.1547
10— 6	—0.1421	+ 0.0534
11— 6	—0.0163	+ 0.0230
5— 7	—0.2103	+ 0.1760
6— 7	—0.7856	— 2.1291
7— 7	+8.383	+ 0.519
8— 7	+5.265	— 2.455
9— 7	+1.224	— 1.821
10— 7	+0.0162	— 0.6112
11— 7	—0.0703	— 0.1186
12— 7	—0.0193	— 0.0139
6— 8	—0.1831	— 0.1359
7— 8	+1.294	— 0.821
8— 8	+0.829	+ 4.607
9— 8	+2.258	+ 2.977
10— 8	+1.566	+ 0.602
11— 8	+0.474	— 0.082
12— 8	+0.096	— 0.072
7— 9	+0.073	— 0.148
8— 9	+0.706	+ 0.724
9— 9	—2.328	+ 1.046
10— 9	—1.541	+ 1.774
11— 9	—0.223	+ 1.078
12— 9	+0.120	+ 0.354
8—10	+0.134	+ 0.044
9—10	—0.355	+ 0.564
10—10	—0.888	— 1.077
11—10	—1.275	— 0.674
12—10	—0.755	+ 0.010
9—11	+0.004	+ 0.099
10—11	—0.378	— 0.135
11—11	+0.444	— 0.617
12—11	+0.237	— 0.854
10—12	—0.051	+ 0.011
11—12	+0.015	— 0.248
12—12	+0.402	+ 0.114

The expressions for the factors C′, D′, E′, and H′ are:

$\text{Arg}=\varkappa\gamma'+i'g'+ig$	C′		D′		E′		H′	
	sin.	cos.	cos.	sin.	sin.	cos.	cos.	sin.
$\varkappa$ i' i	″	″	″	″	″	″	″	″
0 0 0		— 2.2220				+ 0.981		
—1 1 0	— 0.22	+ 1.66	+ 0.35	+ 2.61	+ 0.53	— 1.55	+ 0.85	— 1.63
—1 2 0	+ 3.98	+ 19.20	—20.61	—14.03	— 6.16	+ 4.17	+12.91	+ 8.62
0 1 0	— 8.0924	— 38.5602	+ 9.296	+ 6.423	+ 9.297	— 6.422	+23.605	+16.862
1 0 0	+ 4.0750	+ 19.2066	+ 8.30	+ 5.60	— 6.22	+ 4.40	—43.03	—31.24
—1 3 0	+ 2.02	+ 3.04	— 3.74	— 7.27	— 0.84	+ 1.50	+ 2.64	+ 6.66
0 2 0	— 3.9280	— 5.5262	+ 2.247	+ 4.361	+ 1.124	— 2.181	— 0.293	— 3.182
1 1 0	+ 1.82	+ 1.96	+ 1.09	+ 1.73	— 0.31	+ 1.16	— 1.38	— 1.83
—1 4 0	+ 0.40	+ 0.26	+ 0.06	— 1.75	0.00	+ 0.28	— 0.17	+ 1.48
0 3 0	— 0.7646	— 0.4272	— 0.025	+ 1.171	— 0.008	— 0.390	+ 0.181	— 0.973
1 2 0	+ 0.34	+ 0.08	+ 0.03	+ 0.35	+ 0.06	+ 0.16	— 0.05	— 0.29
—1 5 0	+ 0.06	0.00	+ 0.10	— 0.25	+ 0.02	+ 0.03	— 0.11	+ 0.21
0 4 0	— 0.0998	— 0.0042	— 0.109	+ 0.189	— 0.027	— 0.047	+ 0.105	— 0.147
1 3 0	+ 0.04	0.00	+ 0.04	+ 0.03	+ 0.02	+ 0.01	— 0.02	— 0.05
—1 6 0	+ 0.02	0.00	+ 0.02	— 0.01			— 0.02	+ 0.01
—1— 2— 1	0.00	— 0.02	— 0.04	+ 0.04	— 0.01	— 0.01	+ 0.08	— 0.06
0— 3— 1			— 0.095	+ 0.028	+ 0.032	+ 0.009	— 0.054	+ 0.013
1— 4— 1			+ 0.14	— 0.09	— 0.02	— 0.01	— 0.01	+ 0.03
—1— 1— 1	0.00	0.00	— 0.39	— 0.10	— 0.10	— 0.02	+ 0.60	+ 0.03
0— 2— 1	+ 0.0140	+ 0.2504	— 0.532	— 0.110	+ 0.266	— 0.055	— 0.217	— 0.175
1— 3— 1	0.00	— 0.16	+ 1.05	+ 0.23	— 0.20	+ 0.04	— 0.28	+ 0.09
—1 0— 1	+ 0.08	+ 0.38	— 2.28	— 2.05	— 1.66	+ 2.09	+ 0.84	+ 4.63
0— 1— 1	+ 0.7236	+ 3.6308	— 1.657	— 1.355	+ 1.657	— 1.355	+ 1.388	— 2.805
1— 2— 1	— 0.44	— 2.12	+ 4.48	+ 3.84	— 0.99	+ 0.67	— 2.71	— 0.90
—1 1— 1	— 23.2404	— 110.8630	—14.09	—31.76	+ 7.71	—16.98	+ 7.21	+16.18
0 0— 1	+ 28.0898	+ 133.3682	0.000	0.000	—11.681	+25.334	0.000	0.000
1— 1— 1	— 11.00	— 51.84	+13.98	+31.76	+ 7.89	—16.97	— 7.03	—16.24
—1 2— 1	+219.72	+1046.56	+ 3.61	— 4.00	+ 1.41	+ 0.96	— 4.04	+ 2.62
0 1— 1	—438.3416	—2101.1468	— 1.586	+ 1.440	— 1.586	— 1.440	+ 3.663	+ 0.774
1 0— 1	+219.44	+1053.90	— 1.10	+ 1.70	+ 0.46	+ 2.10	— 1.46	— 3.07
—1 3— 1	— 14.42	+ 78.24	—16.02	+15.91	— 3.30	— 3.31	+22.70	—21.73
0 2— 1	+ 35.4446	— 127.7548	+10.203	—10.000	+ 5.102	+ 5.000	—15.996	+15.204
1 1— 1	— 26.9934	+ 19.8396	+ 2.56	— 2.59	— 3.50	— 3.42	— 1.49	+ 1.56
—1 4— 1	— 5.56	+ 7.96	— 8.89	+ 2.51	— 1.43	— 0.45	+10.49	— 2.70
0 3— 1	+ 10.9366	— 13.1390	+ 6.295	— 1.795	+ 2.099	+ 0.598	— 7.947	+ 2.106
1 2— 1	— 4.88	+ 3.30	+ 0.93	— 0.47	— 1.16	— 0.17	— 0.55	+ 0.45
—1 5— 1	— 0.68	+ 1.26	— 2.23	— 0.55	— 0.30	+ 0.04	+ 2.31	+ 0.62
0 4— 1	+ 1.2096	— 2.0368	+ 1.688	+ 0.382	+ 0.422	— 0.096	— 1.830	— 0.468
1 3— 1	— 0.34	+ 0.68	+ 0.21	— 0.01	— 0.18	+ 0.08	— 0.17	+ 0.06
—1 6— 1	— 0.06	+ 0.14	— 0.36	— 0.33	— 0.04	+ 0.03	+ 0.34	+ 0.28
0 5— 1	+ 0.0442	— 0.2830	+ 0.264	+ 0.236	+ 0.053	— 0.047	— 0.252	— 0.241
1 4— 1	— 0.02	+ 0.10	+ 0.05	+ 0.03	— 0.02	+ 0.03	— 0.06	+ 0.02
—1 7— 1	— 0.02	+ 0.02	— 0.01	— 0.04			+ 0.01	+ 0.04
0 6— 1			+ 0.020	+ 0.059			— 0.016	— 0.055
1 5— 1	0.00	— 0.02	0.00	— 0.03			+ 0.00	+ 0.03

Arg=$x\gamma'+i'g'+ig$	C′		D′		E′		H′	
	sin.	cos.	cos.	sin.	sin.	cos.	cos.	sin.
x i' i	″	″	″	″	″	″	″	″
−1− 1− 2	0.00	+ 0.02	0.00	+ 0.03	0.00	0.00		
0− 2− 2			− 0.003	+ 0.030	+ 0.001	+0.015		
1− 3− 2	0.00	− 0.02	0.00	− 0.06	0.00	−0.01		
−1 0− 2	+ 0.04	+ 0.08	+ 0.10	+ 0.26	− 0.08	+0.13	− 0.03	− 0.16
0− 1− 2	− 0.0464	+ 0.1886	− 0.072	+ 0.108	+ 0.072	+0.108	+ 0.137	− 0.068
1− 2− 2	+ 0.06	− 0.14	+ 0.02	− 0.40	− 0.04	−0.11	− 0.17	+ 0.26
−1 1− 2	− 2.08	− 10.42	− 2.32	+ 0.21	+ 1.01	−2.80	+ 1.39	− 1.01
0 0− 2	+ 1.4308	+ 11.0004	0.000	0.00	− 0.729	+4.353	0.000	0.000
1− 1− 2	− 0.18	− 3.88	+ 2.74	− 0.29	+ 0.39	−2.92	− 1.64	+ 1.07
−1 2− 2	+ 21.74	+121.46	−24.82	+ 5.32	− 7.34	−1.72	+18.24	− 3.99
0 1− 2	− 59.3036	−236.9464	+11.139	− 2.407	+11.137	+2.407	− 7.078	+ 1.707
1 0− 2	+ 32.30	+117.62	+10.07	− 2.00	− 7.49	−1.40	− 8.87	+ 1.64
−1 3− 2	+186.28	− 69.44	− 6.47	− 5.18	− 1.51	+1.14	+ 5.25	+ 4.13
0 2− 2	−374.2492	+146.0158	+ 3.908	+ 2.986	+ 1.954	−1.493	− 2.908	− 2.033
1 1− 2	+186.18	− 78.56	+ 1.41	+ 0.72	− 0.76	+1.08	− 1.22	− 0.71
−1 4− 2	+ 30.54	− 45.48	+ 9.44	+14.30	+ 1.48	−2.24	−13.99	−22.50
0 3− 2	− 55.7896	+ 89.9812	− 6.793	−10.486	− 2.237	+3.495	+10.899	+17.946
1 2− 2	+ 19.98	− 42.10	− 0.26	− 0.45	+ 1.60	−2.42	− 0.63	− 1.19
−1 5− 2	− 0.14	− 10.18	+ 0.71	+ 8.65	+ 0.12	−1.14	− 0.67	−11.39
0 4− 2	+ 1.3874	+ 19.1238	− 0.527	− 6.755	− 0.132	+1.689	+ 0.538	+ 9.408
1 3− 2	− 2.00	− 7.64	− 0.24	− 0.09	− 0.02	−0.97	+ 0.32	− 0.43
−1 6− 2	− 0.94	− 1.30	− 1.05	+ 2.31	− 0.11	−0.27	+ 1.32	− 2.64
0 5− 2	+ 1.8718	+ 2.2764	+ 0.835	− 1.862	+ 0.167	+0.372	− 1.077	+ 2.254
1 4− 2	− 0.96	− 0.68	− 0.06	− 0.09	− 0.12	−0.17	+ 0.11	− 0.01
−1 7− 2	− 0.18	− 0.08	− 0.46	+ 0.33	− 0.04	−0.03	+ 0.46	− 0.33
0 6− 2	+ 0.3954	+ 0.1144	+ 0.372	− 0.279	+ 0.062	+0.047	− 0.422	+ 0.297
1 5− 2	− 0.14	− 0.04	− 0.01	− 0.02	− 0.03	−0.01	+ 0.06	+ 0.01
−1 8− 2	− 0.04	− 0.02	− 0.07	+ 0.01	− 0.01	0.00	+ 0.06	− 0.01
0 7− 2	+ 0.0508	− 0.0164	+ 0.089	− 0.012	+ 0.013	+0.002	− 0.092	+ 0.009
1 6− 2	0.00	0.00	− 0.03	0.00	− 0.01	0.00	+ 0.04	0.00
0 8− 2							− 0.014	− 0.006
−1 0− 3	0.00	+ 0.04						
0− 1− 3			+ 0.005	+ 0.003			+ 0.016	− 0.008
1− 2− 3	0.00	− 0.02						
−1 1− 3	− 0.20	− 0.86	+ 0.10	+ 0.06	+ 0.11	−0.21	− 0.07	− 0.05
0 0− 3	+ 0.1656	+ 0.8434	0.000	0.000	− 0.166	+0.357	0.000	0.000
1− 1− 3	− 0.04	− 0.28	− 0.10	− 0.10	+ 0.11	−0.25	+ 0.07	+ 0.08
−1 2− 3	+ 1.86	+ 10.12	+ 0.22	+ 1.30	− 0.05	−0.02	− 0.12	− 0.98
0 1− 3	− 4.4400	− 20.1678	+ 0.096	− 0.553	+ 0.096	+0.554	− 0.066	+ 0.351
1 0− 3	+ 2.38	+ 10.12	− 0.34	− 1.11	− 0.06	−0.42	+ 0.20	+ 0.86
−1 3− 3	+ 3.86	− 7.50	+ 0.20	+23.54	+ 0.09	−4.87	− 0.16	−16.04
0 2− 3	− 14.5410	+ 5.9098	− 0.068	−14.807	− 0.034	+7.405	+ 0.0305	+ 9.186
1 1− 3	+ 8.32	− 1.80	+ 0.06	− 3.98	+ 0.01	−4.93	+ 0.02	+ 3.94
−1 4− 3	+ 78.80	+110.42	− 6.57	+ 7.31	− 1.09	−1.27	+ 5.05	− 5.79
0 3− 3	−161.2766	−222.5100	+ 4.448	− 5.196	+ 1.483	+1.732	− 3.016	+ 3.902
1 2− 3	+ 80.90	+111.56	+ 0.31	− 0.80	− 0.97	−0.80	− 0.44	+ 0.71

Arg=$\varkappa\gamma'+i'g'+ig$			C′		D′		E′		H′	
			sin.	cos.	cos.	sin.	sin.	cos.	cos.	sin.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	5−	3	+45.04	+19.60	+10.75	−4.21	+1.35	+0.53	−18.45	+6.60
0	4−	3	−88.7194	−35.9720	−8.494	+3.219	−2.123	−0.805	+15.720	−5.424
1	3−	3	+41.10	+13.26	+0.54	−0.30	+1.50	+0.62	−2.32	+0.82
−1	6−	3	+10.94	−1.60	+7.29	+0.92	+0.81	−0.08	−10.36	−1.51
0	5−	3	−20.6194	+4.0212	−5.973	−0.756	−1.195	+0.151	+8.906	+1.326
1	4−	3	+8.42	−2.84	+0.31	+0.15	+0.70	−0.14	−0.87	−0.43
−1	7−	3	+1.52	−1.30	+2.07	+1.40	+0.20	−0.13	−2.43	−1.90
0	6−	3	−2.6574	+2.6124	−1.685	−1.208	−0.281	+0.201	+2.197	+1.653
1	5−	3	+0.88	−1.24	−0.03	+0.17	+0.13	−0.12	−0.17	−0.23
−1	8−	3	+0.08	−0.32	+0.21	+0.58	+0.02	−0.05	−0.29	−0.61
0	7−	3	−0.1210	+0.5698	−0.228	−0.476	−0.033	+0.068	+0.259	+0.582
1	6−	3	0.00	−0.24	+0.03	+0.01	+0.01	−0.04	+0.03	−0.10
−1	9−	3	0.00	−0.06	0.00	+0.08	0.00	−0.01	+0.01	−0.10
0	8−	3	+0.0300	+0.0778	+0.007	−0.112	+0.001	+0.014	−0.012	+0.125
1	7−	3	−0.02	−0.02	−0.01	+0.04	0.00	−0.01	+0.02	−0.04
0	9−	3	+0.0092	+0.0068					−0.013	+0.018
−1	1−	4	−0.02	−0.06			+0.01	−0.01		
0	0−	4			0.000	0.000	−0.012	+0.022	0.000	0.000
1−	1−	4	+0.02	−0.02			+0.01	−0.01		
−1	2−	4	+0.14	+0.78	+0.08	−0.06	0.00	+0.01	−0.05	+0.05
0	1−	4	−0.3170	−1.5314	−0.014	−0.001	−0.014	+0.001	+0.008	+0.002
1	0−	4	+0.16	+0.78	−0.09	+0.04	+0.01	0.00	+0.05	−0.03
−1	3−	4	−0.06	−0.42	+0.55	+0.64	−0.08	−0.16	−0.37	−0.49
0	2−	4	−0.4270	+0.3300	−0.387	−0.597	−0.193	+0.297	+0.228	+0.380
1	1−	4	+0.26	−0.08	−0.53	+0.06	+0.17	−0.21	+0.37	+0.05
−1	4−	4	+9.38	+2.34	+18.52	+3.74	+2.91	−0.57	−11.93	−2.47
0	3−	4	−13.9576	−10.1730	−13.356	−2.842	−4.452	+0.947	+8.037	+1.743
1	2−	4	+6.16	+6.04	−0.88	−0.17	+2.97	−0.63	+1.34	+0.32
−1	5−	4	−56.78	+62.94	+6.27	+7.30	+0.87	−0.98	−4.94	−5.51
0	4−	4	+114.3794	−128.8768	−4.915	−5.487	−1.229	+1.372	+3.749	+3.748
1	3−	4	−57.56	+64.72	−0.18	+0.14	+0.61	−0.87	+0.19	+0.13
−1	6−	4	−8.30	+37.36	−1.09	−6.93	−0.10	+0.71	+1.62	+13.25
0	5−	4	+14.7968	−73.7206	+0.763	+5.729	+0.153	−1.146	−1.289	−11.745
1	4−	4	−4.94	+34.24	−0.16	−0.77	−0.16	+0.83	+0.31	+2.30
−1	7−	4	+3.20	+9.70	+1.84	−5.37	+0.17	+0.50	−2.97	+8.07
0	6−	4	−6.7892	−18.4008	−1.584	+4.553	−0.264	−0.759	+2.677	−7.182
1	5−	4	+3.76	+7.62	+0.26	−0.44	+0.19	+0.45	−0.59	+1.03
−1	8−	4	+1.62	+1.40	+1.62	−1.52	+0.13	+0.13	−2.25	+1.93
0	7−	4	−3.1668	−2.4336	−1.394	+1.288	−0.189	−0.184	+2.004	−1.770
1	6−	4	+1.42	+0.78	+0.16	−0.02	+0.12	+0.08	−0.31	+0.14
−1	9−	4	+0.36	+0.06	+0.53	−0.12	+0.04	+0.01	−0.70	+0.18
0	8−	4	−0.6972	−0.0630	−0.521	+0.131	−0.065	−0.016	+0.676	−0.152
1	7−	4	+0.28	0.00	+0.12	−0.01	+0.03	0.00	−0.12	−0.04
−1	10−	4	+0.06	0.00	+0.08	+0.02	+0.01	0.00	−0.11	−0.03
0	9−	4	−0.0978	+0.0514	−0.120	−0.032	−0.013	+0.004	+0.143	+0.043
1	8−	4	+0.02	−0.02	+0.06	+0.03	+0.01	0.00	−0.06	−0.04
0	10−	4	−0.0084	+0.0142	−0.018	−0.017			+0.020	+0.021

Arg=$\varkappa\gamma'+i'g'+ig$			C′		D′		E′		H′	
			sin.	cos.	cos.	sin.	sin.	cos.	cos.	sin.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	2	−5	0.00	+0.06						
0	1	−5								
1	0	−5	0.00	+0.06						
−1	3	−5	−0.02	−0.02	−0.02	−0.03				
0	2	−5			−0.002	−0.010				
1	1	−5	0.00	0.00	−0.01	+0.06				
−1	4	−5	+0.54	−0.12	+0.98	+0.22	+0.14	−0.12	−0.65	−0.16
0	3	−5	−0.9300	−0.2480	−0.854	−0.062	−0.285	+0.021	+0.519	+0.043
1	2	−5	+0.40	+0.20	−0.01	+0.23	+0.20	+0.01	+0.05	−0.12
−1	5	−5	−0.40	+8.28	+5.63	−12.61	+0.70	+1.61	−3.58	+7.89
0	4	−5	+4.7286	−14.4744	−4.506	+9.828	−1.126	−2.457	+2.704	−5.779
1	3	−5	−3.02	+6.88	+0.14	−0.37	+0.75	+1.65	+0.13	−0.27
−1	6	−5	−44.18	−24.80	+7.22	−4.34	+0.81	+0.51	−5.36	+3.48
0	5	−5	+90.6290	+49.8034	−5.765	+3.652	−1.153	−0.731	+3.984	−2.863
1	4	−5	−45.60	−25.16	+0.43	−0.15	+0.71	+0.38	−0.15	+0.09
−1	7	−5	−27.60	−0.50	−3.74	−0.40	−0.32	+0.05	+8.44	+0.92
0	6	−5	+54.5440	−0.0498	+3.214	+0.412	+0.536	−0.074	−7.710	−0.964
1	5	−5	−25.44	+1.14	−0.67	+0.04	−0.40	+0.02	+1.83	+0.16
−1	8	−5	−7.50	+4 14	−3.47	−2.14	−0.30	+0.17	+5.57	+3.56
0	7	−5	+14.2560	−8.4236	+3.017	+1.917	+0.431	−0.274	−5.064	−3.246
1	6	−5	−5.94	+4.24	−0.40	−0.35	−0.26	+0.18	+0.86	+0.67
−1	9	−5	−1.00	+1.72	−0.87	−1.56	−0.07	+0.11	+1.27	+2.28
0	8	−5	+1.8064	−3.3740	+0.822	+1.374	+0.103	−0.172	−1.176	−2.065
1	7	−5	−0.60	+1.48	−0.12	−0.17	−0.05	+0.10	+0.11	+0.34
−1	10	−5	−0.04	+0.40	+0.01	−0.60	0.00	+0.04	−0.01	+0.71
0	9	−5	−0.0460	−0.7472	+0.022	+0.502	+0.002	−0.056	−0.014	−0.683
1	8	−5	+0.04	+0.30	0.00	−0.03	0.00	+0.03	−0.01	+0.12
−1	11	−5	+0.02	+0.06	+0.04	−0.09	0.00	+0.01	−0.05	+0.12
0	10	−5	−0.0736	−0.1074	−0.056	+0.114	−0.006	−0.011	+0.075	−0.143
1	9	−5	+0.04	+0.02	+0.04	−0.05	0.00	+0.01	−0.05	+0.06
0	11	−5	−0.0198	−0.0104	−0.023	+0.016	−0.002	−0.001	+0.028	−0.018
−1	4	−6	+0.08	−0.06	+0.01	+0.04	0.00	−0.01		
0	3	−6			−0.033	−0.019	−0.011	+0.006	+0.020	+0.012
1	2	−6	+0.04	0.00	+0.01	+0.01	+0.01	0.00		
−1	5	−6	+0.28	+0.54	+0.74	−0.83	+0.14	+0.09	−0.44	+0.58
0	4	−6	−0.0416	−1.0256	−0.452	+0.793	−0.113	−0.198	+0.271	−0.470
1	3	−6	−0.02	+0.52	+0.09	−0.03	+0.07	+0.14	−0.06	−0.04
−1	6	−6	−6.26	+0.96	−7.64	−5.69	−0.81	+0.60	+4.63	+3.50
0	5	−6	+11.9066	+0.5800	+6.203	+4.779	+1.241	−0.956	−3.565	−2.824
1	4	−6	−5.88	−0.72	−0.58	−0.46	−0.84	+0.64	+0.08	+0.12
−1	7	−6	+8.22	−28.22	−2.46	−6.42	−0.24	+0.61	+2.00	+4.72
0	6	−6	−16.2212	+58.0064	+2.129	+5.316	+0.355	−0.886	−1.716	−3.748
1	5	−6	+8.30	−29.26	−0.13	−0.56	−0.17	+0.55	+0.10	+0.35
−1	8	−6	−3.58	−18.54	−0.67	+1.61	−0.06	−0.11	+1.74	−4.74
0	7	−6	+7.7240	+36.7056	+0.685	−1.403	+0.098	+0.200	−1.746	+4.479
1	6	−6	−4.28	−17.18	−0.11	+0.41	−0.05	−0.17	+0.42	−1.27

Arg=$\varkappa\gamma'+i'g'+ig$			C′		D′		E′		H′	
			sin.	cos.	cos.	sin.	sin.	cos.	cos.	sin.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	9	−6	−4.30	−5.10	−1.91	+2.01	−0.13	−0.15	+3.41	−3.36
0	8	−6	+8.6196	+9.7602	+1.725	−1.775	+0.216	+0.222	−3.203	+3.094
1	7	−6	−4.14	−4.10	−0.32	+0.29	−0.14	−0.14	+0.74	−0.59
−1	10	−6	−1.66	−0.62	−1.37	+0.45	−0.09	−0.03	+2.02	−0.62
0	9	−6	+3.2108	+1.0384	+1.200	−0.410	+0.133	+0.046	−1.882	+0.598
1	8	−6	−1.44	−0.30	−0.15	+0.05	−0.08	−0.02	+0.36	−0.08
−1	11	−6	−0.36	+0.04	−0.52	−0.12	−0.03	0.00	+0.63	+0.14
0	10	−6	+0.7100	−0.1642	+0.434	+0.070	+0.043	−0.007	−0.615	−0.114
1	9	−6	−0.30	+0.12	−0.01	0.00	−0.02	0.00	+0.11	+0.04
−1	12	−6	−0.06	+0.06	−0.05	−0.05			+0.10	+0.07
0	11	−6	+0.0996	−0.0982	+0.096	+0.074			−0.125	−0.102
1	10	−6	−0.02	+0.06	−0.05	−0.05			+0.04	+0.06
0	12	−6							−0.014	−0.034
−1	5	−7	+0.02	+0.02	+0.05	0.00	+0.02	0.00		
0	4	−7			−0.046	+0.035	−0.011	−0.009	+0.027	−0.021
1	3	−7	+0.02	+0.02	+0.03	−0.02	+0.01	+0.01		
−1	6	−7	−0.42	+0.34	−0.52	−0.83	−0.04	+0.12	+0.36	+0.51
0	5	−7	+0.9088	−0.3226	+0.555	+0.651	+0.111	−0.130	−0.322	−0.382
1	4	−7	−0.44	+0.12	−0.07	−0.18	−0.08	+0.08	−0.03	+0.08
−1	7	−7	−1.62	−4.22	−4.81	+3.98	−0.43	−0.36	+2.87	−2.36
0	6	−7	+1.7330	+8.4592	+4.150	−3.363	+0.691	+0.560	−2.419	+1.879
1	5	−7	−0.62	−4.24	−0.54	+0.49	−0.46	−0.38	+0.24	−0.13
−1	8	−7	+16.66	+0.78	−5.11	+0.90	−0.44	−0.08	+3.73	−0.81
0	7	−7	−34.30	−1.25	+4.426	−0.772	+0.632	+0.110	−3.114	+0.787
1	6	−7	+17.36	+0.74	−0.67	−0.03	−0.40	−0.04	+0.42	−0.10
−1	9	−7	+11.44	−4.88	+0.31	+0.51	+0.02	−0.03	−2.37	−1.72
0	8	−7	−22.65	+10.12	−0.309	−0.505	−0.039	+0.063	+2.253	+1.678
1	7	−7	+10.66	−5.14	+0.23	+0.09	+0.06	−0.03	−0.68	−0.39
−1	10	−7	+3.04	−3.88	+1.02	+1.55	+0.06	−0.10	−1.78	−2.83
0	9	−7	−5.87	+7.73	−0.874	−1.367	−0.097	+0.152	+1.664	+2.647
1	8	−7	+2.46	−3.70	+0.14	+0.22	+0.06	−0.10	−0.35	−0.59
−1	11	−7	+0.24	−1.44	+0.20	+1.11	+0.01	−0.06	−0.15	−1.66
0	10	−7	−0.38	+2.78	−0.110	−0.948	−0.011	+0.095	+0.150	+1.546
1	9	−7	+0.08	−1.24	−0.08	+0.09	+0.01	−0.05	0.00	−0.32
−1	12	−7	−0.08	−0.32	−0.09	+0.42	−0.01	−0.02	+0.24	+0.15
0	11	−7	+0.2596	+0.6148	+0.131	−0.340	+0.012	+0.031	−0.205	−0.500
1	10	−7	−0.14	−0.26	−0.10	+0.01	−0.01	−0.02	+0.04	+0.57
−1	13	−7	−0.06	−0.06	−0.05	+0.05			+0.08	−0.05
0	12	−7	+0.1208	+0.0774	+0.083	−0.071			−0.121	+0.100
1	11	−7	−0.04	−0.04	−0.04	+0.04			+0.07	−0.07
−1	6	−8	−0.04	+0.06	+0.01	−0.05	+0.01	+0.01		
0	5	−8			+0.022	+0.064	+0.004	−0.013	−0.013	−0.037
1	4	−8	−0.02	+0.02	−0.01	−0.04	0.00	+0.01		
−1	7	−8	−0.36	−0.28	−0.85	+0.21	−0.08	−0.01	+0.51	−0.13
0	6	−8	+0.4538	+0.6688	+0.668	−0.286	+0.111	+0.048	−0.386	+0.162
1	5	−8	−0.18	−0.36	−0.11	+0.05	−0.07	−0.03	+0.04	−0.02

Arg=$\varkappa\gamma'+i'g'+ig$			C′		D′		E′		H′	
			sin.	cos.	cos.	sin.	sin.	cos.	cos.	sin.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	8	−8	+2.55	−1.67	+1.75	+3.55	+0.15	−0.28	−0.95	−2.24
0	7	−8	−5.318	+2.562	−1.484	−3.153	−0.213	+0.450	+0.788	+1.812
1	6	−8	+2.71	−1.12	+0.25	+0.54	+0.16	−0.30	−0.11	−0.26
−1	9	−8	+1.80	+9.11	+0.09	+3.80	−0.01	−0.28	−0.11	+0.50
0	8	−8	−3.950	−18.796	+0.060	−3.322	+0.007	+0.415	+0.107	−2.379
1	7	−8	+1.91	+9.55	−0.24	+0.51	−0.02	−0.26	+0.05	+2.72
−1	10	−8	+4.59	+6.46	0.00	+0.13	+0.01	−0.02	−1.28	+0.89
0	9	−8	−9.462	−12.820	−0.199	−0.204	−0.022	+0.023	+1.258	−0.923
1	8	−8	+4.68	+6.03	+0.23	+0.03	+0.01	0.00	−0.31	+0.24
−1	11	−8	+3.152	+1.554	+0.32	−0.68	+0.06	+0.02	−2.12	+0.72
0	10	−8	−6.282	−3.010	−0.956	+0.323	−0.096	−0.032	+1.998	−0.693
1	9	−8	+2.96	+1.24	+0.93	+0.25	+0.06	+0.02	−0.45	+0.20
−1	12	−8	+1.11	−0.04	+0.22	−0.18	+0.04	0.00	−1.28	−0.06
0	11	−8	−2.220	+0.122	−0.683	−0.071	−0.062	+0.006	+1.162	+0.138
1	10	−8	+0.99	−0.13	+0.66	+0.29	+0.04	0.00	−0.19	−0.13
−1	13	−8	+0.26	−0.16	+0.05	+0.09			−0.41	−0.17
0	12	−8	−0.472	+0.332	−0.242	−0.158	−0.020	+0.013	+0.392	+0.261
1	11	−8	+0.19	−0.15	+0.25	+0.11			−0.07	−0.16
−1	7	−9	−0.04	−0.01	−0.04	−0.02				
0	6	−9			+0.067	−0.002	+0.011	0.000	−0.039	+0.001
1	5	−9	−0.02	−0.04	−0.04	0.00				
−1	8	−9	+0.15	−0.30	−0.02	+0.66	0.00	−0.05	−0.08	−0.54
0	7	−9	−0.400	+0.502	−0.073	−0.569	−0.010	+0.081	+0.038	+0.326
1	6	−9	+0.21	−0.21	+0.04	+0.12	+0.01	−0.05	+0.07	+0.09
−1	9	−9	+1.44	+1.40	+2.36	−0.45	+0.17	+0.03	−1.23	+0.39
0	8	−9	−2.490	−3.018	−2.158	+0.417	−0.270	−0.052	+1.221	−0.184
1	7	−9	+1.16	+1.58	+0.44	−0.15	+0.18	+0.03	−0.33	−0.12
−1	10	−9	−4.61	+2.19	+2.54	+0.51	+0.17	−0.04	−1.52	+0.43
0	9	−9	+9.514	−4.722	−2.303	−0.536	−0.256	+0.060	+1.671	+0.278
1	8	−9	−4.82	+2.34	+0.44	+0.21	+0.16	−0.04	−0.68	−0.85
−1	11	−9	−3.306	+3.732	+1.19	+0.22	+0.02	0.00	+0.66	+1.13
0	10	−9	+6.502	−7.568	−0.354	−0.061	−0.035	+0.006	−0.232	−0.790
1	9	−9	−3.06	+3.69	−0.83	−0.14	+0.01	0.00	−0.31	−0.13
−1	12	−9	−0.62	+2.38	+0.48	−0.52	0.00	+0.04	+0.72	+0.95
0	11	−9	+1.114	−4.718	+0.038	+0.593	+0.004	−0.054	−0.139	−1.378
1	10	−9	−0.44	+2.19	−0.57	−0.26	0.00	+0.04	−0.54	+0.81
−1	13	−9	+0.21	+0.84	+0.22	−0.52	+0.01	+0.03	+0.03	+0.41
0	12	−9	−0.448	−1.624	−0.152	+0.451	−0.013	−0.037	+0.290	−0.896
1	11	−9	+0.24	+0.73	−0.02	−0.06	+0.01	+0.03	−0.42	+0.74
−1	8	−10	+0.02	−0.05	−0.02	+0.04				
0	7	−10			+0.015	−0.059			−0.009	+0.034
1	6	−10	+0.04	−0.04	−0.02	+0.04				
−1	9	−10	+0.26	+0.04	+0.65	+0.01	+0.04	−0.01	−0.21	+0.02
0	8	−10	−0.402	−0.175	−0.424	−0.058	−0.053	+0.007	+0.240	+0.037
1	7	−10	+0.21	+0.13	−0.09	+0.12	+0.04	0.00	−0.11	−0.08

Arg=$x\gamma'+i'g'+ig$			C′		D′		E′		H′	
			sin.	cos.	cos.	sin.	sin.	cos.	cos.	sin.
x	i'	i	″	″	″	″	″	″	″	″
−1	10	−10	−2.09	+1.09	−0.15	−1.39	+0.01	+0.09	−0.44	+0.39
0	9	−10	+1.490	−1.960	−0.091	+1.346	−0.010	−0.149	−0.093	−0.745
1	8	−10	−2.21	+0.97	+0.23	−0.35	+0.01	+0.10	+0.35	+0.57
−1	11	−10	−1.80	−2.07	0.00	−1.27	+0.05	+0.10	−0.57	+0.79
0	10	−10	+3.786	+4.334	−0.699	+1.469	−0.070	−0.147	+0.433	−1.082
1	9	−10	−1.90	−2.24	+0.93	−0.63	+0.05	+0.09	−0.03	+0.63
−1	12	−10	−2.64	−1.41	−0.06	+0.03	+0.01	+0.02	+0.10	+0.55
0	11	−10	+5.426	+2.874	−0.216	+0.318	−0.019	−0.029	−0.420	−0.059
1	10	−10	−2.65	−1.37	+0.33	−0.39	+0.01	+0.01	+0.44	−0.51
−1	13	−10	−1.69	−0.07	−0.12	+0.24			+0.93	+0.38
0	12	−10	+3.264	+0.086	+0.320	+0.074	+0.027	−0.006	−1.100	−0.076
1	11	−10	−1.52	+0.03	−0.31	−0.33			+0.53	−0.28
−1	9	−11	+0.04	−0.01	+0.02	+0.03				
0	8	−11			−0.045	−0.026			+0.025	+0.015
1	7	−11	+0.02	+0.02	+0.03	+0.02				
−1	10	−11	+0.06	+0.18	+0.05	−0.22			−0.18	+0.08
0	9	−11	+0.076	−0.314	−0.118	+0.283	−0.013	−0.031	+0.070	−0.157
1	8	−11	−0.03	+0.14	+0.15	−0.15			+0.06	+0.11
−1	11	−11	−0.77	−0.25	−0.33	−0.47	−0.05	+0.02	+0.26	−0.11
0	10	−11	+1.440	+0.620	+0.763	+0.267	+0.076	−0.027	−0.410	−0.179
1	9	−11	−0.68	−0.33	−0.66	+0.15	−0.05	+0.02	+0.28	+0.32
−1	12	−11	+0.82	−1.23	−0.59	−0.79	−0.05	+0.04	+0.37	+0.13
0	11	−11	−1.672	+2.584	+0.855	+0.672	+0.078	−0.061	−0.637	−0.430
1	10	−11	+0.93	−1.31	−0.54	−0.09	−0.05	+0.04	+0.47	+0.46
−1	13	−11	+0.47	−1.78	−0.36	−0.29			−0.19	−0.29
0	12	−11	−0.922	+3.538	−0.230	+0.275	+0.019	−0.023	+0.033	+0.518
1	11	−11	+0.43	−1.69	+0.10	−0.04			+0.13	−0.43
−1	10	−12	+0.03	+0.01	+0.03	−0.01				
0	9	−12			−0.029	+0.029			+0.017	−0.016
1	8	−12	0.00	+0.01	+0.02	−0.02				
−1	11	−12	−0.118	+0.024	−0.10	−0.18			+0.02	+0.03
0	10	−12	+0.252	−0.008	+0.171	+0.126	+0.017	−0.013	−0.091	−0.074
1	9	−12	−0.12	0.00	−0.11	−0.02			+0.09	+0.08
−1	12	−12	+0.02	−0.49	+0.03	+0.32			+0.26	−0.16
0	11	−12	−0.176	+0.918	+0.260	−0.400	+0.024	+0.036	−0.176	+0.184
1	10	−12	+0.11	−0.46	−0.34	+0.21			−0.02	−0.08
−1	13	−12	+0.81	+0.19	−0.09	+0.21			0.00	−0.04
0	12	−12	−1.594	−0.368	+0.512	−0.367	+0.043	+0.030	−0.066	+0.103
1	11	−12	+0.78	+0.22	−0.58	+0.27			+0.07	−0.09

The second factors of δT and $\delta T'$ have all been given except $\delta\frac{h}{h_0}$, $\frac{u_1}{\cos i}$, and the similar quantities for Saturn. To complete the matter of this chapter these are now given:

Arg=$i'g'+ig$	$\delta\frac{h}{h_0}$		$\frac{u_1}{\cos i}$	
	cos.	sin.	cos.	sin.
i' i	″	″	″	″
0 0	+ 4.7195		−0.0090	
0 − 1	− 0.0380	+0.1732	+0.1248	+0.2825
0 − 2	+ 0.0074	−0.0047	+0.0188	−0.0226
0 − 3	0.0000	−0.0004	+0.0001	−0.0010
1 + 3	− 0.0001	+0.0004		
1 + 2	+ 0.0008	+0.0007	+0.0060	+0.0014
1 + 1	+ 0.0008	−0.0200	+0.1379	−0.0417
1 0	− 0.1638	+0.7967	+0.1799	+0.1231
1 − 1	− 0.7887	+4.0498	+0.0694	−0.0304
1 − 2	− 0.1611	−0.1285	+0.4137	−0.0936
1 − 3	− 0.0076	−0.0085	+0.0104	−0.0101
1 − 4	− 0.0004	−0.0001	+0.0004	+0.0004
2 + 1	+ 0.0011	−0.0019	+0.0088	+0.0179
2 0	− 0.1467	+0.0174	−0.0150	+0.3003
2 − 1	−12.3301	−2.9105	−0.0902	+0.0855
2 − 2	+12.6079	+5.4106	+0.1290	+0.2279
2 − 3	+ 0.1385	+0.3093	+0.0031	−0.1350
2 − 4	− 0.0023	+0.0124	−0.0022	−0.0048
2 − 5	− 0.0001	+0.0004	−0.0008	−0.0004
3 + 1	+ 0.0002	−0.0001	+0.0029	−0.0011
3 0	− 0.0178	−0.0064	+0.0405	−0.0498
3 − 1	+ 1.8188	+1.5474	+0.0394	+0.0061
3 − 2	+ 2.0702	+4.3071	+0.4384	+0.6888
3 − 3	+ 3.5824	−5.0503	+0.0752	−0.0068
3 − 4	+ 0.2743	−0.0696	−0.0575	−0.0112
3 − 5	+ 0.0123	+0.0035	−0.0030	0.0000
3 − 6	+ 0.0004	+0.0003		
4 − 0	− 0.0015	−0.0012	+0.0051	−0.0011
4 − 1	+ 0.0446	+0.1053	+0.0250	+0.0167
4 − 2	− 0.4062	+1.5885	+0.0124	+0.0564
4 − 3	+ 2.3874	−0.7898	−0.3186	+0.1250
4 − 4	− 1.9577	−2.1463	−0.0344	−0.0220
4 − 5	− 0.0114	−0.1996	−0.0125	+0.0254
4 − 6	+ 0.0053	−0.0100	−0.0009	+0.0011
4 − 7	+ 0.0003	−0.0003		
5 0	− 0.0001	−0.0002	−0.0032	−0.0079
5 − 1	− 0.0008	+0.0084	−0.0710	−0.1649
5 − 2	+ 5.5593	−4.2341	+0.0038	+0.0002

Arg=$i'g'+ig$	$\delta\frac{h}{h_0}$		$\frac{u_1}{\cos i}$	
	cos.	sin.	cos.	sin.
i' i	″	″	″	″
5— 3	+0.6908	+0.2297	+3.5980	+0.3498
5— 4	—0.2121	—1.3907	+0.1867	+0.1039
5— 5	—1.2071	+0.6905	—0.0066	+0.0140
5— 6	—0.1285	—0.0185	+0.0100	+0.0088
5— 7	—0.0070	—0.0057	+0.0010	+0.0010
5— 8	—0.0003	—0.0006		
6— 1	—0.0005	+0.0003	0.0000	—0.0010
6— 2	+0.0306	—0.0005	+0.0030	—0.0005
6— 3	+0.1202	+0.1707	+0.0188	+0.0102
6— 4	+0.1892	—0.3905	—0.0223	+0.0698
6— 5	—0.7940	—0.0282	+0.0315	+0.0054
6— 6	+0.1948	+0.6441	+0.0051	+0.0100
6— 7	—0.0281	+0.0760	+0.0050	—0.0041
6— 8	—0.0053	+0.0040	—0.0011	0.0000
6— 9	—0.0005	+0.0002		
7— 2	+0.0015	+0.0009	+0.0024	+0.0010
7— 3	—0.0004	+0.1008	+0.0009	+0.0004
7— 4	+0.0990	—0.0557	—0.0435	+0.0553
7— 5	—0.2214	—0.1557	+0.0216	+0.0161
7— 6	—0.1033	+0.4354	+0.0048	—0.0124
7— 7	+0.3269	—0.0200	+0.0067	—0.0017
7— 8	+0.0405	+0.0260	—0.0010	—0.0026
7— 9	+0.0018	+0.0041		
7—10	+0.0003	+0.0004		
8— 2	0.0000	+0.0003		
8— 3	+0.0056	—0.0092	+0.0002	—0.0002
8— 4	+0.0293	+0.0021	+0.0086	—0.0030
8— 5	—0.0272	—0.0686	+0.0076	+0.0112
8— 6	—0.1205	+0.1199	+0.0086	—0.0086
8— 7	+0.2271	+0.1063	—0.0045	—0.0034
8— 8	+0.0283	—0.1572	0.0000	+0.0033
8— 9	+0.0200	—0.0201	—0.0017	+0.0006
8—10	+0.0029	—0.0006		
8—11	+0.0002	+0.0002		
9— 3	+0.0006	—0.0001		
9— 4	+0.0077	+0.0057	+0.0002	—0.0001
9— 5	+0.0038	—0.0181	—0.0003	+0.0066
9— 6	—0.0485	+0.0107	+0.0059	—0.0019
0— 7	+0.0591	+0.0876	—0.0030	—0.0051
9— 8	+0.0845	—0.1111	—0.0018	+0.0013
9— 9	—0.0709	—0.0323	—0.0011	—0.0005
9—10	—0.0087	—0.0138		
9—11	+0.0001	—0.0020		
9—12	+0.0001	—0.0002		

$\text{Arg}=i'g'+ig$	$\delta\frac{h}{h_0}$		$\frac{u_1}{\cos i}$	
	cos.	sin.	cos.	sin.
i' i	″	″	″	″
10—4	−0.0051	−0.0179		
10—5	+0.0029	−0.0032	+0.0059	−0.0147
10—6	−0.0117	−0.0044	+0.0024	−0.0006
10—7	+0.0014	+0.0335	−0.0006	−0.0033
10—8	+0.0600	−0.0253	−0.0028	+0.0012
10—9	−0.0499	−0.0587	+0.0005	+0.0010
10—10	−0.0239	+0.0297		
10—11	−0.0088	+0.0030		
10—12	−0.0013	−0.0003		
11—5	+0.0014	0.0000		
11—6	−0.0019	−0.0022	+0.0006	+0.0005
11—7	−0.0043	+0.0080	+0.0005	+0.0013
11—8	+0.0220	+0.0031	−0.0018	−0.0004
11—9	−0.0076	−0.0386	+0.0005	+0.0018
11—10	−0.0376	+0.0205		
11—11	+0.0107	+0.0150		
11—12	+0.0006	+0.0051		
12—6	−0.0002	−0.0007		
12—7	−0.0020	+0.0009		
12—8	+0.0050	+0.0041		
12—9	+0.0048	−0.0136		
12—10	−0.0237	−0.0003		
12—11	+0.0056	+0.0224		
12—12	+0.0091	−0.0024		

$\text{Arg}=i'g'+ig$	$\delta\frac{h'}{h_0'}$		$\frac{u_1'}{\cos i'}$	
	cos.	sin.	cos.	sin.
i' i	″	″	″	″
0 0	−173.0111		−0.1347	
1 0	+ 1.9899	− 9.5637	+1.6004	+1.1383
2 0	+ 0.4463	− 0.4873	+0.1788	+0.4386
3 0	+ 0.0494	− 0.0142	−0.0011	+0.0529
4 0	+ 0.0043	+ 0.0011	−0.0031	+0.0037
−3—1	− 0.0003	− 0.0004	−0.0027	+0.0005
−2—1	− 0.0013	− 0.0042	−0.0206	−0.0022
−1—1	− 0.0144	+ 0.0529	−0.1508	−0.0996
0—1	+ 4.6883	− 22.3609	+1.9041	+4.0361
1—1	− 74.5932	+355.4266	+0.9570	−0.6482
2—1	+ 27.6869	+ 20.3654	+0.9911	−1.0278
3—1	− 4.6889	− 3.1427	−0.3682	+0.0102
4—1	− 0.1221	− 0.2223	+0.0769	+0.0576
5—1	+ 0.0009	− 0.0182	+0.0073	+0.0096
6—1	+ 0.0010	− 0.0011	+0.0018	+0.0025

Arg=$i'g'+ig$	$\delta\frac{h'}{h_0'}$		$\frac{u_1'}{\cos i'}$	
	cos.	sin.	cos.	sin.
i' i	″	″	″	″
— 2— 2	— 0.0002	+ 0.0002		
— 1— 2	— 0.0027	+ 0.0060	—0.0036	+0.0078
0— 2	+ 0.2089	— 1.0524	+0.0452	+0.0305
1— 2	— 2.7604	+15.4062	—1.0007	+0.2149
2— 2	—31.2415	—12.8675	—0.2513	—0.2065
3— 2	— 5.1200	—10.6134	+0.4513	+0.2824
4— 2	+ 1.0044	— 3.9387	—1.0240	—8.6226
5— 2	—13.8298	+10.6323	—0.0054	—0.0206
6— 2	— 0.0766	+ 0.0013	+0.2446	—0.0783
7— 2	— 0.0041	— 0.0028	+0.0157	—0.0039
8— 2	— 0.0001	— 0.0004		
— 1— 3	— 0.0002	+ 0.0003		
0— 3	+ 0.0122	— 0.0568	+0.0074	+0.0156
1— 3	— 0.1456	+ 0.7903	—0.0052	+0.0303
2— 3	— 0.3473	— 0.7410	+0.0022	+0.4682
3— 3	— 8.8478	+12.4735	—0.1153	+0.1384
4— 3	— 5.8966	+ 1.9503	+0.2249	—0.0828
5— 3	— 1.7097	— 0.5683	+0.2014	+0.0265
6— 3	— 0.2996	— 0.4225	+0.1305	+0.0854
7— 3	+ 0.0008	— 0.2547	—0.0107	—0.0153
8— 3	— 0.0175	+ 0.0241	—0.0013	+0.0016
9— 3	— 0.0013	+ 0.0004		
0— 4	+ 0.0007	— 0.0032		
1— 4	— 0.0082	+ 0.0434	+0.0009	—0.0009
2— 4	+ 0.0033	— 0.0282	+0.0095	+0.0127
3— 4	— 0.6779	+ 0.1715	+0.2184	+0.0464
4— 4	+ 4.8353	+ 5.3003	+0.0712	+0.0795
5— 4	+ 0.5238	+ 3.4347	—0.0104	—0.0809
6— 4	— 0.4683	+ 0.9656	+0.0240	—0.0692
7— 4	— 0.2448	+ 0.1383	+0.0258	—0.0243
8— 4	— 0.0723	— 0.0069	+0.0133	—0.0048
9— 4	— 0.0183	— 0.0150	—0.0376	—0.0065
10— 4	+ 0.0123	+ 0.0463		
11— 4	— 0.0006	— 0.0004		
2— 5	+ 0.0010	— 0.0008		
3— 5	— 0.0290	— 0.0082	+0.0104	+0.0009
4— 5	+ 0.0278	+ 0.4936	+0.0463	—0.0993
5— 5	+ 2.9806	— 1.7054	+0.0527	—0.0341
6— 5	+ 1.9609	+ 0.0693	—0.0289	—0.0032
7— 5	+ 0.5487	+ 0.3849	—0.0276	—0.0173
8— 5	+ 0.0668	+ 0.1698	—0.0084	—0.0137
9— 5	— 0.0094	+ 0.0437	—0.0003	—0.0058
10— 5	— 0.0080	+ 0.0072	+0.0007	—0.0022
11— 5	— 0.0029	+ 0.0001	+0.0010	—0.0010
12— 5	— 0.0007	— 0.0006		

Arg=$i'g'+ig$	$\delta\frac{h'}{h_0'}$		$\frac{u_1'}{\cos i'}$	
	cos.	sin.	cos.	sin.
i' i	″	″	″	″
3— 6	+0.0026	—0.0010		
4— 6	—0.0136	+0.0252	+0.0044	—0.0065
5— 6	+0.3180	+0.0470	—0.0426	—0.0327
6— 6	—0.4806	—1.5908	—0.0133	—0.0338
7— 6	+0.2549	—1.0753	—0.0047	+0.0095
8— 6	+0.2972	—0.2957	—0.0117	+0.0110
9— 6	+0.1198	—0.0262	—0.0083	+0.0024
10— 6	+0.0290	+0.0109	—0.0029	—0.0005
11— 6	+0.0042	+0.0059	—0.0008	—0.0008
12— 6	+0.0002	+0.0016		
4— 7	—0.0010	+0.0018	+0.0013	—0.0013
5— 7	+0.0170	+0.0142	—0.0037	—0.0037
6— 7	+0.0690	—0.1870	—0.0216	+0.0171
7— 7	—0.8074	+0.0500	—0.0197	+0.0031
8— 7	—0.5610	—0.2616	+0.0009	+0.0028
9— 7	—0.1460	—0.2172	+0.0042	+0.0067
10— 7	—0.0022	—0.0828	+0.0015	+0.0044
11— 7	+0.0110	—0.0186	—0.0006	+0.0032
12— 7	+0.0036	—0.0026	—0.0011	+0.0005
6— 8	+0.0132	—0.0098		
7— 8	—0.1006	—0.0638	+0.0039	+0.0116
8— 8	—0.0698	+0.3884	—0.0012	+0.0107
9— 8	—0.2078	+0.2740		
10— 8	—0.1486	+0.0610		
11— 8	—0.0534	—0.0094	+0.0027	0.0000
12— 8	—0.0122	—0.0092		
7— 9	—0.0048	—0.0096		
8— 9	—0.0492	+0.0504	+0.0057	—0.0014
9— 9	+0.1744	+0.0784	+0.0093	+0.0013
10— 9	+0.1248	+0.1436		
11— 9	+0.0196	+0.0950		
12— 9	—0.0116	+0.0342		
8—10	—0.0080	+0.0026		
9—10	+0.0224	+0.0356		
10—10	+0.0598	—0.0726		
11—10	+0.0922	—0.0488		
12—10	+0.0588	+0.0008		
9—11	—0.0002	+0.0054		
10—11	+0.0218	—0.0078		
11—11	—0.0272	—0.0378		
12—11	—0.0154	—0.0558		
10—12	+0.0026	+0.0010		
11—12	—0.0008	—0.0132		
12—12	—0.0226	+0.0064		

CHAPTER IX.

CALCULATION OF THE TERMS OF δT AND $\delta T'$ WHOSE ARGUMENTS ARE γ AND γ'.

Being now in possession of the several factors of the terms of δT and $\delta T'$, we could proceed immediately to the calculation of the terms, strictly of the second order, which arise from these quantities. But the more important parts of these functions are the terms coming from the secular variations of the elements. This prominence is kept up in the terms of the third, and apparently of all higher orders. And it is, perhaps, the most surprising instance in the planetary theories of a lack of convergence that the secular variations of the eccentricities and places of the perihelia of Jupiter and Saturn are augmented about a fourth part by the terms of the second order with respect to disturbing forces. Since the mass of Jupiter is less than $\frac{1}{1000}$ of that of the Sun it would naturally be supposed that the ratio of the second to the first-order terms would be somewhere in the neighborhood of this fraction. It is, however, 250 times larger.

By far the larger portions of these second-order terms arise from the terms of δT and $\delta T'$, having severally the arguments γ and γ'. By computing these portions at the outset, and annexing them to the first-order terms corresponding to the same arguments before proceeding to the general calculation of δT and $\delta T'$, we shall include in the determination of the second-order terms the more notable portion of the third-order terms. In like manner, on arriving at the general computation of the latter, we shall first compute the terms having the arguments γ and γ', and annexing them to the second-order terms, shall then be able to include the more remarkable portion of the fourth-order terms in that of the third. The modifications which this mode of proceeding requires in the values of the second factors are readily perceived.

In this connection it will be interesting to see how much each set of terms of the second factors of δT and $\delta T'$ contributes to the terms under consideration. Hence, I enter into some details relative to them. Defining these sets of terms by the arguments on which they depend, the general form of which is $i'g' + ig$, I have arranged in the following table the component parts of the coefficients of $\sin \gamma$ and $\cos \gamma$ in δT and of $\sin \gamma'$ and $\cos \gamma'$ in $\delta T'$. The numbers given are in units of the seventh decimal of the second of arc for δT and in units of the sixth decimal for $\delta T'$. They arise from multiplying the terms in the second factors having the argument $i'g' + ig$ by the terms of the first factors having the two arguments $\pm \gamma + i'g' + ig$ in the case of δT, or $\pm \gamma' + i'g' + ig$ in the case of $\delta T'$. Thus, it is plain that, since there are eight terms in both δT and $\delta T'$, the numbers tabulated are the sums of thirty-two component parts. However, in many cases some of these have no significant values:

Arg $= i'g' + ig$	δT		$\delta T'$	
	sin $(-\gamma)$	cos $(-\gamma)$	sin γ'	cos γ'
i' i				
0 0	− 168	+ 155	− 6194	− 9863
0 − 1	+ 1900	− 4860	+ 64527	− 7326
0 − 2	+ 1	− 7	+ 315	− 41
0 − 3			+ 1	0
1 + 1	+ 31	− 64	− 51	+ 56
1 0	+ 10145	− 5731	− 9674	+ 24596
1 − 1	+ 27548	− 24944	− 90608	− 5216
1 − 2	− 508	− 275	+ 1184	+ 6988
1 − 3	+ 1	0	− 1	+ 17
2 − 0	+ 174	− 59	− 23	+ 260
2 − 1	−1618217	+ 717049	+ 63191	−365746
2 − 2	− 45163	+ 10046	+ 28304	+ 3690
2 − 3	+ 5291	− 624	+ 374	+ 1392
2 − 4	+ 16	+ 6	+ 2	+ 4
3 0	− 1	− 7	− 8	+ 1
3 − 1	− 18475	+ 4121	+ 7070	− 2983
3 − 2	− 49846	+ 66600	+ 59124	− 68574
3 − 3	− 9443	+ 12882	+ 5280	− 5075
3 − 4	+ 1050	− 364	+ 53	+ 184
3 − 5	+ 4	0		
4 − 1	+ 9	− 12	− 10	+ 27
4 − 2	− 303245	+1022420	+944393	−508415
4 − 3	+ 19018	− 63009	− 57800	+ 31889
4 − 4	− 2905	+ 4524	+ 1506	− 1688
4 − 5	+ 350	− 189	+ 2	+ 47
4 − 6	+ 2	0		
5 − 1	0	+ 1	0	+ 1
5 − 2	− 236978	+ 700335	+645280	−373622
5 − 3	− 41087	+ 145928	+137249	− 71606
5 − 4	− 1252	− 13744	− 13756	+ 1566
5 − 5	− 948	+ 1581	+ 471	− 573
5 − 6	+ 125	− 73	0	+ 14
6 − 2	+ 33	− 3	+ 3	+ 34
6 − 3	+ 420	− 1516	− 1414	+ 719
6 − 4	+ 811	− 2867	− 2642	+ 1364
6 − 5	+ 716	− 2499	− 2290	+ 1257
6 − 6	− 292	+ 508	+ 136	− 179
6 − 7	+ 42	− 22	0	+ 5

Arg$=i'g'+ig$	δT		δT$'$	
	sin $(-\gamma)$	cos $(-\gamma)$	sin γ'	cos γ'
i' i				
7— 3	−122	+427	+403	−210
7— 4	+165	−601	−563	+283
7— 5	+225	−729	−655	+368
7— 6	+168	−636	−598	+322
7— 7	−93	+163	+40	−55
7— 8	+14	−8	−1	+2
8— 3	−1	+1	+1	−1
8— 4	0	−1	−1	0
8— 5	+29	−105	−97	+49
8— 6	+66	−230	−210	+109
8— 7	+36	−163	−161	+82
8— 8	−27	+51	+9	−18
8— 9	+5	−5	+1	−1
9— 4	−10	+38	+37	−18
9— 5	+5	−10	−11	+6
9— 6	+11	−35	−32	+17
9— 7	+20	−77	−71	+37
9— 8	+9	−41	−45	+20
9— 9	−8	+18	+3	−8
10— 4	−4	+10	+9	−5
10— 5	−2	+5	+5	−3
10— 6	0	−4	−4	0
10— 7	+5	−13	−12	+5
10— 8	+9	−27	−24	+12
10— 9	−1	−10	−14	+7
11— 8	0	−5	−5	+4
11— 9	+1	−8	−8	+5
11—10	+3	−4	−4	+4
12— 9	0	−2	−1	0
12—10	0	−4	−4	0

It will be perceived that the arguments $2g'-g$, $4g'-2g$, $5g'-2g$, and $5g'-3g$ contribute the largest portions to these terms of the second order. In the case of the three latter arguments the cause of the largeness of the portions contributed is the division by the small divisor $5n'-2n$ or its square. But in the case of the first, $2g'-g$, this cause does not operate, and yet for Jupiter this argument contributes the largest quota. Hence, it is hardly correct to say that the superior magnitude of the terms we are considering is due to the smallness of $5n'-2n$. However, $2g'-g$ may be considered as an argument of long period, since $\frac{n}{2n'-n}$ is about 5. But the actual explanation of the magnitude of the coefficients, whose component parts have just been given, appears to be that the terms of T and T$'$ having the arguments γ and γ'

are exceptionally small. This will be apparent when we write them in connection with the maximum terms, as follows:

$$\begin{array}{lll}
 & \quad '' & \qquad\quad '' \\
T = - & 1.142 \sin(-\gamma) & + \quad 1.017 \cos(-\gamma) \\
 & -\ 47.872 \sin(-\gamma + 2g' - g) & + \quad 20.556 \cos(-\gamma + 2g' - g) \\
T' = & 8.631 \sin \gamma' & - \quad 5.350 \cos \gamma' \\
 & +413.905 \sin(\gamma' - g') & +1977.900 \cos(\gamma' - g)
\end{array}$$

With them may also be compared the largest terms of the factors G and G′

$$\begin{array}{lll}
 & \quad '' & \qquad '' \\
G = & 167.70 \sin(-\gamma + 2g' - g) & -\ 71.60 \cos(-\gamma + 2g' - g) \\
G' = & -\ 782.26 \sin(\gamma' - g) & -\ 3729.47 \cos(\gamma' - g)
\end{array}$$

Adding the components of δT and $\delta T'$ given in the preceding table, we obtain

$$\begin{array}{ll}
 & \qquad '' \qquad\qquad\qquad\qquad '' \\
\delta T = & -\ 0.2260338 \sin(-\gamma) + 0.2563282 \cos(-\gamma) \\
\delta T' = & +\ 1.771981 \ \sin \gamma' \ -\ 1.345783 \ \cos \gamma'
\end{array}$$

It is desirable to have the means of readily changing these expressions, so as to correspond to any new values of the masses of Jupiter and Saturn that may be adopted. Thus, by adding the five terms of δT involving the factors A to E, we find that the portion of δT proportional to $\left(\frac{m'}{1+m}\right)^2$ is

$$-\ 0''.0351410 \sin(-\gamma) + 0''.0526059 \cos(-\gamma)$$

and by adding the three terms involving the factors F to H, we find that the portion of δT proportional to $\frac{m}{1+m'} \cdot \frac{m'}{1+m}$ is

$$-\ 0''.1908928 \sin(-\gamma) + 0''.2037223 \cos(-\gamma)$$

In like manner, the portion of $\delta T'$ proportional to $\left(\frac{m}{1+m'}\right)^2$ is

$$+\ 1''.337009 \sin \gamma' - 1''.066713 \cos \gamma'$$

and the portion proportional to $\frac{m}{1+m'} \cdot \frac{m'}{1+m}$ is

$$+\ 0''.434972 \sin \gamma' - 0''.279070 \cos \gamma'$$

By joining the second-order terms, which arise from δT and $\delta T'$, to the principal secular terms of the first order of $n\delta z$ and $n'\delta z'$, obtained in preceding chapters, we have the following exhibit:

	$n\delta z$
	$'' \qquad\qquad ''$
Action of Mercury	$-0.0000059nt\sin(-g) - 0.0000137nt\cos(-g)$
Action of Venus	$-0.0000153nt\sin(-g) - 0.0007144nt\cos(-g)$
Action of the Earth	$-0.0001530nt\sin(-g) - 0.0018372nt\cos(-g)$
Action of Mars	$+0.0002138nt\sin(-g) - 0.0002217nt\cos(-g)$
Action of Saturn	$-1.0173636nt\sin(-g) - 1.1420391nt\cos(-g)$
Action of Uranus	$-0.0021751nt\sin(-g) - 0.0213460nt\cos(-g)$
Action of Neptune	$-0.0000833nt\sin(-g) - 0.0040184nt\cos(-g)$
Terms factored by $\left(\frac{m'}{1+m}\right)^2$	$-0.0526059nt\sin(-g) - 0.0351410nt\cos(-g)$
Terms factored by $\frac{m}{1+m'}\cdot\frac{m'}{1+m}$	$-0.2037223nt\sin(-g) - 0.1908928nt\cos(-g)$
Sum	$-1.2759106nt\sin(-g) - 1.3962243nt\cos(-g)$

	$n'\delta z'$
	$'' \qquad\qquad ''$
Action of Mercury	$0.000000n't\sin g' - 0.000005n't\cos g'$
Action of Venus	$+0.000002n't\sin g' - 0.000235n't\cos g'$
Action of the Earth	$+0.000004n't\sin g' - 0.000579n't\cos g'$
Action of Mars	$-0.000044n't\sin g' - 0.000172n't\cos g'$
Action of Jupiter	$-5.350080n't\sin g' - 8.631067n't\cos g'$
Action of Uranus	$+0.085000n't\sin g' - 0.155652n't\cos g'$
Action of Neptune	$-0.001474n't\sin g' - 0.032936n't\cos g'$
Terms factored by $\left(\frac{m}{1+m'}\right)^2$	$-1.066713n't\sin g' - 1.337009n't\cos g'$
Terms factored by $\frac{m}{1+m'}\cdot\frac{m'}{1+m}$	$-0.279070n't\sin g' - 0.434972n't\cos g'$
Sum	$-6.612375n't\sin g' - 10.592627n't\cos g'$

The expressions actually used in Chapter XI for determining the portion of δT, which is factored by nt, are

$$n\delta z = -\overset{''}{1.2759133}nt\sin(-g) - \overset{''}{1.3962200}nt\cos(-g)$$

$$n'\delta z' = -6.612298n't\sin g' - 10.592645n't\cos g'$$

The terms dependent on the arguments $2g$, $3g$ . . . $2g'$, $3g'$ . . . can be found by the formulæ of page 102.

The secular terms of $\frac{u}{\cos i}$ and $\frac{u'}{\cos i'}$ are, in like manner, summed as follows:

$$\frac{u}{\cos i}$$

	″	″
Action of Mercury	$+ 0.0000080nt \sin(-g)$	$- 0.0000171nt \cos(-g)$
Action of Venus	$+ 0.0002201nt \sin(-g)$	$- 0.0001852nt \cos(-g)$
Action of the Earth	$- 0.0004354nt \sin(-g)$	$+ 0.0000230nt \cos(-g)$
Action of Mars	$- 0.0000186nt \sin(-g)$	$- 0.0001257nt \cos(-g)$
Action of Saturn	$+ 0.2844315nt \sin(-g)$	$+ 0.1253514nt \cos(-g)$
Action of Uranus	$- 0.0019103nt \sin(-g)$	$- 0.0008950nt \cos(-g)$
Action of Neptune	$+ 0.0001987nt \sin(-g)$	$+ 0.0006825nt \cos(-g)$
Sum	$+ 0.2824940nt \sin(-g)$	$+ 0.1248339nt \cos(-g)$

$$\frac{u'}{\cos i'}$$

	″	″
Action of Mercury	$+ 0.000005n't \sin g'$	$+ 0.000002n't \cos g'$
Action of Venus	$+ 0.000067n't \sin g'$	$- 0.000036n't \cos g'$
Action of the Earth	$+ 0.000086n't \sin g'$	$+ 0.000212n't \cos g'$
Action of Mars	$+ 0.000051n't \sin g'$	$+ 0.000022n't \cos g'$
Action of Jupiter	$+ 1.106428n't \sin g'$	$+ 1.552265n't \cos g'$
Action of Uranus	$+ 0.032793n't \sin g'$	$+ 0.042895n't \cos g'$
Action of Neptune	$- 0.001104n't \sin g'$	$+ 0.005024n't \cos g'$
Sum	$+ 1.138326n't \sin g'$	$+ 1.600384n't \cos g'$

The secular term of $\delta\frac{h}{h_0}$ is the same as the non-periodic secular term of -2ν.

CHAPTER X.

CALCULATION OF THE PORTION OF δT NOT FACTORED BY nt.

In determining the portion of δT, which follows, a table of limits for the retention of terms for each argument $\pm\gamma + i'g' + ig$ was computed from the formula

$$\frac{i'n' + in}{n} \cdot \frac{i'n' + (i \pm 1)n}{n} \times 0''.0005$$

and only those combinations were retained in which at least one coefficient exceeded this limit. It has been deemed advisable to give separately the eight products whose sum forms δT:

Arg= $\varkappa\gamma+i'g'+ig$			$An\delta z$		$B(\nu-\sigma)+X\sigma$		$Fn'\delta z'$		$G(\nu'-\sigma')$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		−0.00041718		+0.00039938		+0.00014725		−0.00015832
1	0	−1	+0.0059	−0.0012	+0.0145	−0.0006	−0.0104	−0.0051	+0.0433	−0.0019
−1	0	0	−0.0274051	+0.0386863	−0.0085509	+0.0141722	−0.1400087	+0.1637579	−0.0508789	+0.0399329
0	0	−1	+0.0462	−0.0588	+0.0127	−0.0180	+0.1348	−0.1529	+0.0495	−0.0410
1	0	−2	−0.017	+0.025	−0.005	+0.006	−0.029	+0.048	−0.013	+0.015
−1	0	−1	−0.071	−0.056	−0.010	−0.007	−0.324	−0.248	−0.069	−0.045
0	0	−2	+0.091	+0.068	+0.011	+0.007	+0.317	+0.236	+0.067	+0.040
1	0	−3	−0.035	−0.022	−0.003	−0.002	−0.101	−0.062	−0.020	−0.014
−1	0	−2	+0.108	−0.081			+0.316	−0.244	−0.016	+0.007
0	0	−3	−0.128	+0.101			−0.296	+0.247	+0.018	−0.006
1	0	−4	+0.046	−0.037			+0.094	−0.076		
−1	0	−3	+0.003	−0.019			+0.007	−0.044		
0	0	−4	−0.010	+0.027			−0.019	+0.053		
1	0	−5					+0.01	−0.02		
−1	1	+4	+0.012	−0.008			+0.019	−0.014		
0	1	+3	−0.027	+0.017			−0.052	+0.034		
1	1	+2	+0.017	−0.006			+0.043	−0.015		
−1	1	+3	+0.035	−0.073			+0.071	−0.138		
0	1	+2	−0.094	+0.193			−0.230	+0.465	+0.003	−0.022
1	1	+1	+0.070	−0.152			+0.227	−0.493	−0.007	+0.021
−1	1	+2	+0.029	+0.031	+0.003	+0.004	+0.085	+0.090	+0.022	+0.020
0	1	+1	−0.087	−0.072	−0.012	−0.011	−0.317	−0.271	−0.073	−0.072
1	1	0	+0.0626	+0.0535	+0.0108	+0.0103	+0.3316	+0.2838	+0.0790	+0.0749
−1	1	+1	−0.0196	+0.0057	−0.0083	+0.0042	−0.0215	−0.0002	−0.0124	+0.0043

Arg= $\varkappa\gamma+i'g'+ig$			$An\delta z$		$B(\nu-c)+X\sigma$		$Fn'\delta z'$		$G(\nu'-\sigma')$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	1	0	+0.0405	−0.0241	+0.0186	−0.0109	+0.0698	−0.0502	+0.0209	−0.0218
1	1 −	1	−0.0218	+0.0117	−0.0121	+0.0079	−0.0815	+0.0572	−0.0169	+0.0249
−1	1	0	−0.0039	−0.0141	−0.0032	−0.0178	−0.0143	+0.0450	−0.0274	−0.1010
0	1 −	1	+0.0077	+0.0091	+0.0021	+0.0102	+0.0158	−0.0390	+0.0210	+0.0650
1	1 −	2	−0.0007	−0.0005	+0.0020	+0.0040	+0.0027	+0.0074	+0.0015	+0.0145
−1	1 −	1	−0.0498	−0.0229	−0.0138	−0.0047	−0.2019	−0.1080	−0.0481	−0.0346
0	1 −	2	+0.067	+0.031	+0.019	+0.007	+0.193	+0.104	+0.051	+0.036
1	1 −	3	−0.025	−0.011			−0.061	−0.022	−0.015	−0.008
−1	1 −	2	+0.033	−0.072	+0.004	−0.008	+0.144	−0.309	+0.022	−0.054
0	1 −	3	−0.039	+0.087	−0.004	+0.008	−0.133	+0.303	−0.022	+0.053
1	1 −	4	+0.011	−0.034			+0.035	−0.095	+0.007	−0.017
−1	1 −	3	+0.075	+0.064			+0.214	+0.174	−0.008	−0.012
0	1 −	4	−0.089	−0.071			−0.214	−0.159	+0.007	+0.014
1	1 −	5	+0.04	+0.03			+0.07	+0.05		
−1	2 +	3	+0.012	+0.009			+0.018	+0.014		
0	2 +	2	−0.022	−0.021			−0.040	−0.039		
1	2 +	1	+0.007	+0.014			+0.017	+0.034		
−1	2 +	2	+0.098	+0.023			+0.177	+0.045	−0.007	+0.002
0	2 +	1	−0.251	−0.060	+0.005	0.000	−0.611	−0.145	+0.024	0.000
1	2	0	+0.1755	+0.0385	−0.0035	−0.0002	+0.6443	+0.1391	−0.0231	−0.0041
−1	2 +	1	−0.0250	+0.0311	−0.0031	+0.0060	−0.0608	+0.0774	−0.0142	+0.0228
0	2	0	+0.0486	−0.0838	+0.0111	−0.0197	+0.1744	−0.3078	+0.0600	−0.1063
1	2 −	1	−0.0292	+0.0502	−0.0089	+0.0164	−0.1915	+0.3287	−0.0655	+0.1156
−1	2	0	+0.0017	−0.0074	−0.0030	−0.0053	+0.0087	−0.0052	−0.0010	−0.0206
0	2 −	1	+0.00899	+0.01147	+0.00725	+0.00839	−0.00175	+0.01928	+0.01229	+0.01990
1	2 −	2	−0.0006	−0.0023	−0.0040	−0.0053	+0.0017	−0.0302	−0.0155	−0.0079
−1	2 −	1	+0.0091	−0.0097	+0.0154	−0.0068	−0.0656	−0.0098	+0.0823	−0.0450
0	2 −	2	−0.0016	+0.0097	−0.0098	+0.0055	+0.0603	+0.0108	−0.0611	+0.0378
1	2 −	3	+0.006	−0.007	−0.003	+0.001	−0.019	+0.003	−0.001	−0.002
−1	2 −	2	+0.011	−0.053	+0.001	−0.016	+0.054	−0.203	+0.020	−0.051
0	2 −	3	−0.015	+0.069	−0.002	+0.016	−0.051	+0.199	−0.021	+0.053
1	2 −	4	+0.004	−0.015			+0.007	−0.059	+0.004	−0.016
−1	2 −	3	+0.065	+0.015	+0.005	+0.001	+0.260	+0.055	+0.039	+0.007
0	2 −	4	−0.074	−0.016	−0.006	0.000	−0.251	−0.043	−0.040	−0.007
1	2 −	5	+0.023	+0.004			+0.078	+0.012	+0.012	+0.002
−1	3 +	2	−0.005	+0.013			−0.007	+0.015		
0	3 +	1	+0.013	−0.020			+0.019	−0.033		
1	3 +	0	−0.008	+0.003			−0.019	+0.009		
−1	3 +	1	−0.001	+0.104			0.000	+0.171	0.000	−0.005
0	3	0	+0.0033	−0.2541	+0.0010	+0.0039	+0.0010	−0.6243	−0.0012	+0.0209
1	3 −	1	−0.0006	+0.1351	−0.0001	−0.0029	−0.0005	+0.6625	+0.0016	−0.0227
−1	3 −	0	−0.0110	−0.0164	−0.0065	−0.0020	+0.0132	−0.0215	+0.0096	+0.0051
0	3 −	1	+0.04018	+0.02187	+0.02197	+0.00701	+0.02698	+0.04735	+0.02552	+0.00624
1	3 −	2	−0.0185	−0.0068	−0.0175	−0.0054	−0.0487	−0.0614	−0.0442	−0.0120
−1	3 −	1	+0.0019	−0.0009	+0.0031	−0.0017	−0.0155	−0.0075	+0.0155	−0.0143
0	3 −	2	−0.0019	+0.0064	−0.0051	+0.0030	+0.0110	+0.0072	−0.0119	+0.0186

Arg= $\varkappa\gamma+i'g'+ig$			$An\delta z$		$B(\nu-c)+Xc$		$Fn'\delta z'$		$G(\nu'-c')$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
1	3−	3	−0.0007	+0.0017	+0.0033	−0.0038	+0.0017	+0.0023	+0.0002	−0.0148
−1	3−	2	+0.0092	+0.0019	+0.0075	+0.0090	−0.0057	−0.0714	+0.0806	+0.0989
0	3−	3	−0.009	−0.003	−0.006	−0.004	+0.006	+0.066	−0.072	−0.085
1	3−	4	+0.010	+0.009	+0.003	+0.004	−0.006	−0.022	+0.017	+0.021
−1	3−	3	+0.046	0.000	+0.007	−0.003	+0.176	+0.001	+0.047	+0.004
0	3−	4	−0.056	0.000			−0.167	+0.002	−0.047	−0.004
1	3−	5	+0.013	0.000			+0.051	−0.006	+0.013	−0.001
−1	3−	4	0.000	+0.051			+0.007	+0.191	0.000	+0.027
0	3−	5	0.000	−0.051			−0.013	−0.186	−0.001	−0.026
1	3−	6	0.00	+0.02			0.00	+0.06		
−1	4+	1	+0.001	−0.004			+0.001	−0.003		
0	4	0	−0.003	+0.005			−0.003	+0.006		
1	4−	1	+0.022	−0.023			+0.059	−0.046	−0.005	+0.002
−1	4	0	+0.0187	−0.0034			+0.0228	−0.0042	−0.0006	+0.0005
0	4−	1	+0.0188	−0.0051	−0.0004	−0.0004	+0.0452	−0.0100	−0.0025	−0.0013
1	4−	2	−0.0001	+0.0004	+0.0006	+0.0002	−0.0829	+0.0173	+0.0041	+0.0010
−1	4−	1	−0.0007	+0.0064	−0.0006	+0.0005	−0.0128	+0.0078	−0.0025	+0.0342
0	4−	2	+0.0009	+0.0040	−0.0010	+0.0062	+0.0093	+0.0005	−0.0018	+0.0066
1	4−	3	−0.0018	+0.0048	+0.0007	+0.0083	−0.0015	−0.0073	+0.0052	−0.0442
−1	4−	2	+0.0190	−0.0051	−0.0003	+0.0015	+0.0926	−0.0532	−0.0032	+0.0322
0	4−	3	−0.031	+0.008	0.000	−0.003	−0.089	+0.049	+0.006	−0.025
1	4−	4	+0.014	−0.001	+0.002	+0.001	+0.023	−0.013	+0.002	+0.006
−1	4−	3	+0.018	−0.012	−0.015	+0.019	+0.069	−0.031	−0.067	+0.083
0	4−	4	−0.030	+0.020	+0.021	−0.023	−0.070	+0.032	+0.061	−0.078
1	4−	5	+0.003	−0.004	−0.005	+0.006	+0.014	−0.006	−0.018	+0.020
−1	4−	4	+0.008	+0.030			+0.031	+0.130	+0.004	+0.031
0	4−	5	−0.005	−0.030			−0.032	−0.126	−0.005	−0.030
1	4−	6					+0.014	+0.037	−0.001	+0.004
−1	4−	5	−0.033	+0.006			−0.132	+0.030	−0.017	+0.004
0	4−	6	+0.04	−0.01			+0.13	−0.03	−0.02	0.00
1	4−	7					−0.03	+0.01		
−1	5	0	+0.0018	−0.0003						
0	5−	1	+0.0002	−0.0027			+0.0009	−0.0008		
1	5−	2	−0.000238	+0.000252	−0.000045	+0.000058	−0.001018	+0.000577	−0.000465	+0.000250
−1	5−	1	+0.014627	+0.034556	−0.000364	−0.000530	−0.004851	−0.001226	−0.000659	−0.002451
0	5−	2	+0.0006774	+0.0000259	+0.0004065	+0.0004938	+0.0026506	−0.0011941	+0.0003514	+0.0007744
1	5−	3	+0.014445	+0.034941	−0.000250	−0.000091	+0.000247	+0.003908	+0.000777	+0.001324
−1	5−	2	−0.000892	−0.004138	−0.004603	+0.000805	+0.046018	−0.037480	−0.039562	+0.006578
0	5−	3	−0.0109	+0.0068	−0.0019	−0.0004	−0.0352	+0.0335	+0.0194	−0.0039
1	5−	4	−0.0034	−0.0023	+0.0059	−0.0007	−0.0015	−0.0100	+0.0175	−0.0010
−1	5−	3	+0.009	+0.031	−0.002	−0.002	+0.071	+0.071	−0.029	+0.016
0	5−	4	−0.013	−0.045	−0.001	+0.002	−0.072	−0.070	+0.033	−0.016
1	5−	5	+0.004	+0.016			+0.020	+0.023	−0.007	+0.001
−1	5−	4	+0.018	+0.018	−0.024	−0.016	+0.046	+0.054	−0.067	−0.033
0	5−	5	−0.027	−0.024	+0.033	+0.018	−0.047	−0.057	+0.066	+0.030

Arg= $\varkappa\gamma+i'g'+ig$			$An\delta z$		$B(\nu-c)+Xc$		$Fn'\delta z'$		$G(\nu'-c')$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
1	5	—6	+0.016	+0.010	—0.007	—0.004	+0.002	+0.009	—0.019	—0.010
—1	5	—5	—0.019	+0.007			—0.088	+0.047	—0.020	+0.007
0	5	—6	+0.021	—0.010			+0.086	—0.047	+0.020	—0.007
1	5	—7					—0.02	+0.01		
—1	5	—6	—0.01	—0.03			—0.04	—0.09		
0	5	—7	+0.01	+0.03			+0.04	+0.08		
1	5	—8					—0.01	—0.02		
1	6	—2					—0.0002	—0.0006		
—1	6	—1	+0.0036	+0.0023			—0.0098	—0.0057	—0.0002	—0.0001
0	6	—2	+0.0002	0.0000			+0.0005	—0.0004		
1	6	—3	+0.0032	+0.0027			+0.0079	+0.0074	—0.0001	—0.0003
—1	6	—2	—0.0001	—0.0007	+0.0006	—0.0004	+0.0757	—0.0735	+0.0025	+0.0183
0	6	—3	—0.0167	+0.0124	—0.0009	+0.0006	—0.0412	+0.0468	—0.0024	—0.0070
1	6	—4	—0.0160	+0.0115	+0.0007	—0.0005	—0.0236	+0.0056	+0.0025	+0.0046
—1	6	—3	+0.0061	+0.0093	—0.0031	—0.0070	+0.0857	+0.1768	—0.0430	—0.1255
0	6	—4	—0.009	—0.017	+0.003	+0.004	—0.082	—0.167	+0.040	+0.116
1	6	—5	—0.001	—0.004			+0.021	+0.044	—0.011	—0.028
—1	6	—4	—0.027	+0.013	—0.002	—0.002	—0.030	+0.017	—0.027	—0.024
0	6	—5	+0.034	—0.017	+0.007	—0.002	+0.030	—0.080	+0.027	+0.029
1	6	—6	—0.008	+0.006			—0.018	+0.018	0.000	—0.006
—1	6	—5	—0.011	+0.018	+0.008	—0.021	—0.033	+0.044	+0.011	—0.049
0	6	—6	+0.007	—0.017	—0.008	+0.022	+0.034	—0.044	—0.016	+0.048
1	6	—7	—0.01	+0.01					0.00	—0.01
—1	6	—6	—0.01	0.00			—0.04	—0.05		
0	6	—7	+0.01	0.00			+0.04	+0.05		
—1	7	—1					—0.0023	+0.0003		
0	7	—2					+0.0004	+0.0001		
1	7	—3	+0.00065	+0.00029			+0.00257	+0.00138	—0.00037	—0.00007
—1	7	—2	+0.00006	—0.00012	—0.00006	—0.00026	—0.01954	—0.06615	+0.00007	+0.00057
0	7	—3	+0.00427	+0.00898	+0.00002	+0.00028	+0.02252	+0.04730	—0.00009	—0.00013
1	7	—4	—0.0101	—0.0043	+0.0001	—0.0001	—0.0268	—0.0119	+0.0003	0.0000
—1	7	—3	+0.1015	+0.0949	—0.0004	+0.0011	+0.5323	+0.4937	—0.0083	—0.0035
0	7	—4	—0.1906	—0.1775	—0.0002	—0.0019	—0.5002	—0.4658	+0.0075	+0.0039
1	7	—5	+0.076	+0.073			+0.139	+0.125	—0.003	+0.001
—1	7	—4	—0.017	+0.018	+0.014	—0.007	—0.135	+0.120	+0.094	—0.061
0	7	—5	+0.018	—0.036	—0.019	+0.008	+0.111	—0.133	—0.092	+0.062
1	7	—6	—0.001	+0.016	+0.005	—0.002	—0.027	+0.040	+0.026	—0.018
—1	7	—5	—0.016	—0.015	—0.002	—0.002	—0.063	—0.011	+0.016	—0.025
0	7	—6	+0.020	+0.024	+0.003	+0.010	+0.068	+0.014	—0.020	+0.026
1	7	—7					—0.017	—0.010		
—1	7	—6	—0.017	—0.002	+0.016	+0.003	—0.039	—0.016	+0.034	+0.005
0	7	—7	+0.03	0.00			+0.03	+0.02	—0.03	—0.01
—1	7	—7					+0.02	—0.03		
0	7	—8					—0.01	+0.03		

Arg= $\varkappa\gamma+i'g'+ig$			$An\delta z$		$B(\nu-\upsilon)+X\upsilon$		$Fn'\delta z'$		$G(\nu'-\upsilon')$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	8	−2					−0.0127	−0.0138		
0	8	−3	+0.00141	+0.00134	+0.00006	+0.00008	+0.01082	+0.00974	+0.00005	+0.00006
1	8	−4	−0.0022	−0.0001			−0.0082	−0.0004	−0.0001	−0.0001
−1	8	−3	+0.0290	+0.0069	+0.0003	0.0000	+0.2220	+0.0517	−0.0004	+0.0003
0	8	−4	−0.0522	−0.0139	−0.0003	−0.0002	−0.2024	−0.0519	+0.0001	−0.0001
1	8	−5	+0.0232	−0.0001			+0.0594	−0.0010	0.0000	+0.0005
−1	8	−4	−0.0982	+0.1620	+0.0007	−0.0020	−0.3806	+0.6353	+0.0064	−0.0158
0	8	−5	+0.139	−0.232	−0.001	+0.003	+0.359	−0.612	−0.008	+0.019
1	8	−6	−0.053	+0.090			−0.099	+0.178	0.000	−0.006
−1	8	−5	−0.024	−0.011	+0.011	+0.014	−0.121	−0.067	+0.063	+0.063
0	8	−6	+0.037	+0.003	−0.013	−0.016	+0.124	+0.045	−0.064	−0.061
1	8	−7	−0.012	+0.004			−0.040	−0.007	+0.016	+0.018
−1	8	−6	+0.001	−0.010	+0.008	−0.003	−0.003	−0.048	+0.023	+0.006
0	8	−7	−0.002	+0.013	−0.008	+0.004	+0.002	+0.048	−0.019	−0.011
1	8	−8					0.00	−0.01		
−1	8	−7	−0.01	−0.02	0.00	+0.01			0.00	+0.02
0	8	−8	0.00	+0.02					0.00	−0.01
−1	9	−2					−0.0030	−0.0013		
0	9	−3					+0.0023	+0.0009		
1	9	−4	−0.0003	+0.0001			−0.0013	+0.0006		
−1	9	−3	+0.0043	−0.0014			+0.0436	−0.0142	−0.0001	0.0000
0	9	−4	−0.00774	+0.00210	−0.00008	+0.00010	−0.03947	+0.01114	−0.00009	+0.00004
1	9	−5	+0.0027	−0.0027			+0.0091	−0.0091		
−1	9	−4	−0.0028	+0.0555	−0.0001	−0.0001	−0.0112	+0.2873	−0.0008	−0.0029
0	9	−5	+0.005	−0.079			+0.013	−0.269	+0.001	+0.001
1	9	−6	+0.004	+0.031			+0.009	+0.080	−0.001	−0.002
−1	9	−5	−0.169	−0.058	+0.006	+0.002	−0.595	−0.202	+0.023	+0.004
0	9	−6	+0.217	+0.073	−0.006	−0.002	+0.574	+0.188	−0.020	−0.007
1	9	−7	−0.081	−0.026			−0.170	−0.054	+0.010	+0.001
−1	9	−6	−0.003	−0.017	−0.009	+0.012	+0.017	−0.092	−0.037	+0.060
0	9	−7	+0.013	+0.025	+0.010	−0.013	+0.006	+0.096	+0.033	−0.058
1	9	−8	−0.007	−0.008			−0.008	−0.029	−0.011	+0.015
−1	9	−7	+0.002	−0.007			+0.028	−0.008	−0.004	+0.009
0	9	−8	0.00	+0.01			−0.03	+0.01	+0.01	0.00
−1	10	−3	+0.00035	−0.00041			+0.00491	−0.00586	+0.00002	−0.00002
0	10	−4	−0.0007234	+0.0007572	−0.0000064	+0.0000076	−0.0045060	+0.0047523	−0.0000156	+0.0000140
1	10	−5	−0.00001	−0.00050			+0.00011	−0.00230	+0.00001	−0.00001
−1	10	−4	+0.00504	+0.00965	0.00000	+0.00005	+0.03203	+0.06083	−0.00022	−0.00014
0	10	−5	−0.0065	−0.0133			−0.0274	−0.0567	+0.0001	0.0000
1	10	−6	+0.004	+0.004			+0.013	+0.013		
−1	10	−5	−0.067	+0.010	+0.001	0.000	−0.286	+0.047	+0.004	−0.001
0	10	−6	+0.085	−0.012			+0.274	−0.041	−0.003	0.000
1	10	−7	−0.031	+0.009			−0.081	+0.023		
−1	10	−6	+0.017	−0.142	−0.001	+0.006	+0.053	−0.476	−0.002	+0.021

Arg= $\varkappa\gamma+i'g'+ig$			$An\delta z$		$B(\nu-c)+Xc$		$Fn'\delta z'$		$G(\nu'-c')$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	10—	7	—0.018	+0.175	0.000	—0.007	—0.039	+0.462	+0.003	—0.020
1	10—	8	+0.006	—0.059			+0.011	—0.138	0.000	+0.009
—1	10—	7	+0.007	—0.006	—0.010	—0.005	+0.060	—0.015	—0.045	—0.015
0	10—	8	—0.010	+0.016	+0.011	+0.005	—0.060	+0.034	+0.038	+0.017
1	10—	9	0.00	—0.01			+0.01	—0.01	—0.01	0.00
0	10—	9					0.00	—0.02		
0	11—	4					—0.0002	+0.0009		
—1	11—	4	+0.0014	+0.0009			+0.0109	+0.0068	+0.0001	0.0000
0	11—	5	—0.0019	—0.0013			—0.0097	—0.0065	+0.0002	0.0000
1	11—	6	+0.0010	+0.0002			+0.0037	+0.0004		
—1	11—	5	—0.0123	+0.0096			—0.0632	+0.0500	+0.0003	—0.0003
0	11—	6	+0.016	—0.011			+0.060	—0.046	—0.001	+0.001
1	11—	7	—0.005	+0.006			—0.015	+0.018		
—1	11—	6	—0.023	—0.063			—0.093	—0.241	+0.002	+0.004
0	11—	7	+0.028	+0.073			+0.086	+0.232	—0.002	—0.004
1	11—	8	—0.012	—0.026			—0.034	—0.068		
—1	11—	7	+0.106	—0.011	—0.006	0.000	+0.338	—0.038	—0.019	+0.002
0	11—	8	—0.124	+0.016			—0.327	+0.046	+0.025	—0.003
1	11—	9	+0.04	—0.01			+0.10	—0.01		
—1	11—	8	+0.01	—0.01			+0.02	+0.04	0.00	—0.03
0	11—	9	—0.02	0.00			—0.03	—0.03	0.00	+0.03
0	12—	5	—0.00032	—0.00002			—0.00192	—0.00008	—0.00001	0.00000
—1	12—	5	—0.0011	+0.0029			—0.0070	+0.0171		
0	12—	6	+0.0014	—0.0033			+0.0062	—0.0141		
1	12—	7					0.000	+0.005		
—1	12—	6	—0.013	—0.012			—0.062	—0.054	+0.002	—0.001
0	12—	7	+0.017	+0.014			+0.059	+0.052		
1	12—	8	—0.008	—0.004			—0.020	—0.012		
[illegible]	12—	7	+0.047	—0.031			+0.176	—0.113	—0.003	+0.002
0	[illegible]	8	—0.055	+0.035			—0.171	+0.109	+0.007	—0.004
1	7—	4	+0.019	—0.014			+0.047	—0.034		
—1	7—	3	[illegible]0.024	+0.071			+0.078	+0.218	—0.006	—0.018
0	7—	4	—0.[illegible]	—0.08			—0.07	—0.20	+0.01	+0.02
1	7—	5	+0.076	+0.02			+0.02	+0.05		
—1	7—	4	—0.017							
0	7—	5	+0.018							
1	7—	6	—0.001							
—1	7—	5	—0.016	—[illegible]						
0	7—	6	+0.020	+0.0[illegible]						
1	7—	7								
—1	7—	6	—0.017	—0.002						
0	7—	7	+0.03	0.00						
—1	7—	7								
0	7—	8								

Arg= $\varkappa\gamma+i'g'+ig$			$C\delta\frac{h}{h_0}$		$D\frac{u}{\cos i}$		$E\frac{u_1}{\cos i}$		$H\frac{u'}{\cos i'}$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		+0.00002191		+0.00000013		−0.00000016		−0.00000005
−1	0	0	+0.0008224	−0.0002701	−0.0000073	+0.0000180	−0.0000001	−0.0000005	−0.0000052	+0.0000315
0	0	−1	−0.0020	+0.0004						
−1	1	+1	+0.0002	−0.0002						
0	1	0	−0.0007	+0.0003						
1	1	−1	+0.0001	−0.0001						
−1	1	0	+0.0001	−0.0009						
0	1	−1	0.0000	+0.0010						
1	1	−2	−0.0002	−0.0006						
−1	1	−1	+0.0003	+0.0006						
0	2	−1	−0.00003	+0.00021	+0.00001	+0.00002	−0.00001	−0.00001	+0.00002	+0.00002
−1	2	−1	−0.0001	+0.0002						
0	2	−2	+0.0014	−0.0006						
0	3	−1	0.00000	+0.00009						
−1	3	−1	0.0000	−0.0002						
0	3	−2	−0.0002	+0.0003						
0	4	−2	+0.0002	0.0000						
−1	4	−2	+0.0008	−0.0006						
1	5	−2	+0.000003	−0.000007						
−1	5	−1	+0.000004	+0.000007					−0.000023	−0.000043
0	5	−2	−0.0000096	−0.0000005	−0.0000046	−0.0000113	+0.0000072	+0.0000138	+0.0000032	+0.0000050
1	5	−3	0.000000	+0.000001						
−1	5	−2	+0.000053	−0.000087						
0	7	−3	−0.00011	−0.00006						
−1	7	−3	−0.0005	−0.0001						
0	10	−4	−0.0000002	−0.0000053						

CHAPTER XI.

CALCULATION OF THE PORTION OF δT FACTORED BY nt.

In determining the part of δT having the factor nt a degree of precision 300 times greater than that used in deriving the part not multiplied by nt has been employed. In the following table the factor nt has been omitted, and for convenience all the coefficients have been multiplied by 100000:

Arg= $\varkappa\gamma+i'g'+ig$			$An\delta z$		$B(\nu-c)$		$Fn'\delta z'$		$G(\nu'-\sigma')$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		+ 0.4688		+ 0.5821				
1	0	− 1	+ 0.2857	+ 0.1514	+ 0.5551	− 0.7368	− 0.1425	− 0.0984	+ 0.4297	+ 0.2221
−1	0	0	− 4.3819	− 4.8170	− 4.2330	− 4.7314	+15.2806	+ 5.6172	−36.2745	− 13.3821
0	0	− 1					− 8.4	− 3.1	+18.7	+ 6.9
1	0	− 2	− 4.406	− 4.835	+ 4.307	+ 4.744	− 4.162	− 1.536	+11.244	+ 4.181
−1	0	− 1			+ 1.0	− 0.1	− 1.1	+ 0.6	+ 3.1	− 1.8
0	0	− 2					+ 0.6	− 0.3	− 2.0	+ 1.2
1	0	− 3	0.0	− 0.4	− 0.2	+ 0.2	0.0	− 0.1	+ 0.1	+ 0.5
−1	1	+ 2	+ 0.1	− 0.8	+ 0.1	+ 1.1			− 1.0	+ 0.9
0	1	+ 1							− 0.8	+ 2.8
1	1	0	+ 0.21	− 0.70	+ 0.18	− 0.98	− 0.21	+ 0.08	+ 2.26	− 4.78
−1	1	+ 1			− 7.80	− 4.70	− 0.28	− 0.33	−17.94	− 28.80
0	1	0	+ 2.89	+ 1.82	+ 5.08	+ 3.14				
1	1	− 1	+ 2.91	+ 1.66	+ 1.97	+ 1.01	+ 0.13	+ 0.65	+18.03	+ 28.63
−1	1	0	− 0.53	+ 0.33	− 0.51	− 0.14	+ 7.52	− 9.44	− 4.70	+ 8.14
0	1	− 1	− 0.15	− 1.03	− 0.18	− 0.35	− 4.39	+ 7.94	+ 2.42	− 7.01
1	1	− 2	− 0.7	+ 0.4	+ 0.9	+ 0.5	− 1.0	− 5.0	+ 1.1	+ 4.5
−1	1	− 1			+ 5.4	− 7.3	−19.1	+126.0	+16.6	−110.0
0	1	− 2	+ 2.0	− 2.7	− 3.5	+ 4.8	+17.9	−118.7	−15.1	+ 99.6
1	1	− 3	+ 1.9	− 2.7	− 1.4	+ 1.9	− 5.1	+ 32.4	+ 3.6	− 22.4
−1	1	− 2			− 0.3	+ 0.6	+ 1.6	+ 2.4	− 0.5	− 0.9
0	1	− 3					− 1.0	− 7.0	+ 0.3	+ 4.8
1	1	− 4					+ 1	+ 3	− 1	− 2
1	2	0					0.0	− 0.2	− 0.1	− 0.6
−1	2	+ 1	+ 0.1	− 0.3	− 2.2	+ 0.8	− 1.9	− 0.8	− 5.1	− 2.4
0	2	0	+ 0.90	− 0.81	+ 1.65	− 1.42				
1	2	− 1	+ 0.13	+ 1.72	+ 0.30	+ 0.86	+ 1.71	+ 1.15	+ 4.59	+ 2.78
−1	2	− 0	+ 7.77	−22.53	+ 6.98	−20.37	+12.73	− 14.54	+27.71	− 24.59
0	2	− 1	−14.743	+42.566	− 8.649	+24.918	− 6.893	+ 8.672	−14.455	+ 12.172
1	2	− 2	+ 6.00	−17.48	+ 2.20	− 5.73	− 3.97	+ 1.84	− 8.05	+ 9.01

Arg= $\kappa\gamma+i'g'+ig$			$An\delta z$		$B(\nu-c)$		$Fn'\delta z'$		$G(\nu'-c')$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
κ	i'	i	″	″	″	″	″	″	″	″
−1	2	−1	−1.06	−1.06	−1.08	−2.95	+13.59	+38.64	−11.93	−22.98
0	2	−2	−0.4	+2.8	+1.6	+3.6	−11.6	−35.4	+10.0	+21.0
1	2	−3	+1.7	−2.3	−1.2	−0.4	+5.7	+8.7	−4.8	−4.6
−1	2	−2	−21.8	−9.8	+19.8	+9.0	−127.5	+7.4	+95.4	−5.3
0	2	−3	+41.0	+18.5	−24.1	−10.8	+121.9	−8.5	−88.9	+5.8
1	2	−4	−16.9	−7.7	+5.6	+2.5	−35.2	+2.5	+23.4	−1.4
−1	2	−3	−1.1	+0.9			−4.6	+4.4	+2.7	−2.7
0	2	−4	+4	0			+9	−7	−6	+3
1	2	−5					−4	+2	+2	−1
−1	3	+1			−0.2	+0.5	−0.5	+0.1	−0.6	+0.2
0	3	0	+0.1	−0.3	+0.1	−0.5				
1	3	−1	+0.12	+0.31	+0.13	+0.13	+0.65	+0.17	+0.92	+0.12
−1	3	0	−0.60	−5.02	−1.75	−5.18	−5.07	−11.66	−2.78	−12.19
0	3	−1	+1.035	+9.281	+1.899	+5.889	+5.221	+7.980	+3.382	+7.340
1	3	−2	−1.49	−3.66	−0.92	−1.23	−5.26	−1.34	−4.27	+0.63
−1	3	−1	+31.64	+4.00	+21.74	+2.54	+110.99	+77.57	+87.33	+61.02
0	3	−2	−45.75	−5.36	−25.03	−2.86	−104.07	−72.97	−79.19	−54.96
1	3	−3	+18.0	+1.9	+6.7	+1.1	+29.2	+19.3	+17.5	+13.1
−1	3	−2	−3.3	−1.4	+6.5	−2.3	−39.9	+25.5	+28.0	−18.9
0	3	−3	+4.5	+1.6	−8.4	+2.4	+33.8	−25.9	−29.2	+15.3
1	3	−4	−1.4	+0.4	+2.0	−1.4	−7.7	+10.1	+8.0	−5.3
−1	3	−3	+6.8	−31.2	−4.9	+21.8	−28.4	−102.1	+19.1	+70.1
0	3	−4	−9.3	+44.8	+5.5	−25.0	+28.7	+97.3	−18.5	−65.7
1	3	−5	+4	−18	−1	+6	−8	−29	+5	+19
−1	3	−4	−2	−2			−7	−5	+6	+2
0	3	−5	+1	+5	−1	−2	+9	+8	−7	−5
1	3	−6					−3	−3	+2	+1
1	4	−1					+0.1	0.0		
−1	4	0	−0.5	−0.6	−0.9	−0.6	−2.8	−2.2	−1.9	−1.8
0	4	−1	+0.75	+1.14	+0.85	+0.64	+2.32	+1.57	+1.42	+1.05
1	4	−2	−0.57	−0.28	−0.29	−0.01	−1.62	+0.15	−0.86	+0.46
−1	4	−1	+8.97	−3.21	+6.40	−3.36	+43.91	+4.55	+28.44	+0.76
0	4	−2	−12.80	+4.47	−7.19	+3.65	−40.16	−5.09	−24.73	−1.06
1	4	−3	+4.70	−2.62	+1.81	−1.38	+11.40	−1.69	+5.66	−1.72
−1	4	−2	+1.94	+30.08	+2.20	+17.88	−55.09	+128.13	−34.49	+81.41
0	4	−3	−2.9	−38.8	−2.8	−19.9	+51.5	−122.7	+31.5	−76.6
1	4	−4	+1.3	+14.7	+0.6	+6.0	−13.9	+36.3	−9.2	+19.9
−1	4	−3	−0.2	−7.2	+4.1	+7.9	−32.0	−30.8	+23.6	+25.3
0	4	−4	+0.3	+8.7	−5.1	−9.3	+32.0	+25.1	−21.0	−27.2
1	4	−5	−1	−3	+2	+2	−12	−6	+7	+7
−1	4	−4	+30	0	−19	−1	+69	−37	−45	+23
0	4	−5	−38	0	+21	0	−66	+38	+42	−23
1	4	−6	+14	0	−6	0	+20	−11	−12	+6
−1	4	−5	+2	−2			+2	−8	−1	+6
0	4	−6	−5	+3			−5	+9	+3	−6
1	4	−7					+2	−3		

Arg= $\varkappa\gamma+i'g'+ig$			$An\delta z$		$B(\nu-c)$		$Fn'\delta z'$		$G(\nu'-c')$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	5	0					− 0.6	− 0.2	− 0.3	− 0.1
0	5	−1	+ 0.1	0.0			+ 0.5	+ 0.1	+ 0.2	0.0
1	5	−2	− 0.085	+ 0.022	− 0.034	+ 0.037	− 0.252	+ 0.160	− 0.076	+ 0.133
−1	5	−1	+ 1.222	− 1.348	+ 0.721	− 1.364	+ 8.316	− 3.769	+ 4.320	− 2.772
0	5	−2	− 1.75368	+ 1.85036	− 0.81637	+ 1.45203	− 7.55126	+ 3.03722	− 3.67535	+ 2.20560
1	5	−3	+ 0.434	− 0.971	+ 0.076	− 0.486	+ 1.589	− 1.963	+ 0.424	− 1.100
−1	5	−2	+ 6.034	+ 9.887	+ 4.687	+ 5.942	+ 4.911	+54.892	+ 4.939	+31.254
0	5	−3	− 7.7	−12.7	− 5.2	− 6.5	− 4.0	−51.7	− 4.3	−28.8
1	5	−4	+ 3.6	+ 4.3	+ 1.9	+ 1.7	+ 4.0	+15.0	+ 2.5	+ 7.5
−1	5	−3	−23.9	+ 5.8	−12.2	+ 5.0	−117.4	−23.7	−63.6	−12.1
0	5	−4	+28.9	− 7.4	+13.2	− 5.4	+113.2	+21.2	+60.5	+10.4
1	5	−5	−10.8	+ 3.1	− 4.3	+ 1.4	− 34.3	− 5.1	−16.8	− 3.5
−1	5	−4	+ 8.7	− 2.7	− 7.0	+ 5.9	+ 18.7	−32.2	−18.8	+24.8
0	5	−5	−11	+ 3	+ 8	− 7	− 14	+32	+21	−22
1	5	−6	+ 3	− 1	− 2	+ 2	+ 1	−11	− 5	+ 7
−1	5	−5	+ 5	+23	− 3	−14	+ 35	+42	−21	−26
0	5	−6	− 7	−28	+ 3	+15	− 36	−39	+22	+25
1	5	−7	+ 2	+10	− 1	− 5	+ 11	+11	− 7	− 7
−1	5	−6	+ 3	+ 1			+ 8	+ 1	− 5	0
0	5	−7	− 3	− 4			− 8	− 3	+ 5	+ 1
−1	6	−1	+ 0.03	− 0.29	− 0.07	− 0.29	+ 0.84	− 1.24	+ 0.27	− 0.73
0	6	−2	− 0.11	+ 0.38	+ 0.02	+ 0.29	− 0.80	+ 1.05	− 0.27	+ 0.59
1	6	−3	− 0.06	− 0.18	− 0.06	− 0.08	− 0.02	− 0.45	− 0.11	− 0.15
−1	6	−2	+ 2.34	+ 1.36	+ 1.81	+ 0.65	+ 7.62	+11.08	+ 4.68	+ 5.18
0	6	−3	− 2.91	− 1.76	− 1.95	− 0.74	− 6.80	−10.42	− 4.05	− 4.76
1	6	−4	+ 1.3	+ 0.3	+ 0.7	0.0	+ 3.0	+ 2.4	+ 1.7	+ 0.9
−1	6	−3	− 8.5	+ 7.8	− 4.4	+ 5.1	− 53.5	+15.9	−27.7	+10.2
0	6	−4	+10.0	− 9.5	+ 5.0	− 5.9	+ 51.3	−15.1	+26.3	− 9.2
1	6	−5	− 3.2	+ 4.0	− 1.4	+ 1.9	− 14.6	+ 6.7	− 6.9	+ 4.1
−1	6	−4	− 7.2	−17.1	− 5.5	− 6.7	− 1.1	−93.3	− 1.6	−43.3
0	6	−5	+ 8.7	+19.8	+ 6.2	+ 7.1	+ 2.8	+89.9	+ 2.5	+41.6
1	6	−6	− 4	− 6	− 1	− 3	− 2	−27	− 1	−12
−1	6	−5	+ 5	+ 7	− 5	− 5	+ 28	+ 7	−24	−11
0	6	−6	− 5	− 8	+ 5	+ 6	− 26	− 5	+23	+12
1	6	−7	+ 3	+ 4	− 2	− 2	+ 8	+ 1	− 7	− 3
−1	6	−6	−15	+ 7	+ 9	− 4	− 21	+30	+14	−17
0	6	−7	+18	− 9	− 9	+ 5	+ 20	−29	−12	+17
1	6	−8	− 6	+ 4			− 6	+ 9	+ 3	− 5
−1	7	−1					0.0	− 0.2		
0	7	−2					− 0.02	+ 0.17		
1	7	−3	− 0.02	− 0.02			− 0.05	− 0.05	− 0.03	− 0.01
−1	7	−2	+ 0.49	+ 0.01	+ 0.37	− 0.09	+ 2.32	+ 1.10	+ 1.21	+ 0.33
0	7	−3	− 0.620	− 0.032	− 0.395	+ 0.082	− 2.084	− 1.094	− 1.039	− 0.310
1	7	−4	+ 0.25	− 0.09	+ 0.10	− 0.09	+ 0.76	+ 0.01	+ 0.33	− 0.11
−1	7	−3	− 1.03	+ 2.99	− 0.38	+ 2.01	− 11.08	+11.25	− 4.80	+ 6.14
0	7	−4	+ 1.1	− 3.6	+ 0.2	− 2.3	+ 10.6	−10.5	+ 4.5	− 5.6
1	7	−5	− 0.2	+ 1.5	+ 0.1	+ 0.7	− 2.2	+ 3.7	− 0.7	+ 2.0

Arg= $\varkappa\gamma+i'g'+ig$			$An\delta z$		$B(\nu-\sigma)$		$Fn'\delta z'$		$G(\nu'-\sigma')$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	7	−4	− 8.2	− 5.9	−5.2	−2.5	−23.6	−43.9	−12.7	−21.2
0	7	−5	+ 9.6	+ 6.2	+5.6	+2.9	+23.0	+42.4	+12.1	+20.4
1	7	−6	− 3.7	− 2.0	−1.9	−0.7	− 8.5	−12.0	− 4.6	− 5.6
−1	7	−5	+11.3	− 7.0	+2.8	−4.9	+66.3	−15.1	+26.3	− 6.7
0	7	−6	−12	+ 8	−3	+5	−63	+17	−26	+ 7
1	7	−7	+ 3	− 3	+1	−2	+19	− 6	+ 7	− 3
−1	7	−6	− 6	+ 6	+3	−5	− 1	+20	+ 5	−19
0	7	−7	+ 6	− 6	−4	+5	− 2	−19	− 5	+19
1	7	−8	− 2	+ 4			+ 2	+ 5	0	− 4
−1	7	−7	− 7	−10	+4	+5	−21	− 9	+12	+ 5
0	7	−8	+ 9	+10	−5	−5	+22	+ 8	−12	− 5
1	7	−9	− 3	− 3			− 7	− 3	+ 3	+ 1
−1	8	−2	+ 0.06	− 0.05	+0.04	−0.06	+ 0.38	− 0.06	+ 0.15	− 0.07
0	8	−3	− 0.088	+ 0.042	−0.050	+0.047	− 0.373	+ 0.019	− 0.162	+ 0.049
1	8	−4	+ 0.02	− 0.03			+ 0.09	− 0.08	+ 0.02	− 0.04
−1	8	−3	+ 0.11	+ 0.66	+0.18	+0.43	− 0.88	+ 3.30	− 0.16	+ 1.59
0	8	−4	− 0.19	− 0.81	−0.23	−0.46	+ 0.91	− 3.08	+ 0.15	− 1.46
1	8	−5	+ 0.1	+ 0.3			+ 0.1	+ 0.9	+ 0.1	+ 0.4
−1	8	−4	− 3.2	− 0.3	−2.0	+0.2	−13.4	− 8.7	− 6.7	− 3.4
0	8	−5	+ 3.7	+ 0.3	+2.1	−0.1	+12.9	+ 8.6	+ 6.7	+ 3.5
1	8	−6	− 1.4	+ 0.1			− 4.2	− 1.7	− 2.1	− 0.5
−1	8	−5	+ 2.8	− 7.4	+1.4	−4.4	+31.2	−26.8	+14.1	−13.2
0	8	−6	− 3.1	+ 8.4	−1.5	+4.6	−29.5	+25.6	−12.5	+12.7
1	8	−7	+ 1	− 3			+ 8	− 8	+ 4	− 5
−1	8	−6	+ 5	+ 7	+3	+3	+20	+43	+ 8	+14
0	8	−7	− 7	− 9	−4	−2	−21	−41	−10	−19
1	8	−8	+ 2	+ 1			+ 8	+13	+ 3	+ 5
−1	8	−7	− 7	− 3	+4	+2	−14	+ 3	+14	+ 1
0	8	−8	+ 9	+ 4	−5	−2	+14	− 4	−14	− 1
1	8	−9					− 6	0		
0	9	−3					− 0.04	+ 0.03		
−1	9	−3	+ 0.08	+ 0.09	+0.08	+0.04	+ 0.17	+ 0.61	+ 0.13	+ 0.25
0	9	−4	− 0.10	− 0.11	−0.08	−0.05	− 0.13	− 0.56	− 0.12	− 0.23
1	9	−5					+ 0.13	+ 0.14	+ 0.07	+ 0.04
−1	9	−4	− 0.70	+ 0.32	−0.41	+0.30	− 3.91	− 0.26	− 1.82	+ 0.15
0	9	−5	+ 0.8	− 0.4	+0.5	−0.3	+ 3.6	+ 0.2	+ 1.5	− 0.2
1	9	−6					− 1.1	+ 0.3	− 0.4	+ 0.3
−1	9	−5	− 0.3	− 2.9	−0.4	−1.7	+ 5.4	−13.9	+ 2.0	− 7.0
0	9	−6	+ 0.4	+ 3.3	+0.4	+1.8	− 4.9	+12.9	− 1.6	+ 6.2
1	9	−7					0	− 5	0	− 2
−1	9	−6	+ 6	+ 1	+3	+1	+26	+20	+11	+ 8
0	9	−7	− 7	− 1	−3	0	−25	−19	−11	− 7
1	9	−8					+ 7	+ 4	+ 4	+ 2
−1	9	−7	− 5	+ 4	−1	+3	−25	+20	− 6	+ 6
0	9	−8	+ 1	− 5			+24	−21	+ 6	− 7
1	9	−9					− 5	+ 5	− 3	+ 3
−1	9	−8	+ 2	− 5			− 5	−12	+ 2	+ 7
0	9	−9	− 1	+ 7			+ 5	+ 8	− 2	− 7

Arg= $\varkappa\gamma+i'g'+ig$			$An\delta z$		$B(\nu-c)$		$Fn'\delta z'$		$G(\nu'-c')$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	10	−3	+0.019	+0.005	+0.016	−0.001	+0.074	+0.071	+0.039	+0.023
0	10	−4	−0.02068	−0.00889	−0.01523	−0.00131	−0.06526	−0.07065	−0.03376	−0.02171
1	10	−5	+0.010	−0.002	+0.005	−0.003	+0.031	+0.007	+0.012	−0.005
−1	10	−4	−0.098	+0.120	−0.048	+0.093	−0.727	+0.349	−0.280	+0.205
0	10	−5	+0.1	−0.1			+0.6	−0.3	+0.2	−0.2
1	10	−6					−0.2	+0.2		
−1	10	−5	−0.5	−0.6	−0.4	−0.3	−0.6	−3.7	−0.6	−1.6
0	10	−6	+0.5	+0.7			+0.5	+3.5	+0.5	+1.5
1	10	−7					−0.4	−1.1		
−1	10	−6	+2.3	−0.8	+1.3	−0.6	+12.3	+1.9	+5.7	+0.3
0	10	−7	−3	+1	−1	+1	−11	−2	−6	−1
1	10	−8					+4	0	+1	0
−1	10	−7	0	+4	0	+2	−10	+22	−3	+9
0	10	−8	−1	−4			+10	−22	+4	−9
1	10	−9					−2	+6	−1	+3
−1	10	−8					−17	−14	−4	+1
0	10	−9					+17	+14	+4	+1
−1	11	−4					−0.07	+0.11	−0.01	+0.05
0	11	−5					+0.07	−0.10		
−1	11	−5	−0.1	−0.1			−0.5	−0.2	−0.3	−0.2
0	11	−6					+0.5	+0.7	+0.3	+0.2
−1	11	−6	+0.5	−0.5			+3.3	−1.2	+1.3	−0.8
0	11	−7					−3.2	+1.1	−1.4	+0.7
1	11	−8					+1	−1		
−1	11	−7	+1	+1			0	+9	0	+5
0	11	−8	−2	−1			0	−9	0	−4
1	11	−9					0	+3		
−1	11	−8	−3	+1			−16	−4	−8	−1
0	11	−9					+16	+4	+8	+1
0	11	−10					−5	+12	−1	+5
0	12	−5	−0.001	−0.005			+0.001	−0.020	−0.002	−0.007
−1	12	−5					−0.15	−0.06	−0.06	0.00
−1	12	−6					+0.6	−0.6	+0.2	−0.3
0	12	−7					−0.6	+0.6		
−1	12	−7					+1.6	+2.6	+0.8	+1.1
0	12	−8					−2	−3	−1	−1
−1	12	−8					−7	+2	−3	+1
0	12	−9					+7	−2	+3	−1
−1	12	−9					0	−10	−1	−5
0	12	−10					0	+9	0	+5

Arg= $x\gamma+i'g'+ig$			$C\delta\frac{h}{h_0}$		$D\frac{u}{\cos i}$		$E\frac{u_1}{\cos i}$		$H\frac{u'}{\cos i'}$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
x	i'	i	″	″	″	″	″	″	″	″
0	0	0		+0.0011		+0.1387		−0.1387		
1	0	−1	0.0000	−0.0012	−0.0003	−0.1848	−0.0001	+0.1850	−0.0079	+0.0005
−1	0	0	+0.0011	+0.0051	+0.0479	+0.0077	−0.0021	−0.0169	+0.1239	−0.0250
1	0	−2	+0.001	+0.005	−0.011	0.000	+0.003	−0.007	−0.024	−0.022
−1	1	+1							+0.05	+0.09
1	1	−1			−0.02	−0.03			−0.01	−0.04
−1	1	−0	+0.01	+0.07	+0.10	−0.02	−0.13	+0.02	+0.07	−0.03
0	1	−1	−0.03	−0.14	−0.18	+0.03	+0.19	−0.04	−0.04	+0.01
1	1	−2			+0.1	−0.1			−0.1	0.0
−1	1	−1							0.0	+0.1
−1	1	−2							−0.7	−0.5
−1	2	+1			+0.3	+0.1			+0.4	+0.2
0	2	0			−0.13	−0.05	+0.13	+0.05		
1	2	−1			−0.07	−0.02	−0.09	−0.03	−0.42	−0.16
−1	2	0	+0.05	−0.01	0.00	−0.03			−0.08	−0.07
0	2	−1	−0.111	+0.020	−0.018	+0.031	+0.025	−0.037	+0.033	+0.046
1	2	−2	+0.07	−0.02	+0.03	−0.02			+0.06	−0.04
−1	2	−1	−0.45	+0.19	−0.01	−0.03	−0.05	−0.12	+0.21	+0.46
0	2	−2	+0.9	−0.4	−0.1	−0.2			−0.2	−0.5
1	2	−3	−0.4	+0.2						
1	3	−1							−0.10	+0.01
−1	3	0			−0.04	+0.24	+0.01	−0.06	−0.08	+0.53
0	3	−1	−0.018	+0.013	+0.022	−0.170	−0.011	+0.085	+0.039	−0.224
1	3	−2					+0.01	−0.05	+0.02	−0.20
−1	3	−1	−0.06	+0.09	+0.02	−0.02	−0.02	−0.02	+0.12	0.00
0	3	−2	+0.11	−0.18					−0.06	+0.02
−1	3	−2	−0.2	−0.3	+0.1	0.0			−0.4	+0.4
0	3	−3	+0.4	+0.5					+0.6	−0.4
−1	4	−0							+0.1	+0.1
0	4	−1			−0.02	−0.05			−0.02	−0.06
1	4	−2							−0.02	−0.06
−1	4	−1			−0.20	+0.02	+0.04	0.00	−0.46	+0.02
0	4	−2	0.00	−0.04	+0.15	−0.01	−0.05	0.00	+0.29	−0.01
−1	4	−2	−0.09	−0.04	+0.03	−0.08			+0.01	+0.11
1	5	−2			0.000	−0.003	−0.004	−0.002	−0.011	−0.012
−1	5	−1	+0.001	+0.002	−0.077	+0.044	+0.012	−0.008	−0.164	+0.090
0	5	−2	−0.00305	−0.00359	+0.05809	−0.03320	−0.01824	+0.01012	+0.09935	−0.05494
1	5	−3	+0.002	+0.002	+0.001	+0.003	+0.011	−0.009	+0.032	−0.011
−1	5	−2	−0.019	+0.003	−0.042	−0.158	+0.005	+0.019	−0.072	−0.311
0	5	−3			0.0	+0.1			0.0	+0.2
0	6	−2							+0.01	−0.02
−1	6	−2			−0.05	−0.06			−0.11	−0.12
0	6	−3			+0.04	+0.04			+0.07	+0.08
−1	6	−3							+0.2	+0.1

Arg= $\varkappa\gamma+i'g'+ig$			$C\delta\frac{h}{h_0}$		$D\frac{u}{\cos i}$		$E\frac{u_1}{\cos i}$		$H\frac{u'}{\cos i'}$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	"	"	"	"	"	"	"	"
−1	7	−2			−0.03	0.00			−0.04	−0.01
0	7	−3			+0.018	+0.005			+0.030	+0.010
−1	7	−3			+0.04	−0.05			+0.08	−0.10
0	8	−3							+0.007	−0.001
−1	8	−3			0.00	−0.02			+0.01	−0.05
0	10	−4			+0.00068	+0.00067			+0.00141	+0.00132
−1	10	−4			+0.004	−0.003			+0.009	−0.007

CHAPTER XII.

SECOND-ORDER PERTURBATIONS OF THE MEAN ANOMALY AND RADIUS-VECTOR OF JUPITER, ARISING FROM THE MUTUAL ACTION OF THIS PLANET AND SATURN.

When the eight terms of δT, given in the two preceding chapters, are added the following expression is obtained:

$\text{Arg}=\varkappa\gamma+i'g'+ig$			δT			
			sin.		cos.	
$\varkappa$	i'	i	″	″	″	″
0	0	0			−0.00000704	+1.0520nt
1	0	−1	+0.0533	+1.1197nt	−0.0088	−0.4622nt
−1	0	0	−0.2260338	−29.4310nt	+0.2563282	−17.3424nt
0	0	−1	+0.2412	+10.3nt	−0.2703	+3.8nt
1	0	−2	−0.064	+6.952nt	+0.094	+2.530nt
−1	0	−1	−0.474	+3.0nt	−0.356	−1.1nt
0	0	−2	+0.486	−1.4nt	+0.351	+0.9nt
1	0	−3	−0.159	−0.1nt	−0.100	+0.2nt
−1	0	−2	+0.408		−0.318	
0	0	−3	−0.406		+0.342	
1	0	−4	+0.140		−0.113	
−1	0	−3	+0.010		−0.063	
0	0	−4	−0.03		+0.08	
1	0	−5	+0.01		−0.02	
−1	1	+4	+0.031		−0.022	
0	1	+3	−0.079		+0.051	
1	1	+2	+0.060		−0.021	
−1	1	+3	+0.106		−0.211	
0	1	+2	−0.321		+0.636	
1	1	+1	+0.290		−0.624	
−1	1	+2	+0.139	−0.8nt	+0.145	+1.2nt
0	1	+1	−0.489	−0.8nt	−0.426	+2.8nt
1	1	0	+0.4840	+2.44nt	+0.4225	−6.38nt
−1	1	+1	−0.0616	−25.97nt	+0.0138	−33.74nt
0	1	0	+0.1491	+7.97nt	−0.1067	+4.96nt
1	1	−1	−0.1322	+23.01nt	+0.1016	+31.88nt
−1	1	0	−0.0487	+1.83nt	−0.0888	−1.07nt
0	1	−1	+0.0466	−2.36nt	+0.0463	−0.59nt
1	1	−2	+0.0053	+0.3nt	+0.0248	+0.3nt
−1	1	−1	−0.3133	+2.9nt	−0.1696	+8.8nt

$\text{Arg}=\varkappa\gamma+i'g'+ig$			δT			
			sin.		cos.	
$\varkappa$	i'	i	″	″	″	″
0	1—	2	+0.330	+ 1.3nt	+0.178	— 17.0nt
1	1—	3	—0.101	— 1.0nt	—0.041	+ 9.2nt
—1	1—	2	+0.203	+ 0.1nt	—0.443	+ 1.6nt
0	1—	3	—0.198	— 0.7nt	+0.451	— 2.2nt
1	1—	4	+0.053	0nt	—0.146	+ 1nt
—1	1—	3	+0.281		+0.226	
0	1—	4	—0.296		—0.216	
1	1—	5	+0.11		+0.08	
—1	2+	3	+0.030		+0.023	
0	2+	2	—0.062		—0.060	
1	2+	1	+0.024		+0.048	
—1	2+	2	+0.268		+0.070	
0	2+	1	—0.833		—0.205	
1	2	0	+0.7932	— 0.1nt	+0.1733	— 0.8nt
—1	2+	1	—0.1031	— 8.4nt	+0.1373	— 2.4nt
0	2	0	+0.2941	+ 2.55nt	—0.5176	— 2.23nt
1	2—	1	—0.2951	+ 6.15nt	+0.5109	+ 6.30nt
—1	2	0	+0.0064	+ 55.16nt	—0.0385	— 82.14nt
0	2—	1	+0.02677	— 44.811nt	+0.05928	+ 88.390nt
1	2—	2	—0.0184	— 3.66nt	—0.0457	— 12.44nt
—1	2—	1	+0.0411	— 0.78nt	—0.0711	+ 12.15nt
0	2—	2	—0.0108	+ 0.2nt	+0.0632	— 9.1nt
1	2—	3	—0.017	+ 1.0nt	—0.005	+ 1.6nt
—1	2—	2	+0.086	— 34.1nt	—0.323	+ 1.3nt
0	2—	3	—0.089	+ 49.9nt	+0.337	+ 5.0nt
1	2—	4	+0.015	— 23.1nt	—0.090	— 4.1nt
—1	2—	3	+0.369	— 3.0nt	+0.078	+ 2.6nt
0	2—	4	—0.371	+ 7nt	—0.066	— 4nt
1	2—	5	+0.113	— 2nt	+0.018	+ 1nt
—1	3+	2	—0.012		+0.028	
0	3+	1	+0.032		—0.053	
1	3	0	—0.027		+0.012	
—1	3+	1	—0.001	— 1.3nt	+0.270	+ 0.8nt
0	3	0	+0.0041	+ 0.2nt	—0.8536	— 0.8nt
1	3—	1	+0.0004	+ 1.72nt	+0.7720	+ 0.74nt
—1	3	0	+0.0053	— 10.31nt	—0.0348	— 33.34nt
0	3—	1	+0.11465	+ 11.569nt	+0.08256	+ 30.194nt
1	3—	2	—0.1289	— 11.91nt	—0.0856	— 5.85nt
—1	3—	1	+0.0050	+ 251.76nt	—0.0246	+ 145.18nt
0	3—	2	—0.0081	— 253.99nt	+0.0355	— 136.31nt
1	3—	3	+0.0045	+ 71.4nt	—0.0146	+ 35.4nt
—1	3—	2	+0.0916	— 9.2nt	+0.0384	+ 3.0nt
0	3—	3	—0.081	+ 1.7nt	—0.026	— 6.5nt
1	3—	4	+0.024	+ 0.9nt	+0.012	+ 3.8nt
—1	3—	3	+0.276	— 7.4nt	+0.002	— 41.4nt

Arg=$\kappa\gamma+i'g'+ig$			δT			
			sin.		cos.	
κ	i'	i	″	″	″	″
0	3	−4	−0.270	+6.4nt	−0.002	+51.4nt
1	3	−5	+0.077	0nt	−0.007	−22nt
−1	3	−4	+0.007	−3nt	+0.269	−5nt
0	3	−5	−0.014	+2nt	−0.263	+6nt
1	3	−6	0.00	−1nt	+0.08	−2nt
−1	4	+1	+0.002		−0.007	
0	4	0	−0.006		+0.011	
1	4	−1	+0.0076	+0.1nt	−0.0067	0nt
−1	4	0	+0.0409	−6.0nt	−0.0071	−5.1nt
0	4	−1	+0.0611	+5.30nt	−0.0168	+4.29nt
1	4	−2	−0.0783	−3.36nt	+0.0189	+0.26nt
−1	4	−1	−0.0166	+87.10nt	+0.0489	−1.22nt
0	4	−2	+0.0076	−84.49nt	+0.0173	+1.91nt
1	4	−3	+0.0026	+23.57nt	−0.0550	−7.41nt
−1	4	−2	+0.1089	−85.49nt	−0.0252	+257.49nt
0	4	−3	−0.114	+77.3nt	+0.029	−258.0nt
1	4	−4	+0.041	−21.2nt	−0.007	+76.9nt
−1	4	−3	+0.005	−4.5nt	+0.059	−4.8nt
0	4	−4	−0.018	+6.2nt	−0.049	−2.7nt
1	4	−5	−0.006	−4nt	+0.016	0nt
−1	4	−4	+0.043	+35nt	+0.191	−15nt
0	4	−5	−0.042	−41nt	−0.186	+15nt
1	4	−6	+0.013	+16nt	+0.041	−5nt
−1	4	−5	−0.182	+3nt	+0.040	−4nt
0	4	−6	+0.15	−7nt	−0.04	+6nt
1	4	−7	−0.03	+2nt	+0.01	−3nt
−1	5	0	+0.0018	−0.9nt	−0.0003	−0.3nt
0	5	−1	+0.0011	+0.8nt	−0.0035	+0.1nt
1	5	−2	−0.001763	−0.462nt	+0.001130	+0.335nt
−1	5	−1	+0.008734	+14.351nt	+0.030313	−9.125nt
0	5	−2	+0.0040821	−13.66051nt	+0.0001070	+8.46360nt
1	5	−3	+0.015221	+2.569nt	+0.040083	−4.535nt
−1	5	−2	+0.001014	+20.443nt	−0.034322	+101.528nt
0	5	−3	−0.0286	−21.2nt	+0.0360	−99.4nt
1	5	−4	+0.0185	+12.0nt	−0.0140	+28.5nt
−1	5	−3	+0.049	−217.1nt	+0.116	−25.0nt
0	5	−4	−0.053	+215.8nt	−0.129	+18.8nt
1	5	−5	+0.017	−66.2nt	+0.040	−4.1nt
−1	5	−4	−0.027	+1.6nt	+0.023	−4.2nt
0	5	−5	+0.025	+4nt	−0.033	+6nt
1	5	−6	−0.008	−3nt	+0.005	−3nt
−1	5	−5	−0.127	+16nt	+0.061	+25nt
0	5	−6	+0.127	−18nt	−0.064	−27nt
1	5	−7	−0.02	+5nt	+0.01	+9nt
−1	5	−6	−0.05	+6nt	−0.12	+2nt
0	5	−7	+0.05	−6nt	+0.11	−6nt
1	5	−8	−0.01		−0.02	

Arg=$x\gamma+i'g'+ig$			δT	
			sin.	cos.
x	i'	i	″ ″	″ ″
1	6	−2	−0.0002	−0.0006
−1	6	−1	−0.0064 + 1.07*nt*	−0.0035 − 2.55*nt*
0	6	−2	+0.0007 − 1.15*nt*	−0.0004 + 2.29*nt*
1	6	−3	+0.0110 − 0.25*nt*	+0.0098 − 0.86*nt*
−1	6	−2	+0.0787 + 16.29*nt*	−0.0663 + 18.09*nt*
0	6	−3	−0.0612 − 15.60*nt*	+0.0528 − 17.56*nt*
1	6	−4	−0.0364 + 6.7*nt*	+0.0212 + 3.6*nt*
−1	6	−3	+0.0457 − 93.9*nt*	+0.0536 + 39.1*nt*
0	6	−4	−0.048 + 92.6*nt*	−0.064 − 39.7*nt*
1	6	−5	+0.009 − 26.1*nt*	+0.012 + 16.7*nt*
−1	6	−4	−0.086 − 15.4*nt*	+0.004 −160.4*nt*
0	6	−5	+0.098 + 20.2*nt*	−0.070 +158.4*nt*
1	6	−6	−0.026 − 8*nt*	+0.018 − 48*nt*
−1	6	−5	−0.025 + 4*nt*	−0.008 − 2*nt*
0	6	−6	+0.017 − 3*nt*	+0.009 + 5*nt*
1	6	−7	−0.01 + 2*nt*	0.00 0*nt*
−1	6	−6	−0.05 − 13*nt*	−0.05 + 16*nt*
0	6	−7	+0.05 + 17*nt*	+0.05 − 16*nt*
1	6	−8	− 9*nt*	+ 8*nt*
−1	7	−1	−0.0023 0.0*nt*	+0.0003 − 0.2*nt*
0	7	−2	+0.0004 − 0.02*nt*	+0.0001 + 0.17*nt*
1	7	−3	+0.00285 − 0.10*nt*	+0.00160 − 0.08*nt*
−1	7	−2	−0.01947 + 4.32*nt*	−0.06596 + 1.34*nt*
0	7	−3	+0.02661 − 4.090*nt*	+0.05637 − 1.339*nt*
1	7	−4	−0.0365 + 1.44*nt*	−0.0163 − 0.28*nt*
−1	7	−3	+0.6246 − 17.17*nt*	+0.5861 + 22.24*nt*
0	7	−4	−0.6835 + 16.4*nt*	−0.6413 − 22.0*nt*
1	7	−5	+0.212 − 3.0*nt*	+0.199 + 7.9*nt*
−1	7	−4	−0.044 − 49.7*nt*	+0.070 − 73.5*nt*
0	7	−5	+0.018 + 50.3*nt*	−0.099 + 71.9*nt*
1	7	−6	+0.003 − 18.7*nt*	+0.036 − 20.3*nt*
−1	7	−5	−0.065 +106.7*nt*	−0.053 − 33.7*nt*
0	7	−6	+0.071 −104*nt*	+0.074 + 37*nt*
1	7	−7	−0.017 + 30*nt*	−0.010 − 14*nt*
−1	7	−6	−0.006 + 1*nt*	−0.010 + 2*nt*
0	7	−7	+0.03 − 5*nt*	+0.01 − 1*nt*
1	7	−8	0*nt*	+ 5*nt*
−1	7	−7	+0.02 − 12*nt*	−0.03 − 9*nt*
0	7	−8	−0.01 + 14*nt*	+0.03 + 8*nt*
1	7	−9	− 7*nt*	− 5*nt*
−1	8	−2	−0.0127 + 0.63*nt*	−0.0138 − 0.24*nt*
0	8	−3	+0.01234 − 0.666*nt*	+0.01122 + 0.156*nt*
1	8	−4	−0.0105 + 0.13*nt*	−0.0006 − 0.15*nt*
−1	8	−3	+0.2509 − 0.74*nt*	+0.0589 + 5.91*nt*
0	8	−4	−0.2548 + 0.64*nt*	−0.0661 − 5.81*nt*

Arg=κγ+i′g′+ig			δT			
			sin.		cos.	
κ	i′	i	″	″	″	″
1	8	−5	+0.0826	+ 0.3*nt*	−0.0006	+ 1.6*nt*
−1	8	−4	−0.4717	−25.3*nt*	+0.7795	−12.2*nt*
0	8	−5	+0.489	+25.4*nt*	−0.822	+12.3*nt*
1	8	−6	−0.152	− 7.7*nt*	+0.262	− 2.1*nt*
−1	8	−5	−0.071	+49.5*nt*	−0.001	−51.8*nt*
0	8	−6	+0.084	−46.6*nt*	−0.029	+51.3*nt*
1	8	−7	−0.036	+13*nt*	+0.015	−16*nt*
−1	8	−6	+0.029	+36*nt*	−0.055	+67*nt*
0	8	−7	−0.027	−42*nt*	+0.054	−71*nt*
1	8	−8	0.00	+13*nt*	−0.01	+19*nt*
−1	8	−7	−0.01	− 3*nt*	+0.01	+ 3*nt*
0	8	−8	0.00	+ 4*nt*	+0.01	− 3*nt*
1	8	−9		− 6*nt*		0*nt*
−1	9	−2	−0.0030		−0.0013	
0	9	−3	+0.0023	− 0.04*nt*	+0.0009	+ 0.03*nt*
1	9	−4	−0.0016		+0.0007	
−1	9	−3	+0.0478	+ 0.46*nt*	−0.0156	+ 0.99*nt*
0	9	−4	−0.04738	− 0.43*nt*	+0.01338	− 0.95*nt*
1	9	−5	+0.0118	+ 0.20*nt*	−0.0118	+ 0.18*nt*
−1	9	−4	−0.0149	− 6.84*nt*	+0.3398	+ 0.51*nt*
0	9	−5	+0.019	+ 6.4*nt*	−0.347	− 0.7*nt*
1	9	−6	+0.012	− 1.5*nt*	+0.109	+ 0.6*nt*
−1	9	−5	−0.735	+ 6.7*nt*	−0.254	−25.5*nt*
0	9	−6	+0.765	− 5.7*nt*	+0.252	+24.2*nt*
1	9	−7	−0.241	0*nt*	−0.079	− 7*nt*
−1	9	−6	−0.032	+46*nt*	−0.037	+30*nt*
0	9	−7	+0.062	−46*nt*	+0.050	−27*nt*
1	9	−8	−0.026	+11*nt*	−0.022	+ 6*nt*
−1	9	−7	+0.026	−37*nt*	−0.006	+33*nt*
0	9	−8	−0.02	+31*nt*	+0.02	−33*nt*
1	9	−9		− 8*nt*		+ 8*nt*
−1	9	−8		− 1*nt*		−10*nt*
0	9	−9		+ 2*nt*		+ 8*nt*
−1	10	−3	+0.00528	+ 0.148*nt*	−0.00629	+ 0.098*nt*
0	10	−4	−0.0052516	− 0.13299*nt*	+0.0055258	− 0.10069*nt*
1	10	−5	+0.00011	+ 0.058*nt*	−0.00281	− 0.003*nt*
−1	10	−4	+0.03685	− 1.140*nt*	+0.07039	+ 0.757*nt*
0	10	−5	−0.0338	+ 0.9*nt*	−0.0700	− 0.6*nt*
1	10	−6	+0.017	− 0.2*nt*	+0.017	+ 0.2*nt*
−1	10	−5	−0.348	− 2.1*nt*	+0.056	− 6.2*nt*
0	10	−6	+0.356	+ 1.5*nt*	−0.053	+ 5.7*nt*
1	10	−7	−0.112	− 0.4*nt*	+0.032	− 1.1*nt*
−1	10	−6	+0.067	+21.6*nt*	−0.591	+ 0.8*nt*
0	10	−7	−0.054	−21*nt*	+0.610	− 1*nt*
1	10	−8	+0.017	+ 5*nt*	−0.188	0*nt*
−1	10	−7	+0.012	−13*nt*	−0.041	+37*nt*

Arg=$\varkappa\gamma+i'g'+ig$			δT			
			sin.		cos.	
$\varkappa$	i'	i	″	″	″	″
0	10	−8	−0.021	+13*nt*	+0.072	−35*nt*
1	10	−9	0.00	− 3*nt*	−0.02	+ 9*nt*
−1	10	−8		−21*nt*		−15*nt*
0	10	−9	0.00	+21*nt*	−0.02	+15*nt*
0	11	−4	−0.0002		+0.0009	
−1	11	−4	+0.0124	− 0.08*nt*	+0.0077	+ 0.16*nt*
0	11	−5	−0.0114	+ 0.07*nt*	−0.0078	− 0.10*nt*
1	11	−6	+0.0047		+0.0006	
−1	11	−5	−0.0752	− 0.9*nt*	+0.0593	− 0.9*nt*
0	11	−6	+0.075	+ 0.8*nt*	−0.056	+ 0.9*nt*
1	11	−7	−0.020		+0.024	
−1	11	−6	−0.114	+ 5.1*nt*	−0.300	− 2.5*nt*
0	11	−7	+0.112	− 4.6*nt*	+0.301	+ 1.8*nt*
1	11	−8	−0.046	+ 1*nt*	−0.094	− 1*nt*
−1	11	−7	+0.419	+ 1*nt*	−0.045	+15*nt*
0	11	−8	−0.426	− 2*nt*	+0.059	−14*nt*
1	11	−9	+0.14	0*nt*	−0.02	+ 3*nt*
−1	11	−8	+0.03	−27*nt*	0.00	− 4*nt*
0	11	−9	−0.05	+24*nt*	0.00	+ 5*nt*
0	11	−10		− 6*nt*		+17*nt*
0	12	−5	−0.00225	− 0.002*nt*	−0.00010	− 0.032*nt*
−1	12	−5	−0.0081	− 0.21*nt*	+0.0200	− 0.06*nt*
0	12	−6	+0.0076		−0.0174	
1	12	−7	0.000		+0.005	
−1	12	−6	−0.073	+ 0.8*nt*	−0.067	− 0.9*nt*
0	12	−7	+0.076	− 0.6*nt*	+0.066	+ 0.6*nt*
1	12	−8	−0.028		−0.016	
−1	12	−7	+0.220	+ 2.4*nt*	−0.142	+ 3.7*nt*
0	12	−8	−0.219	− 3*nt*	+0.140	− 4*nt*
1	12	−9	+0.066		−0.048	
−1	12	−8	+0.096	−10*nt*	+0.271	+ 3*nt*
0	12	−9	−0.09	+10*nt*	−0.26	− 3*nt*
1	12	−10	+0.03		+0.04	
−1	12	−9		− 1*nt*		−15*nt*
0	12	−10		0*nt*		+14*nt*

This expression for δT must be subjected to the same treatment as that we employed in Chapter II for deriving $\frac{d\,.\,\delta z}{dt}$ and $\frac{d\nu}{dt}$ from T; that is, we must obtain $\overline{\delta W_0}$ and $-\frac{1}{2}\left(\overline{\frac{d\,.\,\delta W_0}{d\gamma}}\right)$ from δW_0, the latter being given by the equation

$$\delta W_0 = \int \delta T n dt$$

In deriving δW_0 from δT, in the terms whose arguments involve $5g'-2g$ or $10g'-4g$, it seems advantageous to equate the motion of the argument. In this way a more rapid approximation to the correct values of these terms is secured. Joining T and δT together, and supposing that the argument is denoted by χ, let

$$T + \delta T = (a + bnt) \sin \chi + (c + dnt) \cos \chi$$

By means of the equations

$$A \cos K = a \qquad A \sin K = c$$

$$B = \frac{ab + cd}{A} \qquad \varkappa = \frac{ad - bc}{A^2}$$

these terms can be approximately expressed

$$T + \delta T = (A + Bnt) \sin (\chi + K + \varkappa nt)$$

and thence by integrating

$$W_0 + \delta W_0 = - \mu (A + Bnt) \cos (\chi + K + \varkappa nt) + \mu^2 B \sin (\chi + K + \varkappa nt)$$

where μ denotes the integrating factor for the argument $\chi + \varkappa nt$. In the special cases we treat μ will be derived from either of the formulæ

$$\frac{1}{\mu} = \frac{5n' - 2n}{n} + \varkappa \text{ or } \frac{1}{\mu} = \frac{10n' - 4n}{n} + \varkappa$$

By developing the sine and cosine of the argument in powers of $\varkappa nt$, and neglecting in the coefficients terms multiplied by n^2t^2, we get

$$\begin{aligned} W_0 + \delta W_0 = & - \mu[a - \mu B \sin K + (b - \varkappa \mu B \cos K)nt] \cos \chi \\ & + \mu[c + \mu B \cos K + (d - \varkappa \mu B \sin K)nt] \sin \chi \end{aligned}$$

By subtracting from this the value of W_0, found in Chapter II, we have the value of δW_0.

In the five terms we treat by this process there has been found:

Arg.	log A	K	log B	log $\varkappa$	log μ
		° ′ ″			
$\gamma + 5g' - 2g$	7.8385	217 19 24	4.2156	6.8991*n*	1.8983560
$5g' - 2g$	9.2905882	248 8 38.26	5.4424349*n*	6.9088766*n*	1.8989767
$-\gamma + 5g' - 2g$	0.3077026	176 39 35.74	6.1611524*n*	6.7032188*n*	1.8885772
$10g' - 4g$	7.81648	133 13 55	3.24871	6.40327	1.56683
$-\gamma + 10g' - 4g$	8.8359	62 10	4.1369	6.2981	1.56770

By the aid of these quantities the terms of δW_0 in question have been found as follows:

Arg=$\varkappa\gamma+i'g'+ig$			δW_0	
			cos.	sin.
$\varkappa$	i'	i	″ ″	″ ″
1	5	−2	$+0.1508+0.000372nt$	$+0.0564+0.000260nt$
0	5	−2	$+0.20521+0.0107729nt$	$-0.79420+0.00683789nt$
−1	5	−2	$+5.7718-0.016254nt$	$-1.3453+0.078528nt$
0	10	−4	$+0.13735+0.00004901nt$	$+0.14799-0.00003718nt$
−1	10	−4	$-1.3613+0.000421nt$	$+2.6053+0.000279nt$

The expressions for $\overline{\delta W_0}$ and $-\frac{1}{2}\left(\frac{\overline{d\,.\,\delta W_0}}{d\gamma}\right)$ follow:

Arg=$i'g'+ig$		$\overline{\delta W_0}$			
		cos.		sin.	
i'	i	″	″	″	″
0	0	$+0.0525$	$+0.4995nt$		
		$+k_0$	$+0.5260n^2t^2$		
0	−1	$+0.2093$	$+25646.6nt$	$+0.2242$	$-22608.5nt$
		$+k_1$	$-8.6712n^2t^2$	$+k_2$	$-14.7155n^2t^2$
0	−2	-0.2832	$+620.2nt$	$+0.2146$	$-544.4nt$
		$+[8.3821]k_1$	$-0.2090n^2t^2$	$+[8.3822]k_2$	$-0.3548n^2t^2$
0	−3	$+0.0923$	$+22.4nt$	$+0.0818$	$-19.7nt$
		$+[6.9403]k_1$	$-0.0076n^2t^2$	$+[6.9405]k_2$	$-0.0128n^2t^2$
0	−4	$+0.0022$	$+1.0nt$	$+0.0091$	$-0.8nt$
			$-0.0003n^2t^2$		$-0.0005n^2t^2$
1	+4	-0.0007		-0.0006	
1	+3	-0.0148		-0.0088	
1	+2	-0.1337	$-0.1nt$	-0.2170	$-0.4nt$
1	+1	-0.9173	$-4.3nt$	$+0.8004$	$-14.6nt$
1	0	-0.5479	$+37.25nt$	-0.4234	$-65.10nt$
1	−1	$+0.2023$	$-7.86nt$	-0.3128	$-2.53nt$
1	−2	-0.3533	$+5.2nt$	$+0.1841$	$-7.7nt$
1	−3	$+0.0531$	$-0.1nt$	$+0.1508$	$-0.9nt$
1	−4	$+0.0524$		-0.0375	
2	+2	-0.0241		$+0.0141$	
2	+1	-0.6558	$+0.9nt$	$+0.0635$	$-1.8nt$
2	0	-1.8249	$+33.0nt$	-3.1885	$-36.2nt$
2	−1	$+0.13729$	$-301.67nt$	-0.32321	$-545.77nt$
2	−2	$+0.1974$	$-5.7nt$	$+0.3142$	$-58.0nt$
2	−3	$+0.0420$	$-13.0nt$	$+0.1536$	$-3.7nt$
2	−4	$+0.0808$	$-0.4nt$	-0.0123	$-0.2nt$
2	−5	$+0.004$		-0.001	

$\text{Arg}=i'g'+ig$		$\overline{\delta W_0}$			
		cos.		sin.	
i'	i	″	″	″	″
3+	1	+0.0117	− 0.2*nt*	+0.0841	+ 0.1*nt*
3	0	−0.0084	− 8.3*nt*	+3.1295	+ 3.5*nt*
3−	1	−0.7115	− 61.14*nt*	+0.4816	+ 124.44*nt*
3−	2	−0.0002	−1490.72*nt*	−0.1013	+ 849.48*nt*
3−	3	+0.0798	− 39.8*nt*	−0.0398	+ 15.6*nt*
3−	4	+0.0804	− 3.2*nt*	−0.0002	+ 11.0*nt*
3−	5	+0.0026	− 0.9*nt*	−0.0436	+ 1.1*nt*
4	0	−0.0143	− 0.4*n*.	−0.0081	0.0*nt*
4−	1	−0.3265	− 13.18*nt*	−0.0797	+ 3.30*nt*
4−	2	+0.0483	− 342.81*nt*	+0.0721	− 2.44*nt*
4−	3	+0.2313	− 176.3*nt*	+0.0426	− 508.0*nt*
4−	4	+0.0018	− 8.1*nt*	−0.0254	− 11.3*nt*
4−	5	+0.0089	+ 5.8*nt*	−0.0353	+ 2.4*nt*
4−	6	−0.0247	+ 0.1*nt*	−0.0065	+ 0.6*nt*
5	0	+0.0036	+ 0.9*nt*	+0.0013	+ 0.6*nt*
5−	1	+0.1492	+ 36.9*nt*	+0.0518	+ 26.0*nt*
5−	2	+0.21209	+1065.872*nt*	−0.80459	+ 679.033*nt*
5−	3	+5.7511	−1641.6*nt*	−1.3745	+7939.1*nt*
5−	4	+0.1676	− 172.9*nt*	−0.0995	+ 206.5*nt*
5−	5	−0.0011	− 5.4*nt*	−0.0058	+ 8.3*nt*
5−	6	−0.0149	+ 1.9*nt*	−0.0065	− 3.4*nt*
5−	7	−0.0053	+ 0.4*nt*	+0.0111	+ 0.5*nt*
6−	1	+0.0010		−0.0018	
6−	2	+0.0210	+ 1.67*nt*	−0.0206	+ 5.08*nt*
6−	3	−0.3163	− 61.90*nt*	−0.2629	+ 71.07*nt*
6−	4	+0.0478	− 113.5*nt*	−0.0624	− 47.1*nt*
6−	5	−0.0223	− 8.0*nt*	+0.0172	+ 51.8*nt*
6−	6	−0.0084	+ 0.7*nt*	+0.0004	+ 1.7*nt*
6−	7	−0.0033	− 1.5*nt*	+0.0032	− 2.4*nt*
7−	2	+0.0160	− 0.5*nt*	−0.0084	+ 0.5*nt*
7−	3	+0.1416	− 26.66*nt*	−0.3811	+ 9.18*nt*
7−	4	+2.9728	− 82.5*nt*	−2.7899	− 107.5*nt*
7−	5	+0.0547	− 27.0*nt*	−0.1035	+ 33.7*nt*
7−	6	−0.0095	+ 22.3*nt*	−0.0007	+ 8.5*nt*
7−	7	+0.0046	+ 0.3*nt*	+0.0014	− 0.4*nt*
7−	8	+0.0029	− 1.8*nt*	+0.0013	+ 1.4*nt*
8−	3	−0.0577	+ 2.66*nt*	+0.0399	+ 0.67*nt*
8−	4	−1.4136	+ 4.25*nt*	+0.3484	+ 33.27*nt*
8−	5	−0.4131	− 20.8*nt*	−0.6273	+ 10.3*nt*
8−	6	−0.0349	+ 13.6*nt*	−0.0168	+ 15.2*nt*
8−	7	+0.0019	+ 5.3*nt*	+0.0067	− 8.6*nt*
8−	8	−0.0023	− 0.7*nt*	−0.0042	− 0.7*nt*

$\text{Arg}=i'g'+ig$	$\overline{\delta W_0}$	
	cos.	sin.
$i'\quad i$	$''\qquad''$	$''\qquad''$
9— 3	$-0.0066+0.1nt$	$-0.0011\quad 0.0nt$
9— 4	$-0.1940-1.75nt$	$-0.0531+3.98nt$
9— 5	$-0.0242-14.1nt$	$-0.6982-1.2nt$
9— 6	$-0.2846+2.1nt$	$+0.0801+10.4nt$
9— 7	$-0.0139+8.4nt$	$+0.0094-5.6nt$
9— 8	$+0.0022-4.5nt$	$-0.0022-4.1nt$
9— 9	$+0.0002-0.1nt$	$0.0000+0.6nt$
10— 4	$+0.18928+4.812nt$	$+0.20046-3.622nt$
10— 5	$-1.3884+42.8nt$	$+2.6682+28.4nt$
10— 6	$-0.2477-0.5nt$	$+0.0224+4.6nt$
10— 7	$+0.0103+5.0nt$	$+0.1425+0.1nt$
10— 8	$-0.0007-1.4nt$	$+0.0069-5.4nt$
10— 9	$+0.0001-1.2nt$	$+0.0046+0.5nt$
11— 4	$+0.0006$	$+0.0021$
11— 5	$-0.0461+0.3nt$	$+0.0310+0.6nt$
11— 6	$-0.0928-1.0nt$	$-0.0766+0.9nt$
11— 7	$-0.0444+1.7nt$	$+0.0978+1.2nt$
11— 8	$+0.0725-0.1nt$	$+0.0100-2.6nt$
11— 9	$+0.0014-2.4nt$	$+0.0004-0.1nt$
11—10	$-1.1nt$	$-3.1nt$
12— 5	$-0.0134-0.01nt$	$+0.0008+0.19nt$
12— 6	$-0.0420-1.3nt$	$-0.1066+0.4nt$
12— 7	$-0.0370+0.4nt$	$+0.0294+0.5nt$
12— 8	$+0.0465+0.1nt$	$+0.0336-0.4nt$
12— 9	$+0.0172-0.8nt$	$-0.0293-0.3nt$
12—10	$+0.0007-0.3nt$	$-0.0023+0.9nt$

$\text{Arg}=i'g'+ig$	$-\frac{1}{2}\left(\overline{\frac{d\,.\,\delta W_0}{dy}}\right)$	
	sin.	cos.
$i'\quad i$	$''\qquad''$	$''\qquad''$
0 0		$-0.0033-0.2006nt$
0— 1	$-0.0160-12814.7nt$	$+0.0229-11301.1nt$
	$-\frac{1}{2}k_1+4.3356n^2t^2$	$+\frac{1}{2}k_2-7.3577n^2t^2$
0— 2	$+0.2114-619.4nt$	$+0.1606-544.5nt$
	$-[8.3821]k_1+0.2090n^2t^2$	$+[8.3822]k_2-0.3548n^2t^2$
0— 3	$-0.0731-33.6nt$	$+0.0740-29.6nt$
	$-[7.1164]k_1+0.0113n^2t^2$	$+[7.1165]k_2-0.0192n^2t^2$
0— 4	$-0.0049-1.9nt$	$+0.0127-1.7nt$
	$+0.0006n^2t^2$	$-0.0011n^2t^2$

Arg$=i'g'+ig$	$-\frac{1}{2}\left(\overline{\frac{d\,.\,\delta W_0}{d\gamma}}\right)$	
	sin.	cos.
$i'\quad i$	″ ″	″ ″
$1+\;4$	-0.0008	$+0.0008$
$1+\;3$	-0.0155	$+0.0116$
$1+\;2$	$-0.1168 - 0.1nt$	$+0.1662 + 0.4nt$
$1+\;1$	$-0.5766 - 2.3nt$	$-0.4918 + 9.4nt$
$1\quad 0$	$-0.1306 + 10.00nt$	$+0.0914 + 14.78nt$
$1-\;1$	$-0.0607 + 1.90nt$	$-0.1026 - 1.81nt$
$1-\;2$	$+0.2402 - 2.5nt$	$+0.1278 - 5.7nt$
$1-\;3$	$-0.0437 - 0.1nt$	$+0.1249 - 0.8nt$
$1-\;4$	-0.0446	-0.0278
$2+\;2$	-0.0283	-0.0121
$2+\;1$	$-0.4810 + 0.9nt$	$-0.0318 + 1.3nt$
$2\quad 0$	$-0.7837 + 13.4nt$	$+1.3494 + 15.3nt$
$2-\;1$	$-0.00469 + 32.59nt$	$-0.04082 - 56.20nt$
$2-\;2$	$-0.1106 + 3.5nt$	$+0.1799 - 33.4nt$
$2-\;3$	$-0.0383 + 10.7nt$	$+0.1315 - 2.7nt$
$2-\;4$	$-0.0726 + 1.2nt$	$-0.0087 - 0.6nt$
$2-\;5$	-0.004	-0.001
$3+\;1$	$+0.0096 - 0.2nt$	$-0.0902 - 0.1nt$
$3\quad 0$	$-0.0050 - 4.8nt$	$-1.7968 - 1.8nt$
$3-\;1$	$-0.0790 - 10.83nt$	$-0.0651 - 17.00nt$
$3-\;2$	$-0.0032 + 624.74nt$	$-0.0346 + 358.12nt$
$3-\;3$	$-0.0532 + 34.9nt$	$-0.0236 + 15.4nt$
$3-\;4$	$-0.0696 + 3.9nt$	$-0.0023 + 9.4nt$
$3-\;5$	$-0.0050 + 0.5nt$	$-0.0399 + 1.2nt$
$4\quad 0$	$-0.0106 - 0.3nt$	$+0.0054 \quad 0.0nt$
$4-\;1$	$-0.0879 - 5.77nt$	$+0.0211 - 1.35nt$
$4-\;2$	$-0.0116 + 79.48nt$	$+0.0212 - 2.96nt$
$4-\;3$	$-0.1404 + 108.8nt$	$+0.0301 - 314.6nt$
$4-\;4$	$-0.0104 + 7.2nt$	$-0.0171 - 14.2nt$
$4-\;5$	$-0.0081 - 5.1nt$	$-0.0363 + 1.7nt$
$4-\;6$	$+0.0236 - 0.6nt$	$-0.0069 + 0.5nt$
$5\quad 0$	$+0.0036 + 0.9nt$	$-0.0013 - 0.6nt$
$5-\;1$	$+0.0762 + 18.5nt$	$-0.0273 - 13.2nt$
$5-\;2$	$+0.01228 + 8.489nt$	$+0.03518 - 6.460nt$
$5-\;3$	$-2.8808 + 816.3nt$	$-0.6754 + 3933.2nt$
$5-\;4$	$-0.1611 + 138.1nt$	$-0.0849 + 201.2nt$
$5-\;5$	$-0.0024 + 6.6nt$	$-0.0097 + 11.1nt$
$5-\;6$	$+0.0195 - 2.2nt$	$-0.0096 - 3.3nt$
$5-\;7$	$+0.0065 - 0.9nt$	$+0.0130 - 0.5nt$
$6-\;1$	$+0.0007$	$+0.0011$
$6-\;2$	$+0.0066 + 0.27nt$	$+0.0075 - 1.58nt$
$6-\;3$	$+0.0825 + 21.46nt$	$-0.0725 + 22.98nt$

Arg=$i'g'+ig$	$-\frac{1}{2}\left(\frac{d\,.\,\delta W_0}{d\gamma}\right)$	
	sin.	cos.
i' i	″ ″	″ ″
6— 4	—0.0336 +76.2*nt*	—0.0487 —29.6*nt*
6— 5	+0.0223 + 7.6*nt*	—0.0012 +42.3*nt*
6— 6	+0.0049 — 0.2*nt*	+0.0013 + 2.7*nt*
6— 7	+0.0072 + 1.0*nt*	+0.0071 — 1.5*nt*
7— 2	+0.0072 — 0.2*nt*	+0.0042 — 0.3*nt*
7— 3	—0.0250 + 3.22*nt*	—0.0449 + 0.79*nt*
7— 4	—1.6789 +46.7*nt*	—1.5758 —59.7*nt*
7— 5	—0.0641 +20.6*nt*	—0.1025 +24.8*nt*
7— 6	+0.0093 —19.7*nt*	+0.0054 + 7.5*nt*
7— 7	+0.0016 — 1.3*nt*	+0.0022 + 0.9*nt*
7— 8	—0.0024 + 0.9*nt*	+0.0036 + 0.6*nt*
8— 3	—0.0108 + 0.34*nt*	—0.0059 — 0.17*nt*
8— 4	+0.5874 — 1.65*nt*	+0.1348 +13.77*nt*
8— 5	+0.3028 +14.8*nt*	—0.4472 + 7.9*nt*
8— 6	+0.0313 —11.4*nt*	—0.0215 +12.8*nt*
8— 7	—0.0035 — 5.9*nt*	+0.0076 — 9.3*nt*
8— 8	+0.0010 — 0.4*nt*	—0.0008 — 0.9*nt*
9— 3	—0.0028	+0.0003
9— 4	+0.0427 + 0.42*nt*	—0.0157 + 0.86*nt*
9— 5	+0.0224 + 8.8*nt*	—0.4305 — 0.6*nt*
9— 6	+0.2324 — 2.0*nt*	+0.0588 + 8.2*nt*
9— 7	+0.0167 — 8.6*nt*	+0.0085 — 5.2*nt*
9— 8	—0.0028 + 4.5*nt*	+0.0016 — 4.5*nt*
9— 9	—0.0002 + 0.4*nt*	0.0000 + 0.9*nt*
10— 4	+0.00278+ 0.097*nt*	—0.00428+ 0.049*nt*
10— 5	+0.6841 —21.2*nt*	+1.3071 +14.0*nt*
10— 6	+0.1929 + 0.1*nt*	+0.0383 + 3.7*nt*
10— 7	—0.0045 — 4.7*nt*	+0.1281 0.0*nt*
10— 8	—0.0024 + 1.6*nt*	+0.0121 — 5.3*nt*
10— 9	—0.0001 + 2.7*nt*	+0.0007 + 1.6*nt*
11— 4	+0.0001	0.0000
11— 5	+0.0157 — 0.1*nt*	+0.0093 + 0.2*nt*
11— 6	+0.0624 + 0.7*nt*	—0.0476 + 0.8*nt*
11— 7	+0.0338 — 1.5*nt*	+0.0798 + 0.7*nt*
11— 8	—0.0644 — 0.3*nt*	+0.0111 — 2.6*nt*
11— 9	—0.0081 + 3.8*nt*	+0.0004 + 0.4*nt*
12— 6	+0.0239 + 0.6*nt*	—0.0586 + 0.2*nt*
12— 7	+0.0284 — 0.3*nt*	+0.0230 + 0.4*nt*
12— 8	—0.0415 — 0.5*nt*	+0.0280 — 0.8*nt*
12— 9	—0.0146 + 1.6*nt*	—0.0372 — 0.5*nt*
12—10	—0.0007 + 0.2*nt*	—0.0023 + 1.8*nt*

Calling the second-order terms, to be added to $n\delta z$ and ν, $n\delta^2 z$ and $\delta\nu$, they are determined by the equations*

$$\frac{d\,.\,n\delta^2 z}{ndt} = \overline{\delta W_0} + \left(\frac{\overline{dW_0}}{d\gamma}\right) n\delta z + \nu^2$$

$$\frac{d\,.\,\delta\nu}{ndt} = -\frac{1}{2}\left(\frac{\overline{d\,.\,\delta W_0}}{d\gamma}\right) - \frac{1}{2}\left(\frac{\overline{d^2 W_0}}{d\gamma^2}\right) n\delta z$$

It therefore remains to find the three products, $\left(\frac{\overline{dW_0}}{d\gamma}\right) n\delta z$, ν^2, and $-\frac{1}{2}\left(\frac{\overline{d^2 W_0}}{d\gamma^2}\right) n\delta z$. The factor $\left(\frac{\overline{dW_0}}{d\gamma}\right)$ of the first has already been given in Chapter II, being equivalent to $-2\frac{d\nu}{ndt}$, and it is easy to square the value of ν there given. The expressions of the two products to be employed in determining $n\delta^2 z$ follow:

Arg $= i'g' + ig$	$\left(\frac{\overline{dW_0}}{d\gamma}\right) n\delta z$	
	cos.	sin.
i' i	$''$ $''$	$''$ $''$
0 0	$-0.2041 + 0.1395nt$ $- 0.8677n^2t^2$	
0 − 1	$+0.0363 - 0.0523n^2t^2$	$+0.3266$
0 − 2	$-0.0140 - 0.0804n^2t^2$	$+0.0300 - 0.8636n^2t^2$
0 − 3	$+0.0110 - 0.0049n^2t^2$	$+0.0110 - 0.0520n^2t^2$
0 − 4	$- 0.0003n^2t^2$	$- 0.0029n^2t^2$
1 + 2	-0.0180	-0.0310
1 + 1	$-0.0637 + 0.1nt$	$+0.0484 - 5.8nt$
1 0	$+0.0672 + 43.83nt$	$+0.0277 - 26.99nt$
1 − 1	$+0.0200 + 6.05nt$	$-0.0184 - 3.54nt$
1 − 2	$-0.0128 + 30.5nt$	$+0.0102 + 41.1nt$
1 − 3	$+0.0030 + 2.9nt$	$+0.0080 + 2.7nt$
2 + 2	-0.0060	$+0.0030$
2 + 1	$-0.1070 + 1.9nt$	$+0.0160 + 0.6nt$
2 0	$-0.0582 + 43.7nt$	$-0.0971 + 48.3nt$
2 − 1	$-0.0594 + 70.32nt$	$+0.0727 + 197.75nt$
2 − 2	$-0.0175 - 37.2nt$	$-0.0030 + 52.5nt$
2 − 3	$+0.0010 - 192.4nt$	$+0.0070 + 88.4nt$
2 − 4	$+0.0040 - 12.3nt$	$-0.0010 + 4.8nt$
3 + 1	$+0.003$	$+0.034$
3 0	$+0.0073 + 1.8nt$	$+0.6822 + 5.1nt$
3 − 1	$+0.1512 + 12.17nt$	$-0.0371 - 62.55nt$
3 − 2	$+0.0486 + 18.91nt$	$-0.0159 - 2.99nt$
3 − 3	$+0.041 + 64.1nt$	$-0.053 + 6.3nt$
3 − 4	$+0.004 + 9.0nt$	$-0.004 + 25.7nt$
3 − 5	$0.0nt$	$+ 1.2nt$

---

*Auseinandersetzung, Abth. I, s. 98, gl. (40).

Arg=$i'g'+ig$		$\left(\frac{dW_0}{d\gamma}\right)n\delta z$			
		cos.		sin.	
i'	i	″	″	″	″
4	0	−0.008	+ 0.3*nt*	−0.004	− 0.2*nt*
4−	1	−0.0786	+ 6.28*nt*	−0.0155	− 6.71*nt*
4−	2	+0.0085	+ 17.25*nt*	+0.0382	+ 6.65*nt*
4−	3	+0.0577	+ 8.8*nt*	+0.0402	− 2.6*nt*
4−	4	+0.082	− 4.7*nt*	+0.087	+ 17.7*nt*
4−	5	+0.004	+ 7.7*nt*	+0.005	+ 0.6*nt*
5+	1		+ 1.2*nt*		− 0.4*nt*
5	0		+ 22.5*nt*		− 7.9*nt*
5−	1	+0.0003	+464.0*nt*	+0.0020	−167.4*nt*
5−	2	−0.000201	− 86.794*nt*	+0.003721	−110.414*nt*
5−	3	−0.0064	+205.4*nt*	−0.0076	+442.9*nt*
5−	4	−0.014	+109.5*nt*	−0.092	− 76.0*nt*
5−	5	+0.022	+ 11.0*nt*	−0.018	− 2.2*nt*
5−	6		+ 0.8*nt*		− 2.5*nt*
6−	1	−0.0002		+0.0014	
6−	2	−0.0024	− 0.19*nt*	+0.0030	− 1.33*nt*
6−	3	−0.0642	+ 1.92*nt*	−0.0466	− 1.01*nt*
6−	4	−0.0110	0.0*nt*	+0.0321	− 1.6*nt*
6−	5	+0.007	+ 0.7*nt*	−0.002	+ 2.8*nt*
6−	6	−0.002	+ 1.0*nt*	−0.007	− 1.1*nt*
6−	7		− 1.0*nt*		− 0.5*nt*
7−	2	−0.0029	+ 0.2*nt*	+0.0015	− 0.6*nt*
7−	3	+0.0097	+ 1.61*nt*	−0.0161	+ 0.07*nt*
7−	4	+0.4458	+ 0.5*nt*	−0.4738	− 0.4*nt*
7−	5	−0.135	− 0.4*nt*	−0.107	+ 2.0*nt*
7−	6	−0.008	+ 0.7*nt*	−0.008	+ 0.2*nt*
7−	7	−0.002	− 0.4*nt*	0.000	− 0.6*nt*
8−	3	+0.0025	− 0.27*nt*	−0.0015	− 0.18*nt*
8−	4	−0.1461	+ 0.04*nt*	+0.0306	− 0.20*nt*
8−	5	−0.037		−0.048	
8−	6	−0.014		+0.012	
9−	3	+0.0005		+0.0001	
9−	4	−0.0077	+ 0.06*nt*	−0.0027	− 0.05*nt*
9−	5	−0.0035		−0.0612	
9−	6	−0.048		+0.014	
9−	7	0.000		+0.004	
10−	4	−0.00065	− 0.368*nt*	−0.00033	+ 0.106*nt*
10−	5	−0.1824	+ 0.6*nt*	+0.3525	0.0*nt*
10−	6	+0.028		+0.022	
10−	7	+0.004		+0.018	
11−	5	−0.0019		+0.0011	
11−	6	−0.0067		−0.0050	
11−	7	−0.003		+0.008	
11−	8	+0.006		+0.001	
12−	5	−0.0001		0.0000	
12−	6	−0.0023		−0.0057	
12−	7	−0.003		+0.002	
12−	8	+0.004		+0.003	
12−	9	+0.001		−0.003	

Arg $= i'g' + ig$	ν^2			
	cos.		sin.	
i' i	$''$	$''$	$''$	$''$
0 0	$+0.0498$	$-0.0025nt$		
		$+0.2171n^2t^2$		
0 − 1	$+0.0143$	$+1.5nt$	$+0.0094$	$-1.6nt$
		$+0.0200n^2t^2$		$-0.0104n^2t^2$
0 − 2		$-0.0188n^2t^2$		$-0.2163n^2t^2$
0 − 3		$-0.0009n^2t^2$		$-0.0104n^2t^2$
0 − 4		$0.0000n^2t^2$		$-0.0006n^2t^2$
1 + 1	-0.0008	$-0.5nt$	-0.0011	$-0.5nt$
1 0	-0.0180	$-10.26nt$	-0.0074	$+6.30nt$
1 − 1	-0.0043	$-1.09nt$	$+0.0060$	$+1.19nt$
1 − 2	$+0.0015$	$+7.2nt$	-0.0007	$+9.6nt$
1 − 3		$+0.4nt$		$+0.3nt$
2 + 1		$0.0nt$		$-0.4nt$
2 0	-0.0034	$-5.7nt$	-0.0073	$-7.6nt$
2 − 1	$+0.0150$	$-17.16nt$	-0.0178	$-47.44nt$
2 − 2	-0.0023	$-7.7nt$	-0.0026	$+4.0nt$
2 − 3		$-45.7nt$		$+20.7nt$
2 − 4		$-2.5nt$		$+1.0nt$
3 0	-0.0019	$+0.2nt$	$+0.0059$	$+1.6nt$
3 − 1	-0.0384	$-3.16nt$	$+0.0140$	$+15.86nt$
3 − 2	-0.0058	$-3.66nt$	-0.0008	$+1.15nt$
3 − 3	$+0.007$	$+16.1nt$	-0.012	$+1.6nt$
3 − 4		$+1.4nt$		$+5.0nt$
4 − 1	-0.0012	$-1.36nt$	-0.0009	$+1.28nt$
4 − 2	-0.0007	$-3.91nt$	-0.0098	$-1.45nt$
4 − 3	$+0.0047$	$+1.2nt$	$+0.0062$	$+2.4nt$
4 − 4	$+0.019$	$-1.2nt$	$+0.021$	$+4.1nt$
4 − 5		$+1.3nt$		$+0.1nt$
5 − 1		$+5.3nt$		$-0.2nt$
5 − 2	-0.000049	$+21.818nt$	-0.001044	$+27.546nt$
5 − 3	-0.0012	$-0.2nt$	-0.0039	$-3.1nt$
5 − 4	-0.005	$+26.1nt$	-0.019	$-24.0nt$
5 − 5	$+0.005$	$+2.0nt$	-0.004	$-0.8nt$
6 − 2		$+0.04nt$		$+0.32nt$
6 − 3	$+0.0003$	$-0.40nt$	$+0.0007$	$+0.22nt$
6 − 4	-0.0027	$+0.5nt$	$+0.0102$	$+0.1nt$
6 − 5	$+0.003$	$+0.2nt$	0.000	$+0.4nt$
7 − 3	-0.0008	$-0.38nt$	$+0.0007$	$-0.02nt$
7 − 4	-0.0113		$+0.0022$	
7 − 5	-0.038		-0.020	
8 − 3		$+0.06nt$		$+0.05nt$
8 − 4	$+0.0026$		$+0.0006$	
8 − 5	$+0.007$		$+0.012$	
8 − 6	-0.002		$+0.004$	

Arg$=i'g'+ig$	ν^2	
	cos.	sin.
i' i	″ ″	″ ″
9 — 5	0.0000	+0.0015
9 — 6	—0.003	+0.002
10 — 4	—0.00005+0.093nt	—0.00031—0.026nt
10 — 5	+0.0023	—0.0031
10 — 6	+0.014	+0.002

The expression for $-\frac{1}{2}\left(\overline{\frac{d^2W_0}{d\gamma^2}}\right)$ is obtained by differentiating, with respect to γ, the terms of $-\frac{1}{2}\frac{dW_0}{d\gamma}$, which has already been employed in Chapter II. However, in order not to have to return to this term when we compute the third-order terms, I have, before this differentiation, added to the latter quantity the expression for $-\frac{1}{2}\frac{d\,.\,\delta W_0}{d\gamma}$, which has just been obtained. The expression for this factor, together with its product by $n\delta z$, is given in the following table:

Arg$=i'g'+ig$	$-\frac{1}{2}\left(\overline{\frac{d^2W_0}{d\gamma^2}}\right)$	
	cos.	sin.
i' i	″ ″	″ ″
0 0	—7.1170+ 0.7274nt	
0 — 1	—1.8280+63749nt	+ 1.3209—69810nt
	— 4.3356n^2t^2	— 7.3578n^2t^2
0 — 2	—0.4480— 6153nt	+ 0.1175— 6732nt
	— 0.4180n^2t^2	— 0.7096n^2t^2
0 — 3	+0.0828+ 500nt	+ 0.1057— 548nt
	— 0.0340n^2t^2	— 0.0577n^2t^2
0 — 4	+0.0098+ 38nt	+ 0.0205— 42nt
0 — 5	+ 3nt	— 3nt
1 + 3	—0.0327	— 0.0260
1 + 2	—0.2072— 0.2nt	— 0.2038— 0.8nt
1 + 1	—0.9517— 1.0nt	+ 0.5677+ 5.5nt
1 0	—1.8045+ 28.53nt	— 0.8777— 38.79nt
1 — 1	—3.4290— 1.30nt	+17.3976— 2.76nt
1 — 2	—0.0850+ 2.0nt	+ 2.7063— 9.2nt
1 — 3	+0.0850+ 0.2nt	+ 0.4148— 1.4nt
1 — 4	+0.0745	— 0.0188
2 + 2	—0.0689	+ 0.0123
2 + 1	—0.6706+ 1.7nt	— 0.0492— 2.1nt
2 0	—1.2173+ 17.9nt	— 1.7481— 16.2nt
2 — 1	—2.6878— 35.58nt	+ 0.8470— 45.89nt

Arg $=i'g'+ig$	$-\frac{1}{2}\left(\frac{d^2W_0}{dy^2}\right)$	
	cos.	sin.
i' i	$''$ $''$	$''$ $''$
2 − 2	$-126.0048-5.8nt$	$-53.8071-36.5nt$
2 − 3	$-12.0046-17.8nt$	$-5.7615-3.2nt$
2 − 4	$-0.8632-2.3nt$	$-0.5154-1.0nt$
2 − 5	-0.0646	-0.0417
3 + 1	$+0.0177-0.4nt$	$+0.1977+0.2nt$
3 0	$+0.0421-4.6nt$	$+2.0173+2.4nt$
3 − 1	$-0.3719-1.32nt$	$+0.0860-11.05nt$
3 − 2	$+15.7381-584.70nt$	$+21.0388+337.66nt$
3 − 3	$-12.0572-64.5nt$	$+21.4071+31.6nt$
3 − 4	$-1.6844-7.4nt$	$+2.0011+16.9nt$
3 − 5	$-0.1618-0.8nt$	$+0.0908+2.1nt$
3 − 6	-0.0141	$+0.0104$
4 0	$-0.0152-0.5nt$	$0.0000 \quad 0.0nt$
4 − 1	$-0.1436-1.66nt$	$+0.0051-1.74nt$
4 − 2	$+0.1546-63.06nt$	$+1.3338-0.06nt$
4 − 3	$-12.5021-121.0nt$	$+6.2950-346.8nt$
4 − 4	$+4.7176-14.4nt$	$+6.9776-30.1nt$
4 − 5	$+0.4239+8.1nt$	$+1.0265+0.9nt$
4 − 6	$-0.0093+1.4nt$	$+0.0960+1.3nt$
4 − 7	$+0.0010$	$+0.0086$
5 0	$+0.0209+1.8nt$	$-0.0162+1.2nt$
5 − 1	$+0.2148+19.0nt$	$-0.1767+13.1nt$
5 − 2	$-0.0279-5.548nt$	$+0.0780-2.898nt$
5 − 3	$+78.2787-812.1nt$	$+5.0079+3918.6nt$
5 − 4	$+8.7461-199.5nt$	$+4.9450+391.9nt$
5 − 5	$+3.9404-16.9nt$	$-1.4236+30.6nt$
5 − 6	$+0.5961+3.2nt$	$-0.0776-5.1nt$
5 − 7	$+0.0550+1.0nt$	$+0.0236-0.7nt$
5 − 8	$+0.0051$	$+0.0015$
6 − 1	$+0.0028$	-0.0022
6 − 2	$+0.0077-0.39nt$	$-0.0046-0.27nt$
6 − 3	$+0.1920-17.98nt$	$+0.1204+20.19nt$
6 − 4	$-0.4071-87.4nt$	$+1.7113-34.1nt$
6 − 5	$+2.1390-13.7nt$	$+0.0531+54.1nt$
6 − 6	$-0.3128+0.2nt$	$-1.6130+4.9nt$
6 − 7	$+0.0345-2.6nt$	$-0.2965-2.9nt$
6 − 8	$+0.0151$	-0.0352
6 − 9	$+0.0019$	-0.0026
7 − 2	$+0.0082-0.2nt$	$-0.0019+0.1nt$
7 − 3	$+0.0114-2.1nt$	$-0.0168+0.7nt$
7 − 4	$+1.0385-48.5nt$	$-0.9331-62.7nt$
7 − 5	$+0.8054-28.1nt$	$+0.2358+28.5nt$
7 − 6	$+0.2442+25.7nt$	$-1.0804+12.0nt$
7 − 7	$-0.7700+2.5nt$	$-0.0408+0.3nt$

Arg$=i'g'+ig$	$-\frac{1}{2}\left(\frac{\overline{d^2W_0}}{d\gamma^2}\right)$	
	cos.	sin.
i' i	″ ″	″ ″
7— 8	$-0.1502-1.9nt$	$-0.0597+1.4nt$
7— 9	-0.0161	-0.0134
7—10	-0.0013	-0.0017
8— 3	$+0.0007-0.18nt$	$-0.0040-0.05nt$
8— 4	$-0.4192+1.59nt$	$+0.0961+12.93nt$
8— 5	$-0.2288-17.2nt$	$-0.3266+9.6nt$
8— 6	$+0.2274+14.0nt$	$-0.3831+15.8nt$
8— 7	$-0.5384+9.2nt$	$-0.2468-12.6nt$
8— 8	$-0.1121-0.3nt$	$+0.3481-1.4nt$
8— 9	-0.0539	$+0.0721$
8—10	-0.0103	$+0.0070$
9— 4	$-0.0280-0.34nt$	$-0.0087+0.71nt$
9— 5	$-0.0172-9.4nt$	$-0.3615-0.8nt$
9— 6	$-0.1782+1.6nt$	$+0.0150+10.2nt$
9— 7	$-0.1926+11.1nt$	$-0.1846-6.2nt$
9— 8	$-0.1963-5.0nt$	$+0.2559-6.2nt$
9— 9	$+0.1477-0.7nt$	$+0.0952+0.7nt$
9—10	$+0.0305$	$+0.0383$
9—11	$+0.0023$	$+0.0072$
10— 4	$-0.0019-0.1nt$	$-0.0018\quad 0.0nt$
10— 5	$-0.5885+21.0nt$	$+1.1161+14.0nt$
10— 6	$-0.2162+0.9nt$	$+0.0804+4.6nt$
10— 7	$-0.0167+5.8nt$	$+0.0979+0.2nt$
10— 8	$-0.1329-1.9nt$	$+0.0892-7.1nt$
10— 9	$+0.1094-2.8nt$	$+0.1364+1.3nt$
10—10	$+0.0634$	-0.0560
10—11	$+0.0244$	-0.0109
10—12	$+0.0040$	$+0.0001$
11— 5	$-0.0119+0.1nt$	$+0.0073+0.2nt$
11— 6	$-0.0627-0.7nt$	$-0.0478+0.8nt$
11— 7	$-0.0401+1.7nt$	$+0.0823+0.9nt$
11— 8	$+0.0413+0.4nt$	$+0.0186-3.2nt$
11— 9	$+0.0329-3.8nt$	$+0.0870+0.3nt$
11—10	$+0.0846$	-0.0411
11—11	-0.0173	-0.0374
11—12	-0.0029	-0.0137
12— 6	$-0.0216-0.6nt$	$-0.0551+0.2nt$
12— 7	$-0.0326+0.3nt$	$+0.0228+0.4nt$
12— 8	$+0.0423+0.5nt$	$+0.0342-0.8nt$
12— 9	$+0.0150-1.6nt$	$-0.0113-0.5nt$
12—10	$+0.0539-0.3nt$	$-0.0057+1.8nt$
12—11	-0.0082	-0.0495
12—12	-0.0209	$+0.0014$

Arg=$i'g'+ig$	$-\frac{1}{2}\left(\frac{d^2W_0}{d\gamma^2}\right)n\delta z$			
	sin.		cos.	
i' i	″	″	″	″
0 0			$+0.000193$	$+0.2707nt$
$0-1$	-0.0295	$-22.0nt$	$+0.1943$	$-0.3nt$
		$+0.0336n^2t^2$		$+0.0084n^2t^2$
$0-2$	$+0.0065$	$-0.7nt$	$+0.0265$	$+0.9nt$
		$+0.0421n^2t^2$		$-0.4311n^2t^2$
$0-3$	-0.009	$+0.6nt$	$+0.009$	$-0.3nt$
		$+0.0039n^2t^2$		$-0.0460n^2t^2$
$0-4$		$+0.0005n^2t^2$		$-0.0042n^2t^2$
$0-5$		$0.0000n^2t^2$		$-0.0002n^2t^2$
$1+2$	-0.014	$-0.6nt$	$+0.021$	$-1.7nt$
$1+1$	-0.0360	$-1.2nt$	-0.0282	$+7.1nt$
1 0	$+0.0101$	$+8.52nt$	-0.0018	$+5.43nt$
$1-1$	-0.0088	$-2.21nt$	-0.0084	$-0.99nt$
$1-2$	$+0.0073$	$-15.9nt$	$+0.0055$	$+21.3nt$
$1-3$	-0.002	$-2.4nt$	$+0.006$	$+1.9nt$
$1-4$	-0.004		-0.003	
$2+2$	-0.007		-0.003	
$2+1$	-0.072	$+2.4nt$	-0.007	$+5.3nt$
[illegible] 0	-0.0337	$+24.5nt$	$+0.0571$	$-20.9nt$
$2-1$	$+0.0005$	$-6.47nt$	$+0.0071$	$+17.25nt$
$2-2$	-0.0061	$+17.9nt$	$+0.0062$	$+28.1nt$
$2-3$	-0.001	$+99.6nt$	$+0.004$	$+47.0nt$
$2-4$	-0.003	$+9.7nt$	0.000	$+4.7nt$
$3+1$	$+0.002$	$-4.3nt$	-0.039	$-0.2nt$
3 0	$+0.0050$	$-30.0nt$	-0.3992	$+2.9nt$
$3-1$	$+0.0086$	$+1.16nt$	-0.0004	$+6.40nt$
$3-2$	-0.0179	$-4.42nt$	-0.0165	$-0.83nt$
$3-3$	-0.023	$-32.2nt$	-0.032	$+3.1nt$
$3-4$	-0.001	$-6.3nt$	-0.002	$+14.5nt$
$3-5$		$-0.2nt$		$+1.2nt$
4 0	-0.007	$+0.6nt$	$+0.003$	$+0.8nt$
$4-1$	-0.0504	$+3.71nt$	$+0.0101$	$+6.13nt$
$4-2$	-0.0016	$-2.71nt$	$+0.0009$	$+1.08nt$
$4-3$	-0.0329	$-4.6nt$	$+0.0193$	$+0.4nt$
$4-4$	-0.045	$+2.3nt$	$+0.045$	$+9.3nt$
$4-5$	-0.004	$-4.8nt$	$+0.005$	$+0.6nt$
5 0	-0.001	$+25.2nt$	-0.001	$+9.1nt$
$5-1$	-0.0057	$+259.3nt$	-0.0040	$+94.6nt$
$5-2$	-0.015745	$-3.290nt$	-0.038551	$+2.342nt$
$5-3$	$+0.0151$	$-114.5nt$	-0.0092	$+248.1nt$
$5-4$	$+0.006$	$-62.9nt$	-0.046	$-26.8nt$
$5-5$	-0.014	$-8.7nt$	-0.013	$-2.0nt$
$5-6$		$-0.7nt$		$-1.7nt$

Arg$=i'g'+ig$	$-\frac{1}{2}\left(\overline{\frac{d^2W_0}{d\gamma^2}}\right)n\delta z$	
	sin.	cos.
$i'\quad i$	″ ″	″ ″
6— 1	$+0.0004$	-0.0032
6— 2	$-0.0038-0.36nt$	$-0.0035+0.69nt$
6— 3	$+0.0418-2.99nt$	$-0.0297-3.06nt$
6— 4	$+0.0092+0.9nt$	$+0.0158-1.7nt$
6— 5	$-0.005-0.1nt$	$0.000+1.2nt$
6— 6	$+0.001-0.7nt$	$-0.004-0.7nt$
6— 7	$+0.6nt$	$-0.4nt$
7— 2	$-0.0061-0.3nt$	$-0.0013+0.6nt$
7— 3	$-0.0008-0.84nt$	$-0.0081+0.10nt$
7— 4	$-0.2645+17.8nt$	$-0.2776+22.4nt$
7— 5	$+0.057-4.2nt$	$-0.068-2.7nt$
7— 6	$+0.006$	-0.007
8— 3	$-0.0001+0.08nt$	$-0.0009-0.09nt$
8— 4	$+0.0765+0.68nt$	$+0.0162-7.75nt$
8— 5	$+0.030-2.9nt$	$-0.036-1.6nt$
8— 6	$+0.011\quad 0.0nt$	$+0.003-0.6nt$
9— 3	$-0.0002+0.05nt$	$-0.0004-0.10nt$
9— 4	$+0.0038-0.18nt$	$-0.0011-0.39nt$
9— 5	$+0.0023-3.8nt$	$-0.0377+0.3nt$
9— 6	$+0.032+0.2nt$	$+0.008-2.2nt$
9— 7	$+0.001$	$+0.003$
10— 4	$-0.00002+0.181nt$	$+0.00006+0.036nt$
10— 5	$+0.1034+26.0nt$	$+0.2050-16.8nt$
10— 6	$-0.007+2.3nt$	$+0.022+1.3nt$
10— 7	$-0.002+2.0nt$	$+0.015-0.7nt$
11— 5	$+0.0005$	$+0.0004$
11— 6	$+0.0041-0.5nt$	$-0.0031-0.5nt$
11— 7	$+0.002+0.5nt$	$+0.006-0.4nt$
11— 8	-0.004	$+0.001$
12— 5	$+0.02nt$	$+0.03nt$
12— 6	$-0.0023\quad 0.0nt$	$+0.0066+0.1nt$
12— 7	$+0.002$	$+0.002$

If we now add the three portions of $\frac{d\,.\,n\delta^2z}{ndt}$, which have just been given, we shall have the value of this quantity. In $\overline{\delta W_0}$ we give k_0 such a value that $n\delta^2z$ may have no term proportional to t, and k_1 and k_2 are so assumed that the terms having the argument g may vanish. For this it is found necessary to put $k_1=-0''.4862$ and $k_2=-0''.8170$. In integrating the terms depending on the arguments $5g'-2g$ and $10g'-4g$ we have equated the motion of the latter, and have proceeded in a way precisely similar to that followed in deriving δW_0 from δT. By joining the first order with the second order terms, it is found that, as far as these two arguments are concerned, we have

$$\begin{aligned}\frac{d\,.\,n\delta z}{ndt} = {} & [5''.88438 + 0''.01000896nt]\cos(5g' - 2g) + [-14''.13462 + 0''.00596165nt]\sin(5g' - 2g) \\ & + [0.16091 + 0.00004537nt]\cos(10g' - 4g) + [0.17236 - 0.00003542nt]\sin(10g' - 4g)\end{aligned}$$

Setting aside quantities of the third order this expression can be replaced by the following:

$$\begin{aligned}\frac{d(n\delta z)}{ndt} = {} & [(1.1849912) - (7.2193138)nt]\cos[5g' - 2g + 67^\circ\ 23'\ 51''.22 - (6.8768938)nt] \\ & + [(9.37255\quad) + (4.7058\quad)nt]\cos[10g' - 4g + 313\quad 1\quad 58\quad + (6.3858\quad)nt]\end{aligned}$$

The integrating factors for the equated arguments are given by

$$\log\mu = 1.8970002 \qquad\qquad \log\mu = 1.56699$$

After neglecting certain quantities of the third order the integrated expression can be put under the form

$$\begin{aligned}n\delta z = {} & [473''.7148 + 0''.785671nt]\sin(5g' - 2g) + [1111''.0597 - 0''.477027nt]\cos(5g' - 2g) \\ & + [5.9424 + 0.001673nt]\sin(10g' - 4g) + [-6.3550 + 0.001308nt]\cos(10g' - 4g)\end{aligned}$$

By subtracting the corresponding terms of $n\delta z$, found in Chapter II, we get

$$\begin{aligned}n\delta^2 z = {} & [51''.3265 + 0''.785671nt]\sin(5g' - 2g) + [118''.2806 - 0''.477027nt]\cos(5g' - 2g) \\ & + [6.9725 + 0.001673nt]\sin(10g' - 4g) + [-7.3774 + 0.001308nt]\cos(10g' - 4g)\end{aligned}$$

If we add the two portions of $\frac{d\,.\,\delta\nu}{ndt}$ we have the value of this quantity. In integrating we are obliged to derive the constant term of $\delta\nu$ from another equation. It is known that this term is the same as that of the expression*

$$-\frac{1}{6}\left(k_0 + \frac{3}{2}\frac{e}{P_1}k_1\right) + \frac{1}{3}\left(\frac{d\,.\,\delta z}{dt} + \frac{1}{2}\nu\right)^2 + \frac{3}{4}\nu^2$$

k_0 is equivalent to the negative of the sum of the constant terms of $\overline{\delta W_0}$, $\left(\frac{\overline{dW_0}}{d\gamma}\right)n\delta z$, and ν^2; that is, $k_0 = +0''.1018$. It has already been stated that $k_1 = -0''.4862$. In computing the constant term of $\left(\frac{d\,.\,\delta z}{dt} + \frac{1}{2}\nu\right)^2$ it is necessary to take into account only the terms corresponding to the ten arguments having the largest coefficients. We make use of the expressions for $\frac{d\,.\,\delta z}{dt}$ and ν given in Chapter II. It is found that this constant term is $+0''.1225$. The constant term of ν^2 has already been found to be $+0''.0498$. Thus, the constant term of $\delta\nu$ is $+0''.0671$.

---

*American Journal of Mathematics, Vol. IV, p. 258.

The values of $n\delta^2z$ and $\delta\nu$ follow. The proper number of decimals has been restored to the coefficients multiplied by nt and n^2t^2:

Arg=$i'g'+ig$	$n\delta^2z$	
	sin.	cos.
i' i	″ ″	″ ″
0 0		$+.000003182n^2t^2$
		$-.0000004153n^3t^3$
0 — 1	$-.256776nt$	$-.226275nt$
	$+.000087035n^2t^2$	$-.000147259n^2t^2$
0 — 2	$+0.1531-.003108nt$	$+0.1140-.002723nt$
	$+.000001541n^2t^2$	$-.000007173n^2t^2$
0 — 3	$-0.0343-.000075nt$	$+0.0307-.000066nt$
	$+.000000045n^2t^2$	$-.000000251n^2t^2$
0 — 4	$+0.0005-.000002nt$	$+0.0023-.000002nt$
	$+.000000001n^2t^2$	$-.000000010n^2t^2$
1 + 4	-0.0002	$+0.0001$
1 + 3	-0.0043	$+0.0026$
1 + 2	$-0.0631\quad.000000nt$	$+0.1032+.000002nt$
1 + 1	$-0.7000-.000034nt$	$-0.6043+.000149nt$
1 0	$-1.2437+.001758nt$	$+1.0053+.002130nt$
1 — 1	$-0.3651+.000049nt$	$-0.5446-.000082nt$
1 — 2	$+0.2285-.000269nt$	$+0.1214+.000269nt$
1 — 3	$-0.0216-.000012nt$	$+0.0611+.000008nt$
1 — 4	-0.0146	-0.0104
2 + 2	-0.0107	-0.0061
2 + 1	$-0.4225+.000016nt$	$-0.0440+.000009nt$
2 0	$-2.3423+.000882nt$	$+4.0897-.000056nt$
2 — 1	$-0.5817+.012769nt$	$-1.4442-.020319nt$
2 — 2	$-0.1487+.000424nt$	$+0.2580-.000013nt$
2 — 3	$-0.0194+.001144nt$	$+0.0727+.000480nt$
2 — 4	$-0.0265+.000048nt$	$-0.0042+.000018nt$
2 — 5	-0.0010	-0.0002
3 + 1	$+0.0067-.000001nt$	$-0.0535\quad.000000nt$
3 0	$-0.0024-.000052nt$	$-3.1601-.000084nt$
3 — 1	$-2.8597-.002506nt$	$-2.2157-.003737nt$
3 — 2	$-0.0403+.018630nt$	$-0.1725+.010703nt$
3 — 3	$-0.0712-.000225nt$	$-0.0584+.000131nt$
3 — 4	$-0.0301-.000026nt$	$-0.0015+.000149nt$
3 — 5	$-0.0007+.000002nt$	$-0.0115+.000006nt$
4 0	$-0.0138-.000001nt$	$+0.0075+.000001nt$
4 — 1	$-0.6652-.000135nt$	$+0.1571+.000035nt$
4 — 2	$-0.1439+.008464nt$	$+0.2365+.000071nt$
4 — 3	$-0.2140+.001197nt$	$+0.0632-.003658nt$
4 — 4	$-0.0430+.000059nt$	$+0.0346+.000044nt$
4 — 5	$-0.0038-.000044nt$	$-0.0089+.000009nt$
4 — 6	$+0.0056\quad.000000nt$	$-0.0015+.000001nt$

Arg=$i'g'+ig$		$n\delta^2 z$	
		sin.	cos.
i'	i	″ ″	″ ″
5	0	+ 0.0018 + .000116nt	— 0.0005 + .000036nt
5	— 1	+ 0.1461 + .004996nt	— 0.0482 + .001397nt
5	— 2	+51.3265 + .785671nt	+118.2806 — .477027nt
5	— 3	— 5.7355 + .014559nt	— 1.4196 + .084928nt
5	— 4	— 0.0745 + .000188nt	— 0.1061 + .000536nt
5	— 5	— 0.0087 — .000025nt	— 0.0093 + .000018nt
5	— 6	+ 0.0037 — .000007nt	— 0.0016 — .000015nt
5	— 7	+ 0.0011 — .000001nt	+ 0.0022 + .000001nt
6	— 1	+ 0.0006	+ 0.0003
6	— 2	+ 0.0449 + .000037nt	+ 0.0424 — .000098nt
6	— 3	+ 0.6533 + .001034nt	— 0.5307 — .001204nt
6	— 4	— 0.0217 + .000713nt	— 0.0132 — .000307nt
6	— 5	+ 0.0049 + .000027nt	+ 0.0059 + .000213nt
6	— 6	+ 0.0029 — .000005nt	— 0.0018 + .000002nt
6	— 7	+ 0.0007 + .000005nt	+ 0.0007 — .000006nt
7	— 2	+ 0.0160 — .000004nt	+ 0.0084 + .000001nt
7	— 3	— 0.8276 + .001403nt	— 2.1955 + .000509nt
7	— 4	— 2.8854 + .000694nt	— 2.7617 — .000913nt
7	— 5	+ 0.0543 + .000126nt	— 0.1058 + .000164nt
7	— 6	+ 0.0055 — .000072nt	— 0.0027 + .000027nt
7	— 7	— 0.0006 .000000nt	+ 0.0003 — .000002nt
7	— 8	— 0.0006 + .000003nt	+ 0.0003 — .000001nt
8	— 3	— 0.2491 + .000111nt	— 0.1729 — .000024nt
8	— 4	+ 2.0006 — .000055nt	+ 0.4877 + .000425nt
8	— 5	+ 0.2492 + .000117nt	— 0.3730 + .000058nt
8	— 6	+ 0.0183 — .000049nt	— 0.0003 + .000055nt
8	— 7	— 0.0005 — .000014nt	+ 0.0018 — .000023nt
8	— 8	+ 0.0005 + .000001nt	— 0.0009 — .000001nt
9	— 3	— 0.0098 + .000002nt	+ 0.0016 .000000nt
9	— 4	+ 0.5369 + .000045nt	— 0.1485 + .000105nt
9	— 5	+ 0.0201 + .000102nt	— 0.5507 — .000009nt
9	— 6	+ 0.1413 — .000009nt	+ 0.0404 + .000044nt
9	— 7	+ 0.0041 — .000025nt	+ 0.0040 — .000017nt
9	— 8	— 0.0005 + .000010nt	— 0.0005 — .000009nt
10	— 4	+ 6.9725 + .001673nt	— 7.3774 + .001308nt
10	— 5	+ 1.6120 — .000446nt	+ 3.1014 + .000292nt
10	— 6	+ 0.1043 + .000003nt	+ 0.0235 + .000023nt
10	— 7	— 0.0048 — .000017nt	+ 0.0540 .000000nt
10	— 8	— 0.0002 + .000004nt	+ 0.0017 — .000014nt
10	— 9	0.0000 + .000002nt	+ 0.0009 + .000001nt
11	— 4	+ 0.0014	— 0.0049
11	— 5	+ 0.0841 — .000005nt	+ 0.0563 + .000011nt
11	— 6	+ 0.0634 + .000006nt	— 0.0520 + .000006nt

Arg=$i'g'+ig$	$n\delta^2 z$	
	sin.	cos.
i' i	″ ″	″ ″
11— 7	$+0.0184-.000007nt$	$+0.0412+.000005nt$
11— 8	$-0.0220\ \ .000000nt$	$+0.0031-.000007nt$
11— 9	$-0.0003+.000005nt$	$+0.0001\ \ .000000nt$
12— 5	$+0.0806+.000001nt$	$+0.0047+.000011nt$
12— 6	$+0.0379+.000011nt$	$-0.0962+.000003nt$
12— 7	$+0.0185-.000002nt$	$+0.0145+.000002nt$
12— 8	$-0.0159\ \ .000000nt$	$+0.0116-.000001nt$
12— 9	$-0.0044+.000002nt$	$-0.0077-.000001nt$
12—10	$-0.0001+.000001nt$	$-0.0004+.000002nt$

Arg=$i'g'+ig$	δv	
	cos.	sin.
i' i	″ ″	″ ″
0 0	$+0.0671-.003098nt$ $+.000000350n^2t^2$	
0— 1	$+0.0845-.128514nt$ $+.000043692n^2t^2$	$+0.0628+.113101nt$ $+.000073493n^2t^2$
0— 2	$+0.1134-.003104nt$ $+.000001255n^2t^2$	$-0.0852+.002719nt$ $+.000003929n^2t^2$
0— 3	$-0.0272-.000110nt$ $+.000000051n^2t^2$	$-0.0273+.000100nt$ $+.000000217n^2t^2$
0— 4	$-0.0012-.000005nt$ $+.000000003n^2t^2$	$-0.0032+.000004nt$ $+.000000013n^2t^2$
1+ 2	$+0.0544+.000003nt$	$+0.0779+.000005nt$
1+ 1	$+0.4368+.000025nt$	$-0.3703+.000118nt$
1 0	$+0.3004-.000460nt$	$+0.2230+.000502nt$
1— 1	$-0.1165-.000005nt$	$+0.1858+.000047nt$
1— 2	$+0.1551-.000115nt$	$-0.0836-.000098nt$
1— 3	$-0.0176-.000010nt$	$-0.0504-.000004nt$
1— 4	-0.0135	$+0.0086$
2+ 2	$+0.0126$	-0.0054
2+ 1	$+0.3063-.000018nt$	$-0.0215+.000037nt$
2 0	$+1.0147-.000471nt$	$+1.7470-.000070nt$
2— 1	$-0.0342+.001342nt$	$+0.1801+.002001nt$
2— 2	$-0.0977+.000179nt$	$-0.1557+.000044nt$
2— 3	$-0.0178+.000503nt$	$-0.0615-.000202nt$
2— 4	$-0.0237+.000022nt$	$+0.0027-.000007nt$
3+ 1	$-0.0053+.000020nt$	$-0.0585-.000001nt$
3 0	$0.0000+.000288nt$	$-1.8180+.000009nt$
3— 1	$+0.3359+.000465nt$	$-0.3171-.000509nt$

Arg $=i'g'+ig$	δv	
	cos.	sin.
i' i	″ ″	″ ″
3— 2	$-0.0209+.007833nt$	$+0.0744-.004512nt$
3— 3	$-0.0424+.000015nt$	$+0.0310-.000110nt$
3— 4	$-0.0253-.000009nt$	$+0.0015-.000086nt$
3— 5	$-0.0013+.000001nt$	$+0.0105-.000006nt$
4 0	$+0.0109-.000002nt$	$+0.0052+.000005nt$
4— 1	$+0.2265+.000034nt$	$+0.0510+.000078nt$
4— 2	$-0.0340+.001972nt$	$-0.0519+.000048nt$
4— 3	$-0.1263+.000750nt$	$-0.0351+.002262nt$
4— 4	$-0.0232+.000040nt$	$-0.0117+.000021nt$
4— 5	$-0.0036-.000029nt$	$+0.0092-.000007nt$
4— 6	$+0.0054$	$+0.0016$
5 0	$-0.0015-.000130nt$	$-0.0012+.000042nt$
5— 1	$-0.0688-.002741nt$	$-0.0282+.000803nt$
5— 2	$+0.0195-.004003nt$	$+0.0470-.003199nt$
5— 3	$-2.8617+.007113nt$	$+0.7011-.042382nt$
5— 4	$-0.0779+.000379nt$	$+0.0663-.000878nt$
5— 5	$-0.0055-.000007nt$	$+0.0076+.000030nt$
5— 6	$+0.0049-.000007nt$	$+0.0024+.000013nt$
5— 7	$+0.0013-.000002nt$	$-0.0026+.000001nt$
6— 1	-0.0008	-0.0015
6— 2	$-0.0068+.000002nt$	$+0.0096-.000021nt$
6— 3	$+0.2134+.000316nt$	$+0.1756-.000340nt$
6— 4	$-0.0155+.000487nt$	$+0.0211+.000198nt$
6— 5	$+0.0068+.000029nt$	$+0.0005-.000168nt$
6— 6	$+0.0017-.000003nt$	$+0.0010-.000006nt$
6— 7	$+0.0016+.000003nt$	$-0.0015+.000004nt$
7— 2	$-0.0013+.000006nt$	$+0.0035+.000004nt$
7— 3	$-0.1421+.000131nt$	$+0.2932-.000049nt$
7— 4	$-1.6456+.000245nt$	$+1.5693+.000316nt$
7— 5	$-0.0026+.000075nt$	$+0.0782-.000101nt$
7— 6	$+0.0047-.000062nt$	$+0.0005-.000024nt$
7— 7	$+0.0004-.000003nt$	$-0.0005-.000002nt$
7— 8	$-0.0005+.000002nt$	$-0.0007-.000001nt$
8— 3	$+0.0492-.000019nt$	$-0.0306-.000012nt$
8— 4	$+0.8527-.000012nt$	$-0.1940-.000077nt$
8— 5	$+0.1871+.000067nt$	$+0.2716-.000035nt$
8— 6	$+0.0152-.000041nt$	$+0.0067-.000044nt$
8— 7	$-0.0009-.000016nt$	$-0.0020+.000021nt$
8— 8	$+0.0002-.000001nt$	$+0.0002+.000002nt$
9— 3	$+0.0048-.000001nt$	$-0.0002-.000002nt$
9— 4	$+0.1237+.000007nt$	$+0.0447-.000013nt$
9— 5	$+0.0179+.000036nt$	$+0.3402+.000002nt$

$\mathrm{Arg}=i'g'+ig$	δv	
	cos.	sin.
i' i	$''$ $''$	$''$ $''$
9 — 6	$+0.1113-.000008nt$	$-0.0281-.000025nt$
9 — 7	$+0.0052-.000025nt$	$-0.0034+.000015nt$
9 — 8	$-0.0006+.000010nt$	$-0.0004+.000010nt$
10 — 4	$-0.0852-.000103nt$	$-0.1306+.000032nt$
10 — 5	$+0.8092+.000049nt$	$-1.5538+.000029nt$
10 — 6	$+0.0942+.000012nt$	$-0.0306-.000025nt$
10 — 7	$-0.0022-.000009nt$	$-0.0481+.000002nt$
10 — 8	$-0.0006+.000004nt$	$-0.0030+.000013nt$
10 — 9	$0.0000+.000005nt$	$-0.0001-.000003nt$
11 — 5	$+0.0284-.000002nt$	$-0.0170-.000003nt$
11 — 6	$+0.0423+.000001nt$	$+0.0323+.000001nt$
11 — 7	$+0.0139-.000004nt$	$-0.0334-.000001nt$
11 — 8	$-0.0192-.000001nt$	$-0.0034+.000007nt$
11 — 9	$-0.0018+.000008nt$	$-0.0001-.000001nt$
12 — 6	$+0.0185+.000005nt$	$+0.0445-.000003nt$
12 — 7	$+0.0140-.000001nt$	$-0.0115-.000002nt$
12 — 8	$-0.0131-.000002nt$	$-0.0088+.000003nt$
12 — 9	$-0.0035+.000004nt$	$+0.0089+.000001nt$
12 — 10	$-0.0001\quad.000000nt$	$+0.0004-.000003nt$

CHAPTER XIII.

CALCULATION OF THE PORTION OF $\delta T'$ NOT FACTORED BY $n't$.

We have now to go through the same processes for Saturn as, in the three preceding chapters, we have gone through with for Jupiter. In determining the portion of $\delta T'$, which follows, a table of limits for the retention of terms for each argument $\pm\gamma' + i'g' + ig$ was computed from the formulæ

$$\frac{i'n' + in}{n'} \cdot \frac{(i' \pm 1)n' + in}{n'} \times 0''.0005$$

and only those combinations were retained in which at least one coefficient exceeded this limit. It has been deemed advisable to give separately the eight products whose sum forms $\delta T'$:

Arg= $\varkappa\gamma' + i'g' + ig$			$A'n'\delta z'$		$B'(\nu' - c') + X'c'$		$F'n\delta z$		$G'(\nu - c)$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		−0.1065891		+0.1095091		−0.0118202		+0.0118041
−1	1	0	−0.3380	−0.043029	+0.9159	−0.100097	−0.1120	−0.001867	+0.3291	−0.001759
−1	2	0	−0.005	−0.003	+0.288	−0.275	+0.002	+0.721	+0.061	−0.309
0	1	0	−0.2784	−1.1850	−0.5461	+0.9867	−0.2507	−0.4566	−0.1331	+0.3765
1	0	0	+1.018231	−0.376723	+0.337326	−0.723162	+0.320340	−0.088791	+0.114485	−0.190184
−1	3	0	+0.106	−0.112	+0.026	−0.008	+0.050	−0.074	+0.047	+0.018
0	2	0	−1.376	+2.281	−0.409	+0.555	−0.352	+0.666	−0.103	+0.072
1	1	0	+2.220	−3.923	+0.571	−0.890	+0.411	−0.770	+0.084	−0.105
−1	4	0	−0.008	−0.207	+0.048	+0.032	−0.021	−0.175		
0	3	0	+0.377	+3.904	−0.149	−0.183	+0.044	+1.561	−0.005	−0.022
1	2	0	−0.267	−7.003	+0.047	+0.245	−0.019	−1.893	+0.003	+0.031
−1	5	0					−0.97	+0.18		
0	4	0	+1.17	−0.24	+0.03	0.00	+0.95	−0.18		
1	3	0	−0.713	+0.298	−0.006	−0.004	−0.294	+0.120		
0	−4	−1	−0.10	−0.02			−0.12	−0.03		
−1	−2	−1	−0.25	−0.20			−0.11	−0.08		
−0	−3	−1	+0.05	+0.14			+0.05	+0.08		
−1	−1	−1	+5.699	−1.268	−0.206	+0.025	+1.757	−0.396	−0.031	+0.001
0	−2	−1	−3.76	+0.87	+0.14	−0.01	−1.53	+0.35	+0.02	0.00
1	−3	−1	+0.46	−0.11			+0.29	−0.07		
−1	0	−1	+3.321	−2.701	+0.585	−0.602	+0.715	−0.567	+0.080	−0.083

Arg= $\varkappa\gamma'+i'g'+ig$			$A'n'\delta z'$		$B'(\nu'-\sigma')+X'\sigma'$		$F'n\delta z$		$G'(\nu-\sigma)$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0—	1—	1	— 2.373	+1.807	—0.441	+0.495	—0.680	+0.430	—0.091	+0.107
1—	2—	1	+ 0.229	—0.272	+0.068	—0.082	+0.110	+0.025	+0.023	—0.036
—1	1—	1	+ 1.544	—1.833	+0.457	—0.379	+0.369	—0.473	+0.078	—0.143
0	0—	1	— 0.956	+1.234	—0.365	+0.423	—0.403	+0.446	—0.058	+0.227
1—	1—	1	+ 0.041	—0.124	+0.082	—0.174	+0.138	—0.143	+0.004	—0.108
—1	2—	1	+ 0.2334	—0.6170	+0.3490	+1.3286	+0.2540	+0.9889	—0.1740	—0.8917
0	1—	1	— 0.193	+0.198	—0.147	—0.189	—0.204	—0.781	+0.197	+1.037
1	0—	1	+ 0.106	+0.200	—0.239	—1.342	+0.006	+0.076	—0.104	—0.497
—1	3—	1	+ 0.4920	—0.0761	—0.0679	+0.2581	—0.2312	—0.2533	+0.2597	+0.2936
0	2—	1	+ 0.0391	—0.3377	—0.3581	—0.2419	+0.1212	+0.1052	—0.3089	—0.3249
1	1—	1	+ 0.7886	+0.6734	+0.5047	+0.0995	+0.0591	+0.0750	+0.1532	+0.1772
—1	4—	1	+ 0.0283	—0.0704	+0.3058	+0.1104	—0.0986	+0.0204	+0.0549	+0.0067
0	3—	1	+ 2.6870	+0.2307	—2.0576	—0.7217	—0.1070	—0.1974	—0.1688	—0.0505
1	2—	1	+ 0.5773	+0.7208	+1.6494	+0.5672	+0.4009	+0.2232	+0.1629	+0.0441
—1	5—	1	+ 0.003	—0.058	—0.091	+0.025	—5.161	+0.886	—0.098	+0.104
0	4—	1	+10.082	—1.976	+0.431	—0.065	+4.118	—0.677	+0.111	—0.108
1	3—	1	— 1.5994	+0.3047	—0.3264	+0.0360	—0.3432	+0.0126	—0.0557	+0.0386
—1	6—	1	— 0.073	—0.043	+0.002	—0.001	+0.031	+0.015		
0	5—	1	— 0.004	+0.009	—0.016	+0.008	0.000	0.000		
1	4—	1	+ 0.313	—0.122	+0.014	—0.007	+0.125	—0.045	+0.004	—0.002
—1	7—	1	— 0.004	—0.021						
—1—	1—	2	+ 0.22	—0.38			+0.09	—0.16		
0—	2—	2	— 0.18	+0.18			—0.10	+0.10		
1—	3—	2	+ 0.03	—0.02						
—1	0—	2	+ 1.87	+3.95	—0.05	—0.17	+0.61	+1.31		
0—	1—	2	— 1.41	—2.88	+0.03	+0.12	—0.57	—1.19		
1—	2—	2	+ 0.23	+0.49			+0.12	+0.27		
—1	1—	2	+ 2.93	+2.21	+0.49	+0.31	+0.66	+0.52	+0.06	+0.05
0	0—	2	— 2.25	—1.70	—0.39	—0.24	—0.66	—0.50	—0.07	—0.05
1—	1—	2	+ 0.37	+0.23	+0.08	+0.05	+0.13	+0.08	+0.01	+0.01
—1	2—	2	+ 1.977	+1.000	+0.451	+0.271	+0.512	+0.249	+0.107	+0.013
0	1—	2	— 1.463	—0.706	—0.379	—0.200	—0.479	—0.219	—0.119	—0.018
1	0—	2	+ 0.21	+0.05	+0.09	—0.03	+0.09	+0.03	+0.01	0.00
—1	3—	2	+ 0.766	+0.125	—1.500	+0.751	—0.009	+0.088	+0.069	—0.006
0	2—	2	— 0.511	—0.052	+1.361	—0.686	—0.273	+0.046	—0.312	+0.113
1	1—	2	+ 0.032	—0.028	—0.021	+0.022	+0.359	—0.136	+0.327	—0.137
—1	4—	2	+ 0.1255	—0.3283	—0.4340	—0.2016	—0.0636	+0.0545	+0.0266	—0.1177
0	3—	2	+ 0.294	+1.536	+0.453	+0.429	+0.129	+0.416	+0.020	+0.173
1	2—	2	— 0.248	+0.247	—0.076	—0.031	—0.127	—0.571	—0.059	—0.130
—1	5—	2	+ 0.09308	+0.87656	+0.09341	—0.96932	+0.11520	—0.73773	—0.09099	+0.71614
0	4—	2	+ 0.0757	—0.1189	—0.0007	—0.0308	—0.0515	+0.6485	+0.0918	—0.8377
1	3—	2	— 0.3124	+1.6932	—0.1003	+1.0233	—0.0423	—0.1092	—0.0299	+0.4125
—1	6—	2	+ 1.47830	+3.23313	—0.01846	+0.02240	—0.00039	—0.00314	+0.00594	+0.00330
0	5—	2	— 0.0843226	—0.0616656	+0.0460054	+0.0836682	+0.0010169	+0.0091938	—0.0069360	—0.0089510
1	4—	2	+ 1.29547	+3.02602	—0.05540	—0.12780	—0.00279	+0.01556	+0.00668	—0.02291
—1	7—	2	+ 0.850	+0.483	+0.003	+0.003	—0.004	—0.004	+0.004	0.000

Arg= $\varkappa\gamma'+i'g'+ig$			$A'n'\delta z'$		$B'(\nu'-c')+X'c'$		$F'n\delta z$		$G'(\nu-c)$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	6—	2	—0. 2755	—0. 1396	—0. 0070	—0. 0085	+0. 0025	+0. 0034	—0. 0028	—0. 0027
1	5—	2	—0. 00363	—0. 00284	+0. 00553	+0. 00452	+0. 00231	—0. 00128	—0. 00030	+0. 00247
—1	8—	2	+0. 177	—0. 010	+0. 001	0. 000				
0	7—	2	—0. 095	+0. 008	0. 000	—0. 001				
1	6—	2	+0. 003	—0. 002	+0. 003	0. 000				
—1	9—	2	+0. 022	—0. 015						
0	8—	2	—0. 014	+0. 009						
—1	0—	3	+0. 43	+0. 16			+0. 17	+0. 06		
0—	1—	3	—0. 25	—0. 17			—0. 13	—0. 08		
1—	2—	3	+0. 04	+0. 03						
—1	1—	3	—2. 38	+1. 90	+0. 13	—0. 07	—0. 85	+0. 65		
0	0—	3	+1. 85	—1. 50	—0. 10	+0. 05	+0. 78	—0. 62		
1—	1—	3	—0. 37	+0. 29			—0. 19	+0. 15		
—1	2—	3	—1. 17	+2. 62	—0. 13	+0. 38	—0. 30	+0. 65	—0. 02	+0. 05
0	1—	3	+0. 95	—2. 12	+0. 12	—0. 32	+0. 28	—0. 62	+0. 02	—0. 05
1	0—	3	—0. 15	+0. 39	—0. 03	+0. 07	—0. 05	+0. 14		
—1	3—	3	—0. 42	+1. 84	—0. 19	+0. 44	—0. 07	+0. 50	0. 00	+0. 13
0	2—	3	+0. 30	—1. 45	+0. 23	—0. 39	+0. 02	—0. 44	—0. 02	—0. 08
1	1—	3	—0. 02	+0. 23	—0. 03	+0. 07	+0. 06	+0. 05	+0. 02	—0. 01
—1	4—	3	+0. 060	+0. 572	—0. 783	—0. 966	—0. 145	—0. 125	—0. 302	—0. 419
0	3—	3	+0. 01	—0. 27	+0. 58	+0. 70	+0. 53	+0. 76	+0. 64	+0. 95
1	2—	3	+0. 03	+0. 09	—0. 03	—0. 04	—0. 58	—0. 91	—0. 60	—0. 87
—1	5—	3	—1. 062	+0. 528	—0. 204	—0. 372	—0. 209	+0. 071	+0. 111	+0. 036
0	4—	3	+0. 604	—0. 263	+0. 271	+0. 313	—0. 078	—0. 152	—0. 271	—0. 096
1	3—	3	—0. 021	—0. 007	—0. 069	—0. 032	+0. 323	+0. 155	+0. 279	+0. 132
—1	6—	3	—0. 157	+0. 529	+1. 724	—0. 207	—0. 203	+0. 138	+0. 098	—0. 016
0	5—	3	—2. 364	—0. 060	—2. 123	+0. 237	+0. 004	—0. 069	—0. 067	+0. 006
1	4—	3	+0. 020	—0. 061	+0. 247	—0. 025	+0. 062	+0. 036	—0. 024	—0. 001
—1	7—	3	+1. 1923	—0. 5863	+0. 0136	—0. 0131	+0. 2669	—0. 1755	—0. 0212	+0. 0144
0	6—	3	—8. 529	+5. 648	+0. 043	—0. 084	—3. 448	+2. 391	+0. 069	—0. 064
1	5—	3	+0. 043	+0. 005	—0. 039	+0. 029	+4. 336	—3. 021	—0. 074	+0. 072
—1	8—	3	—0. 0188	+1. 4384	—0. 0059	—0. 0013	—0. 0011	+0. 1820	—0. 0036	+0. 0055
0	7—	3	—0. 6786	—0. 3236	+0. 0093	—0. 0028	—0. 1373	—0. 0510	+0. 0052	—0. 0080
1	6—	3	+0. 1335	—0. 0518	—0. 0045	+0. 0035	+0. 0537	—0. 0256	—0. 0007	+0. 0017
—1	9—	3	+0. 1468	+0. 3839	—0. 0012	+0. 0004	+0. 0142	+0. 0372	+0. 0002	+0. 0006
0	8—	3	—0. 1457	—0. 2097	+0. 0007	—0. 0065	—0. 0196	—0. 0285	—0. 0001	—0. 0015
1	7—	3	+0. 0224	—0. 0059	+0. 0001	+0. 0050	+0. 0044	—0. 0002		
—1	10—	3	+0. 054	+0. 051			0. 000	—0. 002		
0	9—	3	—0. 036	—0. 022			—0. 001	+0. 002		
1	8—	3	+0. 024	—0. 041	—0. 005	—0. 006	+0. 005	—0. 003		
0	10—	3	—0. 008	—0. 002						
—1	1—	4	—0. 07	+0. 40			—0. 03	+0. 16		
0	0—	4	+0. 09	—0. 28			+0. 05	—0. 13		
—1	2—	4	—1. 59	—1. 23	+0. 06	+0. 09	—0. 57	—0. 47		
0	1—	4	+1. 32	+0. 99	—0. 02	—0. 04	+0. 55	+0. 43		

Arg= $\varkappa\gamma'+i'g'+ig$			$A'n'\delta z'$		$B'(\nu'-\sigma')+X'\sigma'$		$F'n\delta z$		$G'(\nu-\sigma)$	
			sin.	cos.	sin.	cos	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$
1	0	−4	−0.29	−0.23			−0.14	−0.12		
−1	3	−4	−2.09	−0.42	−0.32	−0.06	−0.54	−0.12	+0.04	0.00
0	2	−4	+1.73	+0.30	+0.25	+0.04	+0.52	+0.11	+0.03	0.00
1	1	−4	−0.34	−0.07	−0.06	−0.01	−0.11	−0.02		
−1	4	−4	−1.50	+0.04	−0.40	−0.02	−0.43	−0.02	−0.05	−0.02
0	3	−4	+1.20	−0.06	+0.32	+0.02	+0.45	+0.11	+0.11	+0.10
1	2	−4	−0.22	+0.02	−0.04	+0.01	−0.10	−0.15	−0.06	−0.08
−1	5	−4	−0.64	+0.34	+0.56	−0.71	−0.25	+0.21	+0.21	−0.22
0	4	−4	+0.50	−0.23	−0.38	+0.50	+0.11	−0.10	−0.26	+0.22
1	3	−4	−0.06	+0.03	+0.07	−0.08	+0.12	−0.02	+0.11	−0.06
−1	6	−4	−0.52	−0.55	+0.33	−0.26	−0.12	−0.32	+0.01	−0.05
0	5	−4	+0.18	+0.38	−0.37	+0.22	+0.06	+0.33	−0.04	+0.08
1	4	−4	−0.04	−0.06	+0.07	−0.04	+0.02	−0.13	+0.02	−0.09
−1	7	−4	−0.232	−2.157	+0.374	+1.039	+0.196	−0.204	+0.119	+0.378
0	6	−4	−0.383	+1.716	−0.249	−0.673	−0.870	+0.073	−0.229	−0.793
1	5	−4	−0.05	−0.06	+0.01	0.00	+1.02	+0.17	+0.20	+0.68
−1	8	−4	−5.631	−5.352	+0.130	+0.080	−1.430	−1.359	+0.029	+0.027
0	7	−4	+3.138	+3.031	−0.108	−0.072	+1.150	+1.108	−0.023	−0.045
1	6	−4	−0.183	−0.155	+0.014	−0.008	−0.110	−0.106		
−1	9	−4	−2.837	−0.704	+0.022	−0.002	−0.561	−0.139	−0.001	0.000
0	8	−4	+1.870	+0.482	−0.029	+0.002	+0.481	+0.124	0.000	−0.001
1	7	−4	−0.130	+0.026	+0.002	−0.005	−0.042	+0.011		
−1	10	−4	−0.66417	+0.19695	+0.00305	−0.00174	−0.10326	+0.03050	−0.00217	+0.00050
0	9	−4	+0.4863	−0.1374	−0.0016	+0.0016	+0.0958	−0.0267	+0.0025	−0.0009
1	8	−4	−0.026	+0.033	−0.003	0.000	−0.006	+0.008		
−1	11	−4	−0.09116	+0.09913	−0.00010	−0.00017	−0.01165	+0.01256	−0.00009	+0.00014
0	10	−4	+0.069759	−0.073644	−0.000101	+0.000255	+0.011093	−0.011609	+0.000080	−0.000113
1	9	−4	−0.00293	+0.01166	+0.00029	−0.00031	−0.00005	+0.00168	+0.00005	0.00000
0	11	−4	+0.0038	−0.0167			+0.0005	−0.0022		
−1	2	−5	−0.33	0.00			−0.13	0.00		
0	1	−5	+0.25	+0.04			+0.12	+0.02		
−1	3	−5	+0.51	−1.16			+0.21	−0.43		
0	2	−5	−0.41	+1.00			−0.19	+0.42		
1	1	−5	+0.10	−0.23			+0.05	−0.11		
−1	4	−5	−0.06	−1.48	−0.01	−0.18	−0.01	−0.41		
0	3	−5	+0.06	+1.28	+0.01	+0.16	−0.01	+0.35		
1	2	−5	−0.02	−0.27			−0.01	−0.09		
−1	5	−5	−0.29	−1.05	−0.05	−0.29	−0.07	−0.24		
0	4	−5	+0.26	+0.90	+0.02	+0.20	+0.04	+0.21		
1	3	−5	−0.06	−0.19						
−1	6	−5	−0.44	−0.49	+0.53	+0.24	−0.22	−0.20	+0.19	+0.11
0	5	−5	+0.37	+0.41	−0.35	−0.17	+0.19	+0.17	−0.21	−0.12
1	4	−5	−0.01	−0.04	+0.08	+0.05	−0.05	−0.03		
−1	7	−5	+0.24	−0.67	+0.26	+0.20	+0.21	−0.09	+0.06	+0.03
0	6	−5	−0.22	+0.55	−0.23	−0.17	−0.33	+0.05	−0.12	−0.07
1	5	−5	+0.06	−0.06	0.00	+0.03	+0.16	+0.04	+0.03	+0.07

Arg= $\kappa\gamma'+i'g'+ig$			$A'n'\delta z'$		$B'(\nu'-\sigma')+X'\sigma'$		$F'n\delta z$		$G'(\nu-e)$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
κ	i'	i	″	″	″	″	″	″	″	″
−1	8	−5	+1.03	−1.10	−0.71	+0.49	+0.21	−0.25	−0.14	+0.07
0	7	−5	−0.76	+0.88	+0.55	−0.38	−0.20	+0.22	+0.12	−0.07
1	6	−5	+0.12	−0.11	−0.08	+0.04	+0.02	−0.01		
−1	9	−5	+3.429	−5.790	−0.084	+0.185	+1.004	−1.672	−0.014	+0.033
0	8	−5	−2.28	+3.89	+0.08	−0.18	−0.85	+1.45	+0.02	−0.02
1	7	−5	+0.25	−0.48	0.00	+0.03	+0.15	−0.25		
−1	10	−5	+0.135	−3.060	+0.009	+0.035	+0.038	−0.710	+0.001	+0.004
0	9	−5	−0.110	+2.248	−0.001	−0.028	−0.038	+0.649	−0.001	−0.004
1	8	−5	−0.044	−0.252			−0.016	−0.096		
−1	11	−5	−0.380	−0.749	+0.003	+0.007	−0.073	−0.146	−0.001	−0.002
0	10	−5	+0.284	+0.579	−0.005	−0.008	+0.066	+0.136		
1	9	−5	−0.052	−0.047			−0.016	−0.015		
−1	12	−5	−0.1488	−0.0959	+0.0003	+0.0002	−0.0244	−0.0161	+0.0004	+0.0001
0	11	−5	+0.109	+0.074	0.000	0.000	+0.018	+0.013		
1	10	−5	−0.015	−0.002			+0.002	+0.003		
−1	13	−5	−0.0341	−0.0006	0.0000	0.0000	−0.0050	−0.0001		
0	12	−5	+0.0275	+0.0006	−0.0001	+0.0001	+0.0046	+0.0001		
1	11	−5	−0.0025	+0.0014			−0.0005	+0.0003		
−1	3	−6	−0.06	−0.25			−0.03	−0.10		
0	2	−6	0.0	+0.2						
−1	4	−6	+0.81	+0.17			+0.29	+0.07		
0	3	−6	−0.71	−0.14			−0.29	−0.05		
1	2	−6	+0.17	+0.04						
−1	5	−6	+1.00	−0.24	+0.11	−0.03	+0.26	−0.05		
0	4	−6	−0.87	+0.21	−0.10	+0.03	−0.26	+0.06		
1	3	−6	+0.16	−0.04						
−1	6	−6	+0.70	−0.38	+0.14	−0.06	+0.14	−0.08		
0	5	−6	−0.60	+0.34	−0.12	+0.06	−0.14	+0.08		
1	4	−6	+0.06	−0.06						
−1	7	−6	+0.28	−0.43	−0.08	+0.34	+0.10	−0.18	−0.05	+0.15
0	6	−6	−0.24	+0.31	+0.04	−0.23	−0.04	+0.11	+0.05	−0.14
−1	8	−6	+0.49	−0.02	−0.10	+0.26	+0.14	+0.14	+0.01	+0.03
0	7	−6	−0.42	−0.05	+0.10	−0.19	−0.12	−0.13	−0.02	−0.07
1	6	−6	+0.07	+0.04						
−1	9	−6	+0.99	+0.41	−0.47	−0.43	+0.20	+0.03	−0.12	−0.13
0	8	−6	−0.81	−0.34	+0.37	+0.34	−0.22	−0.04	+0.08	+0.09
1	7	−6	+0.15	+0.06	−0.06	−0.07	+0.02	+0.03		
−1	10	−6	+4.95	+1.63	−0.24	−0.06	+1.50	+0.50	−0.06	−0.02
0	9	−6	−3.65	−1.17	+0.19	+0.05	−1.37	−0.45	+0.05	+0.02
1	8	−6	+0.58	+0.19			+0.28	+0.09		
−1	11	−6	+2.73	−0.44	−0.04	+0.02	+0.71	−0.10	−0.02	0.00
0	10	−6	−2.12	+0.33	+0.04	−0.01	−0.66	+0.09	+0.01	0.00
1	9	−6	+0.30	−0.10			+0.12	−0.04		
−1	12	−6	+0.683	−0.532	−0.005	+0.004	+0.151	−0.118		
0	11	−6	−0.561	+0.423	+0.004	−0.004	−0.145	+0.109		
1	10	−6	+0.05	−0.07			+0.01	−0.02		

Arg= $\varkappa\gamma'+i'g'+ig$			$A'n'\delta z'$		$B'(\nu'-c')+X'c'$		$F'n\delta z$		$G'(\nu-c)$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	13	−6	+0.073	−0.200	0.000	+0.004	+0.016	−0.037		
0	12	−6	−0.070	+0.160			−0.016	+0.036		
1	11	−6	−0.001	−0.023			0.000	−0.006		
−1	4	−7	+0.2	−0.1						
0	3	−7	−0.1	+0.1						
−1	5	−7	0.0	+0.5			0.0	+0.2		
0	4	−7	0.0	−0.5			0.0	−0.2		
−1	6	−7	+0.29	+0.60			+0.08	+0.17		
0	5	−7	−0.27	−0.53			−0.07	−0.16		
−1	7	−7	+0.38	+0.40			+0.07	+0.06		
0	6	−7	−0.33	−0.35			−0.07	−0.05		
−1	8	−7	+0.32	+0.14	−0.23	−0.02	+0.15	+0.01	−0.11	−0.02
0	7	−7	−0.25	−0.12	+0.20	+0.02	−0.18	−0.02	+0.11	−0.01
−1	9	−7	+0.06	+0.36	−0.22	−0.04	+0.01	+0.08	−0.06	+0.03
0	8	−7	−0.04	−0.31	+0.13	+0.06	−0.01	−0.07	+0.06	−0.03
1	7	−7	−0.02	+0.07						
−1	10	−7	−0.03	+0.68	+0.26	−0.43	+0.02	+0.18	+0.07	−0.10
0	9	−7	+0.04	−0.63	−0.21	+0.37	−0.01	−0.20	−0.06	+0.09
1	8	−7	0.00	+0.12	+0.05	−0.07				
−1	11	−7	−0.37	+3.70	+0.03	−0.19	−0.12	+1.17	+0.01	−0.06
0	10	−7	+0.26	−2.91	−0.02	+0.17	+0.11	−1.09	−0.01	+0.05
1	9	−7	−0.06	+0.54	0.00	−0.04	−0.03	+0.25		
−1	12	−7	+0.82	+2.12	−0.02	−0.04	+0.23	+0.60		
0	11	−7	−0.67	−1.74	+0.01	+0.03	−0.21	−0.57		
1	10	−7	+0.14	+0.26			+0.06	+0.11		
−1	13	−7	+0.61	+0.53	−0.02	−0.01	+0.15	+0.13		
0	12	−7	−0.51	−0.45	−0.02	−0.01	−0.14	−0.12		
1	11	−7	+0.09	+0.06			+0.03	+0.02		
−1	6	−8	−0.3	+0.1						
0	5	−8	+0.3	−0.1						
−1	7	−8	−0.3	+0.2			−0.1	+0.1		
0	6	−8	+0.2	−0.2			+0.1	−0.1		
−1	8	−8	−0.13	+0.30						
0	7	−8	+0.1	−0.2						
−1	9	−8	−0.06	+0.07	−0.01	−0.10	+0.02	+0.13	−0.01	−0.08
0	8	−8			+0.01	+0.09	−0.02	−0.12		
−1	10	−8	−0.21	+0.08	+0.02	−0.11	−0.01	+0.07		
0	9	−8	+0.19	−0.05	−0.02	+0.06	+0.01	−0.06		
−1	11	−8	−0.42	+0.17	+0.32	+0.10	−0.04	+0.07	+0.07	+0.03
0	10	−8	+0.42	−0.16	−0.23	−0.10	+0.07	−0.07	−0.07	−0.03
1	9	−8	−0.07	+0.01	+0.06	+0.03				
−1	12	−8	−2.53	+0.33	+0.16	−0.01	−0.82	+0.10	+0.05	0.00
0	11	−8	+2.07	−0.30	−0.17	+0.02	+0.77	−0.10	−0.05	0.00
1	10	−8	−0.43	+0.05			−0.18	+0.02		
−1	13	−8	−1.48	+0.97	+0.03	−0.02	−0.41	+0.27		
1	11	−8	−0.21	+0.15			−0.09	+0.06		

Arg= $\varkappa\gamma'+i'g'+ig$			$A'n'\delta z'$		$B'(\nu'-c')+X'c'$		$F'n\delta z$		$G'(\nu-c)$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	7	−9	−0.1	−0.2						
−1	8	−9	−0.2	−0.2						
0	7	−9	+0.2	+0.2						
−1	11	−9	−0.10	−0.11						
0	10	−9	+0.02	+0.11						
−1	12	−9	−0.24	−0.20	−0.03	+0.18	−0.07	0.00		
0	11	−9	+0.22	+0.21	+0.03	−0.16	+0.08	+0.02		
−1	13	−9	−0.60	−1.57	+0.04	+0.14	−0.19	−0.52		
0	12	−9	+0.47	+1.29	−0.06	−0.13	+0.18	+0.51		
1	11	−9	−0.09	−0.24			−0.04	−0.12		
−1	13	−10	−0.02	−0.13	−0.11	0.00	−0.06	−0.08		
0	12	−10	0.0	+0.2	+0.1	0.0				

Arg= $\varkappa\gamma'+i'g'+ig$			$C'\delta\frac{h'}{h_0'}$		$D'\frac{u'}{\cos i'}$		$E'\frac{u_1'}{\cos i'}$		$H'\frac{u}{\cos i}$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		−0.0023032		−0.0000019		+0.0000019		+0.0000013
−1	1	0	+0.0011	+0.001441	−0.0006	+0.000003	0.0000	−0.000001	+0.0001	−0.000003
−1	2	0	−0.016	+0.033						
0	1	0	+0.0372	−0.0677						
−1	3	0	−0.012	−0.024						
0	2	0	+0.026	+0.052						
1	1	0	−0.011	−0.029						
−1	4	0	+0.004	+0.008						
0	3	0	−0.009	−0.015						
1	2	0	+0.004	+0.007						
−1	0	−1	+0.004	+0.009						
0	−1	−1	−0.002	−0.020						
1	−2	−1	−0.004	+0.010						
−1	1	−1	+0.008	+0.092						
0	0	−1	−0.004	−0.114						
1	−1	−1	−0.002	+0.049						
−1	2	−1	−0.1625	−0.7965						
0	1	−1	+0.321	+1.601						
1	0	−1	−0.165	−0.808						
−1	3	−1	+0.0124	−0.0553						
0	2	−1	−0.0294	+0.0870	−0.0002	−0.0002				
1	1	−1	+0.0218	−0.0095						
−1	4	−1	+0.0017	−0.0069						
0	3	−1	−0.0077	+0.0146						
1	2	−1	+0.0039	−0.0055						
−1	5	−1	+0.034	−0.029						

Arg= $\varkappa\gamma'+i'g'+ig$			$C'\delta\frac{h'}{h_0'}$		$D'\frac{u'}{\cos i'}$		$E'\frac{u_1'}{\cos i'}$		$H'\frac{u}{\cos i}$	
			sin.	cos.	sin.	cos.	sin	cos.	sin.	cos.
$\varkappa$	i'	i	$''$	$''$	$''$	$''$		$''$	$''$	$''$
0	4	−1	−0.073	+0.063						
1	3	−1	+0.0362	−0.0313						
0	5	−1	+0.004	−0.003						
1	4	−1	−0.004	+0.003						
0	0	−2	+0.02	0.00						
−1	2	−2	−0.159	−0.023						
0	1	−2	+0.268	+0.076						
1	0	−2	−0.13	−0.06						
−1	3	−2	+0.631	−0.295						
0	2	−2	−1.259	+0.573						
1	1	−2	+0.638	−0.279						
−1	4	−2	+0.1033	+0.1164						
0	3	−2	−0.183	−0.241						
1	2	−2	+0.069	+0.135						
−1	5	−2	+0.00157	+0.01056	−0.00006	+0.00019	+0.00003	+0.00001	−0.00001	+0.00012
0	4	−2	0.0000	−0.0180						
1	3	−2	−0.0037	+0.0048						
−1	6	−2	+0.00095	+0.00130	+0.00031	+0.00061	−0.00007	−0.00014	+0.00009	+0.00014
0	5	−2	−0.0017004	−0.0020128	−0.0001555	−0.0002744	+0.0001140	+0.0002169	−0.0000312	−0.0000223
1	4	−2	+0.00080	+0.00049	−0.00010	−0.00019	−0.00007	−0.00014	−0.00007	−0.00013
0	6	−2	+0.0006	+0.0018						
1	5	−2	+0.0015	−0.00080	−0.00004	−0.00002	−0.00002	−0.00001	−0.00004	−0.00002
−1	3	−3	+0.14	−0.06						
0	2	−3	−0.28	+0.10						
1	1	−3	+0.13	−0.07						
−1	4	−3	−0.236	−0.351						
0	3	−3	+0.50	+0.70						
1	2	−3	−0.26	−0.36						
−1	5	−3	−0.124	−0.072						
0	4	−3	+0.234	+0.128						
1	3	−3	−0.115	−0.048						
−1	6	−3	−0.037	+0.010						
0	5	−3	+0.068	−0.018						
1	4	−3	−0.031	+0.012						
−1	7	−3	+0.0157	−0.0381						
0	6	−3	−0.031	+0.076						
1	5	−3	+0.020	−0.041						
−1	8	−3	+0.0021	−0.0022						
0	7	−3	−0.0038	+0.0029						
1	6	−3	+0.0014	0.0000						
0	8	−3	−0.0007	+0.0001						
1	7	−3	+0.0002	0.0000						
−1	5	−4	+0.17	−0.19						
0	4	−4	−0.34	+0.39						
1	3	−4	+0.20	−0.21						

Arg= $\varkappa\gamma'+i'g'+ig$			$C'\delta\frac{h'}{h_0'}$		$D'\frac{u'}{\cos i'}$		$E'\frac{u_1'}{\cos i'}$		$H'\frac{u}{\cos i}$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	6	−4	+0.02	−0.10						
0	5	−4	−0.05	+0.23						
1	4	−4	0.00	−0.11						
−1	7	−4	−0.009	−0.024						
0	6	−4	+0.018	+0.064						
−1	8	−4	−0.008	−0.003						
0	7	−4	+0.017	+0.005						
1	6	−4	−0.008	−0.002						
−1	9	−4	−0.002	+0.001						
0	8	−4	+0.004	−0.002						
−1	10	−4	−0.00041	+0.00030	−0.00005	+0.00001				
0	9	−4	+0.0005	−0.0007						
−1	11	−4	0.00000	+0.00006						
0	10	−4	+0.000013	−0.000131	+0.000003	−0.000004			−0.000001	+0.000004
−1	6	−5	+0.12	+0.08						
0	5	−5	−0.26	−0.14						
1	4	−5	+0.11	+0.06						
−1	7	−5	+0.05	0.00						
0	6	−5	−0.14	0.00						
1	5	−5	+0.03	−0.01						
0	7	−5	−0.02	+0.01						
−1	7	−6	−0.01	+0.04						
0	6	−6	+0.03	−0.14						
0	7	−6	−0.01	−0.05						

CHAPTER XIV.

CALCULATION OF THE PORTION OF $\delta T'$ FACTORED BY $n't$.

In determining the part of $\delta T'$ having the factor $n't$ a degree of precision 200 times greater than that used in deriving the part not multiplied by $n't$ has been employed. In the following table the factor $n't$ has been omitted, and to avoid multiplicity of zeros all the coefficients have been multiplied by 10000:

Arg= $\kappa\gamma'+i'g'+ig$			$A'n'\delta z'$		$B'\nu'$		$F'n\delta z$		$G'\nu$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
κ	i'	i	″	″	″	″	″	″	″	″
0	0	0		− 14.2442		+ 18.8373				
−1	1	0	+ 3.47	+ 44.0178	− 2.39	− 26.0598	+ 2.93	+ 3.5127	− 2.91	− 4.8233
−1	2	0	+ 87.61	+139.28	+107.14	+174.06	− 13.46	− 7.66	+103.02	+ 60.93
0	1	0	− 2.63	− 3.45	+ 2.17	+ 3.04	+158.75	+ 93.79	−217.88	−128.93
1	0	0	+ 83.0235	+132.8585	−110.8643	−177.1608	−198.3620	−117.4360	+187.1815	+111.0051
−1	3	0	+ 43.3	+ 18.6	+ 37.0	+ 14.5	+ 6.8	− 2.2	+ 16.8	+ 1.4
0	2	0	− 14.0	− 5.3	− 16.9	− 7.0	+ 0.8	+ 5.5	− 19.1	− 5.0
1	1	0	+ 1.13	+ 1.82	− 12.41	− 7.22	− 1.99	− 1.58	+ 0.62	+ 0.43
−1	4	0	+ 8.8	− 1.4	+ 6.0	− 1.4	+ 1.1	− 1.5	+ 1.8	− 2.2
0	3	0	− 4.4	+ 1.0	− 3.6	+ 0.8	− 0.9	+ 1.1	− 1.9	+ 0.8
1	2	0	− 0.3	0.0	− 1.5	− 0.1	− 0.2	− 0.1	− 0.2	0.0
−1	5	0	+ 1.0	− 0.9	+ 0.4	− 0.4	+ 0.1	− 0.3	+ 0.1	− 0.3
0	4	0	− 0.5	+ 0.5	− 0.4	+ 0.4	0.0	+ 0.3	− 0.2	+ 0.2
1	3	0	− 0.1	0.0	− 0.1	+ 0.1				
−1−	1−	1	− 0.2	− 0.1	0.0	+ 1.0			− 0.1	+ 0.3
0−	2−	1	− 0.3	+ 0.1	0.0	+ 0.6			− 0.1	+ 0.2
1−	3−	1	+ 0.3	− 0.9	− 0.1	− 1.9	− 0.1	− 0.1	+ 0.4	− 0.7
−1	0−	1	− 12.4	− 7.1	− 1.3	+ 6.9	+ 2.4	− 1.4	− 4.3	+ 4.9
0−	1−	1	− 6.4	− 2.2	+ 6.7	+ 2.3	− 1.9	+ 1.2	+ 4.1	− 0.6
1−	2−	1	+ 3.7	− 1.4	− 6.3	− 9.4	+ 0.1	− 0.1	− 1.2	− 4.1
−1	1−	1	+ 75.7	+ 26.4	+337.2	+124.8	+ 0.9	+ 0.6	+ 12.8	+ 14.2
0	0−	1	−458.7	−168.2	−402.9	−147.7				
1−	1−	1	− 0.6	− 0.6	+ 44.9	+ 16.2	+ 0.3	+ 0.2	− 13.6	− 14.8
−1	2−	1	− 64.24	+ 44.10	+ 70.56	− 16.26	− 10.25	− 9.42	+ 12.56	+ 5.47
0	1−	1	− 8.75	− 32.47	− 71.87	+ 5.80	+ 36.08	+ 22.05	− 23.14	− 18.15
1	0−	1	+ 6.5	− 0.6	+ 8.7	− 1.0	− 37.0	− 22.4	+ 18.8	+ 17.3
−1	3−	1	+ 46.223	− 41.866	−252.421	+239.612	− 28.371	+ 78.277	− 15.666	+ 48.784
0	2−	1	−348.270	−327.391	+301.613	−290.252	+ 24.222	− 63.753	+ 11.406	− 38.695
1	1−	1	+ 0.85	+ 0.63	− 34.03	+ 32.33	− 2.72	+ 6.89	− 0.57	+ 1.72

Arg= $\varkappa\gamma'+i'g'+ig$	$A'n'\delta z'$		$B'\nu'$		$F'n\delta z$		$G'\nu$	
	sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$ i' i	″	″	″	″	″	″	″	″
—1 4— 1	+ 9.31	+ 64.27	— 22.47	+ 72.12	+ 3.33	+ 24.01	+ 4.84	+ 15.94
0 3— 1	— 37.649	— 4.062	+ 28.187	— 59.117	— 2.695	— 21.198	— 4.051	— 13.379
1 2— 1	+ 5.859	— 3.747	+ 0.727	— 2.613	+ 1.095	+ 1.782	+ 0.907	+ 0.408
—1 5— 1	+ 9.3	+ 17.4	+ 3.5	+ 12.0	+ 2.7	+ 3.6	+ 2.8	+ 2.2
0 4— 1	— 8.76	— 8.78	— 1.67	— 9.44	— 2.42	— 3.39	— 2.41	— 1.93
1 3— 1	+ 1.11	+ 0.50	+ 0.22	— 0.89	+ 0.40	+ 0.13	+ 0.27	— 0.09
—1 6— 1	+ 3.1	+ 2.1	+ 1.6	+ 1.1	+ 0.6	+ 0.4	+ 0.6	0.0
0 5— 1	— 2.3	— 1.4	— 1.2	— 0.8	— 0.6	— 0.3	— 0.5	— 0.1
1 4— 1	+ 0.2	— 0.1			0.0	— 0.1		
—1 7— 1	+ 0.5	+ 0.1	+ 0.2	0.0	+ 0.7	+ 0.3	+ 0.7	— 0.1
0 6— 1	— 0.4	0.0	— 0.1	0.0	— 0.1	+ 0.1	— 0.1	+ 0.1
—1 0— 2	— 0.8	— 0.4	— 1.7	— 0.1	0.0	+ 0.2	— 0.3	0.0
0— 1— 2	— 1	0	+ 1	+ 2				
1— 2— 2	+ 1	0						
—1 1— 2	+ 32.0	+ 15.2	+ 9.2	+ 3.6	+ 4.2	— 4.0	+ 2.3	— 5.3
0 0— 2	— 52.8	— 17.6	— 27.7	— 12.8				
1— 1— 2	+ 0.2	— 0.1	+ 11.1	+ 1.0	— 0.1	— 1.6	+ 3.2	— 2.4
—1 2— 2	+ 53.2	—327.9	— 33.5	+231.6	— 10.0	+ 11.5	— 69.5	+ 97.3
0 1— 2	— 32.2	+179.2	+ 19.1	—152.4	+111.8	—146.4	+146.0	—206.6
1 0— 2	+ 2.2	— 10.2	+ 0.9	— 0.5	—139.0	+184.1	—124.9	+177.9
—1 3— 2	— 14.6	—141.7	+ 24.9	+ 79.5	+ 0.9	— 2.5	— 1.4	+ 19.3
0 2— 2	— 11.5	+119.4	+ 0.4	— 73.6	+ 5.7	— 3.9	+ 6.5	— 21.0
1 1— 2	— 4.1	— 4.1	— 2.6	+ 9.2	— 1.7	+ 2.6	— 0.2	+ 1.0
—1 4— 2	—293.69	—212.79	—179.32	—123.93	— 80.05	— 10.97	— 44.37	— 3.65
0 3— 2	+162.6	+122.7	+117.4	+ 77.5	+ 70.4	+ 8.8	+ 38.0	+ 2.5
1 2— 2	— 9.7	— 6.0	+ 1.4	+ 0.6	— 12.8	— 1.4	— 5.8	— 0.9
—1 5— 2	—139.230	— 16.103	— 76.712	— 3.260	— 28.507	+ 10.214	— 16.302	+ 8.259
0 4— 2	+ 92.09	+ 11.50	+ 56.70	+ 2.86	+ 26.12	— 9.22	+ 14.49	— 7.30
1 3— 2	— 5.87	+ 2.05	— 1.02	+ 2.84	— 3.57	+ 2.23	— 1.55	+ 1.44
—1 6— 2	— 30.975	+ 13.247	— 14.433	+ 8.550	— 4.644	+ 5.163	— 2.281	+ 3.959
0 5— 2	+ 22.75472	— 9.41055	+ 11.54684	— 6.59174	+ 4.41582	— 4.79203	+ 2.10408	— 3.58718
1 4— 2	— 0.950	+ 1.643	+ 0.222	+ 1.010	— 0.322	+ 0.839	— 0.018	+ 0.526
—1 7— 2	— 3.82	+ 5.14	— 1.31	+ 2.66	— 0.27	+ 1.21	+ 0.04	+ 0.93
0 6— 2	+ 3.02	— 3.96	+ 1.10	— 2.18	+ 0.29	— 1.16	— 0.07	— 0.86
1 5— 2	+ 0.050	+ 0.413	+ 0.157	+ 0.157	+ 0.040	+ 0.151	+ 0.067	+ 0.083
—1 8— 2	— 0.2	+ 1.0	+ 0.1	+ 0.4	0.0	+ 0.2	+ 0.1	+ 0.1
0 7— 2	+ 0.1	— 0.7	— 0.1	— 0.4	0.0	— 0.2	— 0.1	— 0.1
1 6— 2	+ 0.05	+ 0.05	+ 0.03	0.00	+ 0.02	+ 0.02	+ 0.02	0.00
—1 1— 3	+ 3	+ 2	0	— 1	+ 1	— 1	+ 1	0
0 0— 3	— 4	— 1	— 2	0				
1— 1— 3			+ 1	+ 1				
—1 2— 3	— 7	— 11	+ 4	+ 6	— 10	+ 9	— 5	+ 8
0 1— 3	0	+ 7	0	— 5	+ 28	— 35	+ 16	— 22
1 0— 3			+ 1	+ 1	— 27	+ 37	— 15	+ 23
—1 3— 3	+285.4	— 19.8	—183.6	+ 11.6	+ 75.7	+ 35.4	— 47.2	— 20.6

Arg= $\varkappa\gamma'+i'g'+ig$			$A'n'\delta z'$		$B'\nu'$		$F'n\delta z$		$G'\nu$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	"	"	"	"	"	"	"	"
0	2	—3	—189.4	+ 16.4	+137.3	— 10.9	—61.5	—30.1	+37.4	+15.0
1	1	—3	+ 23	— 1	— 17	+ 2	+ 7	+ 3	— 2	0
—1	4	—3	+ 99.2	— 66.1	— 71.7	+ 45.4	+18.4	+ 3.1	—19.7	+ 5.9
0	3	—3	— 76.7	+ 52.7	+ 55.5	— 32.3	—15.9	— 1.0	+16.6	— 4.0
1	2	—3	+ 7.3	— 8.0	— 5.4	+ 5.7	+ 0.6	— 0.1	— 1.1	+ 1.1
—1	5	—3	+125.9	—292.8	+ 63.2	—151.2	— 2.3	—66.5	— 5.8	—32.9
0	4	—3	— 84.2	+197.1	— 44.8	+110.7	+ 2.7	+60.4	+ 5.4	+29.3
1	3	—3	+ 8.9	— 24.1	+ 6.4	— 13.4	— 1.1	—13.3	— 0.6	— 6.1
—1	6	—3	— 12.5	—145.2	— 10.8	— 73.1	—15.7	—26.2	—10.7	—13.4
0	5	—3	+ 8.6	+106.6	+ 8.3	+ 58.0	+14.5	+24.3	+ 9.6	+12.3
1	4	—3	— 3.7	— 11.6	— 3.0	— 5.7	— 3.4	— 4.0	— 2.1	— 1.9
—1	7	—3	— 22.61	— 33.69	— 12.52	— 14.64	— 7.11	— 4.20	— 4.66	— 1.72
0	6	—3	+ 17.40	+ 26.51	+ 10.24	+ 12.30	+ 6.72	+ 4.06	+ 4.33	+ 1.64
1	5	—3	— 3.0	— 2.0	— 1.7	— 0.6	— 1.3	— 0.3	— 0.8	— 0.1
—1	8	—3	— 7.819	— 3.890	— 3.738	— 1.142	— 1.726	— 0.051	— 1.089	+ 0.230
0	7	—3	+ 6.343	+ 3.269	+ 3.190	+ 1.031	+ 1.665	+ 0.073	+ 1.028	— 0.209
1	6	—3	— 0.80	+ 0.01	— 0.35	+ 0.16	— 0.25	+ 0.08	— 0.13	+ 0.10
—1	9	—3	— 1.59	+ 0.10	— 0.66	+ 0.17	— 0.26	+ 0.14	— 0.15	+ 0.15
0	8	—3	+ 1.351	— 0.065	+ 0.581	— 0.151	+ 0.268	— 0.135	+ 0.147	— 0.141
1	7	—3	— 0.124	+ 0.089	— 0.018	+ 0.053	— 0.028	+ 0.035	— 0.011	+ 0.031
—1	10	—3	— 0.2	+ 0.1	— 0.1	0.0				
0	9	—3	+ 0.18	— 0.12	+ 0.06	— 0.08	+ 0.02	— 0.05	0.00	— 0.03
1	8	—3	0.00	+ 0.02	0.00	+ 0.01				
—1	2	—4	— 1	0	+ 1	0	— 1	+ 1		
0	1	—4					+ 3	— 4	+ 1	— 1
1	0	—4					— 3	+ 4	— 1	+ 1
—1	3	—4	+ 14	— 14	— 8	+ 9	+ 6	— 2	— 3	+ 1
0	2	—4	— 10	+ 5	+ 7	— 3	— 5	— 1	— 2	0
1	1	—4	+ 2	— 1	— 1	+ 1				
—1	4	—4	+ 58	+205	— 36	—128	—17	+78	+10	—44
0	3	—4	— 45	—149	+ 29	+101	+14	—68	— 9	+39
1	2	—4	+ 7	+ 24	— 5	— 18	— 3	+12	+ 1	— 5
—1	5	—4	+ 74.3	+ 68.3	— 52.7	— 57.4	+ 2.4	+23.9	— 9.8	—20.0
0	4	—4	— 63	— 55	+ 40	+ 47	— 4	—21	+ 8	+19
1	3	—4	+ 11	+ 7	— 8	— 5	+ 1	+ 3	— 2	— 2
—1	6	—4	+246.0	+ 49.1	+110.5	+ 19.4	+49.5	— 9.8	+20.5	—10.6
0	5	—4	—182.4	— 35.4	— 86.6	— 13.9	—45.9	+ 9.5	—18.7	+ 9.9
1	4	—4	+ 29.7	+ 4.6	+ 15.9	+ 3.5	+11.0	— 3.1	+ 4.6	— 1.7
—1	7	—4	+126.7	— 37.8	+ 59.4	— 21.3	+20.0	—18.0	+ 8.9	—11.1
0	6	—4	— 99.2	+ 29.1	— 49.1	+ 17.6	—19.1	+17.1	— 8.4	+10.2
1	5	—4	+ 13.9	— 6.6	+ 7.2	— 4.3	+ 3.6	— 4.0	+ 1.6	— 2.2
—1	8	—4	+ 29.7	— 29.7	+ 12.2	— 15.0	+ 2.8	— 7.9	+ 0.8	— 4.6
0	7	—4	— 24.4	+ 24.1	— 10.5	+ 12.7	— 2.7	+ 7.6	— 0.8	+ 4.4
1	6	—4	+ 2.4	— 4.3	+ 0.6	— 2.3	+ 0.1	— 1.5	— 0.1	— 0.8
—1	9	—4	+ 2.74	— 9.77	+ 0.57	— 4.40	— 0.33	— 1.98	— 0.44	— 1.13
0	8	—4	— 2.3	+ 8.1	— 0.5	+ 3.8	+ 0.3	+ 1.9	+ 0.4	+ 1.2

Arg= $\kappa\gamma'+i'g'+ig$			$A'n'\delta z'$		$B'\nu'$		$F'n\delta z$		$G'\nu$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
κ	i'	i	″	″	″	″	″	″	″	″
1	7	−4	−0.1	−1.0	−0.2	−0.4	−0.1	−0.3	−0.2	−0.1
−1	10	−4	−0.502	−2.000	−0.361	−0.783	−0.251	−0.311	−0.204	−0.149
0	9	−4	+0.40	+1.74	+0.31	+0.71	+0.23	+0.31	+0.20	+0.16
1	8	−4	−0.15	−0.17	−0.09	−0.05	−0.06	−0.03	−0.05	0.00
−1	11	−4	−0.253	−0.259	−0.130	−0.082	−0.064	−0.026	−0.049	−0.006
0	10	−4	+0.2240	+0.2395	+0.1174	+0.0789	+0.0639	+0.0268	+0.0460	+0.0048
1	9	−4	−0.040	−0.013	−0.019	+0.005	−0.011	+0.002	−0.008	+0.002
0	11	−4	+0.05	+0.01	+0.01	0.00	+0.02	−0.01	+0.01	−0.01
−1	3	−5	0	−1	−1	+1				
0	2	−5	0	+1						
−1	4	−5	+16	+11	−10	−6	+3	+8	−1	−4
0	3	−5	−9	−10	+6	+7	−1	−6	+1	+3
1	2	−5	−2	0	−2	−1	+1	+1		
−1	5	−5	−134	+71	+80	−43	−64	0	+35	0
0	4	−5	+102	−57	−66	+36	+58	−1	−31	0
1	3	−5	−19	+11	+14	−8	−13	0	+6	0
−1	6	−5	−38	+69	+39	−52	−23	+8	+17	−13
0	5	−5	+32	−61	−34	+41	+21	−8	−17	+11
1	4	−5	−8	+14	+5	−9	0	+3	+2	−3
−1	7	−5	+3	+184	+4	+72	+12	+35	+10	+10
0	6	−5	−4	−145	−4	−58	−12	−32	−10	−9
1	5	−5	+2	+28	−1	+13	+4	+8	+2	+4
−1	8	−5	+53.4	+96.6	+25.9	+42.3	+17.7	+13.4	+10.0	+4.8
0	7	−5	−42.6	−78.8	−22.4	−36.1	−16.8	−12.9	−9.5	−4.4
1	6	−5	+9	+13	+4	+6	+4	+2	+3	+1
−1	9	−5	+32.7	+21.5	+15.4	+8.2	+7.8	+1.0	+4.2	−0.2
0	8	−5	−27.5	−18.3	−13.6	−7.5	−7.6	−1.0	−4.1	+0.1
1	7	−5	+4.9	+2.0	+2.5	+0.7	+1.6	−0.3	+0.9	−0.2
−1	10	−5	+10.4	+0.8	+4.5	−0.2	+1.9	−0.8	+1.0	−0.7
0	9	−5	−9.2	−0.8	−4.0	+0.2	−1.8	+0.8	−1.0	+0.7
1	8	−5	+1.2	−0.2	+0.5	−0.3	+0.3	−0.3	+0.2	−0.2
−1	11	−5	+2.0	−1.0	+0.8	−0.7	+0.3	−0.3	+0.1	−0.3
0	10	−5	−1.8	+0.8	−0.8	+0.6	−0.3	+0.3	−0.1	+0.3
1	9	−5	+0.2	−0.2	+0.1	−0.2	+0.1	−0.1		
−1	12	−5	+0.27	−0.38	+0.07	−0.17	+0.01	−0.09	−0.01	−0.06
0	11	−5	−0.24	+0.33	−0.07	+0.15	−0.01	+0.08	+0.01	+0.06
−1	13	−5	+0.007	−0.080	−0.018	−0.034	−0.003	−0.015	−0.008	−0.010
0	12	−5	−0.008	+0.075	+0.004	+0.029	+0.003	+0.016	+0.007	+0.008
1	11	−5	−0.01	−0.01						
−1	4	−6	+2	0	−1	+1				
0	3	−6	−1	−1	−1	0				
−1	5	−6	−6	+17	+4	−10	−6	+5	+3	−2
0	4	−6	+7	−11	−5	+7	+6	−3	−2	+2
1	3	−6	−2	+4	+1	−2	−1	+1		
−1	6	−6	−67	−77	+40	+44	−10	−45	+5	+25

Arg= $\varkappa\gamma'+i'g'+ig$			$A'n'\delta z'$		$B'\nu'$		$F'n\delta z$		$G'\nu$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	"	"	"	"	"	"	"	"
0	5	− 6	+ 55	+60	−32	−38	+11	+42	− 5	−23
1	4	− 6	− 11	−13	+ 8	+ 9	− 4	−10	+ 1	+ 5
−1	7	− 6	− 55	−14	+46	+20	−11	−19	+13	+13
0	6	− 6	+ 51	+12	−39	−18	+11	+17	−12	−12
1	5	− 6	− 10	0	+ 8	+ 4	− 3	− 3	+ 2	+ 2
−1	8	− 6	−127	+29	−42	+12	−23	+10	− 3	+ 8
0	7	− 6	+104	−26	+35	−11	+22	−12	+ 2	− 9
1	6	− 6	− 22	+ 6	−10	+ 2	− 6	+ 4	− 4	+ 2
−1	9	− 6	− 64	+56	−26	+26	− 8	+15	− 1	+ 7
0	8	− 6	+ 55	−47	+23	−24	+ 8	−15	+ 1	− 7
1	7	− 6	− 9	+ 9	− 5	+ 5	− 1	+ 4	0	+ 2
−1	10	− 6	− 12.5	+31.6	− 4.1	+14.1	+ 0.3	+ 6.9	+ 0.9	+ 3.4
0	9	− 6	+ 11	−27	+ 4	−13	− 1	− 6	− 1	− 3
1	8	− 6	− 1	+ 5	0	+ 3	0	+ 2	+ 1	0
−1	11	− 6	+ 1.2	+ 9.8	+ 1.0	+ 4.1	+ 1.1	+ 1.6	+ 0.8	+ 0.8
0	10	− 6	− 0.9	− 8.8	− 0.9	− 3.6	− 1.1	− 1.6	− 0.8	− 0.7
1	9	− 6	+ 0.5	+ 1.3	+ 0.4	+ 0.6	+ 0.3	+ 0.3	+ 0.2	+ 0.2
−1	12	− 6	+ 1.6	+ 1.9	+ 0.8	+ 0.8	+ 0.4	+ 0.2	+ 0.3	+ 0.1
0	11	− 6	− 1.5	− 1.7	− 0.7	− 0.7	− 0.4	− 0.2	− 0.3	− 0.1
1	10	− 6	+ 0.3	+ 0.2	+ 0.2	+ 0.1	+ 0.1	+ 0.1		
−1	13	− 6	+ 0.4	+ 0.2	+ 0.2	0.0	+ 0.2	− 0.1		
0	12	− 6	− 0.4	− 0.2	− 0.2	0.0	− 0.1	+ 0.1		
−1	6	− 7	− 15	− 3	+ 9	+ 1	− 6	− 4		
0	5	− 7	+ 12	+ 3	− 7	− 1	+ 4	+ 5		
−1	7	− 7	+ 37	−52	−21	+30	+29	−14	−15	+ 7
0	6	− 7	− 29	+44	+19	−26	−27	+14	+14	− 7
1	5	− 7	+ 7	−11	− 4	+ 7	+ 7	− 3		
−1	8	− 7	0	−41	− 9	+37	+13	−12	− 6	+10
0	7	− 7	0	+38	+ 8	−32	−13	+17	+ 6	− 9
1	6	− 7	+ 1	−10	− 1	+ 6	+ 2	− 4		
−1	9	− 7	− 38	−80	−13	−21	−10	−14	− 6	0
0	8	− 7	+ 33	+67	+11	+18	+10	+13	+ 6	− 1
1	7	− 7	− 9	−15	− 4	− 7	− 3	− 2		
−1	10	− 7	− 51	−37	−21	−14	−11	− 1	− 7	0
0	9	− 7	+ 43	+32	+18	+12	+11	+ 1	+ 6	0
1	8	− 7	− 9	− 6	− 5	− 2				
−1	11	− 7	− 27	− 5	−12	− 1	− 5	+ 1	− 3	+ 1
0	10	− 7	+ 24	+ 4	+10	0	+ 5	− 1	+ 3	− 1
1	9	− 7	− 4	0	− 3	0				
−1	12	− 7	− 8	+ 3	− 3	+ 2	− 1	+ 1		
0	11	− 7	+ 7	− 3	+ 3	− 2	+ 1	− 1		
1	10	− 7	− 1	+ 1						
−1	13	− 7	− 1.5	+ 1.6	− 0.5	+ 0.7				
0	12	− 7	+ 1.3	− 1.5						
−1	7	− 8	− 1	−12	+ 1	+ 7				
0	6	− 8	0	+ 9						

Arg= $\varkappa\gamma'+i'g'+ig$			$A'n'\delta z'$		$B'\nu'$		$F'n\delta z$		$G'\nu$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	8	−8	+37	+14	−21	−9	+13	+16	−6	−9
0	7	−8	−32	−12	+19	+7	−14	−15	+7	+8
1	6	−8	+8	+3	−5	−2				
−1	9	−8	+28	−6	−27	−2	+14	+7	−8	−4
0	8	−8	−26	+6	+24	+1	−14	−7	+8	+4
1	7	−8	+8	−1						
−1	10	−8	+46	−35	+10	−10	+9	−7	+2	−4
0	9	−8	−39	+33	−8	+8	−9	+7		
1	8	−8	+6	−5	+5	−4				
−1	11	−8	+19	−42	+6	−16	0	−8	0	−4
0	10	−8	−17	+36	−4	+15	0	+8		
1	9	−8	+2	−7	+1	−4				
−1	12	−8	−1	−19	−1	−9	−2	−4		
0	11	−8	+1	+17	+1	+9	+2	+4		
1	10	−8	−1	−4						
−1	13	−8	−4	−6	−2	−3				
0	12	−8	+3	+5	+2	+2				
−1	8	−9	+8	−3						
−1	9	−9	−2	+23	+2	−13	−8	+11	+4	−6
0	8	−9	+2	−21	−2	+12	+7	−10		
−1	10	−9	+9	+16	−4	−13	−3	+11		
0	9	−9	−8	−15	+4	+12	+3	−11		
−1	11	−9	+30	+23	+5	+2				
0	10	−9	−27	−21	−6	−2				
1	9	−9	+5	+3						
−1	12	−9	+31	+7	+14	+3	+5	−2		
0	11	−9	−28	−6	−12	−2	−5	+1		
1	10	−9	+6	+1						
−1	13	−9	+14	−4	+7	−3				
0	12	−9	−13	+4	−6	+2				
−1	10	−10	−14	+2						
0	9	−10	+13	−2						
−1	11	−10	−15	+8	+8	−5				
0	10	−10	+13	−8	−7	+4				
−1	12	−10	−10	+23	+1	+3				
0	11	−10	+8	−20						
−1	13	−10	+1	+18	0	+5				
0	12	−10	−1	−16	−1	−8				
−1	12	−11	−8	−9						
0	11	−11	+7	+7						
−1	13	−11	−8	0						

Arg= $\varkappa\gamma'+i'g'+ig$	$C'\delta\frac{h'}{h_0'}$		$D'\frac{u'}{\cos i'}$		$E'\frac{u_1'}{\cos i'}$		$H'\frac{u}{\cos i}$	
	sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$ i' i	″	″	″	″	″	″	″	″
0 0 0		+ 0.0200		+0.5438		—0.5438		
—1 1 0	0.00	— 0.0149	+0.01	—0.7200	—0.01	+0.7234	0.00	—0.0005
—1 2 0	+0.04	— 0.17	+0.02	—0.27	+0.11	+0.07	+0.04	—0.03
0 1 0	+0.07	+ 0.35	+0.07	+0.17	—0.14	—0.08	—0.01	0.00
1 0 0	—0.0367	— 0.1728	—0.0931	+0.0882	+0.0826	+0.0011	+0.0191	+0.0678
—1 3 0			—1.2	—0.4	—0.4	—0.1	—0.6	—0.2
0 2 0	0.0	— 0.2	+0.5	+0.2	+0.2	+0.2	+0.4	+0.2
1 1 0	—0.02	— 0.02	+0.45	+0.17	—0.33	—0.12	+0.04	+0.02
—1 4 0			—0.4	+0.1	—0.1	0.0	—0.2	+0.1
0 3 0			+0.3	0.0	+0.1	—0.1	+0.1	—0.1
1 2 0			+0.1	0.0				
—1 5 0			—0.1	0.0				
—1— 1— 1			0.0	—0.2	+0.1	+0.2		
0— 2— 1			—0.1	+0.1	—0.1	—0.1	0.0	—0.1
1— 3— 1			0.0	+0.3			0.0	+0.1
—1 0— 1			—0.8	—1.4	—0.5	—0.8	—0.5	—0.8
0— 1— 1					+0.7	+1.2	+0.3	+0.5
1— 2— 1			+0.8	+1.4	—0.5	—0.8	+0.1	+0.2
—1 1— 1	+0.2	+ 1.0	—0.3	+0.1	—0.1	—0.1		
0 0— 1			0.0	—0.2	0.0	+0.2		
1— 1— 1	+0.1	+ 0.5	+0.1	0.0	+0.1	+0.1		
—1 2— 1	—1.98	— 9.41	—0.52	+0.12	+0.65	—0.13	—0.06	+0.02
0 1— 1	+3.94	+18.90	—0.68	+0.14	—0.97	+0.19	+0.64	—0.14
1 0— 1	—2.0	— 9.5	+1.4	—0.4	+0.7	—0.2	—0.7	+0.1
—1 3— 1	+0.130	— 0.704	+0.138	+0.087	—0.006	+0.115	+0.058	—0.016
0 2— 1	—0.319	+ 1.149	—0.168	+0.027	+0.021	—0.153	—0.010	+0.018
1 1— 1	+0.24	— 0.18	+0.05	—0.08	—0.06	+0.11	—0.02	0.00
—1 4— 1	+0.05	— 0.07	+0.20	—1.11	+0.03	—0.21	+0.08	—0.53
0 3— 1	—0.098	+ 0.118	—0.125	+0.707	—0.044	+0.312	—0.054	+0.415
1 2— 1	+0.044	— 0.030	—0.031	+0.170	+0.032	—0.216	—0.001	—0.020
—1 5— 1			—0.1	—0.4	0.0	—0.1	—0.1	—0.2
0 4— 1	—0.01	+ 0.02	+0.10	+0.32	+0.04	+0.12	+0.05	+0.16
1 3— 1	0.00	— 0.01	+0.01	+0.05	—0.02	—0.06	—0.01	—0.01
—1 6— 1			—0.1	—0.1				
0 5— 1			0.0	+0.1				
—1 0— 2			+0.1	—0.1	—0.1	—0.1	0.0	—0.1
—1 1— 2	0.0	+ 0.1	+0.9	—0.9	—0.3	+0.3	+0.2	—0.2
0 0— 2	—0.4	+ 0.3	+0.4	—0.3				
1— 1— 2			—0.4	+0.3	—0.3	+0.3	—0.2	+0.2
—1 2— 2	—0.2	— 1.1	—0.1	—0.3	+0.1	0.0	—0.1	0.0
0 1— 2	+0.5	+ 2.1	0.0	+0.2	0.0	—0.1	+0.1	0.0
1 0— 2	—0.3	— 1.1	+0.1	+0.1	+0.1	—0.1	0.0	+0.1
—1 3— 2	—1.7	+ 0.6	—0.2	—0.3	—0.2	—0.5	+0.1	+0.3
0 2— 2	+3.4	— 1.3	0.0	—0.1	+0.2	+0.7	—0.1	—0.2
1 1— 2	—1.7	+ 0.7	+0.2	+0.5	—0.2	—0.5	0.0	+0.1

Arg= $\kappa\gamma'+i'g'+ig$			$C'\delta\frac{h'}{h_0'}$		$D'\frac{u'}{\cos i'}$		$E'\frac{u_1'}{\cos i'}$		$H'\frac{u}{\cos i}$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
κ	i'	i	″	″	″	″	″	″	″	″
−1	4	−2	−0.27	+0.41	−0.16	+0.07	−0.13	−0.08	+0.05	+0.09
0	3	−2	+0.5	−0.8	0.0	−0.1	+0.3	+0.1	0.0	0.0
1	2	−2	−0.2	+0.4	0.0	+0.1				
−1	5	−2	+0.001	+0.092	+0.866	−0.015	+0.111	−0.009	+0.421	−0.022
0	4	−2	−0.01	−0.17	−0.65	+0.01	−0.19	+0.01	−0.35	+0.03
1	3	−2	+0.02	+0.07	−0.03	0.00	+0.12	−0.01	+0.05	−0.01
−1	6	−2	+0.008	+0.012	+0.389	−0.214	+0.052	−0.025	+0.177	−0.102
0	5	−2	−0.01684	−0.02048	−0.30202	+0.16809	−0.07626	+0.04104	−0.15174	+0.08781
1	4	−2	+0.009	+0.006	−0.010	−0.008	+0.043	−0.028	+0.011	−0.012
−1	7	−2			+0.07	−0.11	+0.01	−0.01	+0.03	−0.05
0	6	−2			−0.06	+0.07	−0.01	+0.02	−0.03	+0.05
1	5	−2	+0.001	0.000	−0.004	−0.001	+0.006	−0.012	+0.001	−0.005
−1	2	−3			+1	+1				
0	1	−3			−1	0				
−1	3	−3	0.0	+0.1	+0.4	−0.1			+0.1	−0.1
0	2	−3	+0.1	−0.1	−0.2	+0.1	+0.1	−0.1		
−1	4	−3	−0.7	−1.0	+0.4	−0.3	+0.2	0.0	−0.2	+0.2
0	3	−3	+1.5	+2.0	−0.3	+0.2	−0.4	+0.3	+0.3	−0.2
1	2	−3	−0.7	−1.0	−0.2	+0.1	+0.2	−0.2		
−1	5	−3	−0.4	−0.2	−0.1	−0.3			0.0	+0.1
0	4	−3	+0.8	+0.3	+0.1	+0.2	−0.1	+0.1	0.0	−0.1
1	3	−3	−0.4	−0.1						
−1	6	−3	−0.1	0.0	+0.1	+0.6	0.0	−0.1	+0.1	+0.2
0	5	−3	+0.2	0.0	−0.1	−0.5	−0.1	−0.1	−0.1	−0.2
1	4	−3	−0.1	0.0			0.0	+0.1		
−1	7	−3	−0.01	+0.01	+0.24	+0.26	+0.02	+0.03	+0.11	+0.12
0	6	−3	+0.02	−0.02	−0.20	−0.23	−0.04	−0.05	−0.10	−0.10
−1	8	−3	−0.001	+0.003	+0.117	+0.049	+0.011	+0.006	+0.055	+0.018
0	7	−3	−0.001	−0.005	−0.100	−0.040	−0.018	−0.009	−0.047	−0.015
1	6	−3			+0.01	0.00	0.00	+0.01	+0.01	0.00
−1	9	−3			+0.03	−0.01			+0.01	−0.01
0	8	−3			−0.026	+0.003	−0.004	+0.001	−0.012	+0.001
1	7	−3			+0.001	+0.001	+0.002	−0.001	+0.002	−0.002
−1	3	−4			−1	+1				
0	2	−4			0	−1				
−1	5	−4	+0.5	−0.6	+0.3	+0.4	+0.1	+0.1	−0.3	−0.2
0	4	−4	−1	+1	0	−1				
1	3	−4	+1	−1						
−1	6	−4	+0.1	−0.3	+0.2	0.0			−0.2	−0.1
0	5	−4	0.0	+0.7	−0.1	0.0	0.0	−0.1	+0.2	+0.1
1	4	−4	0.0	−0.3						
−1	7	−4	0.0	−0.1	−0.4	+0.3			−0.1	+0.1
0	6	−4	+0.1	+0.2	+0.3	−0.3			+0.1	−0.1
1	5	−4	0.0	−0.1						

Arg= $\kappa\gamma'+i'g'+ig$			$C'\delta\frac{h'}{h_0'}$		$D'\frac{u'}{\cos i'}$		$E'\frac{u_1'}{\cos i'}$		$H'\frac{u}{\cos i}$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
κ	i'	i	″	″	″	″	″	″	″	″
—1	8	—4			—0.1	+0.2			—0.1	+0.1
0	7	—4			+0.2	—0.2			+0.1	—0.1
—1	9	—4			—0.03	+0.10			0.00	+0.05
0	8	—4			+0.1	—0.1				
—1	10	—4			+0.009	+0.026	+0.001	+0.001	+0.004	+0.012
0	9	—4			0.00	—0.02			—0.01	—0.01
—1	11	—4			+0.003	+0.003			+0.002	+0.002
0	10	—4	+0.0001	—0.0001	—0.0046	—0.0045	—0.0006	—0.0005	—0.0021	—0.0019
1	9	—4			+0.003	+0.001			+0.001	—0.001
0	5	—5	—1	0						
—1	8	—5	—0.1	0.0	—0.2	—0.1				
0	7	—5	—0.1	+0.1	+0.2	+0.1				
—1	9	—5			—0.2	0.0				
0	8	—5			+0.2	0.0				
—1	10	—5			—0.1	0.0				
0	9	—5			+0.1	0.0				

CHAPTER XV.

SECOND-ORDER PERTURBATIONS OF THE MEAN ANOMALY AND RADIUS-VECTOR OF SATURN, ARISING FROM THE MUTUAL ACTION OF THIS PLANET AND JUPITER.

When the eight terms of $\delta T'$, given in the two preceding chapters, are added the following expression is obtained:

$\text{Arg}=\varkappa\gamma'+i'g'+ig$			$\delta T'$			
			sin.		cos.	
$\varkappa$	i'	i	$''$	$''$	$''$	$''$
0	0	0			$+0.0000741$	$+\ 4.6131n't$
−1	1	0	$+0.7956$	$+\ 1.1181n't$	-0.145312	$+\ 16.6354n't$
−1	2	0	$+0.330$	$+284.440n't$	$+0.167$	$+366.21n't$
0	1	0	-1.1711	$-\ 59.600n't$	-0.3461	$-\ 35.11n't$
1	0	0	$+1.771981$	$-\ 39.0494n't$	-1.345783	$-\ 50.7489n't$
−1	3	0	$+0.217$	$+101.7n't$	-0.200	$+\ 32.0n't$
0	2	0	-2.214	$-\ 48.1n't$	$+3.626$	$-\ 11.4n't$
1	1	0	$+3.275$	$-\ 12.51n't$	-5.717	$-\ 6.50n't$
−1	4	0	$+0.023$	$+\ 17.0n't$	-0.342	$-\ 6.3n't$
0	3	0	$+0.258$	$-\ 10.3n't$	$+5.245$	$+\ 3.5n't$
1	2	0	-0.232	$-\ 2.1n't$	-8.613	$-\ 0.2n't$
−1	5	0	-0.97	$+\ 1.5n't$	$+0.18$	$-\ 1.9n't$
0	4	0	$+2.15$	$-\ 1.1n't$	-0.42	$+\ 1.4n't$
1	3	0	-1.013	$-\ 0.2n't$	$+0.414$	$+\ 0.1n't$
0	−4	−1	-0.22		-0.05	
−1	−2	−1	-0.36		-0.28	
0	−3	−1	$+0.10$		$+0.22$	
−1	−1	−1	$+7.219$	$-\ 0.2n't$	-1.638	$+\ 1.2n't$
0	−2	−1	-5.13	$-\ 0.6n't$	$+1.21$	$+\ 0.8n't$
1	−3	−1	$+0.75$	$+\ 0.5n't$	-0.18	$-\ 3.2n't$
−1	0	−1	$+4.705$	$-\ 17.4n't$	-3.944	$+\ 0.3n't$
0	−1	−1	-3.587	$+\ 3.5n't$	$+2.819$	$+\ 2.4n't$
1	−2	−1	$+0.426$	$-\ 3.3n't$	-0.355	$-\ 14.2n't$
−1	1	−1	$+2.456$	$+426.4n't$	-2.734	$+167.0n't$
0	0	−1	-1.786	$-861.9n't$	$+2.216$	$-317.1n't$
1	−1	−1	$+0.263$	$+\ 31.3n't$	-0.500	$+\ 1.6n't$
−1	2	−1	$+0.4999$	$+\ 6.72n't$	$+0.0123$	$+\ 14.49n't$
0	1	−1	-0.026	$-\ 64.75n't$	$+1.866$	$-\ 3.68n't$
1	0	−1	-0.396	$-\ 3.6n't$	-2.371	$-\ 16.7n't$
−1	3	−1	$+0.4650$	$-249.915n't$	$+0.1670$	$+324.289n't$
0	2	−1	-0.5363	$-\ 11.505n't$	-0.7127	$-\ 64.268n't$
1	1	−1	$+1.5274$	$-\ 36.26n't$	$+1.0156$	$+\ 41.42n't$

Arg $=\varkappa\gamma'+i'g'+ig$			$\delta T'$			
			sin.		cos.	
$\varkappa$	i'	i	″	″	″	″
—1	4	—1	+ 0.2921	— 4.63$n't$	+0.0602	+174.42$n't$
0	3	—1	+ 0.3459	— 16.529$n't$	—0.7243	— 96.204$n't$
1	2	—1	+ 2.7944	+ 8.632$n't$	+1.5498	— 4.266$n't$
—1	5	—1	— 5.313	+ 18.1$n't$	+0.928	+ 34.5$n't$
0	4	—1	+14.669	— 15.08$n't$	—2.763	— 22.92$n't$
1	3	—1	— 2.2885	+ 1.98$n't$	+0.3606	— 1.38$n't$
—1	6	—1	— 0.040	+ 5.8$n't$	—0.029	+ 3.5$n't$
0	5	—1	— 0.016	— 4.6$n't$	+0.014	— 2.5$n't$
1	4	—1	+ 0.452	+ 0.2$n't$	—0.173	— 0.2$n't$
—1	7	—1	— 0.004	+ 2.1$n't$	—0.021	+ 0.3$n't$
0	6	—1		— 0.7$n't$		+ 0.2$n't$
—1	—1	—2	+ 0.31		—0.54	
0	—2	—2	— 0.28		+0.28	
1	—3	—2	+ 0.03		—0.02	
—1	0	—2	+ 2.43	— 2.8$n't$	+5.09	— 0.6$n't$
0	—1	—2	— 1.95	0.0$n't$	—3.95	+ 2$n't$
1	—2	—2	+ 0.35	+ 1$n't$	+0.76	0$n't$
—1	1	—2	+ 4.14	+ 48.5$n't$	+3.09	+ 8.8$n't$
0	0	—2	— 3.35	— 80.5$n't$	—2.49	— 30.4$n't$
1	—1	—2	+ 0.59	+ 13.5$n't$	+0.37	— 2.3$n't$
—1	2	—2	+ 2.888	— 60.1$n't$	+1.510	+ 11.1$n't$
0	1	—2	— 2.172	+245.3$n't$	—1.067	—324.0$n't$
1	0	—2	+ 0.27	—260.9$n't$	—0.01	+350.3$n't$
—1	3	—2	— 0.043	+ 7.8$n't$	+0.663	— 45.3$n't$
0	2	—2	— 0.994	+ 4.6$n't$	—0.006	+ 20.0$n't$
1	1	—2	+ 1.335	— 10.3$n't$	—0.558	+ 9.5$n't$
—1	4	—2	— 0.2422	—597.94$n't$	—0.4767	—350.85$n't$
0	3	—2	+ 0.713	+389.2$n't$	+2.313	+210.7$n't$
1	2	—2	— 0.441	— 27.1$n't$	—0.350	— 7.2$n't$
—1	5	—2	+ 0.21223	—259.352$n't$	—0.10347	— 0.844$n't$
0	4	—2	+ 0.1153	+188.20$n't$	—0.3569	— 2.28$n't$
1	3	—2	— 0.4886	— 11.85$n't$	+3.0246	+ 8.61$n't$
—1	6	—2	+ 1.46667	— 51.707$n't$	+3.25760	+ 30.590$n't$
0	5	—2	— 0.0460094	+ 40.27460$n't$	+0.0201528	— 24.10504$n't$
1	4	—2	+ 1.24452	— 1.015$n't$	+2.89090	+ 3.976$n't$
—1	7	—2	+ 0.853	— 5.25$n't$	+0.482	+ 9.77$n't$
0	6	—2	— 0.2822	+ 4.24$n't$	—0.1456	— 8.02$n't$
1	5	—2	+ 0.00396	+ 0.318$n't$	+0.00202	+ 0.786$n't$
—1	8	—2	+ 0.178	0.0$n't$	—0.010	+ 1.7$n't$
0	7	—2	— 0.095	— 0.1$n't$	+0.007	— 1.4$n't$
1	6	—2	+ 0.006	+ 0.12$n't$	—0.002	+ 0.07$n't$
—1	9	—2	+ 0.022		—0.015	
0	8	—2	— 0.014		+0.009	
—1	0	—3	+ 0.60		+0.22	
0	—1	—3	— 0.38		—0.25	
1	—2	—3	+ 0.04		+0.03	

Arg=$\varkappa\gamma'+i'g'+ig$			δT′			
			sin.		cos.	
$\varkappa$	i'	i	″	″	″	″
−1	1	−3	−3.10	−5n′t	+2.48	0n′t
0	0	−3	+2.53	−6n′t	−2.07	−1n′t
1—	1	−3	−0.56	+1n′t	+0.44	+1n′t
−1	2	−3	−1.62	−17n′t	+3.70	+13n′t
0	1	−3	+1.37	+43n′t	−3.11	−55n′t
1	0	−3	−0.23	−41n′t	+0.60	+61n′t
−1	3	−3	−0.54	+130.8n′t	+2.85	+6.5n′t
0	2	−3	+0.25	−76.2n′t	−2.26	−9.7n′t
1	1	−3	+0.16	+11n′t	+0.27	+4n′t
−1	4	−3	−1.406	+25.9n′t	−1.289	−12.8n′t
0	3	−3	+2.26	−19.4n′t	+2.84	+17.7n′t
1	2	−3	−1.44	+0.7n′t	−2.09	−2.4n′t
−1	5	−3	−1.488	+180.5n′t	+0.191	−543.8n′t
0	4	−3	+0.760	−120.1n′t	−0.070	+398.0n′t
1	3	−3	+0.397	+13.2n′t	+0.200	−57.0n′t
−1	6	−3	+1.425	−49.6n′t	+0.454	−257.2n′t
0	5	−3	−4.482	+40.9n′t	+0.096	+200.4n′t
1	4	−3	+0.274	−12.3n′t	−0.039	−23.1n′t
−1	7	−3	+1.4673	−46.54n′t	−0.7986	−53.83n′t
0	6	−3	−11.896	+38.37n′t	+7.967	+44.11n′t
1	5	−3	+4.286	−6.8n′t	−2.956	−3.0n′t
−1	8	−3	−0.0273	−14.190n′t	+1.6244	−4.777n′t
0	7	−3	−0.8052	+12.060n′t	−0.3825	+4.095n′t
1	6	−3	+0.1834	−1.51n′t	−0.0722	+0.36n′t
−1	9	−3	+0.1600	−2.62n′t	+0.4221	+0.54n′t
0	8	−3	−0.1654	+2.305n′t	−0.2461	−0.487n′t
1	7	−3	+0.0271	−0.176n′t	−0.0011	+0.206n′t
−1	10	−3	+0.054	−0.3n′t	+0.049	+0.1n′t
0	9	−3	−0.037	+0.26n′t	−0.020	−0.28n′t
1	8	−3	+0.024	0.00n′t	−0.050	+0.03n′t
0	10	−3	−0.008		−0.002	
−1	1	−4	−0.10		+0.56	
0	0	−4	+0.14		−0.41	
−1	2	−4	−2.10	−1n′t	−1.61	+1n′t
0	1	−4	+1.85	+4n′t	+1.38	−5n′t
1	0	−4	−0.43	−4n′t	−0.35	+5n′t
−1	3	−4	−2.91	+8n′t	−0.60	−5n′t
0	2	−4	+2.53	−10n′t	+0.45	0n′t
1	1	−4	−0.51	+1n′t	−0.10	0n′t
−1	4	−4	−2.38	+15n′t	−0.02	+111n′t
0	3	−4	+2.08	−11n	+0.17	−77n′t
1	2	−4	−0.42	0n′t	−0.20	+13n′t
−1	5	−4	+0.05	+14.8n′t	−0.57	+14.5n′t
0	4	−4	−0.37	−20n′t	+0.78	−10n′t
1	3	−4	+0.44	+3n′t	−0.34	+2n′t
−1	6	−4	−0.28	+426.6n′t	−1.28	+47.7n′t

Arg=$\varkappa\gamma'+i'g'+ig$			$\delta T'$			
			sin.		cos.	
$\varkappa$	i'	i	″	″	″	″
0	5	−4	−0.22	$-333.5n't$	+1.24	$-29.0n't$
1	4	−4	+0.07	$+61.2n't$	−0.43	$+3.0n't$
−1	7	−4	+0.448	$+214.5n't$	−0.968	$-87.9n't$
0	6	−4	−1.713	$-175.3n't$	+0.387	$+73.8n't$
1	5	−4	+1.18	$+26.3n't$	+0.79	$-17.2n't$
−1	8	−4	−6.910	$+45.3n't$	−6.607	$-56.9n't$
0	7	−4	+4.714	$-38.1n't$	+4.027	$+48.5n't$
1	6	−4	−0.287	$+3.0n't$	−0.271	$-8.9n't$
−1	9	−4	−3.379	$+2.51n't$	−0.844	$-17.13n't$
0	8	−4	+2.326	$-2.0n't$	+0.605	$+14.9n't$
1	7	−4	−0.170	$-0.6n't$	+0.032	$-1.8n't$
−1	10	−4	−0.76701	$-1.304n't$	+0.22652	$-3.204n't$
0	9	−4	+0.5835	$+1.13n't$	−0.1641	$+2.89n't$
1	8	−4	−0.035	$-0.35n't$	+0.041	$-0.25n't$
−1	11	−4	−0.10300	$-0.491n't$	+0.11172	$-0.368n't$
0	10	−4	+0.080846	$+0.4441n't$	−0.085242	$+0.3430n't$
1	9	−4	−0.00264	$-0.074n't$	+0.01303	$-0.004n't$
0	11	−4	+0.0043	$+0.09n't$	−0.0189	$-0.01n't$
−1	2	−5	−0.46		0.00	
0	1	−5	+0.37		+0.06	
−1	3	−5	+0.72	$-1n't$	−1.59	$0n't$
0	2	−5	−0.60	$0n't$	+1.42	$+1n't$
1	1	−5	+0.15		−0.34	
−1	4	−5	−0.08	$+8n't$	−2.07	$+9n't$
0	3	−5	+0.06	$-3n't$	+1.79	$-6n't$
1	2	−5	−0.03	$-3n't$	−0.36	$0n't$
−1	5	−5	−0.41	$-83n't$	−1.58	$+28n't$
0	4	−5	+0.32	$+63n't$	+1.31	$-22n't$
1	3	−5	−0.06	$-12n't$	−0.19	$+3n't$
−1	6	−5	+0.18	$-5n't$	−0.26	$+12n't$
0	5	−5	−0.26	$+1n't$	+0.15	$-17n't$
1	4	−5	+0.13	$-1n't$	+0.04	$+5n't$
−1	7	−5	+0.82	$+29n't$	−0.53	$+301n't$
0	6	−5	−1.04	$-30n't$	+0.36	$-244n't$
1	5	−5	+0.28	$+7n't$	+0.07	$+53n't$
−1	8	−5	+0.39	$+106.7n't$	−0.79	$+157.0n\,t$
0	7	−5	−0.31	$-91.2n't$	+0.66	$-132.0n\,t$
1	6	−5	+0.06	$+20n't$	−0.08	$+22n\,t$
−1	9	−5	+4.335	$+59.9n't$	−7.244	$+30.5n't$
0	8	−5	−3.03	$-52.6n't$	+5.14	$-26.7n't$
1	7	−5	+0.40	$+9.9n't$	−0.70	$+2.2n't$
−1	10	−5	+0.183	$+17.7n't$	−3.731	$-0.9n't$
0	9	−5	−0.150	$-15.9n't$	+2.865	$+0.9n't$
1	8	−5	−0.060	$+2.2n't$	−0.348	$-1.0n't$
−1	11	−5	−0.451	$+3.2n't$	−0.890	$-2.3n't$
0	10	−5	+0.345	$-3n't$	+0.707	$+2n't$

$\text{Arg}=\varkappa\gamma'+i'g'+ig$			$\delta T'$	
			sin.	cos.
$\varkappa$	i'	i	″ ″	″ ″
1	9	−5	$-0.068 + 0.4n't$	$-0.062 - 0.5n't$
−1	12	−5	$-0.1725 + 0.34n't$	$-0.1117 - 0.70n't$
0	11	−5	$+0.127 - 0.31n't$	$+0.087 + 0.62n't$
1	10	−5	-0.013	$+0.001$
−1	13	−5	$-0.0391 - 0.022n't$	$-0.0007 - 0.139n't$
0	12	−5	$+0.0320 + 0.022n't$	$+0.0008 + 0.128n't$
1	11	−5	$-0.0030 - 0.01n't$	$+0.0017 - 0.01n't$
−1	3	−6	-0.09	-0.35
0	2	−6	0.0	$+0.2$
−1	4	−6	$+1.10 + 1n't$	$+0.24 + 1n't$
0	3	−6	$-1.00 - 2n't$	$-0.19 - 1n't$
1	2	−6	$+0.17$	$+0.04$
−1	5	−6	$+1.37 - 5n't$	$-0.32 + 10n't$
0	4	−6	$-1.23 + 6n't$	$+0.30 - 5n't$
1	3	−6	$+0.16 - 2n't$	$-0.04 + 3n't$
−1	6	−6	$+0.98 - 32n't$	$-0.52 - 53n't$
0	5	−6	$-0.86 + 29n't$	$+0.48 + 41n't$
1	4	−6	$+0.06 - 6n't$	$-0.06 - 9n't$
−1	7	−6	$+0.24 - 7n't$	$-0.08 \quad 0n't$
0	6	−6	$-0.16 + 11n't$	$-0.09 - 1n't$
1	5	−6	$- 3n't$	$+ 3n't$
−1	8	−6	$+0.54 - 195n't$	$+0.41 + 59n't$
0	7	−6	$-0.47 + 163n't$	$-0.49 - 58n't$
1	6	−6	$+0.07 - 42n't$	$+0.04 + 14n't$
−1	9	−6	$+0.60 - 99n't$	$-0.12 + 104n't$
0	8	−6	$-0.58 + 87n't$	$+0.05 - 93n't$
1	7	−6	$+0.11 - 15n't$	$+0.02 + 20n't$
−1	10	−6	$+6.15 - 15.4n't$	$+2.05 + 56.0n't$
0	9	−6	$-4.78 + 13n't$	$-1.55 - 49n't$
1	8	−6	$+0.86 \quad 0n't$	$+0.28 + 10n't$
−1	11	−6	$+3.38 + 4.1n't$	$-0.52 + 16.3n't$
0	10	−6	$-2.73 - 3.7n't$	$+0.41 - 14.7n't$
1	9	−6	$+0.42 + 1.4n't$	$-0.14 + 2.4n't$
−1	12	−6	$+0.829 + 3.1n't$	$-0.646 + 3.0n't$
0	11	−6	$-0.702 - 2.9n't$	$+0.528 - 2.7n't$
1	10	−6	$+0.06 + 0.6n't$	$-0.09 + 0.4n't$
−1	13	−6	$+0.089 + 0.8n't$	$-0.233 + 0.1n't$
0	12	−6	$-0.086 - 0.7n't$	$+0.196 - 0.1n't$
1	11	−6	-0.001	-0.029
−1	4	−7	$+0.2$	-0.1
0	3	−7	-0.1	$+0.1$
−1	5	−7	0.0	$+0.7$
0	4	−7	0.0	-0.7
−1	6	−7	$+0.37 - 12n't$	$+0.77 - 6n't$
0	5	−7	$-0.34 + 9n't$	$-0.69 + 7n't$

$\text{Arg}=\varkappa\gamma'+i'g'+ig$			$\delta T'$	
			sin.	cos.
$\varkappa$	i'	i	″ ″	″ ″
−1	7	−7	$+0.45+30n't$	$+0.46-29n't$
0	6	−7	$-0.40-23n't$	$-0.40+25n't$
1	5	−7	$+10n't$	$-7n't$
−1	8	−7	$+0.13-2n't$	$+0.11-6n't$
0	7	−7	$-0.12+1n't$	$-0.15+14n't$
1	6	−7	$+2n't$	$-8n't$
−1	9	−7	$-0.21-67n't$	$+0.43-115n't$
0	8	−7	$+0.14+60n't$	$-0.35+97n't$
1	7	−7	$-0.02-16n't$	$+0.07-24n't$
−1	10	−7	$+0.32-90n't$	$+0.33-52n't$
0	9	−7	$-0.24+78n't$	$-0.37+45n't$
1	8	−7	$+0.05-14n't$	$+0.05-8n't$
−1	11	−7	$-0.45-47n't$	$+4.62-4n't$
0	10	−7	$+0.34+42n't$	$-3.78+2n't$
1	9	−7	$-0.09-7n't$	$+0.75 \quad 0n't$
−1	12	−7	$+1.07-12n't$	$+2.68+6n't$
0	11	−7	$-0.87+11n't$	$-2.28-6n't$
1	10	−7	$+0.20-1n't$	$+0.37+1n't$
−1	13	−7	$+0.74-2.0n't$	$+0.65+2.3n't$
0	12	−7	$-0.67+1.3n't$	$-0.58-1.5n't$
1	11	−7	$+0.12$	$+0.08$
−1	6	−8	-0.3	$+0.1$
0	5	−8	$+0.3$	-0.1
−1	7	−8	$-0.4 \quad 0n't$	$+0.3-5n't$
0	6	−8	$+0.3 \quad 0n't$	$-0.3+9n't$
−1	8	−8	$-0.13+23n't$	$+0.30+12n't$
0	7	−8	$+0.1-20n't$	$-0.2-12n't$
1	6	−8	$+3n't$	$+1n't$
−1	9	−8	$-0.06+7n't$	$+0.02-5n't$
0	8	−8	$-0.01-8n't$	$-0.03+4n't$
1	7	−8	$+8n't$	$-1n't$
−1	10	−8	$-0.20+67n't$	$+0.04-56n't$
0	9	−8	$+0.18-56n't$	$-0.05+48n't$
1	8	−8	$+11n't$	$-9n't$
−1	11	−8	$-0.07+25n't$	$+0.37-70n't$
0	10	−8	$+0.19-21n't$	$-0.36+59n't$
1	9	−8	$-0.01+3n't$	$+0.04-11n't$
−1	12	−8	$-3.14-4n't$	$+0.42-32n't$
0	11	−8	$+2.62+4n't$	$-0.38+30n't$
1	10	−8	$-0.61-1n't$	$+0.07-4n't$
−1	13	−8	$--1.86-6n't$	$+1.22-9n't$
0	12	−8	$+1.57+5n't$	$-1.00+7n't$
1	11	−8	-0.30	$+0.21$
−1	7	−9	-0.1	-0.2
−1	8	−9	$-0.2+8n't$	$-0.2-3n't$
0	7	−9	$+0.2$	$+0.2$

Arg $=\varkappa\gamma'+i'g'+ig$			$\delta T'$	
			sin.	cos.
$\varkappa$	i'	i	$''$ $''$	$''$ $''$
-1	9	-9	$-4n't$	$+15n't$
0	8	-9	$+7n't$	$-19n't$
-1	10	-9	$+2n't$	$+14n't$
0	9	-9	$-1n't$	$-14n't$
-1	11	-9	$-0.10+35n't$	$-0.11+25n't$
0	10	-9	$+0.02-33n't$	$+0.11-23n't$
1	9	-9	$+5n't$	$+3n't$
-1	12	-9	$-0.34+50n't$	$-0.02+8n't$
0	11	-9	$+0.33-45n't$	$+0.07-7n't$
1	10	-9	$+6n't$	$+1n't$
-1	13	-9	$-0.75+21n't$	$-1.95-7n't$
0	12	-9	$+0.59-19n't$	$+1.67+6n't$
1	11	-9	-0.13	-0.36
-1	10	-10	$-14n't$	$+2n't$
0	9	-10	$+13n't$	$-2n't$
-1	11	-10	$-7n't$	$+3n't$
0	10	-10	$+6n't$	$-4n't$
-1	12	-10	$-9n't$	$+26n't$
0	11	-10	$+8n't$	$-20n't$
-1	13	-10	$-0.19+1n't$	$-0.21+23n't$
0	12	-10	$+0.1-2n't$	$+0.2-24n't$
-1	12	-11	$-8n't$	$-9n't$
0	11	-11	$+7n't$	$+7n't$
-1	13	-11	$-8n't$	$0n't$

In deriving $\delta W_0'$ from $\delta T'$ it has been deemed advantageous to equate the motion of the arguments in the terms involving $5g'-2g$ and $10g'-4g$. The formulæ for this have already been given (p. 275). In the two terms we treat by this process there has been found:

Arg.	log A	K	log B	log $\varkappa$	log μ
		$^\circ$ $'$ $''$			
$5g'-2g$	0.4798525	68 0 23.25	6.8614088n	7.1863943n	1.4973782
$10g'-4g$	9.0047121	313 9 56.7	4.7292	6.7424	1.1722875

By the aid of these quantities the terms of $\delta W_0'$ in question have been found as follows:

Arg.	$\delta W_0'$	
	cos.	sin.
	$''$ $''$	$''$ $''$
$5g'-2g$	$-0.92297-0.1261798n't$	$+4.38773-0.0767907n't$
$10g'-4g$	$-1.20442-0.0006600n't$	$-1.26809+0.0005106n't$

The expressions for $\overline{\delta W_0'}$ and $-\frac{1}{2}\left(\frac{\overline{d \,.\, \delta W_0'}}{d\gamma'}\right)$ follow:

Arg$=i'g'+ig$		$\overline{\delta W_0'}$			
		cos.		sin.	
i'	i	$''$	$''$	$''$	$''$
0	0	-0.7984	$-4.7706n't$		
		$+k_0$	$+2.3065n'^2t^2$		
1	0	$+0.9868$	$-13541.60n't$	-0.2596	$+17867.41n't$
		$+k_1$	$-25.3745n'^2t^2$	$+k_2$	$-19.5247n'^2t^2$
2	0	-2.2418	$-374.62n't$	-3.9742	$+494.75n't$
		$+[8.4472]k_1$	$-0.7106n'^2t^2$	$+[8.4473]k_2$	$-0.5469n'^2t^2$
3	0	-0.0621	$-15.26n't$	-2.8029	$+20.16n't$
		$+[7.0705]k_1$	$-0.0298n'^2t^2$	$+[7.0707]k_2$	$-0.0230n'^2t^2$
4	0	-0.0064	$-0.71n't$	-0.0583	$+1.05n't$
		$+[5.77]k_1$	$-0.0015n'^2t^2$	$+[5.77]k_2$	$-0.0011n'^2t^2$
-4	-1	-0.0336		$+0.0102$	
-3	-1	-0.0018	$+0.01n't$	$+0.0376$	$-0.02n't$
-2	-1	$+1.1202$	$+0.06n't$	$+0.2798$	$-0.07n't$
-1	-1	$+1.0070$	$+1.32n't$	$+0.9110$	$-0.81n't$
0	-1	$+1.0456$	$-49.69n't$	$+1.1018$	$+14.64n't$
1	-1	$+0.8434$	$-17.39n't$	-0.3182	$-3.18n't$
2	-1	-0.8913	$+435.49n't$	$+1.0414$	$+736.12n't$
3	-1	$+4.9768$	$+52.02n't$	-4.5791	$-62.76n't$
4	-1	-2.9691	$-0.66n't$	-0.8452	$-3.82n't$
5	-1	-0.1495	$-0.04n't$	-0.1011	$-0.18n't$
6	-1	-0.0022	$-0.26n't$	-0.0070	$+0.13n't$
-3	-2	$+0.002$	$0.00n't$	$+0.002$	$0.00n't$
-2	-2	$+0.0305$	$-0.01n't$	$+0.0232$	$0.00n't$
-1	-2	$+0.2429$	$-0.10n't$	-0.4942	$+0.06n't$
0	-2	$+0.4969$	$-2.29n't$	-0.3576	$+4.22n't$
1	-2	$+0.4814$	$-11.03n't$	-0.2504	$+8.50n't$
2	-2	-0.0352	$-6.72n't$	-0.2132	$+22.06n't$
3	-2	-0.2412	$-212.09n't$	-1.3520	$+257.39n't$
4	-2	-6.6128	$+7966.41n't$	-27.4774	$-26.42n't$
5	-2	-1.07047	$-1212.911n't$	$+4.50831$	$-742.402n't$
6	-2	-0.1609	$-11.09n't$	$+0.0999$	$+20.41n't$
7	-2	-0.0179	$-0.34n't$	-0.0028	$+0.60n't$
8	-2	-0.0011	$-0.04n't$	-0.0009	$+0.05n't$
-2	-3	$+0.001$		-0.001	
-1	-3	$+0.027$		-0.015	
0	-3	-0.2158	$+0.03n't$	-0.1786	$-0.06n't$
1	-3	-0.1215	$-1.12n't$	-0.2962	$-2.08n't$
2	-3	-0.0638	$+17.27n't$	-0.2591	$-0.17n't$
3	-3	-0.1792	$+5.33n't$	$+0.1154$	$+6.58n't$
4	-3	-0.2802	$+40.74n't$	-0.0963	$+124.48n't$
5	-3	-0.6832	$-23.88n't$	-0.2908	$+105.96n't$
6	-3	-3.2151	$-79.14n't$	-2.4514	$+90.41n't$

Arg=$i'g'+ig$	$\overline{\delta W_0'}$	
	cos.	sin.
i' i	″ ″	″ ″
7— 3	− 1.5666 +51.53$n't$	+3.8956 −17.98$n't$
8— 3	+ 0.2626 − 2.92$n't$	−0.1688 − 1.01$n't$
9— 3	− 0.0390 − 0.06$n't$	−0.0843 − 0.10$n't$
10— 3	+ 0.0020	−0.0033
− 1— 4	0.000	−0.002
0— 4	− 0.004	−0.016
1— 4	− 0.113 − 0.1$n't$	+0.086 0.0$n't$
2— 4	− 0.1704 + 0.10$n't$	+0.0423 + 0.19$n't$
3— 4	− 0.1553 + 1.15$n't$	+0.0081 − 9.33$n't$
4— 4	+ 0.0080 + 3.20$n't$	+0.0435 − 1.88$n't$
5— 4	− 0.1018 +53.25$n't$	+0.1627 − 5.89$n't$
6— 4	− 0.1474 +34.85$n't$	+0.1720 +15.52$n't$
7— 4	− 2.3082 +11.45$n't$	+2.1392 +15.71$n't$
8— 4	− 2.1577 + 2.04$n't$	+0.6759 + 9.96$n't$
9— 4	+12.0354 +20.58$n't$	+3.5243 −51.00$n't$
10— 4	− 1.11130− 6.224$n't$	−1.17797+ 4.770$n't$
11— 4	− 0.0041 − 0.08$n't$	−0.0181 − 0.01$n't$
1— 5	− 0.010	0.000
2— 5	+ 0.031 0.0$n't$	+0.070 − 0.1$n't$
3— 5	− 0.008 0.0$n't$	+0.098 − 0.5$n't$
4— 5	− 0.022 − 5.0$n't$	+0.079 − 1.6$n't$
5— 5	+ 0.0127 − 0.63$n't$	+0.0195 − 1.74$n't$
6— 5	+ 0.0319 + 2.32$n't$	+0.0397 −25.72$n't$
7— 5	+ 0.0774 +10.97$n't$	+0.1305 −15.06$n't$
8— 5	+ 0.6569 + 7.75$n't$	+1.1297 − 3.39$n't$
9— 5	+ 0.0109 + 3.28$n't$	+0.8058 + 0.37$n't$
10— 5	− 0.2064 + 1.17$n't$	+0.3637 + 0.99$n't$
11— 5	− 0.3286 + 0.60$n't$	+0.2073 + 1.23$n't$
12— 5	+ 0.1417 + 0.08$n't$	−0.0043 − 0.47$n't$
2— 6	− 0.004	+0.013
3— 6	+ 0.034 − 0.2$n't$	−0.008 + 0.1$n't$
4— 6	+ 0.041 − 0.3$n't$	+0.009 − 0.5$n't$
5— 6	+ 0.028 − 1.2$n't$	+0.015 + 2.7$n't$
6— 6	+ 0.014 − 0.2$n't$	+0.018 − 0.4$n't$
7— 6	+ 0.0311 −12.81$n't$	−0.0020 − 3.29$n't$
8— 6	+ 0.0681 − 6.29$n't$	−0.0011 − 7.04$n't$
9— 6	+ 0.5948 − 0.96$n't$	−0.1924 − 4.76$n't$
10— 6	+ 0.3925 + 0.56$n't$	+0.0785 − 1.66$n't$
11— 6	+ 0.1216 + 0.47$n't$	+0.1097 − 0.43$n't$
12— 6	+ 0.0172 + 0.17$n't$	+0.0630 − 0.02$n't$
3— 7	+ 0.008	−0.002
4— 7	+ 0.001	−0.008
5— 7	+ 0.006 − 0.3$n't$	−0.013 0.0$n't$
6— 7	+ 0.008 + 1.7$n't$	−0.009 + 1.2$n't$
7— 7	+ 0.001 − 0.1$n't$	+0.001 + 0.4$n't$

$\mathrm{Arg}=i'g'+ig$	$\overline{\delta W_0'}$	
	cos.	sin.
i' i	$''$ $''$	$''$ $''$
8— 7	$-0.011-3.4n't$	$-0.022+5.9n't$
9— 7	$+0.017-4.6n't$	$-0.026+2.5n't$
10— 7	$-0.029-2.7n't$	$-0.315+0.3n't$
11— 7	$+0.095-0.7n't$	$-0.198-0.4n't$
12— 7	$+0.065-0.3n't$	$-0.053-0.2n't$
5— 8	-0.003	-0.001
6— 8	$-0.009+0.1n't$	$-0.002-0.2n't$
7— 8	$-0.003+0.5n't$	$-0.010-0.2n't$
8— 8	$-0.007+0.7n't$	$+0.001+0.5n't$
9— 8	$-0.003+2.6n't$	$0.000+2.3n't$
10— 8	$-0.001+1.0n't$	$-0.011+3.0n't$
11— 8	$-0.174-0.2n't$	$-0.022+1.1n't$
12— 8	$-0.107-0.3n't$	$-0.075+0.4n't$
6— 9	-0.006	$+0.013$
7— 9	$-0.001+0.5n't$	$+0.001+0.2n't$
8— 9	$+0.2n't$	$+0.2n't$
9— 9	$+0.1n't$	$-0.1n't$
10— 9	$-0.007+0.8n't$	$+0.001-0.5n't$
11— 9	$-0.006+1.4n't$	$+0.002-0.3n't$
12— 9	$-0.034+0.4n't$	$+0.080+0.2n't$
9—10	$-0.1n't$	$0.0n't$
10—10	$-0.1n't$	$+0.1n't$
11—10	$-0.1n't$	$-0.6n't$
12—10	$-0.008-0.1n'$	$+0.003\quad 0.0n't$
11—11	$-0.1n't$	$+0.2n't$
12—11	$-0.5n't$	$0.0n't$

$\mathrm{Arg}=i'g'+ig$	$-\frac{1}{2}\left(\overline{\frac{d\,.\,\delta W_0'}{d\gamma'}}\right)$	
	sin.	cos.
i' i	$''$ $''$	$''$ $''$
0 0		$-\ 0.070187+\ 13.4643n't$
1 0	$+\ 0.0570\quad -6657.11n't$	$+\ 0.0397\quad -8767.25n't$
	$+\ \frac{1}{2}k_1\quad -\ 12.6872n'^2t^2$	$-\ \frac{1}{2}k_2\quad +\ 9.7623n'^2t^2$
2 0	$-\ 1.6034\ -\ 353.70n't$	$+\ 2.8239\ -\ 487.77n't$
	$+[8.4472]k_1-\ 0.7106n'^2t^2$	$-[8.4473]k_2+\ 0.5469n'^2t^2$
3 0	$-\ 0.0362\ -\ 20.45n't$	$+\ 2.2717\ -\ 31.83n't$
	$+[7.2466]k_1-\ 0.0447n'^2t^2$	$-[7.2468]k_2+\ 0.0345n'^2t^2$
4 0	$+\ 0.0692\ -\ 1.35n't$	$+\ 0.0797\ -\ 2.30n't$
	$+[6.07]k_1\ -\ 0.0030n'^2t^2$	$-[6.07]k_2\ +\ 0.0023n'^2t^2$

Arg=$i'g'+ig$		$-\frac{1}{2}\left(\frac{d\,.\,\delta W_0'}{d\gamma'}\right)$			
		sin.		cos.	
i'	i	″	″	″	″
−4	−1	-0.0017		$+0.0027$	
−3	−1	-0.0215	$-0.02n't$	$+0.0474$	$-0.02n't$
−2	−1	-1.0238	$-0.14n't$	$+0.2665$	$-0.66n't$
−1	−1	-0.9422	$-4.94n't$	$+0.8056$	$-4.86n't$
0	−1	-0.8188	$-140.51n't$	$+0.8569$	$-55.88n't$
1	−1	-0.5757	$-20.98n't$	-0.4865	$-0.55n't$
2	−1	$+0.9016$	$-254.16n't$	$+0.4326$	$+330.83n't$
3	−1	$+2.9518$	$+6.92n't$	$+1.6489$	$+54.17n't$
4	−1	$+1.3220$	$+2.19n't$	-0.0737	$+8.01n't$
5	−1	-0.0205	$+0.68n't$	$+0.0390$	$+0.61n't$
6	−1	-0.0009	$+0.23n't$	-0.0003	$+0.03n't$
−3	−2	-0.002		$+0.001$	
−2	−2	-0.0396	$0.00n't$	$+0.0138$	$0.00n't$
−1	−2	-0.2504	$+0.05n't$	-0.4805	$-0.01n't$
0	−2	-0.4984	$-4.42n't$	-0.4322	$-0.98n't$
1	−2	-0.4541	$-15.99n't$	-0.2651	$+34.67n't$
2	−2	$+0.2013$	$-1.16n't$	-0.2731	$+24.83n't$
3	−2	$+0.2620$	$+86.66n't$	-0.5827	$+179.81n't$
4	−2	$+3.1325$	$-3892.97n't$	-12.3561	$-9.69n't$
5	−2	$+1.35685$	$-25.805n't$	$+3.11867$	$+17.113n't$
6	−2	$+0.2229$	$-6.10n't$	$+0.1602$	$-9.33n't$
7	−2	$+0.0276$	$-0.33n't$	$+0.0021$	$-0.41n't$
8	−2	$+0.0026$	$-0.05n't$	-0.0017	$-0.06n't$
−2	−3	-0.001		-0.002	
−1	−3	-0.024		-0.025	
0	−3	$+0.2158$	$-0.29n't$	-0.1863	$-0.01n't$
1	−3	$+0.1353$	$-2.02n't$	-0.3148	$+2.87n't$
2	−3	$+0.0845$	$-14.33n't$	-0.2861	$+0.30n't$
3	−3	$+0.0875$	$-5.64n't$	-0.0060	$+8.25n't$
4	−3	$+0.3122$	$-34.21n't$	-0.0312	$+109.73n't$
5	−3	-0.5342	$+18.25n't$	-0.1081	$+88.29n't$
6	−3	-0.7411	$+49.51n't$	$+0.3556$	$+58.74n't$
7	−3	$+0.0912$	$-13.56n't$	$+1.4233$	$-4.25n't$
8	−3	$+0.0889$	$-1.08n't$	$+0.1319$	$+0.41n't$
9	−3	-0.0091	$-0.07n't$	$+0.0547$	$0.00n't$
10	−3	-0.0011		$+0.0025$	
−1	−4	0.000		-0.002	
0	−4	$+0.013$		-0.025	
1	−4	$+0.122$	$-0.1n't$	$+0.085$	$+0.2n't$
2	−4	$+0.1914$	$-0.63n't$	$+0.0370$	$-0.15n't$
3	−4	$+0.1722$	$-1.53n't$	-0.0074	$-8.63n't$
4	−4	$+0.0268$	$-4.45n't$	$+0.0424$	$-1.57n't$
5	−4	$+0.0453$	$-51.14n't$	$+0.1417$	$-4.91n't$
6	−4	$+0.1493$	$-34.27n't$	$+0.3419$	$+14.10n't$
7	−4	$+1.8380$	$-11.29n't$	$+1.7084$	$+13.90n't$

$\text{Arg}=i'g'+ig$	$-\frac{1}{2}\left(\overline{\frac{d\,.\,\delta W_0'}{d\gamma'}}\right)$	
	sin.	cos.
$i'\quad i$	$'' \qquad ''$	$'' \qquad ''$
8— 4	$+1.4611 - 1.96n't$	$+0.5513 + 7.44n't$
9— 4	$-5.7269 - 9.88n't$	$+1.6971 - 24.11n't$
10— 4	$-0.05028 - 0.274n't$	$+0.05993 - 0.179n't$
11— 4	-0.0001	$+0.0004$
1— 5	$+0.020$	$+0.005$
2— 5	-0.031	$+0.076$
3— 5	$+0.006 - 0.3n't$	$+0.111 - 0.6n't$
4— 5	$+0.024 + 5.0n't$	$+0.096 - 1.8n't$
5— 5	$-0.0110 + 0.10n't$	$+0.0248 - 2.21n't$
6— 5	$-0.0615 - 2.90n't$	$+0.0628 - 25.22n't$
7— 5	$-0.0723 - 10.99n't$	$+0.1459 - 16.11n't$
8— 5	$-0.5985 - 7.97n't$	$+1.0395 - 4.15n't$
9— 5	$-0.0329 - 3.42n't$	$+0.7472 + 0.13n't$
10— 5	$+0.1597 - 1.08n't$	$+0.3103 + 0.80n't$
11— 5	$+0.2020 - 0.41n't$	$+0.1337 + 0.83n't$
12— 5	$-0.0347 - 0.02n't$	$0.0000 - 0.12n't$
2— 6	$+0.001$	$+0.014$
3— 6	-0.047	-0.008
4— 6	$-0.065 + 0.3n't$	$+0.016 - 0.2n't$
5— 6	$-0.052 + 1.5n't$	$+0.026 + 2.6n't$
6— 6	$-0.017 + 1.4n't$	$+0.003 \quad 0.0n't$
7— 6	$-0.0403 + 12.26n't$	$-0.0277 - 3.99n't$
8— 6	$-0.0805 + 7.40n't$	$-0.0001 - 7.83n't$
9— 6	$-0.5898 + 1.49n't$	$-0.1847 - 5.04n't$
10— 6	$-0.4024 - 0.54n't$	$+0.0623 - 1.88n't$
11— 6	$-0.1359 - 0.47n't$	$+0.1049 - 0.46n't$
12— 6	$-0.0231 - 0.21n't$	$+0.0571 - 0.03n't$
3— 7	-0.007	$+0.002$
4— 7	-0.001	-0.031
5— 7	$-0.017 + 0.4n't$	$-0.035 + 0.4n't$
6— 7	$-0.022 - 1.0n't$	$-0.022 + 1.1n't$
7— 7	$-0.006 + 0.4n't$	$-0.007 + 0.4n't$
8— 7	$+0.011 + 3.5n't$	$-0.024 + 5.8n't$
9— 7	$-0.017 + 5.6n't$	$-0.039 + 3.1n't$
10— 7	$+0.024 + 3.4n't$	$-0.331 + 0.3n't$
11— 7	$-0.091 + 1.1n't$	$-0.225 - 0.5n't$
12— 7	$-0.074 + 0.2n't$	$-0.067 - 0.3n't$
5— 8	$+0.012$	-0.005
6— 8	$+0.015 - 0.1n't$	$-0.013 + 0.2n't$
7— 8	$+0.005 - 0.9n't$	$-0.013 - 0.5n't$
8— 8	$+0.003 - 0.5n't$	$-0.001 + 0.4n't$
9— 8	$+0.010 - 3.0n't$	$-0.003 + 2.6n't$
10— 8	$+0.015 - 1.3n't$	$-0.020 + 3.5n't$
11— 8	$+0.176 + 0.1n't$	$-0.028 + 1.8n't$
12— 8	$+0.116 + 0.4n't$	$-0.077 + 0.6n't$

Arg$=i'g'+ig$	$-\frac{1}{2}\left(\frac{d\,.\,\delta W_0'}{d\gamma'}\right)$	
	sin.	cos.
i' i	$''$ $''$	$''$ $''$
6— 9	$+0.003$	$+0.006$
7— 9	$+0.007-0.3n't$	$+0.007+0.1n't$
8— 9	$+0.1n't$	$-0.6n't$
9— 9	$-0.1n't$	$-0.6n't$
10— 9	$+0.004-1.3n't$	$+0.005-1.0n't$
11— 9	$+0.018-2.3n't$	$+0.007-0.4n't$
12— 9	$+0.034-1.1n't$	$+0.088+0.4n't$
9—10	$+0.4n't$	$-0.1n't$
10—10	$+0.3n't$	$-0.1n't$
11—10	$+0.3n't$	$-1.0n't$
12—10	$+0.008\quad 0.0n't$	$+0.009-1.0n't$
11—11	$+0.3n't$	$+0.3n't$
12—11	$+0.3n't$	$0.0n't$

In obtaining the products $\left(\frac{\overline{dW_0'}}{d\gamma'}\right)n'\delta z'$ and ν'^2 the secular terms of the second order have been included in the values of the factors, so that, as far as these terms are concerned, we have

$$
\begin{aligned}
2\nu' = {} & \overset{''}{0.1851}n't \\
& + 6.619n't \cos g' - 10.605n't \sin g' \\
& + 0.184n't \cos 2g' - 0.297n't \sin 2g' \\
& + 0.008n't \cos 3g' - 0.013n't \sin 3g'
\end{aligned}
$$

$$
\begin{aligned}
\left(\frac{\overline{dW_0'}}{d\gamma'}\right) = {} & \qquad\qquad\qquad\qquad\qquad - \overset{''}{0.1501} \\
& + [\ 15.2723 + 6.6123n't] \sin g' + [-20.2638 + 10.5926n't] \cos g' \\
& + [\ 0.8186 + 0.3703n't] \sin 2g' + [-\ 3.4692 + 0.5934n't] \cos 2g' \\
& + [-\ 0.0466 + 0.0233n't] \sin 3g' + [-\ 0.3406 + 0.0374n't] \cos 3g' \\
& + [-\ 0.0156 + 0.0015n't] \sin 4g' + [-\ 0.0264 + 0.0025n't] \cos 4g' \\
& + [-\ 0.0018 + 0.0001n't] \sin 5g' + [-\ 0.0012 + 0.0002n't] \cos 5g'
\end{aligned}
$$

The expressions of the two products to be employed in determining $n'\delta^2 z'$ follow:

Arg$=i'g'+ig$	$\left(\frac{\overline{dW_0'}}{d\gamma'}\right)n'\delta z'$	
	cos.	sin.
i' i	$''$ $''$	$''$ $''$
0 0	$-1.1380+2.7711n't$ $-3.7827n'^2t^2$	
1 0	$-3.2193+0.97n't$ $-0.2650n'^2t^2$	$-1.3312-0.79n't$ $0.0000n'^2t^2$

Arg=i'g'+ig		$\left(\frac{dW_0'}{d\gamma'}\right)n'\delta z'$	
		cos.	sin.
i'	i	$'' \quad ''$	$'' \quad ''$
2	0	$+0.2824+7.76n't$ $-1.6748n'^2t^2$	$+0.8468-0.72n't$ $-3.3956n'^2t^2$
3	0	$+0.0407+1.32n't$ $-0.1170n'^2t^2$	$+1.1629+0.36n't$ $-0.2378n'^2t^2$
4	0	$+0.1174+0.11n't$ $-0.0079n'^2t^2$	$+0.0663+0.08n't$ $-0.0160n'^2t^2$
5	0	$-0.0016+0.01n't$ $-0.0004n'^2t^2$	$+0.0031 \quad 0.00n't$ $-0.0010n'^2t^2$
−3	−1	-0.0020	-0.0180
−2	−1	$-0.3850+0.30n't$	$-0.0960-0.44n't$
−1	−1	$-0.2592+8.35n't$	$-0.2343-6.93n't$
0	−1	$-0.2289+145.09n't$	$-0.2146-67.55n't$
1	−1	$-0.0347-80.01n't$	$-0.0075-133.76n't$
2	−1	$+0.0837+121.31n't$	$+0.9138+93.09n't$
3	−1	$+0.5664+85.64n't$	$+0.4607-137.93n't$
4	−1	$+3.3627+16.98n't$	$+0.7575-12.05n't$
5	−1	$+0.0862+1.13n't$	$+0.0728-0.88n't$
6	−1	$+0.0014+0.06n't$	$+0.0070-0.03n't$
−2	−2	-0.0118	-0.0052
−1	−2	$-0.0698-0.04n't$	$+0.1449-0.26n't$
0	−2	$-0.1122+3.99n't$	$+0.0852-5.09n't$
1	−2	$-0.1146-10.07n't$	$+0.0575-64.56n't$
2	−2	$-0.0302-3.35n't$	$+0.0401-20.72n't$
3	−2	$-0.0387-290.00n't$	$+0.5806+212.61n't$
4	−2	$+0.1589-754.93n't$	$+0.1844+112.92n't$
5	−2	$+0.04490-348.440n't$	$-0.00059-162.108n't$
6	−2	$+0.1656-468.79n't$	$-0.0409-658.50n't$
7	−2	$+0.0223-26.84n't$	$+0.0039-36.41n't$
8	−2	$+0.0019-1.71n't$	$+0.0012-2.29n't$
−1	−3	-0.007	$+0.006$
0	−3	$+0.0548+0.25n't$	$+0.0438-0.15n't$
1	−3	$+0.0265-0.68n't$	$+0.0587-0.91n't$
2	−3	$+0.0152-20.81n't$	$+0.0565-1.43n't$
3	−3	$-0.0194-9.22n't$	$+0.0810-6.97n't$
4	−3	$+0.2010-10.09n't$	$+0.0822-23.96n't$
5	−3	$+1.0324+1.16n't$	$+0.1191-15.57n't$
6	−3	$+2.7298+2.02n't$	$+1.0753-2.67n't$
7	−3	$+0.3574+3.37n't$	$-0.7091-1.62n't$
8	−3	$+0.0653-1.82n't$	$-0.0450-0.23n't$
9	−3	$+0.0070-0.16n't$	$-0.0078-0.07n't$
0	−4	$+0.002$	$+0.005$
1	−4	$+0.026$	-0.020
2	−4	$+0.0312-0.65n't$	$-0.0062-0.05n't$
3	−4	$+0.0260-2.42n't$	$-0.0007+7.92n't$

$\text{Arg}=i'g'+ig$	$\left(\frac{\overline{dW_0'}}{d\gamma'}\right)n'\delta z'$	
	cos.	sin.
i' i	$''$ $''$	$''$ $''$
4 — 4	$+0.0254\ -3.56n't$	$+0.0180\ +2.82n't$
5 — 4	$+0.0773\ -9.87n't$	$-0.0414\ +1.70n't$
6 — 4	$+0.2600\ -5.47n't$	$-0.2947\ -1.63n't$
7 — 4	$+0.9499\ -1.56n't$	$-0.9325\ -1.51n't$
8 — 4	$-0.4847\ +0.13n't$	$+0.0015\ +0.45n't$
9 — 4	$-3.7810\ -0.18n't$	$-1.0946\ +0.79n't$
10 — 4	$-0.01472-0.795n't$	$-0.01383+0.744n't$
11 — 4	$-0.0054\ -1.06n't$	$+0.0056\ +0.28n't$
2 — 5	-0.006	-0.014
3 — 5	$-0.001\ -0.2n't$	$-0.013\ +0.3n't$
4 — 5	$-0.001\ +3.1n't$	$-0.005\ +1.7n't$
5 — 5	$+0.0008\ +0.76n't$	$-0.0038\ +1.88n't$
6 — 5	$-0.0117\ -0.12n't$	$-0.0200\ +4.48n't$
7 — 5	$-0.0891\ -1.30n't$	$-0.0735\ +2.39n't$
8 — 5	$-0.2345\ -0.90n't$	$-0.3985\ +0.59n't$
9 — 5	$-0.0131\ -0.35n't$	$-0.2439\ +0.03n't$
10 — 5	$+0.0486\ -0.10n't$	$-0.0976\ -0.06n't$
11 — 5	$+0.0368\ -0.01n't$	$-0.0265\ -0.01n't$
12 — 5	$-0.0084\ \ 0.00n't$	$-0.0006\ -0.02n't$
3 — 6	-0.007	$+0.002$
4 — 6	$+0.1n't$	$+0.2n't$
5 — 6	$+1.1n't$	$-1.1n't$
6 — 6	$+1.1n't$	$-0.2n't$
7 — 6	$-0.0098\ +2.04n't$	$+0.0057\ +0.51n't$
8 — 6	$-0.0387\ +1.06n't$	$+0.0183\ +0.93n't$
9 — 6	$-0.1750\ +0.21n't$	$+0.0589\ +0.54n't$
10 — 6	$-0.1034\ -0.03n't$	$-0.0154\ +0.17n't$
11 — 6	$-0.0309\ -0.02n't$	$-0.0233\ +0.04n't$
12 — 6	$-0.0046\ -0.02n't$	$-0.0107\ +0.01n't$
5 — 7	$+0.1n't$	$-0.1n't$
6 — 7	$-0.4n't$	$-0.6n't$
7 — 7	$0.0n't$	$-0.5n't$
8 — 7	$+0.001\ +0.5n't$	$+0.005\ -0.9n't$
9 — 7	$+0.001\ +0.6n't$	$+0.014\ -0.4n't$
10 — 7	$+0.008\ +0.3n't$	$+0.082\ -0.1n't$
11 — 7	$-0.017\ +0.1n't$	$+0.048\ \ 0.0n't$
12 — 7	-0.014	$+0.012$
6 — 8	$0.0n't$	$-0.1n't$
7 — 8	$-0.3n't$	$+0.1n't$
8 — 8	$-0.3n't$	$-0.2n't$
9 — 8	$-0.5n't$	$-0.4n't$
10 — 8	$+0.006\ -0.1n't$	$+0.002\ -0.4n't$
11 — 8	$+0.037\ \ 0.0n't$	$+0.004\ -0.1n't$
12 — 8	$+0.020\ \ 0.0n't$	$+0.013\ -0.1n't$

Arg=$i'g'+ig$	$\left(\dfrac{d\overline{W_0'}}{d\gamma'}\right)n'\delta z'$	
	cos.	sin
i' i	″ ″	″ ″
8— 9	$0.0n't$	$+0.1n't$
9— 9	$0.0n't$	$+0.2n't$
10— 9	$-0.3n't$	$+0.1n't$
11— 9	$-0.2n't$	$0.0n't$
12— 9	$+0.006-0.1n't$	-0.014 $0.0n't$
10—10	$0.0n't$	$+0.1n't$
12—10	$0.0n't$	$+0.1n't$

Arg=$i'g'+ig$	ν''	
	cos.	sin.
i' i	″ ″	″ ″
0 0	$+0.4048 - 0.6096n't$ $+ 0.9461n'^2t^2$	
1 0	$-0.0981 + 27.53n't$ $+ 0.0826n'^2t^2$	$-0.1633 - 44.23n't$ $- 0.0476n'^2t^2$
2 0	$-0.0268 + 3.22n't$ $- 0.4120n'^2t^2$	$-0.0284 - 0.85n't$ $- 0.8502n'^2t^2$
3 0	$-0.0047 + 0.32n't$ $- 0.0232n'^2t^2$	$+0.0076 + 0.05n't$ $- 0.0477n'^2t^2$
4 0	$0.0000 + 0.02n't$ $- 0.0013n'^2t^2$	$0.0000 + 0.01n't$ $- 0.0027n'^2t^2$
—1— 1	$+0.0079 + 2.09n't$	$+0.0075 - 1.36n't$
0— 1	$+0.0139 + 50.14n't$	$+0.0060 - 20.91n't$
1— 1	$+0.0523 - 17.66n't$	$-0.1469 - 32.69n't$
2— 1	$-0.1163 - 41.53n't$	$-0.2007 - 33.21n't$
3— 1	$-0.2684 - 21.39n't$	$+0.1101 + 27.92n't$
4— 1	$+0.0377 + 2.67n't$	$+0.0124 - 0.61n't$
5— 1	$+ 0.15n't$	$- 0.05n't$
0— 2	$+0.0028 + 0.59n't$	$-0.0019 - 0.75n't$
1— 2	$+0.0916 - 1.44n't$	$-0.0056 - 9.31n't$
2— 2	$-0.0994 - 2.30n't$	$-0.0401 - 4.89n't$
3— 2	$-0.0274 - 82.49n't$	$+0.0868 + 65.78n't$
4— 2	$+0.0724 + 9.85n't$	$+0.2583 + 3.87n't$
5— 2	$+0.00550 + 87.527n't$	$-0.04049 + 40.333n't$
6— 2	$-0.0004 - 5.41n't$	$+0.0012 - 12.35n't$
7— 2	$- 0.26n't$	$- 0.21n't$
8— 2	$- 0.01n't$	$- 0.02n't$
1— 3	$0.00n't$	$- 0.07n't$
2— 3	$- 2.11n't$	$- 0.15n't$
3— 3	$-0.0186 - 1.35n't$	$+0.0271 - 0.67n't$
4— 3	$+0.0037 + 0.34n't$	$+0.0189 + 1.21n't$

Arg$=i'g'+ig$	ν'^2	
	cos.	sin.
i' i	″ ″	″ ″
5— 3	$+0.2831 \; -0.11n't$	$+0.0229 \; +0.71n't$
6— 3	$-0.0492 \; -0.72n't$	$-0.2102 \; +1.04n't$
7— 3	$-0.0241 \; -0.76n't$	$+0.0424 \; +0.39n't$
8— 3	$+0.0027 \; +0.37n't$	$-0.0001 \; +0.07n't$
2— 4	$-0.05n't$	$+0.01n't$
3— 4	$-0.20n't$	$+0.63n't$
4— 4	$+0.0069 \; -0.31n't$	$+0.0075 \; +0.35n't$
5— 4	$+0.0079 \; +0.42n't$	$+0.0018 \; -0.02n't$
6— 4	$+0.0162 \; +0.37n't$	$-0.0443 \; +0.13n't$
7— 4	$+0.0111 \; +0.13n't$	$-0.0200 \; +0.14n't$
8— 4	$-0.2421 \; +0.11n't$	$+0.0622 \; +0.36n't$
9— 4	$+0.0750 \; +0.01n't$	$+0.0280 \; +0.01n't$
10— 4	$-0.00350+0.198n't$	$-0.00620-0.189n't$
11— 4	$-0.04n't$	$+0.01n't$
4— 5	$+0.2n't$	$+0.1n't$
5— 5	$+0.11n't$	$+0.15n't$
6— 5	$+0.03n't$	$-0.10n't$
7— 5	$-0.0095 \; +0.07n't$	$-0.0053 \; -0.12n't$
8— 5	$-0.0041 \; +0.06n't$	$-0.0068 \; -0.03n't$
9— 5	$-0.0005 \; +0.03n't$	$-0.0043 \; -0.01n't$
10— 5	$+0.0023 \; +0.01n't$	$-0.0036 \; +0.01n't$
11— 5	-0.0017	$+0.0010$
12— 5	$+0.0004$	0.0000
7— 6	$-0.05n't$	$-0.02n't$
8— 6	$-0.04n't$	$-0.04n't$
9— 6	$-0.01n't$	$-0.04n't$
10— 6	$+0.01n't$	$-0.01n't$

There follow the expressions of $-\frac{1}{2}\left(\frac{\overline{d^2W_0'}}{d\gamma'^2}\right)$ and its product by $n'\delta z'$, which forms the second part of $\frac{d\,.\,\delta\nu'}{n'dt}$:

Arg$=i'g'+ig$	$-\frac{1}{2}\left(\frac{\overline{d^2W_0'}}{d\gamma'^2}\right)$	
	cos.	sin.
i' i	″ ″	″ ″
0 0	260.121	
1 0	$+\;36.323-3.3061n't$	$+11.655+5.2963n't$
2 0	$+\;2.009-0.3703n't$	$+\;1.121+0.5934n't$
3 0	$+\;0.073-0.0350n't$	$+\;0.067+0.0561n't$
4 0	$-\;0.001-0.0031n't$	$0.000+0.0050n't$

Arg$=i'g'+ig$	$-\frac{1}{2}\left(\frac{d^2\overline{W_0'}}{d\gamma'^2}\right)$	
	cos.	sin.
i' i	"	"
−3 − 1	+ 0.004	+ 0.004
−2 − 1	+ 0.039	+ 0.037
−1 − 1	+ 0.326	+ 0.237
0 − 1	+ 1.962	+ 2.972
1 − 1	+121.718	−535.167
2 − 1	+ 71.015	− 78.902
3 − 1	+ 5.720	− 7.090
4 − 1	+ 0.438	− 0.274
5 − 1	+ 0.020	+ 0.003
6 − 1	− 0.001	+ 0.005
−2 − 2	+ 0.007	− 0.002
−1 − 2	+ 0.074	− 0.012
0 − 2	+ 0.784	+ 0.008
1 − 2	+ 13.625	− 24.844
2 − 2	+ 93.486	+ 36.557
3 − 2	+ 30.517	+ 11.437
4 − 2	− 37.653	−305.118
5 − 2	+ 0.47137	− 1.30389
6 − 2	+ 0.743	+ 0.107
7 − 2	+ 0.086	+ 0.025
8 − 2	+ 0.008	+ 0.003
0 − 3	+ 0.019	− 0.021
1 − 3	+ 0.521	− 1.587
2 − 3	+ 2.949	− 1.210
3 − 3	+ 23.441	− 31.125
4 − 3	+ 18.666	− 8.533
5 − 3	+ 8.008	+ 0.782
6 − 3	+ 3.571	+ 2.373
7 − 3	− 0.230	− 0.460
8 − 3	− 0.006	− 0.015
1 − 4	+ 0.019	− 0.088
2 − 4	+ 0.041	− 0.077
3 − 4	+ 0.526	− 1.531
4 − 4	− 11.114	− 13.018
5 − 4	− 2.259	− 9.639
6 − 4	+ 0.979	− 3.469
7 − 4	+ 0.806	− 0.799
8 − 4	+ 0.292	− 0.127
9 − 4	− 0.802	− 0.212
10 − 4	− 0.00477	− 0.00404
11 − 4	+ 0.0045	+ 0.0052
3 − 5	+ 0.005	− 0.050
4 − 5	− 0.669	− 0.571

Arg=$i'g'+ig$	$-\frac{1}{2}\left(\frac{\overline{d^2W_0'}}{d\gamma'^2}\right)$	
	cos.	sin.
i' i	"	"
5— 5	−7.044	+3.673
6— 5	−5.190	+0.120
7— 5	−1.788	−0.909
8— 5	−0.342	−0.510
9— 5	−0.012	−0.172
10— 5	+0.023	−0.045
11— 5	+0.021	−0.014
4— 6	−0.025	−0.037
5— 6	−0.596	+0.212
6— 6	+0.934	+3.668
7— 6	−0.532	+2.747
8— 6	−0.731	+0.921
9— 6	−0.348	+0.145
10— 6	−0.101	−0.011
11— 6	−0.019	−0.015
5— 7	−0.039	+0.004
6— 7	+0.009	+0.390
7— 7	+1.824	−0.023
8— 7	+1.396	+0.600
9— 7	+0.440	+0.540
10— 7	+0.044	+0.234
11— 7	−0.022	+0.065
12— 7	−0.011	+0.011
6— 8	−0.009	+0.024
7— 8	+0.222	+0.068
8— 8	+0.206	−0.863
9— 8	+0.486	−0.670
10— 8	+0.370	−0.190
11— 8	+0.151	+0.004
12— 8	+0.038	+0.021
7— 9	+0.016	+0.011
8— 9	+0.072	−0.121
9— 9	−0.383	−0.200
10— 9	−0.297	−0.338
11— 9	−0.064	−0.237
12— 9	+0.020	−0.093
8—10	+0.011	−0.009
9—10	−0.055	−0.060
10—10	−0.143	+0.156
11—10	−0.215	+0.115
12—10	−0.141	+0.008

Arg=$i'g'+ig$		$-\frac{1}{2}\left(\frac{d^2W_0'}{d\gamma'^2}\right)n'\delta z'$	
		sin.	cos.
i'	i	$''$ $''$	$''$ $''$
0	0		$+0.088806 - 11.2139n't$
1	0	$-1.8383 - 83.45n't$	$+1.5115 - 134.52n't$
		$- 0.1856n'^2t^2$	$0.0000n'^2t^2$
2	0	$+0.0686 - 10.00n't$	$-0.2856 - 9.33n't$
		$- 0.8493n'^2t^2$	$+ 1.6978n'^2t^2$
3	0	$-0.0537 - 0.67n't$	$-0.6626 - 0.49n't$
		$- 0.1064n'^2t^2$	$+ 0.2140n'^2t^2$
4	0	$-0.1796 - 0.03n't$	$-0.0179 - 0.01n't$
		$- 0.0105n'^2t^2$	$+ 0.0212n'^2t^2$
5	0	$-0.0020 - 0.0010n'^2t^2$	$-0.0046 + 0.0018n'^2t^2$
-3	-1	$+0.014$	-0.014
-2	-1	$+0.2422 - 0.18n't$	$-0.0646 - 0.09n't$
-1	-1	$+0.1606 + 0.61n't$	$-0.1008 - 1.68n't$
0	-1	$+0.1144 + 161.88n't$	$-0.0928 + 47.69n't$
1	-1	$-0.0410 + 67.76n't$	$-0.0061 - 60.56n't$
2	-1	$-0.7129 + 120.84n't$	$+0.1874 - 121.78n't$
3	-1	$-1.5367 + 42.91n't$	$-0.4746 + 23.05n't$
4	-1	$-3.4478 + 9.51n't$	$+0.6212 + 5.14n't$
5	-1	$+0.123 + 0.88n't$	$-0.205 + 0.66n't$
6	-1	$+0.006$	-0.025
-2	-2	$+0.010$	$+0.001$
-1	-2	$+0.0486 + 0.13n't$	$+0.0976 - 0.20n't$
0	-2	$+0.0766 + 9.13n't$	$+0.0532 - 0.38n't$
1	-2	$+0.0446 + 7.33n't$	$+0.0377 - 34.73n't$
2	-2	$-0.0184 + 18.82n't$	$+0.0167 - 12.26n't$
3	-2	$-0.0992 + 173.13n't$	$-0.5991 + 112.58n't$
4	-2	$-0.0240 + 389.46n't$	$-1.1671 + 56.68n't$
5	-2	$-1.32222 - 5.303n't$	$-3.11372 + 2.580n't$
6	-2	$-0.1662 - 236.60n't$	$-0.1938 + 329.73n't$
7	-2	$-0.0116 - 26.79n't$	$-0.0090 + 36.65n't$
8	-2	$- 2.51n't$	$+ 3.49n't$
-1	-3	$+0.002$	$+0.006$
0	-3	$-0.0391 + 0.49n't$	$+0.0321 + 0.12n't$
1	-3	$-0.0180 + 0.76n't$	$+0.0419 - 0.57n't$
2	-3	$-0.0064 + 13.08n't$	$+0.0329 - 1.47n't$
3	-3	$-0.0412 + 5.59n't$	$-0.0514 - 4.77n't$
4	-3	$-0.1540 + 5.12n't$	$+0.0082 - 12.69n't$
5	-3	$+0.8008 - 0.70n't$	$-0.0780 - 7.37n't$
6	-3	$+3.0147 - 0.70n't$	$-2.4513 - 0.78n't$
7	-3	$+0.2875 - 0.54n't$	$-0.6490 - 0.25n't$
8	-3	$+0.0269 - 0.57n't$	$-0.0556 + 0.08n't$
9	-3	$-0.0020 - 0.09n't$	$-0.0072 + 0.01n't$
1	-4	-0.020	-0.014
2	-4	$-0.0234 + 0.48n't$	$-0.0046 + 0.11n't$

Arg=$i'g'+ig$	$-\frac{1}{2}\left(\frac{d^2W_0'}{d\gamma'^2}\right)n'\delta z'$			
	sin.		cos.	
i' i	″	″	″	″
3— 4	−0.0187	+1.64$n't$	−0.0003	+5.22$n't$
4— 4	−0.0081	+2.57$n't$	−0.0023	+1.86$n't$
5— 4	+0.0154	+5.89$n't$	−0.0541	+1.00$n't$
6— 4	+0.0422	+2.98$n't$	−0.3339	−0.97$n't$
7— 4	−0.4838	+0.75$n't$	−0.5260	−0.78$n't$
8— 4	+0.3339	−0.09$n't$	−0.0270	+0.18$n't$
9— 4	+1.8881	+0.11$n't$	−0.5424	+0.45$n't$
10— 4	+0.01006	−0.026$n't$	−0.00958	−0.022$n't$
11— 4	−0.0025	−0.52$n't$	−0.0041	−0.12$n't$
2— 5	+0.005		−0.011	
3— 5	−0.001		−0.010	
4— 5		−2.07$n't$		+1.22$n't$
5— 5		−0.56$n't$		+1.36$n't$
6— 5	+0.0223	+0.13$n't$	−0.0245	+3.01$n't$
7— 5	+0.0604	+0.80$n't$	−0.0633	+1.35$n't$
8— 5	+0.1419	+0.52$n't$	−0.2495	+0.31$n't$
9— 5	+0.0038	+0.18$n't$	−0.1457	−0.01$n't$
10— 5	−0.0280		−0.0515	
11— 5	−0.0142		−0.0101	
12— 5	+0.0042		+0.0003	
5— 6		−0.79$n't$		−0.83$n't$
6— 6		−0.80$n't$		−0.30$n't$
7— 6	+0.0071	−1.45$n't$	+0.0040	+0.39$n't$
8— 6	+0.0313	−0.62$n't$	+0.0134	+0.58$n't$
9— 6	+0.1188	−0.12$n't$	+0.0381	+0.34$n't$
10— 6	+0.0678		−0.0116	
11— 6	+0.0179		−0.0144	
12— 6	+0.0027		−0.0067	
6— 7		+0.3$n't$		−0.5$n't$
7— 7		0.0$n't$		−0.5$n't$
8— 7		−0.3$n't$		−0.5$n't$
9— 7	−0.001	−0.4$n't$	+0.012	−0.3$n't$
10— 7	−0.005		+0.058	
11— 7	+0.012		+0.030	
12— 7	+0.009		+0.008	
10— 8	−0.006		−0.002	
11— 8	−0.024		+0.004	
12— 8	−0.015		+0.010	
12— 9	−0.005		−0.011	

If we add the three portions of $\frac{d\,.\,n'\delta^2z'}{n'dt}$, which have just been given, we shall have the value of this quantity. In $\overline{\delta W_0}'$ we give k_0 such a value that $n'\delta^2z'$ may have no term proportional to t, and k_1 and k_2 are so assumed that the terms having the argument g' may vanish. For this it is necessary to put $k_0 = +1''.5316$, $k_1 = +0''.5432$, and $k_2 = +0''.3989$.

In integrating the terms depending on the arguments $5g' - 2g$ and $10g' - 4g$ we have equated the motion of the latter, and have proceeded in a way precisely similar to that followed in deriving $\delta W_0'$ from $\delta T'$. By joining the first order with the second order terms, it is found that, as far as these two arguments are concerned, we have

$$\begin{aligned}\frac{d(n'\delta z')}{dt} = {} & [\ 35''.34410 - 0''.1473824n't] \cos(\ 5g' - 2g) + [\ 85''.09955 - 0''.0864177n't] \sin(\ 5g' - 2g) \\ & + [-\ 0.96463 - 0.0006821n't] \cos(10g' - 4g) + [-\ 1.03409 + 0.0005325n't] \sin(10g' - 4g)\end{aligned}$$

Setting aside quantities of the third order this expression can be replaced by the following:

$$\begin{aligned}\frac{d(n'\delta z')}{n'dt} = {} & [(1.9644830) - (8.3669461)n't] \cos[\ 5g' - 2g + 247^\circ\ 26'\ 44''.14 - (7.2640621)n't] \\ & + [(0.1504991) + (5.88018\ \ \ )n't] \cos[10g' - 4g + 133^\circ\ 0'\ 34''.7 + (6.78501\ \ \ )n't]\end{aligned}$$

The integrating factors for the equated arguments are given by

$$\log\mu = 1.5015038 \qquad\qquad \log\mu = 1.1719199$$

After neglecting certain quantities of the third order the integrated expression can be put under the form

$$\begin{aligned}n'\delta z' = {} & [-1143''.2024 - 4''.660293n't] \sin(\ 5g' - 2g) + [-2691''.4278 + 2''.782008n't] \cos(\ 5g' - 2g) \\ & + [-\ \ 14.3434 - 0.010128n't] \sin(10g' - 4g) + [\ \ \ 15.3517 - 0.007920n't] \cos(10g' - 4g)\end{aligned}$$

By subtracting the corresponding terms of $n'\delta z'$, found in Chapter II, we get

$$\begin{aligned}n'\delta^2 z' = {} & [-114''.0044 - 4''.660293n't] \sin(\ 5g' - 2g) + [-273''.6868 + 2''.782008n't] \cos(\ 5g' - 2g) \\ & + [-\ \ 16.8155 - 0.010128n't] \sin(10g' - 4g) + [\ \ \ 17.8091 - 0.007920n't] \cos(10g' - 4g)\end{aligned}$$

If we add the two portions of $\frac{d\,.\,\delta v'}{n'dt}$ we have the value of this quantity. In integrating we get the constant term of $\delta v'$ by the aid of the formula given at page 289. The constant term of $\frac{1}{3}\left(\frac{d\,.\,\delta z'}{dt} + \frac{1}{2}v'\right)^2$ is $+0''.2374$. Thus, the constant term of $\delta v'$ is found to be $+0''.2781$.

In integrating the term depending on the argument $5g' - 2g$ the motion of the latter has been equated. The first-order terms being included, we have

$$\begin{aligned}\frac{dv'}{n'dt} & = [-0''.76834 - 0''.0031108n't \sin(5g' - 2g) + [-1''.51690 + 0''.0019693n't] \cos(5g' - 2g) \\ & = [(0.2305490) - (6.54556)n't] \sin[5g' - 2g + 243^\circ\ 8'\ 12''.78 - (7.33352)n't]\end{aligned}$$

The logarithm of the integrating factor is 1.5059163, and we obtain

$$\nu' = [24''.9523 + 0''.099369n't] \cos(5g' - 2g) + [-48''.4634 + 0''.063826n't] \sin(5g' - 2g)$$

By subtracting the corresponding terms of ν', in Chapter II, we get

$$\delta\nu' = [0''.8754 + 0''.099369n't] \cos(5g' - 2g) + [-2''.8311 + 0.''063826n't] \sin(5g' - 2g)$$

The values of $n'\delta^2z'$ and $\delta\nu'$ follow:

Arg$=i'g'+ig$		$n'\delta^2z'$	
i'	i	sin.	cos.
		$''$ $''$	$''$ $''$
0	0		$- .00013045n'^2t^2$ $- .00001767n'^3t^3$
1	0	$-1.355224n't$ $- .00255569n'^2t^2$	$-1.787350n't$ $+ .00195723n'^2t^2$
2	0	$- 0.9731 - .018422n't$ $- .00013987n'^2t^2$	$+ 1.5631 - .024799n't$ $+ .00023963n'^2t^2$
3	0	$- 0.0087 - .000461n't$ $- .00000567n'^2t^2$	$+ 0.5428 - .000690n't$ $+ .00001028n'^2t^2$
4	0	$+ 0.0277 - .000016n't$ $- .00000027n'^2t^2$	$- 0.0020 - .000028n't$ $+ .00000049n'^2t^2$
5	0	$- 0.0003$	$- 0.0006$
$-4-$	1	$+ 0.0052$	$+ 0.0016$
$-3-$	1	$+ 0.0007$	$+ 0.0036$
$-2-$	1	$- 0.1640 - .000008n't$	$+ 0.0410 - .000011n't$
$-1-$	1	$- 0.2170 - .000338n't$	$+ 0.1965 - .000261n't$
$0-$	1	$- 0.3357 - .005861n't$	$+ 0.3621 - .002973n't$
$1-$	1	$- 0.5882 + .007757n't$	$- 0.3238 - .011436n't$
$2-$	1	$+ 2.2522 - .106611n't$	$+ 3.8506 + .164691n't$
$3-$	1	$+ 10.1442 + .022503n't$	$+ 7.8013 + .033438n't$
$4-$	1	$+ 0.2837 + .001252n't$	$+ 0.0504 + .001087n't$
$5-$	1	$- 0.0252 + .000049n't$	$+ 0.0112 + .000044n't$
$6-$	1	$- 0.0002 - .000006n't$	$0.0000 - .000003n't$
$-2-$	2	$- 0.0027$	$+ 0.0026$
$-1-$	2	$- 0.0290 + .000002n't$	$- 0.0585 - .000003n't$
$0-$	2	$- 0.0780 - .000046n't$	$- 0.0552 - .000033n't$
$1-$	2	$- 0.0933 + .000568n't$	$- 0.0501 - .001648n't$
$2-$	2	$+ 0.0555 + .000417n$	$- 0.0720 - .000120n't$
$3-$	2	$+ 0.1701 + .029724n't$	$- 0.3633 + .027242n't$
$4-$	2	$+ 6.6113 - .747047n't$	$- 27.1947 + .009349n't$
$5-$	2	$-114.0044 - 4.660293n't$	$-273.6868 + 2.782008n't$
$6-$	2	$- 0.0567 - .046962n't$	$- 0.1037 + .062944n't$
$7-$	2	$+ 0.0013 - .001350n't$	$- 0.0012 + .001771n't$
$8-$	2	$+ 0.0003 - .000058n't$	$- 0.0001 + .000074n't$

Arg=$i'g'+ig$		$n'\delta^2 z'$	
		sin.	cos.
i'	i	″ ″	″ ″
-1	-3	-0.0024	-0.0011
0	-3	$+0.0216-.000004n't$	$-0.0181-.000003n't$
1	-3	$+0.0147+.000028n't$	$-0.0368-.000047n't$
2	-3	$+0.0089+.000104n't$	$-0.0372-.000032n't$
3	-3	$+0.0488+.000118n't$	$+0.0502-.000024n't$
4	-3	$+0.0228-.000898n't$	$+0.0017+.002949n't$
5	-3	$-0.2566+.000928n't$	$-0.0611+.003718n't$
6	-3	$+0.3728+.005367n't$	$-1.0977+.006121n't$
7	-3	$+2.7313-.012031n't$	$+7.2025-.004269n't$
8	-3	$+0.6006-.000794n't$	$+0.3875+.000213n't$
9	-3	$-0.0206-.000014n't$	$+0.0594+.000011n't$
10	-3	$+0.0008$	$+0.0013$
0	-4	$+0.0002$	-0.0011
1	-4	$+0.0097+.000001n't$	$+0.0074\ \ .000000n't$
2	-4	$+0.0175+.000008n't$	$+0.0045+.000002n't$
3	-4	$+0.0187+.000021n't$	$+0.0011-.000011n't$
4	-4	$-0.0068+.000011n't$	$+0.0116+.000022n't$
5	-4	$+0.0034-.000888n't$	$+0.0252-.000085n't$
6	-4	$-0.0326-.000774n't$	$-0.0422+.000356n't$
7	-4	$+0.4595-.000342n't$	$+0.4047+.000489n't$
8	-4	$+1.4923-.000118n't$	$+0.3826+.000557n't$
9	-4	$-8.9304-.002187n't$	$+2.6357-.005379n't$
10	-4	$-16.8155-.010128n't$	$+17.8091-.007920n't$
11	-4	$-0.0089-.000111n't$	$+0.0116-.000026n't$
1	-5	$+0.0009$	0.0000
2	-5	$-0.0024\ \ .000000n't$	$+0.0054-.000001n't$
3	-5	$+0.0009+.000002n't$	$+0.0090-.000002n't$
4	-5	$+0.0027+.000020n't$	$+0.0088-.000002n't$
5	-5	$-0.0018-.000003n't$	$+0.0021+.000004n't$
6	-5	$-0.0032-.000035n't$	$+0.0031-.000333n't$
7	-5	$+0.0039-.000180n't$	$+0.0095-.000236n't$
8	-5	$-0.0947-.000154n't$	$+0.1640-.000064n't$
9	-5	$+0.0008-.000087n't$	$+0.1632+.000011n't$
10	-5	$+0.0643-.000045n't$	$+0.1086+.000039n't$
11	-5	$+0.2071-.000042n't$	$+0.1283+.000086n't$
12	-5	$-0.3212-.000019n't$	$-0.0118-.000118n't$
2	-6	$+0.0003$	$+0.0010$
3	-6	$-0.0023+.000002n't$	$-0.0005+.000001n't$
4	-6	$-0.0038+.000002n't$	$+0.0008-.000003n't$
5	-6	$-0.0028+.000001n't$	$+0.0015+.000016n't$
6	-6	$-0.0016-.000010n't$	$+0.0020-.000007n't$
7	-6	$-0.0027+.000137n't$	$+0.0005-.000035n't$
8	-6	$-0.0043+.000076n't$	$+0.0025-.000089n't$
9	-6	$-0.0711+.000013n't$	$-0.0226-.000072n't$
10	-6	$-0.0590-.000011n't$	$+0.0129-.000031n't$
11	-6	$-0.0233-.000012n't$	$+0.0221-.000010n't$
12	-6	$-0.0043-.000005n't$	$+0.0180\ \ .000000n't$

Arg=$i'g'+ig$	$n'\delta^2z'$	
	sin.	cos.
i' i	″ ″	″ ″
3— 7	-0.0005	-0.0001
4— 7	-0.0001	-0.0006
5— 7	$-0.0005+.000002n't$	$-0.0010-.000001n't$
6— 7	$-0.0007-.000011n't$	$-0.0008+.000005n't$
7— 7	$-0.0001+.000001n't$	$+0.0001-.000001n't$
8— 7	$+0.0011+.000031n't$	$-0.0018+.000053n't$
9— 7	$-0.0021+.000048n't$	$-0.0014+.000025n't$
10— 7	$+0.0028+.000033n't$	$-0.0316+.000003n't$
11— 7	$-0.0122+.000009n't$	$-0.0235-.000006n't$
12— 7	$-0.0095+.000006n't$	$-0.0076-.000004n't$
5— 8	$+0.0002$	-0.0001
6— 8	$+0.0006-.000001n't$	$-0.0001-.000002n't$
7— 8	$+0.0002+.000002n't$	$-0.0008-.000001n't$
8— 8	$+0.0006-.000003n't$	$+0.0001+.000003n't$
9— 8	$+0.0003-.000019n't$	$0.0000+.000017n't$
10— 8	$-0.0005-.000009n't$	$-0.0009+.000026n't$
11— 8	$+0.0154+.000002n't$	$-0.0020+.000011n't$
12— 8	$+0.0111+.000004n't$	$-0.0079+.000004n't$
6— 9	$+0.0004$	$+0.0008$
7— 9	$+0.0001-.000003n't$	$+0.0001+.000001n't$
8— 9	$0.0000-.000001n't$	$0.0000+.000002n't$
9— 9	$0.0000-.000001n't$	$0.0000+.000001n't$
10— 9	$+0.0006-.000004n't$	$+0.0001-.000003n't$
11— 9	$+0.0005-.000011n't$	$+0.0002-.000003n't$
12— 9	$+0.0027-.000003n't$	$+0.0064+.000002n't$
12—10	$+0.0006+.000001n't$	$+0.0002+.000001n't$

Arg=$i'g'+ig$	$\delta\nu'$	
	cos.	sin.
i' i	″ ″	″ ″
0 0	$+0.2781+.018619n't$ $+.00011252n'^2t^2$	
1 0	$+0.6169+.676008n't$ $+.00128728n'^2t^2$	$+0.6757-.892752n't$ $+.00097623n'^2t^2$
2 0	$+0.7470+.018297n't$ $+.00007799n'^2t^2$	$+1.2544-.024933n't$ $+.00011223n'^2t^2$
3 0	$+0.0292+.000710n't$ $+.00000504n'^2t^2$	$+0.5359-.001080n't$ $+.00000828n'^2t^2$
4 0	$+0.0276+.000034n't$ $+.00000034n'^2t^2$	$+0.0154-.000058n't$ $+.00000059n'^2t^2$
5 0	$+0.0004+.000001n't$ $+.00000002n'^2t^2$	$-0.0009-.000003n't$ $+.00000004n'^2t^2$

Arg=$i'g'+ig$	$\delta v'$	
	cos.	sin.
i' i	" "	" "
— 4— 1	—0.0003	— 0.0004
— 3— 1	—0.0014	— 0.0061
— 2— 1	—0.1744—.000007$n't$	— 0.0450+.000017$n't$
— 1— 1	—0.2245—.000124$n't$	— 0.2023+.000188$n't$
0— 1	—0.2838+.000861$n't$	— 0.3074+.000330$n't$
1— 1	—0.4186+.003154$n't$	+ 0.3343+.004120$n't$
2— 1	+0.4800—.027584$n't$	— 1.3399—.043252$n't$
3— 1	—2.7100—.009645$n't$	+ 2.2915+.014945$n't$
4— 1	+1.4022—.000771$n't$	+ 0.3615+.000867$n't$
5— 1	+0.0033—.000062$n't$	+ 0.0074+.000050$n't$
6— 1	+0.0001—.000007$n't$	— 0.0008+.000001$n't$
— 2— 2	—0.0042	— 0.0021
— 1— 2	—0.0338+.000003$n't$	+ 0.0642+.000004$n't$
0— 2	—0.0849+.000095$n't$	+ 0.0763+.000027$n't$
1— 2	—0.1032—.000218$n't$	+ 0.0572+.000002$n't$
2— 2	+0.0618+.000595$n't$	+ 0.0866—.000424$n't$
3— 2	+0.0904+.013210$n't$	+ 0.6076—.014867$n't$
4— 2	+3.2207—.362438$n't$	+13.6148—.004861$n't$
5— 2	+0.8754+.099369$n't$	— 2.8311+.063826$n't$
6— 2	—0.0249+.023487$n't$	— 0.0552+.031006$n't$
7— 2	—0.0070+.001334$n't$	— 0.0041+.001782$n't$
8— 2	—0.0009+.000084$n't$	— 0.0006+.000113$n't$
— 1— 3	—0.0026	+ 0.0022
0— 3	+0.0237+.000003$n't$	+ 0.0207—.000001$n't$
1— 3	+0.0182—.000020$n't$	+ 0.0423—.000036$n't$
2— 3	+0.0143—.000023$n't$	+ 0.0464+.000021$n't$
3— 3	+0.0104—.000001$n't$	+ 0.0129—.000078$n't$
4— 3	+0.0467—.000843$n't$	+ 0.0065—.002813$n't$
5— 3	+0.1102+.000716$n't$	+ 0.0762—.003302$n't$
6— 3	+1.5708+.003366$n't$	+ 1.4476—.003996$n't$
7— 3	+0.8394—.003134$n't$	— 1.7277+.001000$n't$
8— 3	—0.2103+.000300$n't$	+ 0.1382+.000089$n't$
9— 3	+0.0072+.000010$n't$	+ 0.0306+.000001$n't$
10— 3	+0.0004	+ 0.0010
0— 4	+0.0013	+ 0.0025
1— 4	+0.0114—.000001$n't$	— 0.0079—.000002$n't$
2— 4	+0.0212—.000002$n't$	— 0.0041+.000001$n't$
3— 4	+0.0221+.000002$n't$	+ 0.0011+.000049$n't$
4— 4	+0.0032—.000032$n't$	— 0.0068—.000005$n't$
5— 4	+0.0123—.000917$n't$	— 0.0180+.000079$n't$
6— 4	+0.0488—.000795$n't$	— 0.0022—.000334$n't$
7— 4	+0.4619—.000359$n't$	— 0.4032—.000447$n't$
8— 4	+0.9286—.000106$n't$	— 0.2713—.000394$n't$
9— 4	—4.1159—.001047$n't$	— 1.2383+.002535$n't$
10— 4	+0.5985+.000450$n't$	+ 0.7483—.000301$n't$
11— 4	+0.0024+.000049$n't$	— 0.0035—.000011$n't$

Arg=$i'g'+ig$	$\delta v'$	
	cos.	sin.
i' i	″ ″	″ ″
1— 5	$+0.0018$	-0.0004
2— 5	-0.0025	-0.0062
3— 5	$+0.0005-.000003n't$	$-0.0107+.000006n't$
4— 5	$+0.0029+.000035n't$	$-0.0114+.000007n't$
5— 5	$-0.0015-.000006n't$	$-0.0033+.000011n't$
6— 5	$-0.0055-.000043n't$	$-0.0056+.000346n't$
7— 5	$-0.0023-.000188n't$	$-0.0153+.000272n't$
8— 5	$-0.1034-.000169n't$	$-0.1788+.000087n't$
9— 5	$-0.0852-.000095n't$	$-0.1760-.000004n't$
10— 5	$+0.0545-.000045n't$	$-0.1071-.000033n't$
11— 5	$+0.1325-.000029n't$	$-0.0872-.000059n't$
12— 5	$-0.0733-.000006n't$	$-0.0007+.000029n't$
2— 6	$+0.0001$	-0.0011
3— 6	-0.0039	$+0.0007$
4— 6	$-0.0060+.000003n't$	$-0.0015+.000002n't$
5— 6	$-0.0053+.000007n't$	$-0.0026-.000018n't$
6— 6	$-0.0019+.000007n't$	$-0.0003+.000003n't$
7— 6	$-0.0042+.000137n't$	$+0.0030+.000046n't$
8— 6	$-0.0071+.000098n't$	$-0.0019+.000105n't$
9— 6	$-0.0798+.000023n't$	$+0.0248+.000080n't$
10— 6	$-0.0683-.000011n't$	$-0.0103+.000038n't$
11— 6	$-0.0303-.000012n't$	$-0.0232+.000012n't$
12— 6	$-0.0070-.000007n't$	$-0.0174+.000001n't$
3— 7	-0.0005	-0.0001
4— 7	-0.0001	$+0.0023$
5— 7	$-0.0014+.000003n't$	$+0.0028-.000003n't$
6— 7	$-0.0019-.000011n't$	$+0.0019-.000005n't$
7— 7	$-0.0006+.000004n't$	$+0.0007+.000001n't$
8— 7	$+0.0012+.000034n't$	$+0.0026-.000056n't$
9— 7	$-0.0021+.000062n't$	$+0.0032-.000033n't$
10— 7	$+0.0026+.000046n't$	$+0.0370-.000004n't$
11— 7	$-0.0124+.000017n't$	$+0.0305+.000008n't$
12— 7	$-0.0121+.000004n't$	$+0.0110+.000006n't$
5— 8	$+0.0008$	$+0.0003$
6— 8	$+0.0011-.000001n't$	$+0.0009-.000001n't$
7— 8	$+0.0004-.000007n't$	$+0.0010+.000004n't$
8— 8	$+0.0003-.000004n't$	$+0.0001-.000003n't$
9— 8	$+0.0009-.000028n't$	$+0.0003-.000024n't$
10— 8	$+0.0009-.000013n't$	$+0.0022-.000035n't$
11— 8	$+0.0171+.000001n't$	$+0.0027-.000020n't$
12— 8	$+0.0128+.000005n't$	$+0.0085-.000008n't$
6— 9	$+0.0002$	-0.0002
7— 9	$+0.0005-.000002n't$	$-0.0005-.000001n't$
8— 9	$+.000001n't$	$+.000004n't$
9— 9	$-.000001n't$	$+.000004n't$
10— 9	$+0.0003-.000011n't$	$-0.0003+.000008n't$
11— 9	$+0.0016-.000020n't$	$-0.0006+.000004n't$
12— 9	$+0.0028-.000011n't$	$-0.0074-.000004n't$
12—10	$+0.0006\ \ .000000n't$	$-0.0007+.000008n't$

CHAPTER XVI.

PERTURBATIONS OF THE THIRD ORDER WITH RESPECT TO DISTURBING FORCES IN THE LONGITUDES AND RADII-VECTORES, ARISING FROM THE MUTUAL ACTION OF JUPITER AND SATURN—DETERMINATION OF THE FACTORS OF δ^2T AND δ^2T′.

The corrections to be applied to T and T′, on account of terms of three dimensions with respect to disturbing forces, will be denoted by δ^2T and δ^2T′. In determining them it is important to reject all parts of the formulæ which afford insignificant terms, and thus reduce the very onerous labor involved. In this connection it will be seen, on referring to the previous elaboration of the second-order terms, that those involving the factors u and u' are generally insignificant and in the remaining cases quite small. As, in the third-order terms, these factors should give rise to still smaller terms, it is thought allowable to entirely neglect them. For like reasons, all terms arising through the consideration of δh and $\delta h'$ will be neglected. Thus, we will consider only the four augmentations, $n\delta z$, ν, $n'\delta z'$, and ν'.

Then, applying Taylor's theorem, extended to any number of variables, to the functions T and T′, we have

$$\begin{aligned}\delta^2\mathrm{T} = \quad & \mathrm{A}n\delta^2 z + \mathrm{B}\delta\nu + \mathrm{F}n'\delta^2 z' + \mathrm{G}\delta\nu' \\ & + \frac{1}{2}\frac{d\mathrm{A}}{dg}(n\delta z)^2 + \frac{d\mathrm{A}}{dg'}(n\delta z)(n'\delta z') + \frac{1}{2}\frac{d\mathrm{F}}{dg'}(n'\delta z')^2 \\ & + \frac{d\mathrm{B}}{dg}(n\delta z)\nu + \frac{d\mathrm{B}}{dg'}(n'\delta z')\nu + \frac{d\mathrm{G}}{dg}(n\delta z)\nu' + \frac{d\mathrm{G}}{dg'}(n'\delta z')\nu' \\ & + \frac{1}{2}r^2\frac{d^2\mathrm{T}}{dr^2}\nu^2 + rr'\frac{d^2\mathrm{T}}{dr\,dr'}\nu\nu' + \frac{1}{2}r'^2\frac{d^2\mathrm{T}}{dr'^2}\nu'^2\end{aligned}$$

$$\begin{aligned}\delta^2\mathrm{T}' = \quad & \mathrm{A}'n'\delta^2 z' + \mathrm{B}'\delta\nu' + \mathrm{F}'n\delta^2 z + \mathrm{G}'\delta\nu \\ & + \frac{1}{2}\frac{d\mathrm{A}'}{dg'}(n'\delta z')^2 + \frac{d\mathrm{A}'}{dg}(n\delta z)(n'\delta z') + \frac{1}{2}\frac{d\mathrm{F}'}{dg}(n\delta z)^2 \\ & + \frac{d\mathrm{B}'}{dg'}(n'\delta z')\nu' + \frac{d\mathrm{B}'}{dg}(n\delta z)\nu' + \frac{d\mathrm{G}'}{dg'}(n'\delta z')\nu + \frac{d\mathrm{G}'}{dg}(n\delta z)\nu \\ & + \frac{1}{2}r'^2\frac{d^2\mathrm{T}}{dr'^2}\nu'^2 + rr'\frac{d^2\mathrm{T}'}{dr\,dr'}\nu\nu' + \frac{1}{2}r^2\frac{d^2\mathrm{T}'}{dr^2}\nu^2\end{aligned}$$

The four factors, A, B, F, and G, of the first line of each of these two expressions are the same as the factors, denoted by these symbols, employed in the computation of the second order terms, and the first factors of the two following lines are obtained from those just mentioned by partial differentiation with respect to g or g'. Thus, eleven out of fourteen quantities involved in each of these expressions are determined in the easiest manner possible; a notable advantage resulting from the employment of HANSEN'S variables. But the first factors of the three terms in the last line of each expression must be specially computed. They all have the form

$$\mathrm{H}_1\left(\frac{\mathrm{a}'}{\triangle}\right)^7 + \mathrm{H}_2\left(\frac{\mathrm{a}'}{\triangle}\right)^5 + \mathrm{H}_3\left(\frac{\mathrm{a}'}{\triangle}\right)^3 + \mathrm{H}_4\frac{\mathrm{a}'}{\triangle} + \mathrm{H}_5$$

The readiest method of obtaining them appears to be the following: Derive, in the first place, the two quantities

$$a_0\left(r\frac{d}{dr}\right)^3\Omega = \mu\left\{-\frac{15}{8}\left(\frac{\mathrm{a}'}{\triangle}\right)^7\left(\alpha^2\frac{r^2}{\mathrm{a}^2}-\frac{r'^2}{\mathrm{a}'^2}\right)^3 + \left(\frac{\mathrm{a}'}{\triangle}\right)^5\left[\frac{9}{8}\left(\alpha^2\frac{r^2}{\mathrm{a}^2}-\frac{r'^2}{\mathrm{a}'^2}\right)^2 + \frac{9}{2}\frac{r'^2}{\mathrm{a}'^2}\left(\alpha^2\frac{r^2}{\mathrm{a}^2}-\frac{r'^2}{\mathrm{a}'^2}\right)\right]\right.$$
$$\left. - \frac{1}{2}\left(\frac{\mathrm{a}'}{\triangle}\right)^3\frac{r'^2}{\mathrm{a}'^2} - \frac{1}{8}\left(\frac{\mathrm{a}'}{\triangle}\right)^3\left(\alpha^2\frac{r^2}{\mathrm{a}^2}-\frac{r'^2}{\mathrm{a}'^2}\right) - \frac{1}{8}\frac{\mathrm{a}'}{\triangle} - \frac{\mathrm{a}'r}{r'^2}\mathrm{H}\right\}$$

$$a_0'\left(r'\frac{d}{dr'}\right)^3\Omega' = \mu'\left\{\frac{15}{8}\left(\frac{\mathrm{a}'}{\triangle}\right)^7\left(\alpha^2\frac{r^2}{\mathrm{a}^2}-\frac{r'^2}{\mathrm{a}'^2}\right)^3 + \left(\frac{\mathrm{a}'}{\triangle}\right)^5\left[\frac{9}{8}\left(\alpha^2\frac{r^2}{\mathrm{a}^2}-\frac{r'^2}{\mathrm{a}'^2}\right)^2 - \frac{9}{2}\alpha^2\frac{r^2}{\mathrm{a}^2}\left(\alpha^2\frac{r^2}{\mathrm{a}^2}-\frac{r'^2}{\mathrm{a}'^2}\right)\right]\right.$$
$$\left. - \frac{1}{2}\left(\frac{\mathrm{a}'}{\triangle}\right)^3\alpha^2\frac{r^2}{\mathrm{a}^2} + \frac{1}{8}\left(\frac{\mathrm{a}'}{\triangle}\right)^3\left(\alpha^2\frac{r^2}{\mathrm{a}^2}-\frac{r'^2}{\mathrm{a}'^2}\right) - \frac{1}{8}\frac{\mathrm{a}'}{\triangle} - \frac{\mathrm{a}'r'}{r^2}\mathrm{H}\right\}$$

To save labor, it must be remembered that

$$\left(\frac{\mathrm{a}'}{\triangle}\right)^5\left(\alpha^2\frac{r^2}{\mathrm{a}^2}-\frac{r'^2}{\mathrm{a}'^2}\right)^2 \text{ and } \left(\frac{\mathrm{a}'}{\triangle}\right)\left(\alpha\frac{r^2}{\mathrm{a}^2}-\frac{r'^2}{\mathrm{a}'^2}\right)$$

have been computed in our work on the second-order terms, and that

$$\left(\frac{\mathrm{a}'}{\triangle}\right)^3\left(\alpha^2\frac{r^2}{\mathrm{a}^2}-\frac{r'^2}{\mathrm{a}'^2}\right),\quad \left(\frac{\mathrm{a}'}{\triangle}\right)^3\alpha^2\frac{r^2}{\mathrm{a}^2},\quad \left(\frac{\mathrm{a}'}{\triangle}\right)^3\frac{r'^2}{\mathrm{a}'^2},\quad \frac{\mathrm{a}'}{\triangle},\quad \frac{\mathrm{a}'r}{r'^2}\mathrm{H}, \text{ and } \frac{\mathrm{a}'r'}{r^2}\mathrm{H}$$

have been computed for the first-order terms.

In the second place, derive Y and Y′ from the equations

$$\mathrm{Y} = \mathrm{M}\frac{d}{dg}\left(a_0 r\frac{d\Omega}{dr}\right) + \mathrm{N}a_0\left(r\frac{d}{dr}\right)^2\Omega$$
$$\mathrm{Y}' = \mathrm{M}'\frac{d}{dg'}\left(a_0' r'\frac{d\Omega'}{dr'}\right) + \mathrm{N}'a_0'\left(r'\frac{d}{dr'}\right)^2\Omega'$$

M, N, M′, and N′ have been used in deriving the quantities denoted X and X′ in the second-order terms. Also, $\frac{d}{dg}\left(a_0 r\frac{d\Omega}{dr}\right)$, etc., have already been obtained in computing V and V′ in the second-order terms.

Then

$$r^3\frac{d^3\mathrm{T}}{dr^3} = A\frac{d}{dg}\left[a_0\left(r\frac{d}{dr}\right)^2\Omega\right] + Ba_0\left(r\frac{d}{dr}\right)^3\Omega - \mathrm{B} - \mathrm{X} + 2\mathrm{Y}$$
$$r'^3\frac{d^3\mathrm{T}'}{dr^3} = A'\frac{d}{dg'}\left[a_0'\left(r'\frac{d}{dr'}\right)^2\Omega'\right] + B'a_0'\left(r'\frac{d}{dr'}\right)^3\Omega' - \mathrm{B}' - \mathrm{X}' + 2\mathrm{Y}'$$

We have already used

$$a_0\left(r\frac{d}{dr}\right)^2\Omega \text{ and } a_0'\left(r'\frac{d}{dr'}\right)^2\Omega'$$

in computing V and V′. The quantities, for the sake of distinction, denoted $\mathcal{A}$, $\mathcal{B}$, $\mathcal{A}'$, and $\mathcal{B}'$ are the same as A, B, A′, and B′ in the formulæ of the first-order terms

$$\mathrm{T} = \mathrm{A}a_0\frac{d\Omega}{dg} + \mathrm{B}a_0 r\frac{d\Omega}{dr}$$

$$\mathrm{T}' = \mathrm{A}'a_0'\frac{d\Omega'}{dg'} + \mathrm{B}'a_0 r'\frac{d\Omega'}{dr'}$$

After $r^2\frac{d^2\mathrm{T}}{dr^2}$ and $r'^2\frac{d^2\mathrm{T}'}{dr'^2}$ have been obtained, we get the four remaining factors from the equations

$$r^2\frac{d^2\mathrm{T}}{dr'^2} + rr'\frac{d^2\mathrm{T}}{drdr'} = -2\mathrm{B} - \mathrm{X} + \mathrm{Y}$$

$$rr'\frac{d^2\mathrm{T}}{drdr'} + r'^2\frac{d^2\mathrm{T}}{dr'^2} = -2\mathrm{G} - \mathrm{X} - \mathrm{Y}$$

$$r'^2\frac{d^2\mathrm{T}'}{dr'^2} + rr'\frac{d^2\mathrm{T}'}{drdr'} = -2\mathrm{B}' - \mathrm{X}' + \mathrm{Y}'$$

$$rr'\frac{d^2\mathrm{T}}{drdr'} + r^2\frac{d^2\mathrm{T}'}{dr^2} = -2\mathrm{G}' - \mathrm{X}' - \mathrm{Y}'$$

These equations result very simply from the consideration that T and T′ are homogeneous functions of r and r'.

If, then, we make the two following multiplications, viz.,

$$\left(\frac{\mathrm{a}'}{\triangle}\right)^7 \text{ by } \frac{15}{8}\left(\frac{r'^2}{\mathrm{a}'^2} - \alpha^2\frac{r^2}{\mathrm{a}^2}\right)^3$$

$$\left(\alpha^2\frac{r^2}{\mathrm{a}^2} - \frac{r'^2}{\mathrm{a}'^2}\right)\left(\frac{\mathrm{a}'}{\triangle}\right)^5 \text{ by } -\frac{9}{2}\alpha^2\frac{r^2}{\mathrm{a}^2}$$

we shall be able, by the aid of the products, together with expressions previously obtained, to get the quantities $a_0\left(r\frac{d}{dr}\right)^3\Omega$ and $a_0'\left(r'\frac{d}{dr'}\right)^3\Omega'$, and that through simple additions or subtractions.

By raising the value of $\frac{r'^2}{\mathrm{a}'^2} - \alpha^2\frac{r^2}{\mathrm{a}}$, given on page 201, to the third power, we have

$$\begin{aligned}\frac{15}{8}\left(\frac{r'^2}{\mathrm{a}'^2} - \alpha^2\frac{r^2}{\mathrm{a}^2}\right)^3 = [9.83735] &- 2[9.19885]\cos g' + 2[8.0107]\cos 2g' - 2[5.61]\cos 3g' \\ &+ 2[8.6105\ ]\cos g + 2[7.1181]\cos 2g + 2[5.57]\cos 3g \\ &- 2[7.8045\ ]\cos(g' - g) - 2[7.8045]\cos(g' + g) \\ &- 2[6.1481\ ]\cos(g' - 2g) - 2[6.1481]\cos(g' + 2g) \\ &+ 2[6.2142\ ]\cos(2g' - g) + 2[6.2142]\cos(2g' + g)\end{aligned}$$

Also

$$-\frac{9}{2}\alpha^2\frac{r^2}{\mathrm{a}^2} = -[0.12783] + 2[8.8096]\cos g + 2[6.89]\cos 2g$$

The derived products are:

Arg $= i'g'+ig$	$\frac{15}{8}\left(\frac{a'}{\Delta}\right)^7\left(\frac{r'^2}{a'^2}-\alpha^2\frac{r^2}{a^2}\right)^3$		$\frac{9}{2}\left(\frac{a'}{\Delta}\right)^5\left(\frac{r'^2}{a'^2}-\alpha^2\frac{r^2}{a^2}\right)\alpha^2\frac{r^2}{a^2}$	
	cos.	sin.	cos.	sin.
i' i				
0 0	+17.678		+6.785	
0 − 1	− 2.853	− 3.058	−1.404	− 1.582
0 − 2	− 0.149	+ 0.294	−0.073	+ 0.162
0 − 3	+ 0.017	+ 0.007	+0.011	+ 0.002
0 − 4	+ 0.001	0.000		
1 + 4	0.000	+ 0.001		
1 + 3	+ 0.006	+ 0.002	+0.001	+ 0.001
1 + 2	+ 0.003	− 0.045	+0.003	− 0.024
1 + 1	− 0.591	+ 0.127	−0.315	+ 0.062
1 0	+ 4.033	+ 4.930	+1.949	+ 2.190
1 − 1	+ 6.615	−32.805	+2.397	−11.966
1 − 2	− 1.728	+ 1.241	−1.073	+ 0.698
1 − 3	+ 0.133	+ 0.128	+0.068	+ 0.061
1 − 4	+ 0.006	− 0.007	+0.003	− 0.005
1 − 5	− 0.010	+ 0.001		
2 + 3	− 0.003	− 0.009		
2 + 2	+ 0.003	− 0.007	+0.002	− 0.002
2 + 1	− 0.081	− 0.035	−0.042	− 0.018
2 0	− 0.003	+ 1.021	+0.020	+ 0.509
2 − 1	+ 6.925	− 4.278	+2.782	− 1.947
2 − 2	−26.604	−11.248	−8.932	− 3.743
2 − 3	+ 0.333	+ 0.459	+0.256	+ 0.588
2 − 4	+ 0.097	− 0.066	+0.041	− 0.021
2 − 5	+ 0.008	− 0.006	−0.001	− 0.002
3 + 2	+ 0.001	− 0.001		
3 + 1	− 0.008	− 0.007	−0.003	− 0.005
3 0	− 0.089	+ 0.122	−0.044	+ 0.066
3 − 1	+ 1.493	+ 0.234	+0.681	+ 0.053
3 − 2	− 3.576	− 8.298	−1.531	− 3.020
3 − 3	−13.154	+19.067	−4.000	+ 5.853
3 − 4	− 0.464	− 0.034	+0.226	− 0.053
3 − 5	− 0.062	− 0.087	−0.004	− 0.025
3 − 6	− 0.006	− 0.002	−0.001	+ 0.001
4 + 1	+ 0.002	− 0.006		
4 0	− 0.021	+ 0.007	−0.010	+ 0.004
4 − 1	+ 0.171	+ 0.175	+0.088	+ 0.079
4 − 2	+ 0.569	− 1.868	+0.157	− 0.774
4 − 3	− 8.723	+ 2.196	−2.884	+ 0.914
4 − 4	+12.065	+12.722	+3.396	+ 3.539
4 − 5	− 0.092	+ 0.964	−0.002	− 0.011
4 − 6	− 0.085	+ 0.059	−0.015	+ 0.001
4 − 7	+ 0.002	+ 0.004		

Arg=$i'g'+ig$	$\frac{15}{8}\left(\frac{a'}{\Delta}\right)^7\left(\frac{r'^2}{a'^2}-\alpha^2\frac{r^2}{a^2}\right)^3$		$\frac{9}{2}\left(\frac{a'}{\Delta}\right)^5\left(\frac{r'^2}{a'^2}-\alpha^2\frac{r^2}{a^2}\right)\alpha^2\frac{r^2}{a^2}$	
	cos.	sin.	cos.	sin.
i' i				
5 0	−0.003	+0.001	−0.001	−0.001
5 −1	+0.006	+0.039	+0.004	+0.018
5 −2	+0.290	−0.201	+0.114	−0.101
5 −3	−2.032	−0.959	−0.767	−0.271
5 −4	+0.611	+8.191	+0.326	+2.471
5 −5	+10.806	−6.581	+2.766	−1.709
5 −6	+1.087	+0.290	+0.087	+0.017
5 −7	+0.010	+0.099	+0.001	+0.012
5 −8	−0.005	+0.025	0.000	+0.001
6 −1	−0.004	+0.003	−0.001	+0.002
6 −2	+0.058	+0.001	+0.025	−0.003
6 −3	−0.212	−0.406	−0.099	−0.144
6 −4	−1.317	+1.951	−0.364	+0.672
6 −5	+6.967	+0.757	+1.927	+0.106
6 −6	−2.838	−8.305	−0.691	−1.967
6 −7	+0.463	−0.966	+0.043	−0.109
6 −8	+0.112	−0.024	+0.010	−0.002
6 −9	+0.019	+0.013		
7 −2	+0.007	+0.004	+0.003	+0.002
7 −3	+0.015	−0.080	+0.001	−0.033
7 −4	−0.505	+0.169	−0.164	+0.081
7 −5	+1.662	+1.556	+0.528	+0.410
7 −6	+1.655	−5.417	+0.350	−1.382
7 −7	−5.855	+0.623	−1.291	+0.150
7 −8	−0.723	−0.549	−0.094	−0.065
7 −9	−0.010	−0.109	−0.002	−0.010
8 −3	+0.005	−0.010	+0.004	−0.004
8 −4	−0.097	−0.032	−0.036	−0.006
8 −5	+0.094	+0.567	+0.053	+0.169
8 −6	+1.651	−1.246	+0.411	−0.369
8 −7	−3.853	−2.049	−0.914	−0.438
8 −8	−0.432	+3.777	−0.088	+0.785
8 −9	−0.589	+0.478	−0.067	+0.066
9 −3	+0.003	−0.001	+0.001	−0.001
9 −4	−0.015	−0.015	−0.005	−0.005
9 −5	−0.055	+0.126	−0.012	+0.038
9 −6	+0.653	−0.005	+0.158	−0.022
9 −7	−0.832	−1.861	−0.226	−0.372
9 −8	−2.604	+2.770	−0.423	+0.554
9 −9	+2.698	+1.384	+0.446	+0.163

Thence are derived the following expressions for $\frac{a_0}{\mu}\left(r\frac{d}{dr}\right)^3\Omega$ and $\frac{a_0'}{\mu'}\left(r'\frac{d}{dr'}\right)^3\Omega'$:

Arg=$i'g'+ig$	$\frac{a_0}{\mu}\left(r\frac{d}{dr}\right)^3\Omega$		$\frac{a_0'}{\mu'}\left(r'\frac{d}{dr'}\right)^3\Omega'$	
	cos.	sin.	cos.	sin.
i' i				
0 0	−1.820		− 7.663	
0 − 1	−0.184	−0.308	+ 1.171	+ 0.888
0 − 2	−0.023	+0.043	+ 0.068	− 0.135
0 − 3	+0.002	+0.003	− 0.004	− 0.004
1 + 3	+0.004	+0.001	− 0.005	− 0.001
1 + 2	0.000	−0.009	0.000	+ 0.017
1 + 1	−0.065	+0.021	+ 0.219	− 0.054
1 0	+0.151	+0.137	− 1.579	− 2.043
1 − 1	−0.339	+1.593	− 3.685	+18.109
1 − 2	−0.296	+0.198	+ 0.499	− 0.132
1 − 3	+0.024	+0.028	− 0.058	− 0.029
1 − 4	+0.001	−0.001	− 0.002	+ 0.004
2 + 2	0.000	−0.003	− 0.001	+ 0.005
2 + 1	−0.015	−0.008	+ 0.032	+ 0.014
2 0	−0.015	+0.078	+ 0.018	− 0.396
2 − 1	+0.477	+0.070	− 3.137	+ 1.798
2 − 2	−1.625	−0.663	+13.083	+ 5.559
2 − 3	+0.114	+0.120	− 0.090	+ 0.045
2 − 4	+0.027	−0.010	− 0.047	+ 0.034
2 − 5	+0.008	−0.002	− 0.009	+ 0.003
3 + 1	−0.003	0.000	+ 0.004	+ 0.001
3 0	−0.013	+0.021	+ 0.036	− 0.044
3 − 1	+0.129	+0.097	− 0.625	− 0.146
3 − 2	−0.027	−1.038	+ 1.489	+ 4.058
3 − 3	−1.941	+2.882	+ 7.037	−10.183
3 − 4	−0.109	−0.069	+ 0.508	+ 0.008
3 − 5	−0.019	−0.033	+ 0.045	+ 0.052
3 − 6	−0.002	−0.001	+ 0.004	+ 0.002
4 0	−0.005	+0.001	+ 0.009	− 0.002
4 − 1	+0.013	+0.027	− 0.065	− 0.076
4 − 2	+0.199	−0.224	− 0.344	+ 0.848
4 − 3	−1.622	+0.029	+ 4.601	− 0.928
4 − 4	+2.794	+2.881	− 6.912	− 7.300
4 − 5	−0.083	+0.308	+ 0.087	− 0.768
4 − 6	−0.035	+0.017	+ 0.058	− 0.045
4 − 7	+0.005	+0.001	− 0.003	− 0.003
5 0	−0.001	+0.001	+ 0.002	− 0.002
5 − 1	0.000	+0.008	− 0.001	− 0.017
5 − 2	+0.063	−0.009	− 0.143	+ 0.075
5 − 3	−0.335	−0.329	+ 0.996	+ 0.577

Arg$=i'g'+ig$	$\frac{a_0}{\mu}\left(r\frac{d}{dr}\right)^3\Omega$		$\frac{a_0'}{\mu'}\left(r'\frac{d}{dr'}\right)^3\Omega'$	
	cos.	sin.	cos.	sin.
i' i				
5— 4	−0.146	+2.011	−0.167	−4.613
5— 5	+3.176	−1.994	−6.566	+4.000
5— 6	+0.403	+0.153	−0.814	−0.234
5— 7	−0.028	+0.047	+0.003	−0.074
5— 8	−0.006	+0.019	+0.005	−0.023
6— 1	−0.003	0.000	+0.003	0.000
6— 2	+0.011	+0.005	−0.027	−0.004
6— 3	−0.015	−0.105	+0.086	+0.216
6— 4	−0.463	+0.418	+0.803	−1.026
6— 5	+2.096	+0.446	−4.150	−0.583
6— 6	−1.044	−2.925	+1.808	+5.294
6— 7	+0.226	−0.398	−0.359	+0.713
6— 8	+0.060	+0.003	−0.088	+0.014
6— 9	+0.015	+0.013	−0.018	−0.013
7— 2	+0.001	0.000	−0.003	−0.002
7— 3	+0.013	−0.018	−0.012	+0.041
7— 4	−0.152	+0.001	+0.284	−0.064
7— 5	+0.435	+0.577	−0.925	−0.972
7— 6	+0.728	−1.902	−1.127	+3.383
7— 7	−2.358	+0.285	+3.884	−0.415
7— 8	−0.318	−0.266	+0.531	+0.415
7— 9	+0.006	−0.057	+0.003	+0.086
8— 3	−0.001	−0.002	0.000	+0.005
8— 4	−0.024	−0.018	+0.050	+0.024
8— 5	−0.017	+0.195	−0.023	−0.336
8— 6	+0.658	−0.374	−1.060	+0.725
8— 7	−1.524	−0.901	+2.502	+1.390
8— 8	−0.145	+1.673	+0.283	−2.582
8— 9	−0.323	+0.225	+0.459	−0.353
9— 4	−0.005	−0.006	+0.009	+0.009
9— 5	−0.029	+0.047	+0.039	−0.076
9— 6	+0.294	+0.042	−0.434	−0.023
9 7	−0.293	−0.954	+0.511	+1.324
9— 4	−1.498	+1.357	+1.968	−1.948
9— 5	+1.523	+0.921	−2.023	−1.126
9— 6				
9— 7	−			
9— 8	− 2.			
9— 9	+ 2.69			

In the next place we obtain the expression of Y:

Arg=$\kappa\gamma+i'g'+ig$			Y	
			sin.	cos.
κ	i'	i	"	"
0	0	0		+ 0.29
1	0	−1	+42.30	− 0.13
−1	0	0	+ 4.11	− 3.93
0	0	−1	− 3.07	− 0.02
1	0	−2	− 0.32	+ 2.57
−1	0	−1	+ 0.20	+ 0.41
0	0	−2	+ 0.02	− 0.19
1	0	−3	− 0.14	− 0.04
−1	0	−2	− 0.02	+ 0.01
0	0	−3	+ 0.01	+ 0.01
1	0	−4	+ 0.01	− 0.01
−1	1	+3	+ 0.02	− 0.01
0	1	+2	− 0.01	− 0.04
1	1	+1	+ 0.01	+ 0.05
−1	1	+2	+ 0.22	+ 0.26
0	1	+1	+ 0.36	− 0.24
1	1	0	− 0.71	− 0.17
−1	1	+1	− 5.05	+ 3.33
0	1	0	+ 0.64	+ 3.00
1	1	−1	+ 4.79	− 4.52
−1	1	0	− 8.13	−41.32
0	1	−1	− 0.44	+ 0.10
1	1	−2	+ 5.15	+24.23
−1	1	−1	+ 1.23	+ 3.13
0	1	−2	− 0.35	− 1.77
1	1	−3	− 0.98	+ 0.58
−1	1	−2	− 0.20	+ 0.22
0	1	−3	+ 0.08	− 0.05
1	1	−4	+ 0.01	− 0.04
−1	1	−3	− 0.01	− 0.01
0	1	−4	0.00	+ 0.02
1	1	−5	+ 0.01	− 0.01
−1	2	+2	+ 0.04	+ 0.01
0	2	+1	+ 0.03	− 0.06
1	2	0	− 0.09	+ 0.02
−1	2	+1	− 0.32	+ 0.76
0	2	0	+ 0.75	+ 0.35
1	2	−1	+ 0.19	− 0.96
−1	2	0	−10.24	− 4.81
0	2	−1	− 6.12	+ 2.66
1	2	−2	+ 4.49	+ 3.75
−1	2	−1	+84.45	−35.85
0	2	−2	− 0.20	− 0.15
1	2	−3	−13.17	+ 5.88
−1	2	−2	− 1.83	− 1.77
0	2	−3	+ 0.97	− 0.42
1	2	−4	− 0.65	− 0.14
−1	2	−3	− 0.25	− 0.16
0	2	−4	+ 0.05	+ 0.02
1	2	−5	0.00	+ 0.01
−1	2	−4	0.00	− 0.01
0	2	−5	0.00	0.00
1	2	−6	0.00	+ 0.01
−1	3	+1	+ 0.01	+ 0.09
0	3	0	+ 0.15	− 0.03
1	3	−1	− 0.06	− 0.13
−1	3	0	− 2.04	+ 0.30
0	3	−1	− 0.81	+ 1.45
1	3	−2	+ 1.08	+ 0.10
−1	3	−1	+11.27	−19.84
0	3	−2	− 3.53	− 4.98
1	3	−3	− 2.34	+ 3.94
−1	3	−2	+47.54	+68.68
0	3	−3	− 0.12	− 0.19
1	3	−4	− 4.93	− 6.64
−1	3	−3	+ 3.97	− 1.27
0	3	−4	+ 0.34	+ 0.50
1	3	−5	− 0.22	− 0.44
−1	3	−4	+ 0.17	− 0.29
0	3	−5	+ 0.01	+ 0.03
1	3	−6	− 0.02	− 0.01
−1	4	0	− 0.24	+ 0.17
0	4	−1	+ 0.03	+ 0.30
1	4	−2	+ 0.16	− 0.09
−1	4	−1	− 0.48	− 4.03
0	4	−2	− 1.87	− 0.69
1	4	−3	+ 0.07	+ 1.01
−1	4	−2	+25.61	+ 9.49
0	4	−3	+ 3.28	− 3.57
1	4	−4	− 3.12	− 1.16
−1	4	−3	−45.37	+48.01
0	4	−4	+ 0.13	− 0.27
1	4	−5	+ 3.07	− 3.54
−1	4	−4	+ 1.47	+ 4.85
0	4	−5	− 0.23	+ 0.24
1	4	−6	+ 0.24	− 0.31
−1	4	−5	+ 0.35	+ 0.19
0	4	−6	− 0.01	+ 0.02
1	4	−7	+ 0.01	− 0.03

Arg=$\varkappa\gamma+i'g'+ig$		Y	
		sin.	cos.
$\varkappa$	i' i	″	″
−1	5— 0	− 0.01	+ 0.04
0	5— 1	+ 0.03	+ 0.04
1	5— 2	+ 0.01	− 0.02
−1	5— 1	− 0.41	− 0.47
0	5— 2	− 0.43	+ 0.11
1	5— 3	+ 0.12	+ 0.15
−1	5— 2	+ 5.61	− 1.47
0	5— 3	+ 0.35	− 1.96
1	5— 4	− 0.83	+ 0.19
−1	5— 3	− 4.78	+26.74
0	5— 4	+ 3.04	+ 1.79
1	5— 5	+ 0.38	− 2.26
−1	5— 4	−40.99	−24.83
0	5— 5	+ 0.32	+ 0.01
1	5— 6	+ 2.32	+ 1.22
−1	5— 5	− 4.62	+ 1.93
0	5— 6	− 0.15	− 0.11
1	5— 7	+ 0.26	+ 0.09
−1	5— 6	− 0.19	+ 0.39
0	5— 7	− 0.03	0.00
1	5— 8	+ 0.02	0.00
−1	5— 7	+ 0.01	+ 0.04
0	5— 8	+ 0.01	0.00
1	5— 9	0.00	− 0.02
−1	6— 1	− 0.08	− 0.03
0	6— 2	− 0.05	+ 0.06
1	6— 3	+ 0.03	0.00
−1	6— 2	+ 0.66	− 0.77
0	6— 3	− 0.20	− 0.45
1	6— 4	− 0.13	+ 0.14
−1	6— 3	+ 2.72	+ 6.25
0	6— 4	+ 1.74	− 0.03
1	6— 5	− 0.26	− 0.64
−1	6— 4	−23.94	+ 0.30
0	6— 5	− 0.75	+ 2.29
1	6— 6	+ 1.52	− 0.04
−1	6— 5	+10.51	−31.00
0	6— 6	+ 0.06	+ 0.28
1	6— 7	− 0.35	+ 1.41
−1	6— 6	− 2.29	− 3.70
0	6— 7	+ 0.05	− 0.12
1	6— 8	0.00	+ 0.18
−1	6— 7	− 0.39	− 0.13
0	6— 8	+ 0.01	− 0.01
1	6— 9	0.00	+ 0.02
−1	6— 8	− 0.03	0.00
0	6— 9	0.00	+ 0.01
1	6—10	− 0.01	− 0.01
−1	7— 2	+ 0.01	− 0.16
0	7— 3	− 0.09	− 0.05
1	7— 4	− 0.01	+ 0.03
−1	7— 3	+ 1.16	+ 0.73
0	7— 4	+ 0.43	− 0.29
1	7— 5	− 0.14	− 0.10
−1	7— 4	− 5.85	+ 3.86
0	7— 5	+ 0.30	+ 1.40
1	7— 6	+ 0.45	− 0.28
−1	7— 5	− 4.16	−18.98
0	7— 6	− 1.58	− 0.14
1	7— 7	0.00	+ 0.94
−1	7— 6	+21.27	+ 2.17
0	7— 7	− 0.20	+ 0.10
1	7— 8	− 0.80	− 0.01
−1	7— 7	+ 2.61	− 2.34
0	7— 8	+ 0.06	+ 0.04
1	7— 9	− 0.12	+ 0.06
−1	7— 8	+ 0.06	− 0.37
0	7— 9	+ 0.01	0.00
1	7—10	− 0.01	+ 0.02
−1	8— 2	0.00	− 0.02
0	8— 3	− 0.02	0.00
1	8— 4	0.00	0.00
−1	8— 3	+ 0.22	+ 0.01
0	8— 4	+ 0.04	− 0.11
1	8— 5	− 0.03	0.00
−1	8— 4	− 0.64	+ 1.44
0	8— 5	+ 0.33	+ 0.34
1	8— 6	+ 0.05	− 0.11
−1	8— 5	− 4.52	− 4.71
0	8— 6	− 0.97	+ 0.44
1	8— 7	+ 0.30	+ 0.26
−1	8— 6	+13.57	− 5.95
0	8— 7	− 0.14	− 0.98
1	8— 8	− 0.55	+ 0.27
−1	8— 7	+ 1.73	+13.38
0	8— 8	− 0.12	− 0.14
1	8— 9	− 0.10	− 0.41
−1	8— 8	+ 2.07	+ 1.60
0	8— 9	+ 0.01	+ 0.03
1	8—10	− 0.02	− 0.07

Arg=$\varkappa\gamma+i'g'+ig$			Y		Arg=$\varkappa\gamma+i'g'+ig$			Y	
			sin.	cos.				sin.	cos.
$\varkappa$	i'	i	"	"	$\varkappa$	i'	i	"	"
−1	9	−3	+0.03	−0.02	−1	9	−6	+3.24	−4.58
0	9	−4	−0.01	−0.02	0	9	−7	−0.06	−0.64
1	9	−5	−0.01	0.00	1	9	−8	−0.13	+0.22
−1	9	−4	+0.03	+0.31	−1	9	−7	+6.27	+8.76
0	9	−5	+0.11	+0.02	0	9	−8	+0.57	−0.23
1	9	−6	−0.01	−0.03	1	9	−9	−0.23	−0.30
−1	9	−5	−1.60	−0.41	−1	9	−8	−7.76	+2.89
0	9	−6	−0.24	+0.33	0	9	−9	−0.51	+0.03
1	9	−7	+0.11	+0.02	1	9	−10	+0.17	−0.03

In like manner we have the expression of Y′:

Arg=$\varkappa\gamma'+i'g'+ig$			Y′		Arg=$\varkappa\gamma'+i'g'+ig$			Y′	
			sin.	cos.				sin.	cos.
$\varkappa$	i'	i	"	"	$\varkappa$	i'	i	"	"
0	0	0		+4.6	−1	4	−1	−45.4	−3.9
−1	1	0	−433.8	−5.8	0	3	−1	+16.1	+14.4
−1	2	0	−120.3	+101.6	1	2	−1	+10.7	−0.1
0	1	0	+73.0	+1.1	−1	5	−1	−6.1	+3.5
1	0	0	+33.7	−54.4	0	4	−1	+3.8	+0.2
−1	3	0	−6.0	+23.2	1	3	−1	+1.1	−0.8
0	2	0	+10.3	−8.6	−1	6	−1	−0.4	+0.8
1	1	0	−0.6	−8.0	0	5	−1	+0.5	−0.3
−1	4	0	+1.2	+3.1	1	4	−1	+0.1	−0.2
0	3	0	+0.5	−2.0					
1	2	0	−0.5	−0.7	−1	0	−2	0.0	+0.2
−1	5	0	+0.4	+0.3	0	−1	−2	+0.3	+0.1
0	4	0	−0.1	−0.3	1	−2	−2	−0.2	−0.3
1	3	0	−0.1	0.0	−1	1	−2	+1.3	+2.6
					0	0	−2	+4.7	+10.4
−1	−1	−1	+0.5	+0.2	1	−1	−2	−3.4	−1.9
0	−2	−1	+0.6	−0.5	−1	2	−2	−24.1	+16.1
1	−3	−1	−1.2	+0.1	0	1	−2	+21.5	−9.7
−1	0	−1	+4.5	−0.7	1	0	−2	−55.1	−122.3
0	−1	−1	+2.0	−5.3	−1	3	−2	+865.8	−365.0
1	−2	−1	−7.6	+5.4	0	2	−2	+4.7	−6.2
−1	1	−1	+37.5	−6.2	1	1	−2	−255.3	+112.6
0	0	−1	+14.3	+69.9	−1	4	−2	+163.7	−260.1
1	−1	−1	−25.0	+63.3	0	3	−2	−73.1	+29.8
−1	2	−1	−200.0	−1012.4	1	2	−2	−31.0	+57.9
0	1	−1	−8.2	−3.1	−1	5	−2	−3.1	−65.3
1	0	−1	−171.9	−827.9	0	4	−2	−13.8	+21.8
−1	3	−1	−192.3	−169.7	1	3	−2	+1.7	+11.5
0	2	−1	+16.1	+85.4	−1	6	−2	−7.0	−9.1
1	1	−1	+60.9	+42.2					

Arg=$\varkappa\gamma+i'g'+ig$			Y′	
			sin.	cos.
$\varkappa$	i'	i	″	″
0	5	−2	+0.3	+5.5
1	4	−2	+1.2	+1.2
−1	7	−2	−1.7	−0.6
0	6	−2	+0.6	+0.7
1	5	−2	+0.3	+0.1
−1	8	−2	−0.2	0.0
0	7	−2	+0.2	0.0
1	6	−2	+0.1	−0.1
−1	1	−3	−0.1	+0.1
0	0	−3	+0.2	+0.9
1—	1	−3	+0.1	−0.1
−1	2	−3	−2.2	+1.8
0	1	−3	+0.3	+0.7
1	0	−3	−1.8	−11.6
−1	3	−3	−4.9	−46.7
0	2	−3	+8.1	+10.9
1	1	−3	−8.6	−9.4
−1	4	−3	+424.0	+610.3
0	3	−3	+4.5	+5.2
1	2	−3	−95.1	−131.5
−1	5	−3	+283.3	+114.2
0	4	−3	−34.7	−51.8
1	3	−3	−48.9	−16.6
−1	6	−3	+75.2	−15.7
0	5	−3	−23.7	−9.9
1	4	−3	−10.5	+3.1
−1	7	−3	+10.8	−10.9
0	6	−3	−6.3	+1.3
1	5	−3	−1.3	+1.6
−1	8	−3	+0.5	−2.5
0	7	−3	−1.0	+1.0
1	6	−3	−0.1	+0.3
−1	9	−3	−0.2	−0.4
0	8	−3	0.0	+0.2
1	7	−3	0.0	0.0
−1	2	−4	−0.1	+0.1
0	1	−4	−0.1	+0.1
1	0	−4	−0.1	−0.8
−1	3	−4	−2.3	−2.6
0	2	−4	+0.1	+0.6
1	1	−4	+0.5	−0.1
−1	4	−4	+57.0	−5.2
0	3	−4	−5.0	+6.0
1	2	−4	+0.1	−7.4
−1	5	−4	−367.8	+393.9
0	4	−4	−5.3	+3.5

Arg=$\varkappa\gamma'+i'g'+ig$			Y′	
			sin.	cos.
$\varkappa$	i'	i	″	″
1	3	−4	+62.0	−69.5
−1	6	−4	−51.9	+264.4
0	5	−4	+31.4	−32.4
1	4	−4	+5.6	−37.1
−1	7	−4	+28.8	+73.3
0	6	−4	+4.6	−22.3
1	5	−4	−4.2	−8.6
−1	8	−4	+14.5	+10.4
0	7	−4	−2.4	−6.1
1	6	−4	−1.7	−1.0
−1	9	−4	+3.5	+0.1
0	8	−4	−1.2	−0.8
1	7	−4	−0.8	−0.1
−1	10	−4	+0.5	−0.3
0	9	−4	−0.2	0.0
1	8	−4	−0.1	+0.1
−1	3	−5	−0.1	−0.1
0	2	−5	−0.1	0.0
1	1	−5	−0.1	+0.1
−1	4	−5	+3.0	−3.1
0	3	−5	−0.5	+0.3
1	2	−5	−0.2	+0.1
−1	5	−5	+10.8	+55.4
0	4	−5	−4.1	−1.9
1	3	−5	+4.1	−3.3
−1	6	−5	−317.3	−186.9
0	5	−5	−3.1	−4.6
1	4	−5	+46.3	+25.6
−1	7	−5	−218.5	+0.6
0	6	−5	+26.2	+16.1
1	5	−5	+25.8	−1.1
−1	8	−5	−62.1	+38.6
0	7	−5	+18.3	+0.2
1	6	−5	+6.1	−4.4
−1	9	−5	−8.0	+16.6
0	8	−5	+5.2	−3.2
1	7	−5	+0.7	−1.6
−1	10	−5	+0.5	+3.9
0	9	−5	+0.7	−1.3
1	8	−5	−0.1	−0.4
−1	4	−6	+0.1	−0.2
0	3	−6	−0.1	0.0
1	2	−6	0.0	0.0
−1	5	−6	+3.7	+3.0
0	4	−6	−0.4	−0.2
1	3	−6	+0.1	−0.1

Arg=$\kappa\gamma'+i'g'+ig$			Y'		Arg=$\kappa\gamma'+i'g'+ig$			Y'	
			sin.	cos.				sin.	cos.
κ	i'	i	″	″	κ	i'	i	″	″
−1	6	−6	−46.3	+16.6	−1	10	−7	+30.2	−42.5
0	5	−6	+0.3	−2.6	0	9	−7	−9.3	+4.0
1	4	−6	+3.8	+1.9	1	8	−7	−2.3	+3.4
−1	7	−6	+73.2	−229.1	−1	6	−8	0.0	+0.4
0	6	−6	+3.6	−2.6	0	5	−8	+0.1	0.0
1	5	−6	−8.1	+28.4	1	4	−8	0.0	0.0
−1	8	−6	−35.1	−163.1	−1	7	−8	−3.9	−1.4
0	7	−6	−6.7	+18.9	0	6	−8	+0.2	0.0
1	6	−6	+2.9	+16.6	1	5	−8	+0.2	+0.2
−1	9	−6	−42.9	−46.3	−1	8	−8	+22.5	−19.4
0	8	−6	+2.9	+13.6	0	7	−8	+0.4	+0.9
1	7	−6	+4.4	+3.9	1	6	−8	−2.1	+0.5
−1	10	−6	−17.2	−4.7	−1	9	−8	+14.9	+91.7
0	9	−6	+3.6	+3.9	0	8	−8	−1.6	+2.0
1	8	−6	+1.4	+0.3	1	7	−8	−1.6	−8.9
−1	5	−7	+0.3	+0.1	−1	10	−8	+51.1	+69.3
0	4	−7	0.0	−0.1	0	9	−8	−1.2	−7.7
1	3	−7	+0.1	0.0	1	8	−8	−3.8	−5.5
−1	6	−7	−2.4	+3.9	−1	7	−9	−0.4	+0.1
0	5	−7	0.0	−0.3	0	6	−9	−0.1	0.0
1	4	−7	+0.2	0.0	1	5	−9	+0.1	+0.1
−1	7	−7	−19.4	−34.1	−1	8	−9	+0.7	−3.6
0	6	−7	+1.6	−0.1	0	7	−9	+0.2	+0.1
1	5	−7	−0.2	+3.0	1	6	−9	−0.2	+0.2
−1	8	−7	+151.1	+11.9	−1	9	−9	+16.6	+13.2
0	7	−7	+2.6	+2.6	0	8	−9	−0.4	+0.4
1	6	−7	−16.4	−0.7	1	7	−9	−0.8	−1.3
−1	9	−7	+111.0	−49.0	−1	10	−9	−52.4	+23.7
0	8	−7	−12.5	−1.2	0	9	−9	−1.5	−1.0
1	7	−7	−9.9	+5.0	1	8	−9	+4.0	−1.3

The developments of the three factors, which must be specially computed for δ^2T, follow:

Arg=$\kappa\gamma+i'g'+ig$			$r^2\frac{d^2T}{dr^2}$		$rr'\frac{d^2T}{drdr'}$		$r'^2\frac{d^2T}{dr'^2}$	
			sin.	cos.	sin.	cos.	sin.	cos.
κ	i'	i	″	″	″	″	″	″
0	0	0		+0.46		−0.35		
1	0	−1	+216.02	−0.46	−131.94	+0.53	−37.81	−0.54
−1	0	0	−5.41	+5.60	+13.88	−16.22	−29.36	+32.28
0	0	1	+3.01	−7.78	−8.18	+11.86	+19.36	−16.93
1	0	−2	+0.95	+2.74	−2.60	+2.06	+3.58	−9.59

Arg=$\varkappa\gamma+i'g'+ig$			$r^2\frac{d^2T}{dr^2}$		$rr'\frac{d^2T}{dr\,dr'}$		$r'^2\frac{d^2T}{dr'^2}$	
			sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″
−1	0	−1	−0.56	−1.49	+1.19	+2.57	−2.07	−4.12
0	0	−2	+0.87	+1.06	−1.20	−1.65	+1.67	+2.51
1	0	−3	−0.10	+0.02	−0.13	−0.14	+0.40	+0.23
−1	0	−2	+0.11	−0.09	−0.19	+0.12	+0.28	−0.17
0	0	−3	−0.11	+0.09	+0.16	−0.10	−0.21	+0.12
1	0	−4	+0.07	+0.05	−0.06	−0.06	+0.04	+0.07
−1	1	+3	+0.01	+0.08	+0.01	−0.11	−0.05	+0.14
0	1	+2	−0.02	+0.13	+0.03	−0.23	−0.04	+0.35
1	1	+1	0.00	−0.31	−0.01	+0.45	+0.02	−0.66
−1	1	+2	−0.16	+0.07	+0.78	+0.38	−1.52	−1.10
0	1	+1	−0.90	−0.70	+1.68	+0.75	−3.09	−0.63
1	1	0	+1.33	+0.45	−3.02	−1.13	+5.66	+2.06
−1	1	+1	−7.49	+4.47	−3.01	+4.96	+22.00	−19.12
0	1	0	+0.96	+4.43	−0.80	−3.79		
1	1	−1	+6.58	−8.66	+4.21	+0.55	−23.30	+12.98
−1	1	0	−19.95	−96.21	−1.75	−12.11	+36.97	+190.20
0	1	−1	−15.45	−73.41	+25.07	+124.14	−37.07	−189.36
1	1	−2	+35.53	+170.25	−27.45	−132.20	+12.68	+63.22
−1	1	−1	−11.34	−10.15	+15.46	+17.80	−17.57	−31.29
0	1	−2	+6.91	+4.65	−6.51	−10.31	+5.01	+19.46
1	1	−3	+2.16	+3.25	−5.22	−1.99	+9.60	−0.68
−1	1	−2	+1.02	−0.98	−1.62	+1.67	+2.49	−2.67
0	1	−3	−0.77	+1.07	+1.32	−1.71	−2.09	+2.55
1	1	−4	+0.14	+0.01	−0.25	+0.03	+0.44	−0.13
−1	1	−3	0.00	+0.07	−0.05	−0.10	+0.10	+0.13
0	1	−4	−0.10	0.00	+0.14	+0.02	−0.19	−0.02
1	1	−5	+0.11	0.00	−0.10	−0.01	+0.10	+0.01
−1	2	+2	−0.20	+0.17	+0.32	−0.16	−0.45	+0.14
0	2	+1	−0.14	−0.03	+0.22	−0.02	−0.36	+0.12
1	2	0	+0.40	−0.21	−0.63	+0.23	+0.99	−0.28
−1	2	+1	−0.60	−0.34	+0.24	+2.41	+0.63	−5.40
0	2	0	+1.03	+0.58	−0.98	−0.41		
1	2	−1	−0.36	−0.20	+1.41	−1.88	−3.17	+5.14
−1	2	0	+3.54	−7.26	−33.38	−4.40	+85.91	+20.53
0	2	−1	−28.62	−4.87	+48.86	+6.66	−66.99	−12.18
1	2	−2	+23.91	+13.31	−25.23	−2.98	+26.11	−13.87
−1	2	−1	−25.48	+9.13	+251.14	−104.85	−703.79	+297.96
0	2	−2	+186.78	−79.19	−398.87	+169.60	+701.32	−298.81
1	2	−3	−142.94	+61.66	+181.31	−77.64	−234.17	+99.53
−1	2	−2	+9.77	−7.13	−14.07	+1.45	+19.13	+11.32
0	2	−3	−1.27	−4.48	−3.03	+15.30	+10.18	−30.64
1	2	−4	−4.22	+7.69	+6.63	−12.83	−10.22	+19.84
−1	2	−3	+1.25	+0.36	−2.02	−0.90	+3.08	+1.79
0	2	−4	−1.33	−0.96	+1.65	+1.96	−1.93	−3.34
1	2	−5	+0.09	+0.64	−0.03	−1.05	−0.10	+1.64

Arg=κγ+i′g′+ig			$r^2\frac{d^2T}{dr^2}$		$rr'\frac{d^2T}{drdr'}$		$r'^2\frac{d^2T}{dr'^2}$	
			sin.	cos.	sin.	cos.	sin.	cos.
κ	i′	i	″	″	″	″	″	″
−1	2	−4	+ 0.25	+ 0.06	− 0.25	− 0.09	+ 0.28	+ 0.14
0	2	−5	− 0.04	− 0.08	+ 0.06	+ 0.16	− 0.08	− 0.26
1	2	−6	− 0.24	+ 0.02	+ 0.24	− 0.05	− 0.24	+ 0.08
−1	3	+2	− 0.10	− 0.04	+ 0.10	+ 0.04	− 0.11	− 0.03
0	3	+1	− 0.02	+ 0.02	+ 0.02	− 0.02	− 0.03	+ 0.03
1	3	0	+ 0.11	+ 0.01	− 0.13	+ 0.01	+ 0.15	− 0.03
−1	3	+1	− 0.21	− 0.47	+ 0.32	+ 0.72	− 0.44	− 1.09
0	3	0	+ 0.21	− 0.04	− 0.19	+ 0.04		
1	3	−1	− 0.01	+ 0.50	− 0.01	− 0.84	+ 0.02	+ 1.37
−1	3	0	+ 1.94	− 2.24	− 7.74	+ 3.78	+ 17.67	− 6.51
0	3	−1	− 5.00	+ 3.62	+ 8.58	− 5.05	− 12.08	+ 5.34
1	3	−2	+ 2.89	− 1.55	− 2.09	+ 3.60	+ 0.86	− 6.59
−1	3	−1	+ 2.87	+ 29.16	+ 23.87	− 88.52	− 83.24	+190.18
0	3	−2	+ 12.96	− 68.50	− 39.23	+116.27	+ 82.22	−174.71
1	3	−3	− 12.33	+ 39.74	+ 11.88	− 50.83	− 11.83	+ 64.69
−1	3	−2	− 81.31	−122.95	+226.15	+332.17	−465.38	−675.68
0	3	−3	+170.11	+246.40	−299.38	−429.37	+467.82	+666.44
1	3	−4	− 85.07	−120.85	+115.71	+164.38	−154.50	−219.21
−1	3	−3	− 3.27	+ 8.82	+ 16.85	− 11.97	− 39.13	+ 15.20
0	3	−4	+ 21.21	+ 0.62	− 39.24	− 5.00	+ 62.48	+ 11.34
1	3	−5	− 13.53	− 3.74	+ 20.36	+ 5.98	− 29.04	− 8.99
−1	3	−4	− 0.53	+ 1.74	+ 1.21	− 2.70	− 2.30	+ 3.98
0	3	−5	+ 1.69	− 1.83	− 3.12	+ 2.37	+ 4.98	− 2.93
1	3	−6	− 0.85	+ 0.25	+ 1.47	− 0.32	− 2.28	+ 0.36
−1	3	−5	− 0.08	+ 0.05	+ 0.11	− 0.10	− 0.16	+ 0.17
0	3	−6	+ 0.08	− 0.13	− 0.19	+ 0.20	+ 0.32	− 0.28
1	3	−7	0.00	+ 0.11	+ 0.04	− 0.11	− 0.10	+ 0.11
−1	4	0	+ 0.26	− 0.56	− 0.89	+ 1.20	+ 1.97	− 2.22
0	4	−1	− 0.28	+ 1.04	+ 0.66	− 1.40	− 1.14	+ 1.58
1	4	−2	− 0.11	− 0.56	+ 0.44	+ 0.74	− 0.89	− 1.00
−1	4	−1	+ 6.40	+ 6.66	− 9.55	− 18.63	+ 11.69	+ 38.70
0	4	−2	− 9.02	− 13.25	+ 10.13	+ 22.55	− 8.98	− 33.96
1	4	−3	+ 4.36	+ 6.33	− 6.48	− 7.50	+ 8.99	+ 8.90
−1	4	−2	− 67.54	− 2.47	+149.09	+ 28.01	−274.98	− 75.63
0	4	−3	+114.79	+ 15.00	−181.43	− 40.08	+262.95	+ 77.15
1	4	−4	− 53.17	− 7.01	+ 70.89	+ 8.60	− 92.45	− 11.07
−1	4	−3	+149.90	−152.38	−297.70	+308.83	+514.56	−540.04
0	4	−4	−226.88	+239.96	+350.01	−374.56	−500.86	+540.59
1	4	−5	+ 91.46	− 98.39	−123.79	+133.83	+162.62	−176.63
−1	4	−4	− 9.71	− 14.64	+ 14.04	+ 31.84	− 19.24	− 57.03
0	4	−5	+ 2.92	+ 35.33	− 1.80	− 55.84	0.00	+ 80.98
1	4	−6	+ 1.09	− 17.18	− 2.02	+ 24.54	+ 3.27	− 33.45
−1	4	−5	− 2.13	− 0.44	+ 3.33	+ 1.22	− 4.91	− 2.40
0	4	−6	+ 2.77	+ 2.40	− 3.66	− 4.00	+ 4.68	+ 5.99
1	4	−7	− 0.88	− 1.36	+ 1.11	+ 2.05	− 1.37	− 2.92

Arg=$\kappa\gamma+i'g'+ig$			$r^2\frac{d^2T}{dr^2}$		$rr'\frac{d^2T}{dr\,dr'}$		$r'^2\frac{d^2T}{dr'^2}$	
			sin.	cos.	sin.	cos.	sin.	cos.
κ	i'	i	″	″	″	″	″	″
−1	4	−6	+0.15	−0.06	−0.07	+0.09	−0.03	−0.15
0	4	−7	−0.02	−0.04	−0.08	−0.07	+0.20	+0.20
1	4	−8	−0.32	−0.16	+0.36	+0.20	−0.41	−0.25
−1	5	0	+0.03	−0.20	−0.05	+0.33	+0.10	−0.51
0	5	−1	+0.06	+0.16	−0.03	−0.21	−0.03	+0.24
1	5	−2	−0.12	+0.07	+0.17	−0.07	−0.25	+0.09
−1	5	−1	+1.99	+0.30	−3.54	−1.57	+5.53	+3.82
0	5	−2	−2.90	−0.80	+3.77	+1.87	−4.40	−3.33
1	5	−3	+1.27	+0.12	−1.68	+0.04	+2.17	−0.29
−1	5	−2	−14.44	+12.90	+32.11	−19.22	−59.36	+26.05
0	5	−3	+23.73	−17.02	−38.11	+20.42	+55.83	−22.39
1	5	−4	−9.72	+7.98	+12.65	−10.72	−16.20	+13.86
−1	5	−3	−2.88	−103.08	−10.00	+191.92	+33.07	−319.08
0	5	−4	−1.92	+150.54	+15.35	−223.01	−35.05	+309.81
1	5	−5	−1.59	−60.95	+1.57	+81.13	−1.17	−105.01
−1	5	−4	+190.17	+121.17	−329.26	−205.25	+520.72	+319.87
0	5	−5	−263.84	−159.18	+380.46	+225.99	−519.19	−304.80
1	5	−6	+98.32	+57.77	−130.66	−76.16	+168.54	+97.54
−1	5	−5	+21.64	−13.33	−38.42	+19.72	+61.40	−27.81
0	5	−6	−41.86	+10.55	+60.77	−13.37	−83.19	+16.37
1	5	−7	+18.21	−2.55	−24.88	+3.07	+32.72	−3.62
−1	5	−6	−0.92	−2.77	+0.15	+4.17	+0.93	−5.94
0	5	−7	−2.89	+3.69	+4.34	−4.93	−6.09	+6.34
1	5	−8	+3.07	−1.14	−3.69	+1.54	+4.44	−2.00
−1	5	−7	−0.28	−0.77	+0.29	+0.89	−0.30	−1.05
0	5	−8	−0.02	+0.55	+0.11	−0.69	−0.21	+0.85
1	5	−9	+0.26	+0.19	−0.32	−0.17	+0.38	+0.13
−1	6	−1	+0.36	−0.15	−0.65	+0.11	+1.05	0.00
0	6	−2	−0.46	+0.19	+0.64	−0.12	−0.80	−0.01
1	6	−3	+0.12	−0.17	−0.15	+0.23	+0.17	−0.30
−1	6	−2	−0.90	+4.22	+2.82	−7.02	−6.01	+10.66
0	6	−3	+1.48	−5.72	−3.22	+7.56	+5.56	−9.33
1	6	−4	+0.02	+2.45	−0.03	−3.18	0.01	+4.04
−1	6	−3	−21.36	−22.37	+31.77	+42.83	−44.45	−72.37
0	6	−4	+27.31	+32.50	−34.13	−49.39	+40.66	+69.88
1	6	−5	−12.14	−11.79	+15.66	+15.65	−19.66	−20.24
−1	6	−4	+122.64	−19.71	−204.58	+22.65	+315.61	−24.03
0	6	−5	−163.95	+22.95	+230.39	−23.57	−308.33	+22.24
1	6	−6	+61.07	−11.58	−79.93	+14.42	+101.78	−17.46
−1	6	−5	−69.52	+189.75	+105.96	−297.82	−152.95	+439.39
0	6	−6	+81.42	−244.11	−108.35	+332.83	+139.12	−435.70
1	6	−7	−27.54	+86.15	+35.13	−111.51	−43.68	+140.56
−1	6	−6	+17.44	+23.11	−25.41	−36.86	+35.41	+54.85
0	6	−7	−18.67	−40.32	+24.19	+55.29	−30.44	−72.62
1	6	−8	+6.07	+16.57	−7.65	−21.85	+9.43	+27.92

Arg=$\varkappa y+i'g'+ig$			$r^3\frac{d^3T}{dr^3}$		$rr'\frac{d^2T}{drdr'}$		$r'^3\frac{d^3T}{dr'^3}$	
			sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″
−1	6	− 7	+ 3.51	− 0.32	− 4.97	− 0.22	+ 6.79	+ 0.96
0	6	− 8	− 4.58	− 2.23	+ 6.01	+ 3.34	− 7.62	− 4.65
1	6	− 9	+ 1.34	+ 1.79	− 1.84	− 2.29	+ 2.40	+ 2.87
−1	6	− 8	+ 0.62	− 0.58	− 0.75	+ 0.60	+ 0.90	− 0.62
0	6	− 9	− 0.54	+ 0.13	+ 0.70	− 0.08	− 0.88	+ 0.02
1	6	−10	− 0.14	+ 0.34	+ 0.07	− 0.39	+ 0.01	+ 0.44
−1	7	− 2	+ 0.40	+ 0.72	− 0.40	− 1.25	+ 0.32	+ 1.98
0	7	− 3	− 0.40	− 0.92	+ 0.32	+ 1.30	− 0.15	− 1.71
1	7	− 4	+ 0.20	+ 0.24	− 0.29	− 0.33	+ 0.41	+ 0.45
−1	7	− 3	− 7.21	− 0.89	+ 11.34	+ 3.10	− 16.68	− 6.56
0	7	− 4	+ 9.32	+ 1.89	− 12.23	− 3.98	+ 15.29	+ 6.71
1	7	− 5	− 3.72	− 0.21	+ 4.79	+ 0.28	− 6.00	− 0.42
−1	7	− 4	+ 26.86	− 30.38	− 46.53	+ 44.57	+ 73.55	− 62.26
0	7	− 5	− 36.08	+ 38.18	+ 52.37	− 48.31	− 71.80	+ 59.03
1	7	− 6	+ 11.97	− 16.05	− 15.79	+ 20.26	+ 20.26	− 25.00
−1	7	− 5	+ 38.84	+121.90	− 54.75	−188.25	+ 74.00	+274.58
0	7	− 6	− 47.86	−153.55	+ 60.27	+206.96	− 73.27	−268.55
1	7	− 7	+ 19.08	+ 53.95	− 25.35	− 69.25	+ 32.02	+ 86.67
−1	7	− 6	−162.06	− 21.37	+237.62	+ 28.94	−333.04	− 37.99
0	7	− 7	+197.79	+ 18.97	−258.58	− 22.68	+327.73	+ 26.61
1	7	− 8	− 67.44	− 5.55	+ 85.12	+ 6.46	−105.05	− 7.41
−1	7	− 7	− 19.75	+ 20.23	+ 29.58	− 28.61	− 42.02	+ 38.95
0	7	− 8	+ 33.12	− 24.40	− 43.60	+ 31.08	+ 55.51	− 38.60
1	7	− 9	− 13.62	+ 8.84	+ 17.31	− 10.86	− 21.46	+ 13.12
−1	7	− 8	+ 0.49	+ 3.61	− 0.22	− 5.02	− 0.15	+ 6.73
0	7	− 9	+ 1.49	− 5.19	− 2.18	+ 6.59	+ 2.97	− 8.16
1	7	−10	− 1.28	+ 1.91	+ 1.61	− 2.39	− 1.96	+ 2.93
−1	8	− 3	− 1.25	+ 0.73	+ 2.05	− 0.79	− 3.12	+ 0.78
0	8	− 4	+ 1.60	− 0.80	− 2.21	+ 0.76	+ 2.89	− 0.62
1	8	− 5	− 0.53	+ 0.49	+ 0.68	− 0.61	− 0.87	+ 0.74
−1	8	− 4	+ 0.37	− 10.41	− 2.30	+ 15.57	+ 5.25	− 22.16
0	8	− 5	− 1.37	+ 12.73	+ 3.26	− 16.54	− 5.73	+ 20.75
1	8	− 6	− 0.15	− 4.60	+ 0.07	+ 5.94	+ 0.05	− 7.46
−1	8	− 5	+ 38.62	+ 26.01	− 55.06	− 42.11	+ 75.44	+ 63.42
0	8	− 6	− 47.25	− 33.29	+ 59.45	+ 46.64	− 72.75	− 62.32
1	8	− 7	+ 18.87	+ 10.15	− 23.27	− 13.30	+ 28.20	+ 16.93
−1	8	− 6	−104.69	+ 56.36	+152.82	− 78.30	−213.49	+105.13
0	8	− 7	+126.47	− 69.18	−164.87	+ 86.30	+208.51	−105.06
1	8	− 8	− 42.74	+ 27.18	+ 53.74	− 33.39	− 66.09	+ 40.30
−1	8	− 7	− 10.46	−121.53	+ 16.84	+169.68	− 25.01	−228.78
0	8	− 8	+ 19.86	+143.20	− 26.75	−181.20	+ 34.66	+223.75
1	8	− 9	− 8.65	− 48.52	+ 10.82	+ 59.69	− 13.29	− 72.11
−1	8	− 8	− 22.09	− 14.33	+ 29.73	+ 20.43	− 38.98	− 27.97
0	8	− 9	+ 25.80	+ 23.32	− 32.25	− 29.79	+ 39.43	+ 37.06
1	8	−10	− 8.03	− 9.45	+ 10.13	+ 11.73	− 12.45	− 14.29

Arg $=\varkappa\gamma+i'g'+ig$			$r^2\frac{d^2T}{dr^2}$		$rr'\frac{d^2T}{drdr'}$		$r'^2\frac{d^2T}{dr'^2}$	
			sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	"	"	"	"	"	"
−1	9	−3	−0.20	+0.21	+0.30	−0.29	−0.45	+0.38
0	9	−4	+0.17	−0.24	−0.26	+0.28	+0.36	−0.32
1	9	−5	+0.06	+0.09	−0.07	−0.13	+0.07	+0.17
−1	9	−4	−1.25	−2.29	+1.48	+3.36	−1.67	−4.73
0	9	−5	+1.37	+2.26	−1.45	−3.07	+1.45	+3.99
1	9	−6	−0.78	−0.21	+0.93	+0.42	−1.09	−0.67
−1	9	−5	+15.37	−0.68	−21.12	−0.50	+28.31	+2.33
0	9	−6	−15.56	+0.19	+19.89	+1.08	−24.68	−2.77
1	9	−7	+3.09	−0.99	−4.49	+1.05	+6.07	−1.09
−1	9	−6	−21.76	+51.70	+32.97	−68.45	−47.44	+88.92
0	9	−7	+26.15	−51.55	−34.98	+64.29	+45.35	−78.28
1	9	−8	−5.52	+10.84	+7.66	−15.22	−10.09	+20.10
−1	9	−7	−80.16	−87.80	+104.62	+119.20	−132.53	−157.83
0	9	−8	+77.19	+92.04	−95.38	−116.86	+115.40	+144.73
1	9	−9	−11.58	−21.12	+17.73	+28.19	−24.53	−36.03
−1	9	−8	+95.51	−44.99	−123.61	+55.82	+157.31	−69.05
0	9	−9	−94.56	+37.38	+115.58	−47.08	−138.90	+57.98
1	9	−10	+18.20	+3.15	−24.59	−0.03	+31.61	−3.43

And in a similar manner we have the developments of the three factors specially computed for δ^2T':

Arg $=\varkappa\gamma'+i'g'+ig$			$r'^2\frac{d^2T'}{dr'^2}$		$rr'\frac{d^2T'}{drdr'}$		$r^2\frac{d^2T'}{dr^2}$	
			sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	"	"	"	"	"	"
0	0	0		+11.7		−3.3		
−1	1	0	−5796.1	−27.0	+2077.7	+10.1	−1460.6	−2.9
−1	2	0	−979.6	+954.2	+446.3	−497.0	−272.6	+264.6
0	1	0	+449.2	−304.5	−130.9	+160.5	+61.8	−72.3
1	0	0	+409.4	−415.9	−277.9	+257.3	+190.0	−169.3
−1	3	0	−28.5	+226.5	+6.3	−128.4	−6.1	+74.4
0	2	0	+24.7	−113.2	+1.1	+59.0	−2.0	−29.2
1	1	0	+18.2	−77.0	−13.1	+53.5	+9.2	−38.2
−1	4	0	+16.6	+30.4	−11.6	−18.0	+7.0	+11.0
0	3	0	−8.5	−17.9	+6.9	+9.3	−4.2	−4.8
1	2	0	−2.9	−9.8	+2.3	+7.2	−1.9	−5.4
−1	5	0	+4.7	+2.3	−3.3	−1.3	+2.1	+0.8
0	4	0	−2.7	−1.6	+1.8	+0.8	−1.1	−0.3
1	3	0	−1.2	−0.7	+0.9	+0.5	−0.8	−0.3
−1	6	0	+0.6	−0.2	−0.6	+0.2	+0.6	−0.2
0	5	0	−0.4	−0.3	+0.2	+0.3	−0.1	−0.3
1	4	0	−0.3	+0.6	+0.3	−0.6	−0.3	+0.6

Arg $=\varkappa\gamma'+i'g'+ig$			$r'^2\frac{d^2T'}{dr'^2}$		$rr'\frac{d^2T'}{drdr'}$		$r^2\frac{d^2T'}{dr^2}$	
			sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″
−1	−2	−1	+ 1.0	0.0	− 0.8	0.0	+ 0.7	− 0.1
0	−3	−1	+ 0.6	+ 0.6	− 0.4	− 0.4	+ 0.3	+ 0.3
1	−4	−1	− 1.6	− 0.8	+ 1.2	+ 0.6	− 1.0	− 0.4
−1	−1	−1	+ 7.4	+ 1.9	− 5.6	− 1.7	+ 4.3	+ 1.5
0	−2	−1	+ 5.8	+ 0.1	− 3.7	− 0.7	+ 2.2	+ 0.7
1	−3	−1	− 14.7	− 2.7	+ 10.3	+ 2.7	− 7.3	− 2.4
−1	0	−1	+ 54.1	− 15.8	− 38.5	+ 11.9	+ 28.8	− 2.9
0	−1	−1	+ 23.7	− 17.8	− 14.9	+ 6.2	+ 9.1	− 0.1
1	−2	−1	− 85.3	+ 38.9	+ 57.7	− 22.3	− 38.8	+ 12.0
−1	1	−1	+ 354.9	− 222.2	− 194.0	+ 316.2	+ 66.1	− 492.1
0	0	−1	− 22.8	− 104.6	− 40.0	− 192.1		
1	−1	−1	− 295.8	+ 494.7	+ 205.2	− 258.0	− 153.3	+ 73.7
−1	2	−1	−1685.8	−8498.2	+ 463.4	+2442.1	+ 503.5	+ 2298.6
0	1	−1	+ 601.5	+3035.7	+ 535.7	+2487.1	−2344.6	−11174.2
1	0	−1	+ 595.8	+2887.3	− 591.9	−2865.5	+2632.4	+12603.3
−1	3	−1	−1856.7	−1392.6	+ 969.5	+ 515.0	− 430.9	+ 40.8
0	2	−1	+ 954.9	+ 808.5	− 488.3	− 102.8	+ 122.8	− 551.6
1	1	−1	+ 328.6	+ 378.9	− 225.8	− 279.0	+ 181.8	+ 275.0
−1	4	−1	− 445.3	+ 19.6	+ 246.3	− 45.9	− 127.2	+ 52.7
0	3	−1	+ 276.2	+ 1.7	− 149.4	+ 43.1	+ 66.3	− 70.5
1	2	−1	+ 76.3	+ 29.8	− 55.4	− 21.4	+ 42.9	+ 17.0
−1	5	−1	− 57.2	+ 46.2	+ 31.4	− 32.2	− 16.6	+ 21.0
0	4	−1	+ 38.9	− 30.4	− 19.7	+ 23.8	+ 8.2	− 17.5
1	3	−1	+ 12.7	− 0.7	− 9.8	+ 0.6	+ 7.7	− 0.7
−1	6	−1	− 2.7	+ 10.9	+ 1.0	− 7.4	− 0.3	+ 4.8
0	5	−1	+ 2.1	− 7.5	− 0.5	+ 5.2	− 0.3	− 3.3
1	4	−1	+ 1.7	− 1.5	− 1.4	+ 1.3	+ 1.1	− 1.1
−1	7	−1	+ 0.6	+ 1.1	− 0.6	− 0.7	+ 0.5	+ 0.4
0	6	−1	0.0	− 1.3	0.0	+ 0.9	+ 0.1	− 0.8
1	5	−1	− 0.6	+ 0.3	+ 0.6	+ 0.3	− 0.7	+ 0.3
−1	−1	−2	− 0.1	+ 1.0	+ 0.1	− 1.0	− 0.1	+ 1.0
0	−2	−2	− 0.1	+ 0.2	+ 0.1	− 0.2	− 0.1	+ 0.2
1	−3	−2	+ 0.3	− 1.3	− 0.3	+ 1.1	+ 0.3	− 1.0
−1	0	−2	+ 0.3	+ 4.1	− 0.3	− 3.2	+ 0.3	+ 3.0
0	−1	−2	+ 0.9	+ 1.3	− 0.3	− 1.0	+ 0.1	+ 0.9
1	−2	−2	− 1.5	− 5.6	+ 0.8	+ 4.3	− 0.5	− 3.4
−1	1	−2	+ 20.0	+ 31.9	− 11.8	− 12.0	+ 3.4	− 15.3
0	0	−2	+ 5.8	− 0.6	− 6.5	− 21.1		
1	−1	−2	− 27.5	− 29.1	+ 18.9	+ 25.9	− 14.1	− 27.1
−1	2	−2	− 63.6	+ 186.3	− 74.7	− 262.7	+ 168.6	+ 462.3
0	1	−2	+ 47.7	− 38.3	+ 80.1	+ 404.2	− 257.6	− 1143.4
1	0	−2	− 228.5	− 54.9	+ 126.8	− 72.1	+ 131.9	+ 1080.8
−1	3	−2	+8208.7	−3452.1	−4262.6	+1766.8	+1969.8	− 779.9
0	2	−2	−4284.6	+1827.5	+2447.3	−1015.9	−1160.0	+ 420.7
1	1	−2	− 913.7	+ 393.2	+ 580.1	− 250.9	− 436.5	+ 196.4

Arg=$\varkappa\gamma'+i'g'+ig$			$r'^2\frac{d^2T'}{dr'^2}$		$rr'\frac{d^2T'}{drdr'}$		$r^2\frac{d^2T'}{dr^2}$	
			sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″
−1	4	−2	+1421.5	−2704.8	−693.4	+1550.5	+297.2	−817.3
0	3	−2	−943.1	+1685.1	+455.0	−1030.8	−166.4	+560.3
1	2	−2	−219.1	+136.2	+153.8	−99.4	−115.6	+90.7
−1	5	−2	−126.9	−676.6	+114.0	+393.6	−85.7	−212.6
0	4	−2	+74.1	+470.6	−86.1	−281.6	+75.9	+151.2
1	3	−2	−38.8	+46.4	+27.9	−36.6	−18.9	+31.9
−1	6	−2	−94.4	−85.3	+65.0	+46.7	−41.7	−23.2
0	5	−2	+67.8	+64.5	−50.3	−34.7	+34.3	+15.4
1	4	−2	−2.9	+11.3	+1.7	−9.1	−0.5	+7.5
−1	7	−2	−21.3	−2.3	+14.1	−0.1	−8.9	+1.1
0	6	−2	+16.1	+1.9	−11.0	+0.4	+7.0	−1.5
1	5	−2	+0.9	+2.2	−0.8	−1.7	+0.9	+1.3
−1	8	−2	−2.7	+1.3	+1.9	−1.1	−1.2	+0.8
0	7	−2	+2.4	−1.4	−1.6	+1.2	+0.9	−1.0
1	6	−2	+0.1	+0.2	0.0	−0.3	−0.1	+0.3
−1	0	−3	−1.0	−0.2	+1.0	+0.2	−0.9	−0.1
0	−1	−3	−0.1	0.0	+0.1	0.0	−0.1	0.0
1	−2	−3	+1.0	+0.2	−1.0	−0.2	+0.9	+0.2
−1	1	−3	−1.3	+0.8	+1.2	+0.3	−1.2	−2.2
0	0	−3	0.0	+0.2	−0.4	−1.7		
1	−1	−3	+1.4	−1.1	−0.9	+1.0	+0.4	−1.4
−1	2	−3	−22.6	+23.6	+11.6	−26.3	−2.2	+40.9
0	1	−3	+5.6	−2.0	+3.3	+32.8	−17.3	−93.5
1	0	−3	+8.6	−14.9	−9.3	+0.7	+24.9	+83.6
−1	3	−3	−80.2	−352.3	+78.0	+143.2	−60.5	−21.9
0	2	−3	−22.1	+151.7	+1.4	−67.7	+10.6	+10.5
1	1	−3	−0.4	−109.3	−6.3	+77.1	+5.3	−52.2
−1	4	−3	+4530.9	+6572.2	−2632.4	−3853.4	+1384.5	+2050.9
0	3	−3	−2857.9	−4072.6	+1832.6	+2630.2	−1043.8	−1512.9
1	2	−3	+27.9	+40.2	−17.5	−22.7	−24.0	−36.7
−1	5	−3	+3225.4	+1044.1	−1986.2	−545.1	+1134.6	+240.9
0	4	−3	−2234.6	−756.8	+1477.2	+403.7	−896.5	−167.9
1	3	−3	+82.1	−100.2	−53.1	+79.5	+14.4	−67.7
−1	6	−3	+839.3	−292.4	−516.2	+225.3	+294.6	−162.5
0	5	−3	−627.6	+204.0	+403.7	−177.0	−237.6	+142.1
1	4	−3	−1.7	−53.9	+5.3	+41.1	−10.8	−29.4
−1	7	−3	+105.6	−153.6	−60.2	+107.5	+30.6	−71.4
0	6	−3	−83.3	+117.2	+47.1	−87.3	−22.5	+61.5
1	5	−3	−10.5	−10.4	+9.2	+7.6	−8.3	−4.8
−1	8	−3	−1.0	−33.7	+3.1	+23.0	−3.8	−14.9
0	7	−3	+0.1	+26.7	−2.7	−18.6	+3.9	+12.2
1	6	−3	−3.1	+0.3	+2.6	−0.4	−2.0	+0.6
−1	9	−3	−1.9	−4.9	+1.3	+3.2	−0.7	−2.1
0	8	−3	+2.1	+3.7	−1.7	−2.5	+1.2	+1.5
1	7	−3	−1.1	+0.3	+0.9	−0.3	−0.7	+0.3

Arg=$\kappa\gamma'+i'g'+ig$			$r'^2\frac{d^2T'}{dr'^2}$		$rr'\frac{d^2T'}{drdr'}$		$r^2\frac{d^2T'}{dr^2}$	
			sin.	cos.	sin.	cos.	sin.	cos.
κ	i'	i	″	″	″	″	″	″
−1	2	−4	−0.8	−0.1	+0.3	−0.6	+0.3	+2.0
0	1	−4	+0.1	+0.3	+0.4	+2.2	−1.3	−6.9
1	0	−4	+0.7	+0.4	−0.6	−1.0	+1.7	+7.2
−1	3	−4	−29.3	−23.8	+20.1	+12.3	−12.9	−5.4
0	2	−4	+6.2	+11.7	−5.3	−6.4	+4.2	+2.6
1	1	−4	+7.5	+0.1	−5.8	−0.6	+4.3	+0.9
−1	4	−4	+556.0	−80.0	−305.8	+72.0	+140.6	−56.8
0	3	−4	−298.8	−38.1	+182.8	+15.3	−94.9	+1.0
1	2	−4	+81.0	−1.4	−58.0	−0.9	+38.2	−0.1
−1	5	−4	−4480.7	+4737.3	+2870.5	−3002.6	−1712.0	+1766.4
0	4	−4	+3058.4	−3307.0	−2143.3	+2296.9	+1392.9	−1474.2
1	3	−4	−344.7	+369.5	+248.5	−266.4	−149.8	+159.8
−1	6	−4	−438.8	+3303.9	+216.3	−2161.1	−76.4	+1328.4
0	5	−4	+342.6	−2451.2	−171.0	+1719.4	+52.9	−1131.3
1	4	−4	+67.5	+252.7	−57.9	−183.8	+50.7	+115.6
−1	7	−4	+477.5	+885.1	−354.1	−573.0	+251.7	+347.3
0	6	−4	−362.3	−694.5	+291.4	+472.6	−224.5	−299.5
1	5	−4	+77.1	+36.4	−60.7	−23.4	+45.3	+10.7
−1	8	−4	+211.7	+103.2	−150.9	−59.7	+102.8	+30.1
0	7	−4	−169.0	−86.4	+127.6	+51.2	−92.1	−25.8
1	6	−4	+20.4	−6.3	−15.5	+6.0	+11.1	−5.9
−1	9	−4	+46.1	−8.1	−31.9	+8.6	+21.0	−8.2
0	8	−4	−37.9	+6.0	+27.3	−7.4	−18.6	+7.7
1	7	−4	+1.2	−4.4	−0.8	+3.7	+0.6	−3.0
−1	10	−4	+7.1	−5.8	−4.9	+4.5	+3.4	−3.5
0	9	−4	−5.3	+4.6	+3.7	−3.8	−2.4	+3.0
1	8	−4	−0.8	−0.5	+0.7	+0.4	−0.6	−0.3
−1	3	−5	−2.7	−1.1	+2.2	+0.6	−1.8	−0.4
0	2	−5	0.0	+0.6	−0.1	−0.4	+0.1	+0.2
1	1	−5	+1.4	+0.6	−1.5	−0.5	+1.6	+0.4
−1	4	−5	+30.7	−39.2	−17.2	+27.0	+8.5	−17.6
0	3	−5	−19.5	+11.5	+11.9	−9.1	−6.2	+6.9
1	2	−5	+1.6	+3.6	−0.4	−2.9	−0.5	+2.3
−1	5	−5	+148.8	+637.4	−112.0	−396.1	+79.6	+223.7
0	4	−5	−9.5	−401.7	+13.8	+274.3	−16.4	−171.4
1	3	−5	+3.3	+86.4	−2.5	−63.0	+3.2	+41.9
−1	6	−5	−4256.3	−2569.7	+2876.8	+1762.1	−1838.0	−1146.6
0	5	−5	+3176.5	+1857.6	−2332.6	−1381.7	+1621.3	+975.9
1	4	−5	−538.0	−318.3	+407.2	+241.4	−283.0	−168.6
−1	7	−5	−2992.0	+170.8	+2056.5	−165.6	−1344.1	+149.2
0	6	−5	+2328.2	−126.8	−1709.9	+140.3	+1196.0	−140.4
1	5	−5	−337.6	+94.6	+256.6	−78.3	−180.6	+63.5
−1	8	−5	−809.6	+633.7	+547.1	−469.3	−349.4	+335.7
0	7	−5	+658.0	−506.1	−467.8	+400.9	+314.1	−307.4
1	6	−5	−58.0	+104.2	+41.3	−83.7	−25.7	+64.5

Arg=$\varkappa\gamma'+i'g'+ig$			$r'^2\frac{d^2T'}{dr'^2}$		$rr'\frac{d^2T'}{dr\,dr'}$		$r^2\frac{d^2T'}{dr^2}$	
			sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″
−1	9	−5	− 76.8	+ 255.9	+ 43.3	− 186.3	− 19.9	+ 130.8
0	8	−5	+ 68.1	− 210.4	− 39.9	+ 161.2	+ 18.9	− 119.2
1	7	−5	+ 5.1	+ 28.7	− 4.9	− 22.1	+ 4.8	+ 16.1
−1	10	−5	+ 19.0	+ 59.2	− 16.9	− 43.2	+ 14.6	+ 30.5
0	9	−5	− 14.5	− 47.5	+ 14.1	+ 35.2	− 13.1	− 25.1
1	8	−5	+ 5.1	+ 0.7	− 4.2	+ 0.3	+ 3.6	− 1.2
−1	4	−6	+ 1.2	− 2.4	− 0.7	+ 1.6	+ 0.4	− 0.9
0	3	−6	− 1.2	+ 0.9	+ 0.7	− 0.7	− 0.3	+ 0.5
1	2	−6	− 0.3	− 0.1	+ 0.3	+ 0.1	− 0.3	− 0.1
−1	5	−6	+ 48.6	+ 31.6	− 33.8	− 18.4	+ 22.3	+ 9.3
0	4	−6	− 20.0	− 24.8	+ 15.3	+ 16.4	− 11.2	− 9.6
1	3	−6	+ 1.1	+ 5.4	− 0.8	− 3.6	+ 0.6	+ 1.9
−1	6	−6	− 603.8	+ 236.4	+ 403.5	− 171.9	− 252.7	+ 119.6
0	5	−6	+ 425.6	− 89.4	− 309.9	+ 70.9	+ 212.5	− 54.6
1	4	−6	− 94.2	+ 15.9	+ 71.0	− 12.3	− 50.0	+ 9.7
−1	7	−6	+1141.1	−3407.0	− 828.9	+2419.6	+ 579.2	−1645.8
0	6	−6	− 844.7	+2665.7	+ 661.1	−2040.6	− 498.7	+1499.9
1	5	−6	+ 180.3	− 548.1	− 142.3	+ 429.0	+ 107.1	− 318.2
−1	8	−6	− 631.8	−2437.8	+ 478.5	+1744.9	− 350.6	−1199.3
0	7	−6	+ 509.2	+1964.2	− 412.5	−1495.9	+ 322.8	+1096.9
1	6	−6	− 148.8	− 340.2	+ 116.7	+ 266.8	− 85.4	− 198.6
−1	9	−6	− 726.6	− 646.5	+ 545.7	+ 452.2	− 397.7	− 301.4
0	8	−6	+ 601.0	+ 539.9	− 479.4	− 396.7	+ 372.4	+ 278.2
1	7	−6	− 124.4	− 59.1	+ 102.2	+ 44.1	− 82.0	− 30.2
−1	10	−6	− 294.9	− 35.4	+ 223.6	+ 15.8	− 165.7	− 2.2
0	9	−6	+ 234.7	+ 34.5	− 183.3	− 16.9	+ 138.9	+ 3.7
1	8	−6	− 20.9	+ 8.0	+ 13.4	− 7.5	− 6.6	+ 6.8
−1	5	−7	+ 2.5	+ 1.2	− 1.2	− 0.7	+ 0.3	+ 0.3
0	4	−7	− 1.6	− 1.8	+ 1.2	+ 1.3	− 0.8	− 0.9
1	3	−7	+ 1.6	+ 0.2	− 1.3	− 0.2	+ 1.3	+ 0.2
−1	6	−7	− 19.1	+ 56.9	+ 8.3	− 40.9	− 0.9	+ 28.2
0	5	−7	+ 26.6	− 28.2	− 18.8	+ 21.7	+ 12.3	− 16.0
1	4	−7	− 16.2	+ 2.9	+ 14.4	− 1.9	− 12.7	+ 1.0
−1	7	−7	− 299.3	− 496.0	+ 220.3	+ 348.6	− 158.4	− 233.5
0	6	−7	+ 163.6	+ 381.2	− 128.8	− 289.7	+ 98.5	+ 210.6
1	5	−7	− 32.6	− 91.3	+ 26.2	+ 71.1	− 20.5	− 52.7
−1	8	−7	+2473.7	+ 241.4	−1827.3	− 193.0	+1304.9	+ 150.4
0	7	−7	−2003.5	− 157.2	+1584.4	+ 136.2	−1214.6	− 115.2
1	6	−7	+ 462.5	+ 42.7	− 371.9	− 35.8	+ 287.9	+ 29.3
−1	9	−7	+1800.0	− 876.8	−1331.8	+ 668.6	+ 953.1	− 496.2
0	8	−7	−1488.8	+ 733.5	+1167.2	− 595.8	− 887.7	+ 472.6
1	7	−7	+ 288.2	− 186.9	− 231.6	+ 154.6	+ 179.3	− 124.5
−1	10	−7	+ 449.3	− 801.4	− 323.9	+ 625.8	+ 223.9	− 480.3
0	9	−7	− 378.1	+ 625.4	+ 283.7	− 504.2	− 204.2	+ 397.0
1	8	−7	+ 37.4	− 79.7	− 26.6	+ 56.2	+ 16.8	− 34.5

Arg $=\varkappa\gamma'+i'g'+ig$			$r'^2\frac{d^2T'}{dr'^2}$		$rr'\frac{d^2T'}{dr\,dr'}$		$r^2\frac{d^2T'}{dr^2}$	
			sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″
−1	6	−8	+ 1.6	+ 8.5	− 1.6	− 6.9	+ 1.5	+ 5.7
0	5	−8	+ 1.0	− 3.3	− 0.5	+ 2.7	+ 0.1	− 2.2
1	4	−8	− 1.2	− 2.6	+ 1.2	+ 2.6	− 1.1	− 2.6
−1	7	−8	− 61.8	− 14.0	+ 45.9	+ 7.1	− 32.9	− 2.1
0	6	−8	+ 35.6	+ 21.1	− 28.0	− 15.1	+ 21.3	+ 10.0
1	5	−8	− 4.5	− 9.8	+ 3.3	+ 8.0	− 2.2	− 5.9
−1	8	−8	+ 357.5	− 320.7	− 260.5	+ 241.0	+ 182.5	− 175.6
0	7	−8	− 296.8	+ 209.1	+ 233.1	− 167.9	− 177.0	+ 131.4
1	6	−8	+ 78.8	− 48.6	− 63.8	+ 40.1	+ 49.8	− 32.6
−1	9	−8	+ 219.8	+1639.8	− 155.4	−1249.9	+ 103.3	+ 926.9
0	8	−8	− 216.3	−1367.9	+ 166.3	+1110.5	− 123.2	− 879.9
1	7	−8	+ 53.4	+ 346.0	− 43.4	− 285.4	+ 34.1	+ 229.0
−1	10	−8	+1060.6	+1264.7	− 848.5	− 975.7	+ 671.0	+ 737.0
0	9	−8	− 798.7	−1015.4	+ 654.6	+ 814.9	− 525.5	− 637.9
1	8	−8	+ 96.6	+ 157.5	− 63.2	− 119.3	+ 32.0	+ 84.1
−1	7	−9	− 7.5	+ 3.9	+ 6.1	− 3.8	− 4.9	+ 3.5
0	6	−9	+ 3.4	+ 0.1	− 2.7	+ 0.1	+ 2.2	− 0.3
1	5	−9	+ 1.6	− 2.3	− 1.7	+ 2.4	+ 1.7	− 2.3
−1	8	−9	+ 5.0	− 59.6	− 1.6	+ 45.0	− 1.0	− 33.1
0	7	−9	− 13.9	+ 39.1	+ 10.3	− 31.4	− 7.2	+ 24.6
1	6	−9	+ 7.7	− 7.4	− 6.7	+ 6.0	+ 5.6	− 4.6
−1	9	−9	+ 306.7	+ 230.1	− 237.2	− 173.7	+ 179.3	+ 127.1
0	8	−9	− 215.8	− 201.2	+ 176.2	+ 162.0	− 140.6	− 126.9
1	7	−9	− 46.6	+ 54.1	− 37.7	− 44.1	+ 29.5	+ 34.8
−1	10	−9	−1103.5	+ 524.7	+ 886.3	− 429.3	− 702.4	+ 349.9
0	9	−9	+ 848.0	− 357.5	− 704.1	+ 289.2	+ 573.2	− 228.4
1	8	−9	− 145.0	− 11.2	+ 108.2	+ 27.7	− 73.8	− 43.1

We have in the next place to attend to the following factors of δ^2T and δ^2T'. As the multiplications by which these quantities are formed naturally divide themselves into three sections independent of each other, according as the terms produced are independent of the factors nt or $n't$, or involve the first power of these factors, or in the last place their squares, we will divide the consideration of the second factors of δ^2T and δ^2T' into corresponding portions.

Of these fourteen factors four, viz., $n\delta^2z$, $n'\delta^2z'$, $\delta\nu$, $\delta\nu'$, have already been given; but from $\delta\nu$ and $\delta\nu'$ must be subtracted the constants which have virtually been supposed to belong to these quantities in Chapter I, when we derived the value of α. Now, from the present investigation, it appears that the value $+1''.622$ of the constant term of $l(r')$, given by HANSEN,[*] is six times too large. Thus it is necessary to attribute to the constant terms of $\delta\nu$ and $\delta\nu'$ severally the values $+0''.0665$ and $-1''.387$.

---

[*] Gegenseitige Störungen des Jupiter und Saturn, s. 167.

We have next to form the ten squares and products of the four quantities $n\delta z$, $n'\delta z'$, ν, and ν'. In doing this it will be more accurate to employ for $n\delta z$, $n'\delta z'$, etc., not their values, as given by considering first-order terms only, but as they are after $n\delta^2 z$, $n'\delta^2 z'$, etc., have been added to them. In this connection it is plain that the values, which ought to be attributed to the constant terms of ν and ν', are the same as those just given in the case of $\delta\nu$ and $\delta\nu'$. It is convenient to give these squares and products the factor $\frac{1''}{2}$ (log=4.38454), as, on multiplication by the first factor of $\delta^2\mathrm{T}$ or $\delta^2\mathrm{T}'$, the resulting product is in seconds of arc as we desire it.

The following tables contain the developments of the portions of these squares and products which are independent of the factors nt or $n't$ or their powers. The coefficients are uniformally carried to nine places of decimals; and the decimal point is omitted:

Arg= $i'g'+ig$		$(n\delta z)^2\times\frac{1''}{2}$		$(n\delta z)(n'\delta z')\times\frac{1''}{2}$		$(n'\delta z')^2\times\frac{1''}{2}$		$(n\delta z)\nu\times\frac{1''}{2}$		$(n'\delta z')\nu\times\frac{1''}{2}$	
		cos.	sin.	cos.	sin.	cos.	sin.	sin.	cos.	sin.	cos.
i'	i										
0	0	+9121		−21153		+54062			− 11		+ 26
1	0	+ 73	+ 160	− 4805	−3011	+20252	+11864	+ 17	− 20	−264	+ 205
2	0	+ 540	+ 967	− 228	− 379	− 454	− 1119	+121	−213	− 85	+ 144
3	0	− 6	−2764	− 2	+3576	+ 24	− 1115	0	+789	− 3	−1884
4	0	+ 21	+ 8	− 45	− 9	+ 95	+ 5	+ 7	− 3	− 18	+ 6
−3	− 1	− 3	+ 40	+ 5	− 49			+ 2	+ 21	− 3	− 51
−2	− 1	+ 222	+ 35	− 315	− 64	+ 216	+ 50	− 81	+ 12	+189	− 39
−1	− 1	+ 199	+ 132	− 310	− 195	+ 167	+ 122	− 64	+ 42	+176	− 109
0	− 1	− 496	−1997	+ 444	+2305	+ 203	+ 193	+137	−508	−205	+1145
1	− 1	− 120	+ 27	+ 56	+ 778	− 155	+ 113	+ 16	+ 2	− 1	+ 398
2	− 1	+ 82	− 434	+ 357	+ 993	− 1149	− 4427	+ 3	− 25	− 14	− 12
3	− 1	+ 411	−1468	− 2395	+4739	+ 5976	−13326	+ 38	+124	−240	− 352
4	− 1	+1105	+ 244	− 1375	− 253	+ 28	− 201	+183	− 42	−437	+ 96
5	− 1	− 15	− 12	+ 103	+ 47	− 392	− 142	− 5	+ 8	+ 15	− 25
6	− 1	− 3	− 16	+ 6	+ 22	− 2	− 12	− 1	+ 5	+ 1	− 13
7	− 1	+ 2	− 4	− 3	+ 6	+ 1	− 4	+ 1	+ 2	− 2	− 3
−2	− 2	+ 5	+ 3	− 6	− 4	+ 3	+ 2	− 3	+ 1	+ 7	− 3
−1	− 2	+ 26	− 53	− 35	+ 68	+ 28	− 59	− 12	− 21	+ 24	+ 41
0	− 2	+ 7	− 40	− 48	+ 71	+ 48	− 35	+ 1	− 17	+ 32	+ 57
1	− 2	+ 21	− 23	− 47	+ 51	+ 47	− 29	− 5	− 6	+ 30	+ 35
2	− 2	− 18	− 4	− 53	+ 24	− 16	− 87	+ 3	+ 3	− 30	+ 17
3	− 2	− 169	+ 220	+ 347	− 285	− 94	− 103	+ 30	+ 46	−176	− 99
4	− 2	− 103	+ 22	+ 298	− 273	− 1007	+ 300	+ 15	+ 1	− 56	− 40
5	− 2	− 65	+ 94	+ 154	− 207	− 434	+ 589	− 1	+ 1	+ 1	− 17
6	− 2	+ 122	− 97	− 135	+ 114	− 151	+ 101	+ 9	+ 12	− 31	− 35
7	− 2	+ 80	− 16	− 77	+ 17	− 81	+ 14	+ 17	+ 4	− 38	− 10
8	− 2	− 24	− 9	+ 33	+ 12	− 11	− 4	− 6	+ 3	+ 17	− 7
0	− 3	− 15	− 10	+ 18	+ 16	− 17	− 14	+ 5	− 5	− 11	+ 11
1	− 3	− 4	− 14	+ 6	+ 21	− 7	− 20	+ 4	− 4	− 5	+ 15
2	− 3	− 4	0	0	+ 27	− 4	− 22	+ 2	0	− 2	+ 18
3	− 3	− 104	+ 150	+ 40	− 29	− 13	+ 6	+ 44	+ 64	− 22	− 12
4	− 3	− 190	− 140	+ 504	+ 166	− 264	− 43	+ 66	− 54	−274	+ 89

Arg= $i'g'+ig$	$(n\delta z)^2\times\frac{1''}{2}$		$(n\delta z)(n'\delta z')\times\frac{1''}{2}$		$(n'\delta z')^2\times\frac{1''}{2}$		$(n\delta z)\nu\times\frac{1''}{2}$		$(n'\delta z')\nu\times\frac{1''}{2}$	
	cos.	sin.	cos.	sin.	cos.	sin.	sin.	cos.	sin.	cos.
i' i										
5— 3	+121	+70	−398	−100	−457	−230	−27	+27	+163	−63
6— 3	+930	+659	−1216	−1363	+452	+3576	−144	+103	+376	−341
7— 3	−651	+1621	+1907	−4801	−5497	+13674	+54	+109	−122	−250
8— 3	−176	+76	+433	−152	−1052	+278	−18	−7	+43	+19
9— 3	−4	−3	+4	0	−1	−1	−2	0	+2	−1
10— 3	+2	+7	−2	−9	+2	−1	0	−1	−2	+4
1— 4	−4	+3	+7	−5	−7	+5	+2	+2	−4	−3
2— 4	−4	+1	+6	−2	−9	+3	+2	+1	−5	−1
3— 4	−3	+6	+9	−4	−9	+2	+4	+3	−8	−3
4— 4	−144	−150	+26	+25	−8	−2	+81	−86	−21	+13
5— 4	+39	+219	+28	−44	−5	+22	−21	+106	−14	−26
6— 4	+34	−128	+211	−811	−26	+320	−15	−54	−123	−449
7— 4	−1756	+1909	+1951	−2259	−938	+929	+472	+533	−1058	−1210
8— 4	+1160	−273	−1291	+279	+1820	−362	−240	−55	+614	+133
9— 4	+235	+60	−4961	−1384	+22658	+6332	−27	+9	+30	−6
10— 4	+5932	+6184	−14390	−14957	+34894	+36150	+42	−62	−102	+147
11— 4	+1	+4	−7	+2	+32	−27			0	+1
2— 5	+1	+2	−1	−3	+1	+3	0	+1	+1	−2
3— 5	0	+1	0	−4	0	+4	0	+1	0	−2
4— 5	−5	−3	+1	−2	−1	+3	+3	−4	−1	−2
5— 5	−27	+22	+9	−6	−2	+1	+17	+13	−6	−6
6— 5	−12	+1	−1	−17	+7	+10	+9	−2	−4	−17
7— 5	+318	+202	−51	−80	+39	+33	−168	+106	+21	−63
8— 5	+43	+64	−231	−341	+131	+225	−8	+7	+141	−203
9— 5	+14	+179	+20	+360	+3	+181	−5	+51	−3	+164
10— 5	+1021	−2039	−1221	+2451	−51	+120	−248	−497	+613	+1225
11— 5	+6	−7	+20	−1	−143	+31	−2	0	+2	+3
12— 5	−11	−34	+32	+98	−89	−275	+1	−2	−3	+6
13— 5	0	−5	−2	+13	+5	−28				
6— 6	+3	+8	0	−2			−2	+6	+1	−1
7— 6	+9	+7	−3	−1	+3	−2	−6	+8	+2	−2
8— 6	+19	−20	−18	+6	+13	−7	−11	−12	+15	+3
9— 6	+83	−27	−87	+34	+73	−23	−36	−14	+57	+24
10— 6	−107	−37	−78	+19	+49	+10	+57	−21	+54	+24
11— 6	+18	+13	−12	−9	+13	+12	−5	+4	+6	−3
12— 6	−24	−44	+29	+45	−12	−29	+7	−15	−15	+28
13— 6	−6	+32	+6	−29	−2	+6	+1	+6	−4	−16
7— 7	+2	0	−1	0			−2	0		
8— 7	+2	−1	−2	+1	+1	−1	−1	0	+1	+1
9— 7	−1	−5	+2	+5	−2	−5	+2	−3	−1	+6
10— 7	−6	−21	+1	+28	−2	−26	+4	−10	0	+21
11— 7	+5	−15	−6	+18	+7	−17	−2	−6	+4	+11
12— 7	−5	+5	−5	+2	+5	−5	+5	+3	+2	+4
13— 7	0	−1	+12	−1	−5	−3	−2	+1	−8	−1
10— 8	−2	0	+2	+1	−1	0	+1	0	−1	+1
11— 8	−8	−1	+10	+1	−11	−1	+3	0	−6	+1
12— 8	−5	−3	+5	+4	−6	−4	+2	−1	−4	+3
12— 9	−1	+3	+1	−3	−1	+4	0	+1	−1	−2

Arg= $i'g'+ig$		$(n\delta z)\nu'\times\frac{1''}{2}$		$(n'\delta z')\nu'\times\frac{1''}{2}$		$\nu^2\times\frac{1''}{2}$		$\nu\nu'\times\frac{1''}{2}$		$\nu'^2\times\frac{1''}{2}$	
		sin.	cos.	sin.	cos.	cos.	sin.	cos.	sin.	cos.	sin.
i'	i										
0	0		+17		−33	+122		−12		+945	
1	0	−1846	+922	+5028	−2572	−45	−19	−47	−14	−255	−426
2	0	−169	+198	−89	+421	−8	−19	+116	+195	−75	−76
3	0	+1	−278	−15	+535	−5	+14	+1	−36	−11	+22
4	0	+17	−10	−38	+24					+1	0
−3	−1	+1	−2	−2	+4			0	+1		
−2	−1	+63	−7	−122	+28			+11	−4	−6	+1
−1	−1	+16	+7	−86	+69	−4	+3	−13	−16	+12	+22
0	−1	−141	+95	−77	+63	+35	+23	−95	−70	+33	+26
1	−1	−83	−549	+63	−28	−10	+11	−48	+279	+67	−3
2	−1	−7	+355	+120	−525	+41	−47	+38	+8	−46	−496
3	−1	−84	−765	+10	+1798	−96	+36	−24	−1	−698	+261
4	−1	+1223	−275	−3002	+698	−4	−2	−2	−3	+95	+30
5	−1	+18	−18	−54	+48			+2	+2	+2	−1
6	−1	+1	−2	−3	+9			0	+1	0	−1
−1	−2	+7	+17	−16	−33	−1	+2	0	−1	−1	+1
0	−2	+4	+9	−24	−20	+6	−2	−4	−2	+6	−2
1	−2	−8	−5	−33	−13	+4	−1	−11	+11	+2	−6
2	−2	−79	+17	+11	−38	+1	−7	−23	+2	−173	−70
3	−2	+131	+48	−2	−486	−16	−1	+63	+17	−30	+234
4	−2	−122	+215	+351	−70	−1	−21	−22	−88	+95	−30
5	−2	+3	+11	−23	+21	0	−2	+3	+4	−45	+17
6	−2	+80	+20	−195	−35	−1	0	−4	+2	+2	+5
7	−2	+9	0	−39	0			+3	0		
8	−2	+3	0	−6	+2			−1	0		
0	−3	−4	+3	+10	−8						
1	−3	−1	+1	+4	−11	+1	0	+1	+2	−1	−3
2	−3	−2	+2	+1	−11	+1	−1	0	+2	−7	−4
3	−3	+105	+157	−9	−34	+17	−27	+63	−89	−38	+58
4	−3	−227	+16	+112	−42	+8	+14	−105	−13	+14	+42
5	−3	+140	−57	+834	−69	−6	−9	+63	+28	+717	+52
6	−3	−1060	+488	+2429	−417	0	+2	0	+33	−120	−526
7	−3	−375	−720	+861	+1668	−2	0	+11	−15	−67	+99
8	−3	+78	+26	−188	−62			−1	+1	+7	−2
9	−3	+3	+2	−6	−6					−1	0
1	−4	−2	−1	+3	+3						
2	−4	−1	−1	+5	+1			+1	0	−1	0
3	−4	+3	+6	+4	+1	+1	−2	+3	−5	−2	+2
4	−4	−37	+37	+8	−7	+45	+48	−24	−23	+14	+15
5	−4	−17	+2	+24	+15	−11	−49	−8	−4	+22	+6
6	−4	−175	−546	+112	+167	−4	+25	−84	+297	+36	−110
7	−4	+112	−16	+452	+461	−29	+6	+133	−65	+29	−53
8	−4	−125	−58	−1000	−168	+6	+2	−4	−4	−642	+164

Arg= $i'g'+ig$	$(n\delta z)\nu'\times\frac{1''}{2}$		$(n'\delta z')\nu'\times\frac{1''}{2}$		$\nu^2\times\frac{1''}{2}$		$\nu\nu'\times\frac{1''}{2}$		$\nu'^2\times\frac{1''}{2}$	
	sin.	cos.	sin.	cos.	cos.	sin.	cos.	sin.	cos.	sin.
i' i										
9— 4	+2271	−645	−5306	+1480			−19	− 9	+198	+74
10— 4	− 247	+298	+ 598	− 721	0	− 1	+ 2	+ 3	− 9	−16
11— 4	− 4	− 3	+ 9	+ 8						
4— 5			+ 1	+ 2	+ 2	+ 3				
5— 5	− 11	− 5	+ 3	+ 4	+10	− 9	− 7	+ 6	+ 7	− 3
6— 5	− 8	− 13	+ 1	+ 7	+ 8	+ 2	− 8	+ 12	+ 4	− 3
7— 5	+ 53	− 33	− 33	+ 28	−95	−48	+32	+ 16	− 22	−14
8— 5	+ 54	− 80	− 76	+ 127	+19	+29	+16	+ 21	− 8	−14
9— 5	− 15	+288	− 8	+ 101	− 1	+ 5	− 8	−161	− 1	− 9
10— 5	− 34	− 61	+ 15	+ 41	+ 6	− 8	−13	+ 21	− 5	+ 6
11— 5	+ 3	+ 4	+ 9	− 19					+ 9	+ 5
12— 5	− 4	+ 15	+ 8	− 35					− 1	− 3
13— 5	− 1	− 1	0	+ 5						
6— 6	+ 1	− 4	0	+ 1	− 2	− 5	+ 1	+ 2	− 1	− 2
7— 6	+ 1	− 3	− 2	− 1	− 4	− 3	+ 2	+ 3	− 1	− 1
8— 6	+ 11	+ 8	− 10	− 8	− 7	+ 9	+ 6	− 7	− 6	+ 5
9— 6	+ 21	+ 12	− 43	− 15	− 9	+ 3	+ 4	− 6	− 5	+ 2
10— 6	+ 17	− 3	− 30	+ 4	+38	+ 5	+ 2	+ 1	− 2	0
11— 6	− 4	+ 4	− 7	+ 5			− 5	− 5	+ 2	+ 1
12— 6	0	− 6	+ 5	− 16			− 1	+ 3	+ 2	+ 3
13— 6	0	+ 6	+ 2	+ 14					+ 2	−13
7— 7	+ 1	0			− 1	0			− 1	0
8— 7	+ 1	0	0	− 1	− 1	0				
9— 7	− 2	+ 4	+ 1	− 6	+ 2	+ 2	− 2	− 3	+ 1	+ 3
10— 7	0	+ 8	+ 2	− 15	+ 2	+ 2	0	− 2	0	+ 2
11— 7	+ 2	+ 5	− 4	− 10	0	+ 1	0	− 1	0	+ 1
12— 7	+ 2	+ 2	− 4	− 3	+ 3	− 3				
13— 7	− 3	0	+ 3	0			− 1	0	+ 1	0
14— 7	+ 7	− 4	+ 3	0			+ 5	+ 1		
10— 8	− 2	0	+ 1	− 1	+ 1	0	− 1	0	+ 1	0
11— 8	− 4	+ 1	+ 6	− 1	+ 1	0			+ 1	0
12— 8	− 2	+ 1	+ 4	− 3						
12— 9	0	− 1	+ 1	+ 2						

We are now in possession of all the quantities needed for the calculation of the portions of δ^2T and δ^2T′ which are not multiplied by the factors *nt*, *n′t* or their powers. Hence, proposing to stop the approximation with the terms of three dimensions, we avail ourselves of the possibility of obtaining a higher degree of precision by computing the terms of these quantities which depend severally on the arguments γ and γ', and applying the resulting corrections to $n\delta z$, $n\delta^2 z$, ν, $\delta\nu$, $n'\delta z'$, $n'\delta^2 z'$, ν', and $\delta\nu'$. This course of proceeding is exactly similar to that we employed with the second-order terms in Chapter IX

There has been found

$$\begin{aligned}
\delta^2\mathrm{T} &= -0''.0038635 \sin(-\gamma) + 0''.0080094 \cos(-\gamma)\\
\delta^2\mathrm{T}' &= +0.100156 \sin\gamma' \quad -0.059616 \cos\gamma'
\end{aligned}$$

On integration these equations give

$$\begin{aligned}
n\delta^3 z &= -0''.0080094nt \sin(-g) - 0''.0038635nt \cos(-g)\\
n'\delta^3 z' &= -0.059616n't \sin g' \quad -0.100156n't \cos g'
\end{aligned}$$

If these corrections are applied to the complete values of $n\delta z$ and $n'\delta z'$, obtained previously, we get

$$\begin{aligned}
n\delta z &= -1''.284368nt \sin(-g) - 1''.400329nt \cos(-g)\\
n'\delta z' &= -6.681432n't \sin g' \quad -10.708152n't \cos g'
\end{aligned}$$

The values used in the computation of the third-order terms, however, were

$$\begin{aligned}
n\delta z = &-1''.284370nt \sin(-g) - 1''.400324nt \cos(-g)\\
&-0.015494nt \sin(-2g) - 0.016877nt \cos(-2g)\\
&-0.000373nt \sin(-3g) - 0.000407nt \cos(-3g)\\
&-0.000012nt \sin(-4g) - 0.000013nt \cos(-4g)\\
\nu = &-0.015507nt\\
&-0.642311nt \cos(-g) + 0.700126nt \sin(-g)\\
&-0.015490nt \cos(-2g) + 0.016872nt \sin(-2g)\\
&-0.000557nt \cos(-3g) + 0.000612nt \sin(-3g)
\end{aligned}$$

$$\begin{aligned}
n'\delta z' = &-6''.681439n't \sin g' \quad -10''.708170n't \cos g'\\
&-0.092997n't \sin 2g' \quad -0.149736n't \cos 2g'\\
&-0.002549n't \sin 3g' \quad -0.004189n't \cos 3g'\\
&-0.000094n't \sin 4g' \quad -0.000158n't \cos 4g'\\
\nu' = &+0.093349n't\\
&+3.339074n't \cos g' \quad -5.352962n't \sin g'\\
&+0.092870n't \cos 2g' \quad -0.149869n't \sin 2g'\\
&+0.003852n't \cos 3g' \quad -0.006330n't \sin 3g'\\
&+0.000191n't \cos 4g' \quad -0.000319n't \sin 4g'
\end{aligned}$$

In order to have the proper values of $n\delta^2 z$, $n'\delta^2 z'$, $\delta\nu$, and $\delta\nu'$ the values employed for $n\delta z$, $n'\delta z'$ ν, and ν' in computing the second-order terms ought to be subtracted from those just given. With regard to $n\delta z$ and ν the values were not quite the same in computing $\delta\mathrm{T}'$ as in computing $\delta\mathrm{T}$. Hence, in deriving $\delta^2\mathrm{T}$ we employ the terms

$$\begin{aligned}
n\delta^2 z = &-0''.008457nt \sin(-g) - 0''.004104nt \cos(-g)\\
&-0.000116nt \sin(-2g) - 0.000046nt \cos(-2g)\\
&-0.000002nt \sin(-3g) - 0.000001nt \cos(-3g)\\
\delta\nu = &-0.000105nt\\
&-0.004354nt \cos(-g) + 0.002016nt \sin(-g)\\
&-0.000112nt \cos(-2g) + 0.000041nt \sin(-2g)
\end{aligned}$$

$$\begin{aligned} n'\delta^2z' = &- 0''.069141n't\sin g' - 0''.115525n't\cos g' \\ &- 0.000416n't\sin 2g' - 0.001386n't\cos 2g' \\ &+ 0.000044n't\sin 3g' - 0.000034n't\cos 3g' \\ \delta v' = &+ 0.000574n't \\ &+ 0.032925n't\cos g' - 0.056640n't\sin g' \\ &+ 0.000289n't\cos 2g' - 0.001519n't\sin 2g' \\ &- 0.000037n't\cos 3g' - 0.000098n't\sin 3g' \end{aligned}$$

But in deriving $\delta^2\mathrm{T}'$ we substitute for the first two the following:

$$\begin{aligned} n\delta^2z = &- 0''.008008nt\sin(-g) - 0''.003859nt\cos(-g) \\ &- 0.000097nt\sin(-2g) - 0.000047nt\cos(-2g) \\ &- 0.000002nt\sin(-3g) - 0.000001nt\cos(-3g) \\ \delta v = &- 0.000097nt \\ &- 0.004004nt\cos(-g) + 0.001930nt\sin(-g) \\ &- 0.000097nt\cos(-2g) + 0.000047nt\sin(-2g) \\ &- 0.000003nt\cos(-3g) + 0.000002nt\sin(-3g) \end{aligned}$$

The remaining terms of these quantities are the same as those given at pages 290–294, 335–339. In computing $\delta\mathrm{T}$ it is convenient to have n always accompany t as a factor, and with $\delta\mathrm{T}'$ to have n' accompany the same variable. Hence, we take occasion to make some multiplications by $\frac{n'}{n}$, $\frac{n'^2}{n^2}$ or the reciprocals of these factors.

The following tables contain the portions of the ten factors which are multiplied by nt or $n't$. The coefficients are expressed in units of the eleventh decimal:

Arg= $i'g'+ig$	$(n\delta z)^2\times\frac{1''}{2}$		$(n\delta z)(n'\delta z')\times\frac{1''}{2}$		$(n'\delta z')^2\times\frac{1''}{2}$		$(n\delta z)v\times\frac{1''}{2}$		$(n'\delta z')v\times\frac{1''}{2}$	
	nt cos.	nt sin.	nt cos.	nt sin.	$n't$ cos.	$n't$ sin.	nt sin.	nt cos.	nt sin.	nt cos.
i' i										
0 0	−186		+421		−2572					−6
0 −1	−25	+45	−193	+33	+129	+144	−51	+10	+60	+4
0 −2	−3	−1	−11	−14	+36	−6	−4	+1	+6	−8
1 +2	−2	+3	+6	−2	−14	+2			+3	+1
1 +1	+7	+36	−10	−18	−10	−67	−4	−13	−2	+7
1 0	−164	+96	+176	−98	−1897	+1367	−62	−8	+6	−7
1 −1	−30	+10	−200	−316	+3402	+5127			+29	−51
1 −2	−107	−143	−92	−562	+67	+468	+43	−13	+52	−318
1 −3	−5	−3	+1	−6					−2	−8
2 +1	−6	+32	+10	−10	+1	+34	−1	−11	+1	+4
2 0	−249	−188	+339	+344	−2	+48	−38	+33	+175	−171
2 −1	−144	−407	−171	−159	+38	+481	−82	−13	+75	−62
2 −2	+178	−210	−401	+583	+69	+316	−63	−39	+197	+273
2 −3	+398	−181	−85	+18	+99	+9	−211	+95	+55	+11
2 −4	+11	−5					−9	+3		

Arg= $i'g'+ig$	$(n\delta z)^2\times\frac{1''}{2}$		$(n\delta z)(n'\delta z')\times\frac{1''}{2}$		$(n'\delta z')^2\times\frac{1''}{2}$		$(n\delta z)\nu\times\frac{1''}{2}$		$(n'\delta z')\nu\times\frac{1''}{2}$	
	nt cos.	*nt* sin.	*nt* cos.	*nt* sin.	*n't* cos.	*n't* sin.	*nt* sin.	*nt* cos.	*nt* sin.	*nt* cos.
i' i										
3 + 1	+ 10	− 6	− 7	+ 2					− 5	0
3 0	+ 241	+ 29	− 293	+ 1	+ 167	+ 30	− 60	+ 2	− 154	+ 1
3 − 1	+ 60	+ 232	− 78	− 431	− 1274	+ 6239	− 21	− 7	− 52	− 40
3 − 2	− 40	− 30	+ 442	− 293	+ 8065	− 6246	− 30	− 1	− 240	− 177
3 − 3	− 183	− 17	− 18	+ 3	+ 52	+ 22	+ 85	0	+ 20	− 22
3 − 4	− 12	− 35	− 2	+ 17	+ 8	− 27	+ 7	+ 2	+ 2	+ 12
4 0	0	+ 2	+ 13	− 7	− 3	+ 18			+ 10	+ 7
4 − 1	− 48	+ 104	+ 564	− 700	− 456	+ 170	− 2	− 16	+ 266	+ 325
4 − 2	− 34	− 15	−3786	+ 590	+42475	− 6746	− 21	0	+ 70	+ 18
4 − 3	− 28	− 12	+ 834	+ 829	+ 60	+ 120	+ 8	+ 1	− 411	+ 406
4 − 4	+ 9	− 32	+ 4	+ 18	+ 14	− 12	− 5	+ 3	− 6	+ 16
4 − 5	− 8	0	+ 5	+ 2			+ 5	+ 1		
5 0	− 33	+ 11	+ 37	− 13			− 15	− 5	+ 37	+ 13
5 − 1	−2541	+ 902	+ 880	−1084	+ 45	− 79	−623	−227	+1537	+ 545
5 − 2	+ 238	+ 280	− 71	− 21	+ 9414	+ 4247	+108	− 21	+ 33	+ 44
5 − 3	− 917	−2449	− 832	+2920	+ 62	+ 31	+278	−626	− 680	+1454
5 − 4	− 274	+ 221	0	+ 41	+ 39	− 7	+134	− 14	− 4	+ 37
5 − 5	− 12	+ 3					+ 8	0	− 3	+ 3
5 − 6	− 1	+ 3								
6 − 2	+ 9	+ 16	−2077	−2971	+25050	+35598			− 16	+ 30
6 − 3	− 61	+ 64	− 174	+ 312	− 485	+ 76	+ 7	− 11	+ 113	+ 170
6 − 4	+ 5	+ 3	+ 27	+ 19	− 19	+ 4			− 12	+ 12
6 − 5	0	− 5								
7 − 2	+ 2	+ 10	− 33	− 50	+ 367	+ 469			− 3	+ 5
7 − 3	− 144	− 60	+ 416	+ 185	− 3095	− 1229	+ 13	+ 6	− 18	+ 15
7 − 4	− 123	− 149	+ 145	+ 158	− 167	− 188	+ 35	+ 50	− 77	+ 84
7 − 5	− 12	+ 5					+ 7	+ 1		
8 − 2	+ 2	− 3			+ 12	+ 11				
8 − 3	− 10	− 21	+ 14	+ 58	− 91	− 447				
8 − 4	+ 12	+ 121	− 5	− 113	+ 9	+ 404	− 3	− 13	+ 5	− 56
8 − 5	− 3	− 2	+ 29	− 15	− 43	+ 20			− 17	− 9
9 − 3	+ 1	0			+ 2	− 18				
9 − 4	− 9	+ 22	+ 257	− 595	− 2850	+ 6635			− 3	− 5
9 − 5	− 18	− 3	− 48	+ 2	− 47	− 6	+ 6	− 1	+ 22	+ 1
9 − 6	+ 2	+ 7								
10 − 3	0	− 13	− 1	+ 15					+ 1	− 7
10 − 4	−1065	+ 762	+2535	−1834	−15188	+11120	− 12	0	+ 20	+ 10
10 − 5	+ 264	+ 165	− 315	− 195	− 41	− 27	− 50	− 7	+ 155	− 97
10 − 6	+ 4	− 19							+ 3	− 6
11 − 4			+ 47	− 10	− 566	+ 132				
11 − 5			− 3	+ 10	− 20	− 6			+ 4	− 5
12 − 5					+ 30	+ 43				

Arg= $i'g'+ig$	$(n\delta z)\nu'\times\frac{1''}{2}$		$(n'\delta z')\nu'\times\frac{1''}{2}$		$\nu^2\times\frac{1''}{2}$		$\nu\nu'\times\frac{1''}{2}$		$\nu'^2\times\frac{1''}{2}$	
	nt sin.	nt cos.	$n't$ sin.	$n't$ cos.	nt cos.	nt sin.	nt cos.	nt sin.	$n't$ cos.	$n't$ sin.
i' i										
0 0		+69		−360	−2		+1		−43	
0−1	+103	+38	+1187	+554			+33	−14	+1226	−516
0−2	+7	−8	−2	+16	−1	+1	+3	+4	+13	−19
1+2			−6	−1	0	+1				
1+1	−6	−3	−38	+20	−2	−3	0	−6	+53	+37
1 0	+243	+165	−426	−358	−24	+15	−87	+58	+12	−35
1−1	+92	−140	−1231	+2069	−4	+3	+6	+23	−425	−146
1−2	+166	−443	−55	+345	+17	+23	+87	+239	−36	−228
1−3	+4	−8			+2	+1	+3	+7		
2+1	−89	+81	+1	−24	+1	−3	+1	−1	+3	+4
2 0	−87	+98	−49	−13	−14	−19	+37	+52	+81	−4
2−1	−7	+13	+934	−905	−45	−115	−21	−13	−934	−829
2−2	+145	+202	−45	+156	−20	+9	+70	−92	−62	−114
2−3	+56	+15	−76	+6	−111	+51	+34	−8	−52	−4
2−4					−6	+2				
3+1					−2	+1				
3 0	−33	−11	+73	−11	−2	+4	+8	−5	+13	+1
3−1	+42	+193	+13	−707	−6	+34	+16	−13	−567	−52
3−2	+281	+187	−4114	−3169	−9	+1	+135	−95	−2121	+1701
3−3	+41	+1	−45	+27	+39	+4	+24	+1	−33	−15
3−4	0	+12	−6	−22	+4	+13	+2	−8	−5	+15
4 0	−1	−8	−1	+9			+1	−5	+4	+2
4−1	−252	−363	−7	+481	−3	+3	+117	−142	+57	+28
4−2	+1673	+278	−10889	−2104	−8	−4	−59	−3	−335	+483
4−3	−421	+450	−25	+52	+2	+6	−217	−233	+6	+30
4−4	−1	+14	−10	−12	−3	+10	−2	−11	−6	+9
4−5	−4	+1			+3	+1	−4	−1	+5	+3
5−1	−22	−61			+12	0	+11	−28	+8	+3
5−2	−3	+58	−53	−317	+56	+69	−5	−3	+2243	+1036
5−3	−67	+33	−30	−66	0	−9	−32	−20	−25	+109
5−4			−14	−4	+66	−61	−2	−2	+11	−1
5−5					+5	−3	−2	−2	+2	+4
5−6					+1	−1				
6−2	−1035	+1476	+5861	−8835			+7	+14	+466	+62
6−3	−92	−121	−178	−419			−71	+96	+153	−5
6−4	−30	+13	+11	−15	−1	+1	−19	−5	+25	+10
6−5									+1	−3
6−6					0	−1				
7−2	−29	+39	+182	−237					+10	+5
7−3	−70	+26	+341	−186	+2	0	−2	+3	−18	−20
7−4	−3	−10	+85	−100	0	−1	+4	+12	+20	+11
7−5	−3	−3			+3	−7	−2	+3	+3	−6

Arg= $i'g'+ig$	$(n\delta z)\nu'\times\frac{1''}{2}$		$(n'\delta z')\nu'\times\frac{1''}{2}$		$\nu^2\times\frac{1''}{2}$		$\nu\nu'\times\frac{1''}{2}$		$\nu'^2\times\frac{1''}{2}$	
	nt sin.	nt cos.	$n't$ sin.	$n't$ cos.	nt cos.	nt sin.	nt cos.	nt sin.	$n't$ cos.	$n't$ sin.
i' i										
8− 2			+8	−8						
8− 3	+5	−16	−25	+78	−1	−1			−1	−15
8− 4	−6	+12	−3	+244			+1	+2	−26	−170
8− 5	−10	−6	+20	+12	−3	+2			+5	−3
9− 3			−4	+8						
9− 4	−117	−270	+680	+1591			+2	−3	−32	+24
9− 5	+33	0	+22	−2			+19	−1	+3	0
9− 6			−1	+6						
10− 4	+26	−9	−161	+60	−2	0			−31	+35
10− 5	−4	+4	+14	−9			−1	−2		
10− 6			+2	+6	−1	+6				
11− 4	+24	+3	−144	−31					+5	+2
11− 5	−4	+6	+4	−8			−1	−3		
12− 5			−5	+6						

In like manner the portions of the ten factors multiplied by n^2t^2 or n'^2t^2 follow; the coefficients are expressed in units of the thirteenth decimal when multiplied by n^2t^2, but in units of the twelfth decimal when multiplied by n'^2t^2:

Arg= $i'g'+ig$	$(n\delta z)^2\times\frac{1''}{2}$		$(n\delta z)(n'\delta z')\times\frac{1''}{2}$		$(n'\delta z')^2\times\frac{1''}{2}$		$(n\delta z)\nu\times\frac{1''}{2}$		$(n'\delta z')\nu\times\frac{1''}{2}$	
	n^2t^2 cos.	n^2t^2 sin.	n^2t^2 cos.	n^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n^2t^2 sin.	n^2t^2 cos.	n^2t^2 sin.	n^2t^2 cos.
i' i										
0 0	+261		−59		+1112					
0− 1	+1	−9	+4	+11			+3	+1	−2	+6
0− 2	+18	+211					−10	+106		
0− 3	+1	+6								
1+ 2									+6	+1
1+ 1			+258	−52					+279	+52
1 0	0	−1			+71	+24			+5	+8
1− 1			+92	+275	0	+17			−76	+275
1− 2	−1	+1	−11	+6					+1	+8
2+ 1									+3	0
2 0	+3	−5	−12	+4	+412	+840	+1	+2	−3	−6
2− 1	+3	−2	+16	+12			−1	−1	−1	+3
2− 2	+4	−1			−5	−1	−1	0	+4	+3
3 0	+1	−1			+11	+24				
3− 1	−5	−2	+6	+7	−31	−16	−1	+1		
3− 2	+1	+1	+8	+12	+28	+80				
3− 3	+1	−3	−11	+12			−1	−1		

Arg= $i'g'+ig$	$(n\delta z)^2\times\frac{1''}{2}$		$(n\delta z)(n'\delta z')\times\frac{1''}{2}$		$(n'\delta z')^2\times\frac{1''}{2}$		$(n\delta z)\nu\times\frac{1''}{2}$		$(n'\delta z')\nu\times\frac{1''}{2}$	
	n^2t^2 cos.	n^2t^2 sin.	n^2t^1 cos.	n^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n^2t^2 sin.	n^2t^2 cos.	n^2t^2 sin.	n^2t^2 cos.
i' i										
4 − 1	−1	−1	+6	+3	−3	−7			+14	−13
4 − 2	+1	−1	+18	−73	−12	+690				
4 − 3	+1	−1	+3	−6					+20	+15
5 0	−2	−4					−1	+1	+2	−3
5 − 1	−55	−220	+42	+143			−14	+55	+34	−131
5 − 2	−15	+14	−7	−5	−71	+130	−5	−1		
5 − 3	+173	−54	−88	+32			−44	−13	+101	+33
5 − 4	−9	−13					+1	+5		
6 − 2			+187	−98	−815	+425				
6 − 3			−20	−15					+12	−9
7 − 2			+8	−1	−32	+7				
7 − 3	0	−3			−2	−13				
8 − 4	−3	0	+8	+4						
9 − 4	−1	0			−39	−25				
10 − 4	−23	−44	+27	+52	−82	−153				
10 − 5	−6	+7	+4	−5			+1	+2	−4	−5

Arg= $i'g'+ig$	$(n\delta z)\nu'\times\frac{1''}{2}$		$(n'\delta z')\nu'\times\frac{1''}{2}$		$\nu^2\times\frac{1''}{2}$		$\nu\nu'\times\frac{1''}{2}$		$\nu'^2\times\frac{1''}{2}$	
	n^2t^2 sin.	n^2t^2 cos.	n'^2t^2 sin.	n'^2t^2 cos.	n^2t^2 cos.	n^2t^2 sin.	n^2t^2 cos.	n^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.
i' i										
0 0		−1			+53				+234	
0 − 1	−8	+2	−2	+3	+4	−3	−3	+3	−2	−2
0 − 2	+2	+1			−5	−53				
0 − 3					0	−2				
1 + 2							−3	0		
1 + 1	+279	+52					−140	+26		
1 0	−5	+4	+11	−19			−3	+4	+17	−12
1 − 1	+84	−271	0	+3			+38	+136	0	−3
1 − 2	+21	+2					+3	+4		
2 + 1	+8	+2					−3	0		
2 0	0	+4	+206	−420			−1	−1	−102	−210
2 − 1	+11	−3	−3	−3	+1	−1	+1	+4	+2	−2
2 − 2	−4	+1	+4	+1			+3	+2	+1	0
2 − 3					+1	+1				
3 0			+8	−18					−6	−12
3 − 1	+13	−8	−2	+6	−1	−1			−5	−2
3 − 2	+4	−11	−19	+41			+1	+2	−10	−18
3 − 3					−1	+1				

Arg= $i'g'+ig$	$(n\delta z)\nu'\times\frac{1''}{2}$		$(n'\delta z')\nu'\times\frac{1''}{2}$		$\nu^2\times\frac{1''}{2}$		$\nu\nu'\times\frac{1''}{2}$		$\nu'^2\times\frac{1''}{2}$	
	n^2t^3 sin.	n^2t^3 cos.	n'^2t^3 sin.	n'^2t^3 cos.	n^2t^3 cos.	n^2t^2 sin.	n^2t^3 cos.	n^2t^3 sin.	n'^2t^3 cos.	n'^2t^2 sin.
i' i										
4 0									0	− 1
4 − 1	− 13	+ 10	− 2	+ 4			+6	+7	0	+ 1
4 − 2	− 26	− 16	+ 1	+164			0	−2	0	+ 8
4 − 3	+ 9	+ 8					+9	−6		
5 − 1	− 1	− 5					−1	−3		
5 − 2	+ 11	+ 15	− 8	− 2	−4	+3			−18	+31
5 − 3	− 6	− 2					−3	+1		
5 − 4					−2	+3				
6 − 1							+1	0		
6 − 2	+222	+118	−209	−108			0	+1	+ 7	− 1
6 − 3	− 23	+ 19					−6	−5		
7 − 2	+ 13	+ 4	− 14	− 2					+ 3	0
7 − 3	− 2	+ 2	0	− 1						
8 − 4			+ 2	0						
9 − 4	− 7	+ 4	+ 10	− 6						
10 − 4	+ 1	− 2	− 2	+ 4						
10 − 5										
11 − 4			0	+ 1						

CHAPTER XVII.

CALCULATION OF THE SEVERAL PORTIONS OF δ^2T.

The fourteen parts of the portion of δ^2T not factored by nt or n^2t^2 are as follows:

Arg= $\varkappa\gamma+i'g'+ig$			$An\delta^2 z$		$B\delta\nu$		$Fn'\delta^2 z'$		$G\delta\nu'$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		−0.0000551		−0.0000128		−0.0001186		−0.0000415
1	0−	1		+0.0000052		+0.0000008	−0.0006	−0.0000791		+0.0000109
−1	0	0	−0.0004507	+0.0013445	−0.0000076	+0.0003655	−0.0027567	+0.0076775	−0.0003125	+0.0013968
0	0−	1	+0.0007	−0.0016			+0.0028	−0.0069	+0.0002	−0.0015
1	0−	2	−0.000093	+0.000746		+0.000122	−0.000056	+0.002351		+0.000490
−1	0−	1	−0.006	−0.005			−0.026	−0.020	−0.004	−0.002
0	0−	2	+0.008	+0.006			+0.028	+0.021	+0.004	+0.002
1	0−	3					−0.008	−0.004		
−1	1+	2	+0.002	+0.003						
0	1+	1	−0.007	−0.006			−0.025	−0.021	−0.003	−0.004
1	1	0	+0.0051	+0.0048	+0.0003	+0.0005	+0.0255	+0.0226	+0.0040	+0.0051
−1	1+	1	−0.0005	+0.0001			−0.0020	+0.0001	−0.0003	−0.0001
0	1	0	+0.0012	−0.0006	+0.0002	0.0000	+0.0054	−0.0033	+0.0014	−0.0004
1	1−	1	−0.0008	+0.0002	−0.0002	0.0000	−0.0062	+0.0033	−0.0014	+0.0002
−1	1	0	0.0000	+0.0002			+0.0003	+0.0016	+0.0001	+0.0008
0	1−	1	0.0000	−0.0003			−0.0004	−0.0013	−0.0002	−0.0003
−1	1−	1	−0.0019	−0.0003			−0.0091	−0.0016	−0.0016	+0.0001
0	1−	2	+0.002	0.000			+0.011	+0.001	+0.001	0.000
1	1−	3					−0.002	0.000		
−1	1−	2	+0.003	−0.007			+0.012	−0.030		
0	1−	3	−0.004	+0.009			−0.012	+0.028		
1	1−	4					+0.003	−0.007		
−1	2+	2					+0.020	+0.005		
0	2+	1					−0.069	−0.017		
1	2	0					+0.0728	+0.0164		
−1	2+	1	−0.0019	+0.0020			−0.0055	+0.0060	−0.0008	+0.0010
0	2	0	+0.0042	−0.0065	+0.0006	−0.0005	+0.0149	−0.0248	+0.0033	−0.0045
1	2−	1	−0.0027	+0.0038	−0.0006	+0.0005	−0.0163	+0.0262	−0.0038	+0.0053
−1	2	0	+0.0002	−0.0003			+0.0010	−0.0006	0.0000	−0.0003
0	2−	1	+0.00026	+0.00056	+0.00016	+0.00027	−0.00024	+0.00185	+0.00006	+0.00051
1	2−	2	−0.0001	−0.0001	−0.0001	−0.0002	0.0000	−0.0026	0.0000	−0.0006
−1	2−	1	−0.0003	+0.0001			−0.0020	+0.0005	−0.0012	+0.0004
0	2−	2					+0.0024	−0.0010	+0.0010	−0.0004

Arg= $\varkappa\gamma+i'g'+ig$			$An\delta^2 z$		$B\delta\nu$		$Fn'\delta^2 z'$		$G\delta\nu'$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
—1	2	—2	0.000	—0.002			—0.002	—0.010		
0	2	—3					0.000	+0.008		
—1	2	—3	+0.006	+0.002			+0.021	+0.005		
0	2	—4	—0.008	—0.002			—0.020	—0.005		
0	3	+1					+0.002	—0.003		
1	3	0					+0.002	+0.001		
—1	3	+1	0.000	+0.013			0.000	+0.019		
0	3	0	—0.0001	—0.0306			+0.0008	—0.0705		
1	3	—1	+0.0001	+0.0163			—0.0013	+0.0744	+0.0002	—0.0007
—1	3	0	0.0000	—0.0016	—0.0002	—0.0001	+0.0013	—0.0026	+0.0006	+0.0003
0	3	—1	+0.00193	+0.00206	+0.00086	+0.00063	+0.00235	+0.00528	+0.00153	+0.00054
1	3	—2	—0.0009	—0.0006	—0.0005	—0.0004	—0.0043	—0.0066	—0.0026	—0.0011
—1	3	—1	0.0000	+0.0002			—0.0004	+0.0009	0.0000	+0.0001
0	3	—2								
1	3	—3								
—1	3	—2					—0.0017	—0.0024		
0	3	—3								
—1	3	—3					+0.008	—0.002		
0	3	—4					—0.008	+0.002		
—1	3	—4					0.000	+0.016		
0	3	—5					0.000	—0.016		
1	4	—1					—0.0022	—0.0024	—0.0007	—0.0001
—1	4	0	+0.0022	—0.0004			+0.0026	—0.0005		
0	4	—1	+0.0023	—0.0005			+0.0051	—0.0008	+0.0006	—0.0002
1	4	—2					—0.0092	+0.0017	—0.0003	—0.0001
—1	4	—1	—0.0001	+0.0011			—0.0003	+0.0010	—0.0005	+0.0018
0	4	—2	+0.0001	—0.0008	—0.0001	—0.0001	0.0000	+0.0002	+0.0001	+0.0004
1	4	—3	0.0000	+0.0005			+0.0002	—0.0012	+0.0004	+0.0019
—1	4	—2					—0.0013	—0.0009	+0.0004	+0.0004
0	4	—3					0.000	+0.001		
—1	4	—3					+0.002	—0.002		
—1	4	—4					+0.003	+0.006		
—1	4	—5					—0.012	+0.002		
0	5	—1					—0.0008	+0.0025	0.0000	+0.0007
1	5	—2	+0.000485	—0.000695	+0.000241	—0.000215	+0.000877	—0.002763	+0.000039	—0.000867
—1	5	—1	+0.001427	+0.003366	—0.000211	—0.000510	—0.000005	—0.000172	—0.000241	—0.000659
0	5	—2	+0.0006399	+0.0015326	+0.0004766	+0.0011381	—0.0000225	+0.0002158	+0.0004096	+0.0009613
1	5	—3	+0.001489	+0.003311	—0.000269	—0.000639	+0.000108	+0.000036	—0.000207	—0.000504
—1	5	—2	—0.001379	—0.000076	—0.000648	—0.000031	—0.002065	—0.000908	—0.002835	—0.000527
0	5	—3	+0.0019	+0.0001			+0.0029	+0.0007	+0.0025	+0.0005
1	5	—4	—0.0008	—0.0001			—0.0010	—0.0002	+0.0006	+0.0001
—1	5	—3								
0	5	—4								
1	5	—5								
—1	5	—4								
0	5	—5								

Arg= $\varkappa\gamma+i'g'+ig$			$An\delta^2z$		$B\delta\nu$		$Fn'\delta^2z'$		$G\delta\nu'$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
1	6—	2								
—1	6—	1	+0.0005	+0.0003			—0.0010	—0.0006		
0	6—	2	—0.0004	—0.0002			—0.0007	—0.0004	—0.0001	+0.0001
1	6—	3	+0.0008	+0.0006			+0.0006	+0.0006	—0.0001	0.0000
—1	6—	2	+0.0002	—0.0002	+0.0004	—0.0002	+0.0076	—0.0058	+0.0002	+0.0004
0	6—	3	—0.0021	+0.0018	—0.0004	+0.0003	—0.0035	+0.0032	—0.0002	—0.0001
1	6—	4	—0.0018	+0.0014			—0.0022	+0.0015		
—1	6—	3	—0.0006	—0.0013			+0.0005	+0.0038	—0.0009	—0.0074
0	6—	4					—0.003	—0.004	+0.001	+0.006
—1	6—	4								
0	6—	5								
—1	6—	5								
0	6—	6								
1	7—	3	+0.00085	+0.00011						
—1	7—	2	+0.00004	—0.00022	+0.00004	—0.00014	—0.00212	—0.00685	+0.00005	—0.00016
0	7—	3	+0.00048	+0.00115	—0.00004	+0.00010	+0.00270	+0.00489	0.00000	+0.00026
1	7—	4	—0.0012	—0.0005			—0.0030	—0.0010	—0.0001	+0.0003
—1	7—	3	+0.0119	+0.0109	+0.0002	+0.0001	+0.0549	+0.0528	—0.0002	—0.0003
0	7—	4	—0.0223	—0.0211			—0.0513	—0.0497	+0.0002	0.0000
1	7—	5	+0.010	+0.009			+0.014	+0.013		
—1	7—	4					—0.003	+0.001	+0.005	—0.002
0	7—	5	—0.002	—0.002			+0.002	—0.004	—0.005	+0.002
—1	8—	2	0.0000	+0.0004			—0.0013	—0.0014		
0	8—	3	+0.00017	—0.00071	—0.00003	—0.00046	+0.00146	+0.00056	—0.00004	—0.00034
1	8—	4	—0.0002	+0.0005			—0.0012	+0.0006	+0.0001	+0.0006
—1	8—	3	+0.0036	+0.0008	+0.0001	0.0000	+0.0236	+0.0059	+0.0004	—0.0003
0	8—	4	—0.0063	—0.0016			—0.0219	—0.0061		
1	8—	5	+0.0027	0.0000			+0.0064	0.0000		
—1	8—	4	—0.0113	+0.0186			—0.0406	+0.0649	+0.0003	—0.0005
0	8—	5	+0.016	—0.027			+0.040	—0.063		
1	8—	6	—0.006	+0.010			—0.011	+0.017		
—1	8—	5					—0.001	—0.001	+0.002	+0.004
0	8—	6	0.000	—0.004			+0.003	—0.001		
—1	9—	3	+0.0005	—0.0002			+0.0049	—0.0013	+0.0006	—0.0002
0	9—	4	—0.00092	+0.00027			—0.00432	+0.00099	+0.00010	—0.00003
1	9—	5	+0.0003	—0.0003			+0.0010	—0.0010	—0.0006	+0.0002
—1	9—	4	—0.0003	+0.0065			—0.0030	+0.0291	+0.0006	+0.0006
0	9—	5	+0.001	—0.010			+0.004	—0.027		
1	9—	6	0.000	+0.003			+0.001	+0.009		
—1	9—	5	—0.019	—0.007			—0.061	—0.023		
0	9—	6	+0.025	+0.008			+0.056	+0.020		
1	9—	7	—0.010	—0.003			—0.017	—0.006		
—1	9—	6	—0.002	+0.001			—0.002	0.000		
0	9—	7	+0.003	0.000			+0.003	+0.002		

Arg= $\kappa\gamma+i'g'+ig$			$An\delta^2 z$		$B\delta\nu$		$Fn'\delta^2 z'$		$G\delta\nu'$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
κ	i'	i	″	″	″	″	″	″	″	″
—1	10	—3	+0.00029	—0.00032			+0.00059	—0.00057	—0.00005	+0.00001
0	10	—4	—0.0001146	+0.0000983	—0.0000091	+0.0000053	—0.0005380	+0.0004872	—0.0000143	+0.0000247
1	10	—5	+0.00001	—0.00006			+0.00003	—0.00025	+0.00004	—0.00006
—1	10	—4	+0.00055	+0.00124			+0.00248	+0.00618	+0.00034	+0.00052
0	10	—5	—0.0008	—0.0016			—0.0029	—0.0062		
1	10	—6					+0.002	+0.002		
—1	10	—5	—0.008	+0.001			—0.029	+0.002		
0	10	—6	+0.010	—0.002			+0.029	—0.004		
1	10	—7	—0.004	+0.002			—0.008	+0.002		
—1	10	—6	+0.002	—0.016			+0.006	—0.046		
0	10	—7	—0.002	+0.020			—0.005	+0.044		
1	10	—8	0.000	—0.007			+0.001	—0.013		
—1	11	—4	+0.0002	+0.0002			+0.0002	+0.0004	+0.0002	0.0000
0	11	—5	—0.0002	—0.0001			—0.0006	—0.0005		
—1	11	—5	—0.0015	+0.0011			—0.0067	+0.0044		
0	11	—6	+0.002	—0.002			+0.006	—0.005		
1	11	—7					—0.002	+0.002		
—1	11	—6	—0.003	—0.007			—0.009	—0.025		
0	11	—7	+0.004	+0.009			+0.009	+0.024		
1	11	—8					—0.003	—0.007		
—1	11	—7	+0.011	—0.001			+0.032	—0.003		
0	11	—8	—0.013	+0.002			—0.031	+0.003		
0	12	—5	+0.00002	—0.00008			+0.00012	—0.00041	0.00000	0.00000
—1	12	—5	+0.0003	—0.0009			—0.0007	+0.0017		
0	12	—6					+0.0007	—0.0016		
—1	12	—6	—0.002	—0.002			—0.007	—0.006		
0	12	—7					+0.006	+0.006		
—1	12	—7	+0.006	—0.004			+0.019	—0.011		
0	12	—8	—0.006	+0.004			—0.018	+0.011		
—1	12	—8	+0.002	+0.007			+0.007	+0.019		
0	12	—9					—0.01	—0.02		

Arg= $ny+i'g'+ig$			$\frac{1}{2}\frac{dA}{dg}(n\delta z)^2$		$\frac{dA}{dg'}(n\delta z)(n'\delta z')$		$\frac{1}{2}\frac{dF}{dg'}(n'\delta z')^2$		$\frac{dB}{dg}(n\delta z)\nu$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
n	i'	i	″	″	″	″	″	″	″	″
0	0	0		+0.0000064		+0.0000019		−0.0000921		+0.0000037
1	0	−1		−0.0000052		−0.0000098		+0.0001557		−0.0000018
1	0	−2		−0.000057		−0.000209		−0.000170		−0.000051
0	1	+1					0.000	+0.001		
1	1	0			−0.0003	−0.0009	+0.0005	−0.0016		
−1	1	+1								
0	1	0			−0.0004	+0.0001				
1	1	−1					+0.0001	+0.0001		
−1	1	0					−0.0007	−0.0010		
0	1	−1					+0.0001	+0.0004		
−1	1	−1					+0.0038	−0.0010		
1	2	0					+0.0023	−0.0008		
1	2	−1					+0.0004	+0.0008		
−1	2	0					−0.0013	−0.0002		
0	2	−1	+0.00010	+0.00012	+0.00045	+0.00032	+0.00105	−0.00005	+0.00003	+0.00004
1	2	−2					−0.0003	+0.0003		
−1	2	−1			+0.0043	−0.0017	+0.0117	−0.0054		
0	2	−2			−0.0076	+0.0033	−0.0107	+0.0046		
0	3	0					−0.0009	−0.0016		
1	3	−1			+0.0005	+0.0003	+0.0014	+0.0028		
−1	3	0					−0.0006	+0.0003		
0	3	−1	−0.00014	−0.00005	−0.00014	−0.00027	+0.00076	−0.00044	−0.00011	−0.00004
1	3	−2					−0.0005	+0.0003		
−1	3	−1			+0.0016	−0.0019	+0.0049	−0.0067		
0	3	−2			−0.0028	+0.0035	−0.0044	+0.0059		
1	3	−3					+0.0006	−0.0013		
−1	3	−2	+0.0007	+0.0010			+0.0109	+0.0154		
0	3	−3	−0.002	−0.002			−0.010	−0.013		
1	4	−1					+0.0041	+0.0041		
0	4	−1					+0.0009	−0.0010		
1	4	−2					−0.0014	+0.0013		
−1	4	−1			+0.0001	−0.0007	+0.0002	−0.0027		
0	4	−2	0.0000	+0.0002	−0.0001	+0.0008	−0.0003	+0.0023		
1	4	−3					−0.0002	−0.0004		
−1	4	−2	+0.0003	+0.0002	+0.0048	+0.0022	+0.0120	+0.0062		
0	4	−3			−0.006	−0.002	−0.011	−0.005		
0	5	−1			+0.0005	−0.0014	+0.0022	−0.0035		
1	5	−2	−0.000156	+0.000169	−0.001197	+0.001571	−0.002609	+0.003926	−0.000079	+0.000079
−1	5	−1	+0.000080	+0.000194	+0.000263	+0.000440	−0.000161	−0.000688	+0.000057	+0.000141
0	5	−2	−0.0001827	−0.0004691	−0.0003418	−0.0009403	+0.0003366	+0.0007807	−0.0001234	−0.0003257
1	5	−3	+0.000087	+0.000225	+0.000075	+0.000468	−0.000408	−0.000546	+0.000050	+0.000123

Arg= $\kappa\gamma+i'g'+ig$			$\frac{1}{2}\frac{dA}{dg}(n\delta z)^2$		$\frac{dA}{dg'}(n\delta z)(n'\delta z')$		$\frac{1}{2}\frac{dF}{dg'}(n'\delta z')^2$		$\frac{dB}{dg}(n\delta z)\nu$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
κ	i'	i	″	″	″	″	″	″	″	″
—1	5	—2	+0.000217	+0.000026	+0.002108	+0.000288	+0.005902	+0.000863	+0.000075	+0.000005
0	5	—3			—0.0027	—0.0006	—0.0054	—0.0007		
1	5	—4					+0.0007	—0.0003		
—1	5	—3			—0.001	+0.005	—0.003	+0.015		
0	5	—4			+0.002	—0.007	+0.004	—0.015		
1	5	—5					0.000	+0.003		
—1	5	—4			—0.009	—0.005	—0.015	—0.009		
0	5	—5			+0.011	+0.006	+0.015	+0.008		
1	6	—2					—0.0025	+0.0075		
0	6	—2	+0.0001	0.0000	+0.0007	+0.0005	+0.0014	+0.0010		
1	6	—3			—0.0002	—0.0002	—0.0018	—0.0013		
—1	6	—2	—0.0002	+0.0001	—0.0003	+0.0003	+0.0016	—0.0005	—0.0001	+0.0001
0	6	—3	+0.0003	—0.0002	+0.0002	—0.0004	—0.0017	+0.0004	+0.0002	—0.0002
—1	6	—3			+0.0001	+0.0032	+0.0005	+0.0081		
0	6	—4			0.000	—0.002	0.000	—0.008		
—1	6	—4			—0.006	0.000	—0.016	—0.001		
0	6	—5			+0.008	0.000	+0.012	+0.001		
—1	6	—5			+0.003	—0.009	+0.004	—0.014		
0	6	—6			—0.003	+0.011	—0.004	+0.013		
0	7	—2			+0.0029	+0.0003	+0.0045	+0.0006		
1	7	—3	—0.00015	—0.00002	—0.00181	—0.00025			—0.00011	—0.00001
—1	7	—2			—0.00013	+0.00027	+0.00030	—0.00030		
0	7	—3	+0.00003	—0.00006	—0.00008	—0.00006	—0.00052	+0.00019	+0.00001	—0.00006
1	7	—4					+0.0004	—0.0001		
—1	7	—3			—0.0002	+0.0005	+0.0010	+0.0025	+0.0001	—0.0001
0	7	—4	+0.0003	+0.0003						
—1	7	—4			—0.002	+0.001	—0.009	+0.002		
0	7	—5					+0.004	—0.002		
0	8	—3	+0.00001	+0.00022	—0.00009	+0.00058	—0.00029	+0.00033	+0.00001	+0.00012
—1	8	—3	—0.0001	0.0000	—0.0004	0.0000	+0.0004	+0.0004		
—1	8	—4					—0.0028	+0.0022		
—1	8	—6			+0.005	—0.002	+0.009	—0.004		
—1	9	—3					+0.0001	+0.0001		
0	9	—4			+0.00016	—0.00004	—0.00004	—0.00010		
—1	10	—3	—0.00004	+0.00004			+0.00002	+0.00005		
0	10	—4	+0.0000014	—0.0000006	+0.0000029	—0.0000301	—0.0000468	—0.0000532	+0.0000011	—0.0000019
1	10	—5	+0.00004	—0.00004			+0.00004	—0.00001		
—1	10	—4	—0.00003	—0.00004	—0.00029	—0.00009	—0.00101	—0.00021	—0.00002	—0.00002
0	12	—5			+0.00010	—0.00012	+0.00033	—0.00035		
—1	12	—5			+0.0008	—0.0020	+0.0021	—0.0063		
0	12	—6			—0.0014	+0.0038				

Arg= $\varkappa\gamma+i'g'+ig$			$\frac{dB}{dg'}(n'\delta z')\nu$		$\frac{dG}{dg}(n\delta z)\nu'$		$\frac{dG}{dg'}(n'\delta z')\nu'$		$\frac{1}{2}r^2\frac{d^2T}{dr^2}\nu^2$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		+0.0000153		+0.0000173		−0.0000316		+0.0000001
1	0	−1		−0.0000119		−0.0000068		−0.0000258		+0.0000012
1	0	−2		−0.000059		−0.000210		−0.000317		−0.000012
1	1	0					−0.0011	−0.0006		
−1	1	+1					+0.0002	−0.0004		
0	1	0	+0.0001	0.0000						
−1	1	0					+0.0002	+0.0002		
−1	1	−1					−0.0021	−0.0002		
1	2	−1	+0.0002	−0.0003			+0.0007	−0.0003		
−1	2	0	−0.0001	−0.0002			−0.0002	−0.0003		
0	2	−1	+0.00003	+0.00013	−0.00010	−0.00009	−0.00009	−0.00003		
−1	2	−1					−0.0006	+0.0004		
1	3	−1	+0.0002	−0.0004			+0.0006	+0.0009		
−1	3	0					−0.0002	0.0000		
0	3	−1	−0.00023	−0.00015	−0.00023	−0.00015	−0.00045	−0.00041		
1	3	−2	+0.0002	+0.0003			+0.0005	+0.0004		
−1	3	−1	+0.0003	−0.0002			+0.0012	−0.0015		
0	3	−2	−0.0003	+0.0003			−0.0012	+0.0014		
−1	3	−2					−0.0008	−0.0009		
1	4	−1					+0.0014	+0.0013		
0	4	−1					−0.0006	−0.0002		
1	4	−2					+0.0006	+0.0006		
−1	4	−1	0.0000	−0.0001			0.0000	−0.0008		
0	4	−2	0.0000	−0.0001			−0.0001	+0.0005		
−1	4	−2					+0.0023	+0.0012		
0	4	−3					−0.002	−0.001		
0	5	−1					+0.0008	−0.0006		
1	5	−2	−0.000359	+0.000330	−0.000212	+0.000325	−0.001096	+0.001563		
−1	5	−1	+0.000197	+0.000415	+0.000011	−0.000028	+0.000221	+0.000203		
0	5	−2	−0.0003058	−0.0007671	−0.0000245	−0.0001270	−0.0001538	−0.0003436	+0.0000055	+0.0000080
1	5	−3	+0.000130	+0.000395	−0.000047	−0.000038	−0.000053	+0.000113		
−1	5	−2	+0.000297	+0.000016	+0.000364	+0.000088	+0.001013	+0.000539		
0	5	−3					−0.0013	−0.0004		
−1	5	−3					−0.001	+0.003		
0	6	−2	+0.0001	+0.0001			+0.0007	+0.0006		
1	6	−3					−0.0008	−0.0007		
−1	6	−2	−0.0004	+0.0003			−0.0006	+0.0006		
0	6	−3	+0.0005	−0.0004			+0.0004	−0.0005		
−1	6	−3					−0.0003	+0.0018		

Arg= $\varkappa\gamma+i'g'+ig$			$\frac{dB}{dg'}(n'\delta z')\nu$		$\frac{dG}{dg}(n\delta z)\nu'$		$\frac{dG}{dg'}(n'\delta z')\nu'$		$\frac{1}{2}r^2\frac{d^2T}{dr^2}\nu^2$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	7	—2					+0.0018	+0.0003		
1	7	—3	—0.00040	—0.00005	—0.00043	—0.00005	—0.00187	—0.00024		
—1	7	—2	—0.00007	+0.00023			—0.00011	+0.00039		
0	7	—3	+0.00001	—0.00014	—0.00003	—0.00003	—0.00006	—0.00037		
1	7	—4					+0.0002	—0.0002		
—1	7	—3					—0.0011	—0.0009		
0	8	—3	0.00000	+0.00024	+0.00001	+0.00007	—0.00002	+0.00009		
—1	8	—3	—0.0003	0.0000			—0.0007	—0.0001		
—1	8	—4					+0.0015	—0.0016		
—1	9	—3					—0.0002	0.0000		
0	9	—4	+0.00010	—0.00005			+0.00013	—0.00006		
0	10	—4	+0.0000101	—0.0000165	+0.0000082	—0.0000114	+0.0000323	—0.0000411		
—1	10	—4	—0.000004	—0.000010	+0.000001	—0.000003	+0.00013	+0.00008		

Arg= $\varkappa\gamma+i'g'+ig$			$rr'\frac{d^2T}{drdr'}\nu\nu'$		$\frac{1}{2}r'^2\frac{d^2T}{dr'^2}\nu'^2$	
			sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″
0	0	0		+0.0000088		+0.0000219
1	0	—1		—0.0000079		—0.0000268
1	0	—2		—0.000048		—0.000058
—1	1	0			0.0000	+0.0002
—1	2	0			+0.0001	0.0000
0	2	—1	—0.00007	—0.000008	—0.00009	—0.00008
—1	2	—1			—0.0007	+0.0002
0	3	—1	—0.00011	—0.00004	—0.00025	—0.00013
—1	3	—1			0.0000	—0.0002
1	5	—2			—0.000181	+0.000126
—1	5	—1	—0.000029	—0.000039	—0.000043	—0.000045
0	5	—2	+0.0000524	+0.0000648	+0.0000575	+0.0000446
1	5	—3	—0.000038	—0.000062	—0.000058	—0.000055
—1	5	—2	+0.000029	+0.000006	+0.000292	—0.000055
0	7	—3	—0.00003	+0.00006		
0	10	—4	+0.0000004	—0.0000015	+0.0000001	—0.0000008

The fourteen parts of the portion of δ^2T factored by nt follow; for convenience the coefficients have been multiplied by 100000:

Arg= $\kappa\gamma+i'g'+ig$			$An\delta^2z$		$B\delta\nu$		$Fn'\delta^2z'$		$G\delta\nu'$	
			nt sin.	nt cos.	nt sin.	nt cos.	nt sin.	nt cos.	nt sin.	nt cos.
κ	i'	i	″	″	″	″	″	″	″	″
0	0	0		− 0.3108		−0.1193		− 0.9327		−0.1447
1	0	− 1	+ 0.0850	+ 0.3178	+0.0079	+0.1385	+ 0.2281	+ 1.3868	−0.0081	+0.2255
−1	0	0	− 1.8586	− 0.9437	−0.5523	−0.2297	− 8.6483	− 4.8762	−2.2695	−1.1106
0	0	− 1	+ 1.7	+ 0.9			+ 7.9	+ 4.7	+2.2	+1.0
1	0	− 2	− 1.15	− 0.39	−0.14	−0.03	− 2.83	− 0.87	−0.34	−0.13
−1	0	− 1	+ 4	− 4			+20	−21	+3	−3
0	0	− 2	− 6	+ 5			−20	+20	−3	+3
1	0	− 3					+ 4	− 6		
−1	0	− 2	+ 6	+10			+16	+30		
0	0	− 3	− 7	−12			−17	−28		
1	0	− 4					+ 6	+ 9		
−1	1	+ 3	− 7	− 2			−12	− 5		
0	1	+ 2	+17	+ 6			+42	+13		
1	1	+ 1	−13	− 4			−44	−13		
−1	1	+ 2	+ 2	− 2			+ 5	− 7		
0	1	+ 1	− 4	+ 7			−15	+24	−3	+4
1	1	0	+ 3.4	− 5.0	+0.5	−0.7	+16.4	−25.3	+2.7	−4.0
−1	1	+ 1					+ 0.5	+ 1.6	−0.2	0.0
0	1	0	− 1.2	− 1.5	−0.2	−0.4	− 3.4	− 4.2	−0.7	−0.9
1	1	− 1	+ 0.5	+ 0.7	+0.4	+0.5	+ 3.7	+ 4.6	+1.2	+1.6
−1	1	0					− 2.3	+ 0.2	−0.7	+0.1
0	1	− 1					+ 2.3	0.0	+0.3	−0.1
−1	1	− 1	+ 0.5	− 1.8			+ 3.3	− 9.5	+0.7	−3.5
0	1	− 2	− 1	+ 1			− 3	+11	−1	+2
−1	1	− 2	+ 5	+ 3			+21	+13	+3	+2
0	1	− 3	− 6	− 3			−20	−13		
1	1	− 4					+ 5	+ 3		
−1	1	− 3	− 6	+ 6			−18	+15		
0	1	− 4	+ 7	− 7			+16	−16		
1	2	+ 1					+ 3	− 2		
−1	2	+ 2	0	− 9			+ 2	−16		
0	2	+ 1	− 2	+22			− 5	+53		
1	2	0	+ 1.0	−15.4			+ 4.2	−55.1		
−1	2	+ 1	+ 1.7	+ 1.1			+ 5.5	+ 3.6	+1.2	+0.5
0	2	0	− 6.2	− 2.5	−1.0	−0.5	−21.9	− 8.6	−5.3	−2.2
1	2	− 1	+ 3.7	+ 1.9	+0.9	+0.5	+23.6	+ 9.7	+5.5	+2.1
−1	2	0	− 0.24	− 0.10	+0.07	−0.08	− 1.00	− 0.40	+0.40	−0.80
0	2	− 1	+ 0.36	− 0.16	+0.12	−0.05	+ 1.88	− 0.17	+0.26	+0.09
1	2	− 2	− 0.2	+ 0.1	−0.2	+0.1	− 2.2	+ 0.4	−0.9	+0.5
−1	2	− 1	− 0.1	− 0.3			− 0.3	− 2.4		
0	2	− 2					+ 0.5	+ 2.0		
−1	2	− 2					+ 6	+ 1		
0	2	− 3					− 6	− 1		
−1	2	− 3					− 6	+14		
0	2	− 4					+ 6	−14		

Arg= $x\gamma+i'g'+ig$			$An\delta^2z$		$B\delta\nu$		$Fn'\delta^2z'$		$G\delta\nu'$	
			nt sin.	nt cos.	nt sin.	nt cos.	nt sin.	nt cos.	nt sin.	nt cos.
x	i'	i	″	″	″	″	″	″	″	″
1	3	0					+1	+1		
−1	3	+1	+9	−1			+14	−2		
0	3	0	−21.7	+3.0			−51.5	+6.8	+1.9	−0.3
1	3	−1	+11.5	−1.6			+54.7	−7.3	−2.0	+0.3
−1	3	0	−1.1	+0.7	−0.1	+0.3	−2.1	−0.8	+0.1	−0.7
0	3	−1	+1.20	−2.35	+0.29	−1.13	+3.63	−2.29	+0.11	−0.99
1	3	−2	−0.2	+1.1	−0.2	+1.0	−4.4	+4.2	−0.2	+2.0
−1	3	−1					+0.9	+0.4	+1.4	+0.9
0	3	−2					−0.9	−0.7	−0.9	−0.6
−1	3	−2					+2.3	−1.0		
−1	3	−3					+1	+6		
0	3	−4					−1	−6		
−1	3	−4					−12	−2		
0	3	−5					+11	+2		
1	4	−1					−0.5	−0.4		
−1	4	0	−0.5	−1.6			−0.6	−1.8		
0	4	−1	−0.5	−1.6			−1.4	−3.7	+0.1	+0.3
1	4	−2					+2.4	+6.7	−0.1	−0.5
−1	4	−1	+0.5	−0.1			+1.1	+0.1	+2.3	0.0
0	4	−2	+0.15	0.00	+0.27	+0.01	−0.30	−0.10	−0.24	−0.01
1	4	−3					−0.9	0.0	−2.0	0.0
−1	4	−2					−0.1	−0.9	−0.5	+1.8
0	4	−3					0	+1		
1	5	−2	+0.030	+0.018	+0.014	+0.005	+0.179	+0.185	−0.167	−0.145
−1	5	−1	+2.749	−1.662	−0.007	+0.008	−0.002	+0.111	−0.313	+0.172
0	5	−2	−0.0328	−0.0050	+0.0068	−0.0094	−0.0701	−0.0348	−0.0689	+0.0230
1	5	−3	+2.734	−1.656	+0.001	+0.007	+0.133	−0.050	+0.404	−0.249
−1	5	−2	+0.005	+0.267	+0.083	+0.377	−0.695	−1.330	+0.341	+2.407
0	5	−3					+0.1	+0.5	−0.2	−1.2
−1	5	−3					+2	0		
−1	6	−1					−0.3	+0.8		
1	6	−3	+0.2	−0.3			+0.4	−0.7		
−1	6	−2					−5.5	−5.4	+0.2	−0.3
0	6	−3	+1.1	+1.2			+3.1	+2.9	−0.2	+0.3
1	6	−4	+1.1	+1.2			+1.4	+1.4		
−1	6	−3					+7.8	−2.7	−5.9	+2.6
0	6	−4					−7	+3	+5	−2
1	7	−3					+0.08	−0.28		
−1	7	−2					−5.04	+2.17	−0.31	−0.07
0	7	−3	+0.69	−0.47			+3.46	−2.27	+0.17	+0.02
1	7	−4	−0.3	+1.0			−0.6	+2.1		
−1	7	−3	+6.7	−9.4			+33.8	−46.9	+0.4	−0.4
0	7	−4	−12.7	+17.8			−31.9	+44.0	−0.4	+0.4
1	7	−5	+5	−7			+8	−12		
−1	7	−4					+3	+5	−3	−4
0	7	−5					−4	−6	+3	+4

Arg= $\varkappa\gamma+i'g'+ig$			$An\delta^2z$		$B\delta\nu$		$Fn'\delta^2z'$		$G\delta\nu'$	
			nt sin.	*nt* cos.	*nt* sin.	*nt* cos.	*nt* sin.	*nt* cos.	*nt* sin.	*nt* cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
—1	8	—2					— 0.9	+ 1.1		
0	8	—3	+ 0.08	— 0.13			+ 0.66	— 1.00	+0.03	—0.04
1	8	—4	0.0	+ 0.1			+ 0.1	+ 0.6		
—1	8	—3	+ 0.3	— 2.5			+ 1.9	—19.1	+0.2	—0.9
0	8	—4	— 0.5	+ 4.6			— 2.4	+17.4	—0.1	+1.0
1	8	—5	— 0.3	— 1.9			— 0.6	— 4.6		
—1	8	—4	+14.2	+ 6.2			+53.2	+23.8	0.0	+0.1
0	8	—5	—20	— 8			—52	—22	+2	+1
1	8	—6	+ 8	+ 3			+14	+ 6		
—1	8	—5					+ 3	— 2		
—1	9	—3	— 0.2	— 0.3			— 1.7	— 3.5		
0	9	—4	+ 0.27	+ 0.63			+ 1.33	+ 3.22	+0.06	+0.13
1	9	—5	— 0.3	— 0.2			— 0.8	— 0.6		
—1	9	—4	+ 4.7	— 0.5			+23.7	— 2.1		
0	9	—5	— 7	0			—21	+ 2		
1	9	—6	+ 2	0			+ 6	— 1		
—1	9	—5	— 3	+14			—11	+48		
0	9	—6	+ 3	—18			+ 9	—46		
1	9	—7	— 1	+ 6			— 2	+12		
—1	10	—3	— 0.04	— 0.03			— 0.55	— 0.36	—0.02	—0.02
0	10	—4	+ 0.0646	+ 0.0489	—0.0026	—0.0013	+ 0.4684	+ 0.3523	+0.0281	+0.0240
1	10	—5	— 0.05	0.00			— 0.18	+ 0.02		
—1	10	—4	+ 0.73	— 0.53			+ 4.79	— 3.33	+0.20	—0.14
0	10	—5	— 1.1	+ 0.8			— 4.2	+ 2.8		
1	10	—6					+ 1	— 1		
—1	10	—5	+ 2	+ 6			+ 6	+22		
0	10	—6	— 2	— 7			— 6	—21		
1	10	—7					+ 2	+ 6		
—1	10	—6	—12	0			—34	0		
0	10	—7	+13	0			+33	— 1		
1	10	—8					—10	+ 1		
—1	11	—4					+ 0.4	— 0.9		
0	11	—5					— 0.4	+ 0.8		
—1	11	—5	+ 1.0	+ 0.9			+ 4.6	+ 4.4		
0	11	—6	— 1	+ 1			— 4	— 4		
—1	11	—6	— 5	+ 3			—17	+ 9		
0	11	—7	+ 6	— 3			+16	— 9		
1	11	—8					— 5	+ 3		
—1	11	—7	— 2	— 8			— 6	—23		
0	11	—8	+ 3	+ 9			+ 6	+22		
0	12	—5	0.00	+ 0.03			+ 0.01	+ 0.15		
—1	12	—5	+ 0.2	0.0			+ 1.3	+ 0.3		
0	12	—6					— 1.3	— 0.4		
—1	12	—6					— 4	+ 5		
0	12	—7					+ 4	— 5		
—1	12	—7					—10	—12		
0	12	—8					+10	+12		
—1	12	—8					+13	— 7		
0	12	—9					—13	+ 8		

Arg= $\varkappa\gamma+i'g'+ig$			$\frac{1}{2}\frac{dA}{dg}(n\delta z)^2$		$\frac{dA}{dg}(n\delta z)(n'\delta z')$		$\frac{1}{2}\frac{dF}{dg'}(n'\delta z')^2$		$\frac{dB}{dg}(n\delta z)\nu$	
			nt sin.	*nt* cos.	*nt* sin.	*nt* cos.	*nt* sin.	*nt* cos.	*nt* sin.	*nt* cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		+0.0899		+0.3707		+0.4147		+0.0664
1	0	—1	—0.0005	—0.0845	—0.0306	—0.3789	—0.0550	—0.5776	+0.0043	—0.0623
—1	0	0	+0.1116	—0.0786	+0.9086	+0.2789	+2.2683	+0.7933	+0.0841	—0.0005
0	0	—1					—1.3	—0.4		
1	0	—2	+0.08	—0.08	+0.48	+0.07	+0.66	+0.14	+0.03	—0.01
—1	0	—1					—2	+2		
0	0	—2					+2	—2		
0	1	+1					+2	—2		
1	1	0			—1.1	+1.2	—2.3	+2.6		
0	1	0			+0.5	+0.8	+0.6	+1.0		
1	1	—1			—0.2	—0.4	—0.6	—1.1		
—1	1	0					+0.9	—0.1		
0	1	—1					—0.9	+0.1		
—1	1	—1					—0.1	+2.4		
0	1	—2					0	—2		
0	2	0			+1.5	+0.7	+1.4	+0.9		
1	2	—1			—1.0	—0.6	—1.9	—1.2	—0.1	—0.1
—1	2	0			—0.09	+0.09	+0.29	—0.14		
0	2	—1	—0.09	—0.01	—0.37	+0.23	—0.38	+0.07	+0.01	0.00
—1	2	—1			+0.2	+0.3	+0.1	+1.5		
1	3	—1					—1.4	0.0		
—1	3	0			0.0	—0.2				
0	3	—1	—0.18	+0.16	—0.07	+0.56	—0.13	+0.12	—0.11	+0.07
1	3	—2			0.0	—0.3	0.0	—0.1		
—1	3	—1					+0.1	+0.6		
1	4	—2					+0.1	—0.1		
0	4	—2			+0.09	+0.01				
1	5	—2	—0.003	+0.008	+0.198	+0.132	+0.409	+0.219	—0.017	—0.006
—1	5	—1			+0.028	+0.005	—0.031	+0.053		
0	5	—2	—0.0328	+0.0118	—0.0566	—0.0211	+0.0271	—0.0663	+0.0160	—0.0196
1	5	—3	+0.012	—0.008	+0.040	0.000	—0.018	+0.027		
—1	5	—2	—0.023	—0.035	—0.162	—0.067	—0.361	—0.318	+0.008	—0.017
—1	6	—2					—0.2	0.0		
—1	6	—3					+2.0	—0.7		
1	7	—3					—0.01	+0.69		
—1	7	—2			—0.12	—0.06	—0.22	—0.02		
0	7	—3	+0.26	—0.02	+0.28	+0.13	+0.16	+0.05	+0.15	—0.01
0	8	—3	+0.04	—0.03	+0.08	0.00	+0.12	—0.08	+0.02	—0.02
—1	8	—3					+0.4	—1.8		
0	8	—4					—0.3	+1.7		
0	9	—4			+0.09	+0.38	+0.23	+0.78		
—1	10	—3			—0.04	—0.03	—0.19	—0.16		
0	10	—4	+0.0202	+0.0082	+0.0608	+0.0697	+0.1725	+0.1730	+0.0148	+0.0027
1	10	—5			—0.02	0.00	—0.06	—0.02		
—1	10	—4			+0.45	—0.25	+1.48	—0.75	+0.003	—0.006

Arg= $\varkappa\gamma+i'g'+ig$			$\frac{dB}{dg'}(n'\delta z')\nu$		$\frac{dG}{dg}(n\delta z)\nu'$		$\frac{dG}{dg'}(n'\delta z')\nu'$		$\frac{1}{2}r^2\frac{d^2T}{dr^2}\nu^2$	
			nt sin.	*nt* cos.	*nt* sin.	*nt* cos.	*nt* sin.	*nt* cos.	*nt* sin.	*nt* cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		+0.1647		+0.1924		+0.3172		+0.0132
1	0	−1	+0.0264	−0.1789	−0.0127	−0.1857	−0.0527	−0.4351	+0.0014	−0.0130
−1	0	0	+0.3933	−0.0976	+0.4761	+0.0617	+1.5418	+0.3467	+0.0065	−0.0035
0	0	−1					−1.8	−0.5		
1	0	−2	+0.11	−0.03	+0.20	+0.03	+0.43	+0.09	0.00	+0.01
0	1	+1					+1	−2		
1	1	0			−0.4	+0.4	−1.5	+1.5		
0	1	0	0.0	+0.2	0.0	+0.2	+0.5	+1.0		
1	1	−1					−0.5	−1.2		
−1	1	0					+0.9	0.0		
0	1	−1					−0.6	+0.1		
−1	1	−1					−0.1	+2.0		
0	1	−2					+1	−2		
0	2	0	+0.3	+0.5	+0.5	+0.3	+1.2	+0.9		
1	2	−1	−0.2	−0.4	−0.3	−0.2	−1.3	−0.8		
−1	2	0			−0.08	+0.12	+0.06	+0.07		
0	2	−1	−0.23	+0.06	−0.15	−0.04	−0.22	−0.02	+0.01	−0.02
1	2	−2					+0.2	0.0		
−1	2	−1					+0.2	+1.2		
1	3	−1					+0.6	+0.1		
0	3	−1	−0.27	+0.21	−0.02	−0.02	−0.01	+0.04		
1	3	−2	+0.2	−0.2			+0.1	−0.3		
−1	3	−1					+0.1	+0.1		
1	4	−2					−0.1	+0.1		
0	4	−2					+0.09	−0.02		
1	5	−2			+0.042	+0.031	+0.385	+0.315		
−1	5	−1	+0.009	+0.029	+0.027	+0.021	+0.060	−0.003		
0	5	−2	−0.0417	−0.0079	−0.0480	−0.0168	+0.0102	−0.0294	+0.0016	−0.0012
1	5	−3	+0.020	+0.004	+0.003	+0.025	−0.060	+0.018		
−1	5	−2	−0.068	−0.049	−0.026	−0.094	−0.067	+0.096		
−1	6	−2					+0.2	0.0		
−1	6	−3					−0.2	+0.3		
0	6	−4					+1	−1		
−1	7	−2	−0.31	+0.01			−0.25	−0.11		
0	7	−3	+0.40	−0.04	+0.06	+0.02	+0.09	+0.07		
−1	7	−3					−0.4	+0.4		
0	8	−3	+0.13	−0.11			+0.07	−0.10		
−1	8	−3					+0.3	−1.4		
0	8	−4			−0.1	+0.5				
−1	9	−3					−0.2	−0.5		
0	9	−4	+0.19	+0.15	+0.06	+0.13	+0.22	+0.43		
−1	9	−4					+1.9	−0.1		
0	9	−5					−2	0		
−1	10	−3	−0.06	0.00			−0.11	−0.06		
0	10	−4	+0.0619	+0.0067	+0.0169	+0.0111	+0.0945	+0.0607		
1	10	−5	−0.01	+0.01			−0.03	+0.01		
−1	10	−4	+0.14	−0.27	+0.15	−0.10	+0.71	−0.50		

Arg= $\varkappa\gamma+i'g'+ig$			$rr'\frac{d^2T}{drdr'}\nu\nu'$		$\frac{1}{2}r'^2\frac{d^2T}{dr'^2}\nu'^2$	
			nt sin.	nt cos.	nt sin.	nt cos.
$\varkappa$	i'	i	″	″	″	″
0	0	0		+0.1031		+0.0950
1	0	−1	+0.0131	−0.1147	+0.0138	−0.1315
−1	0	0	+0.1481	−0.0014	+0.6303	+0.1561
1	0	−2	+0.10	+0.02	+0.19	+0.06
0	1	0	+0.2	−0.1		
−1	2	0	−0.06	+0.08		
0	2	−1	−0.06	−0.06	+0.05	−0.04
0	3	−1	−0.01	−0.04		
1	5	−2			+0.040	+0.046
−1	5	−1	−0.007	+0.012	−0.005	+0.003
0	5	−2	+0.0163	−0.0023	+0.0324	−0.0279
1	5	−3	+0.016	−0.019	−0.017	+0.008
−1	5	−2	−0.009	−0.058	+0.020	−0.209
0	10	−4	−0.0028	−0.0007	−0.0040	−0.0037
−1	10	−4			−0.005	+0.001

The fourteen parts of the portion of δ^2T having the factor n^2t^2 follow; for convenience the coefficients are multiplied by 100000000:

Arg= $\varkappa\gamma+i'g'+ig$			$An\delta^2z$		$B\delta\nu$		$Fn'\delta^2z'$		$G\delta\nu$	
			n^2t^2 sin.	n^2t^2 cos.	n^2t^2 sin.	n^2t^2 cos.	n^2t^2 sin.	n^2t^2 cos.	n^2t^2 sin.	n^2t^2 cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		+0.022		+0.006				
1	0	−1	+0.020	+0.025	+0.126	−0.015	−0.417	−0.031	+1.503	+0.065
−1	0	0	+0.301	−0.515	+0.299	−0.506	−0.726	+1.644	+1.821	−3.871
1	0	−2	+0.3	−0.5	−0.3	+0.5	+0.2	−0.5	−0.5	+1.2
1	1	−1							+3	−2
−1	1	−1					−13	−3	+12	+2
−1	2	0	−2	−1	−2	−2	−1	−1	−3	−3
0	2	−1	+3.3	+2.4	+1.9	+1.4	+0.7	+0.6	+1.6	+1.5
−1	2	−1					−4	+1	+2	−1
0	3	−1	+0.8	+0.4	+0.6	−0.1	+0.7	−0.6	+0.7	−0.5
−1	3	−1	+1	−2	0	−2	+7	−11	+6	−10
0	3	−2	−2	+4			−6	+11	−6	+9
−1	4	−1					0	−5	0	−3
0	4	−2	+1	+1			0	+3	0	+3
−1	4	−2					+13	+4	+9	+3
−1	5	−1	−0.10	−0.13	−0.11	−0.09	−0.61	−1.08	−0.44	−0.56
0	5	−2	+0.069	+0.203	+0.085	+0.107	+0.489	+1.022	+0.352	+0.546
1	5	−3	−0.08	−0.06			−0.27	−0.20		
−1	5	−2	+0.99	−0.20	+0.65	−0.26	+6.08	−1.06	+4.09	−0.97
−1	7	−3					+1	+1		
0	10	−4					−0.009	+0.010		

Arg= $\varkappa\gamma+i'g'+ig$	$\frac{1}{2}\frac{dA}{dg}(n\delta z)^2$		$\frac{dA}{dg'}(n\delta z)(n'\delta z')$		$\frac{1}{2}\frac{dF}{dg'}(n'\delta z')^2$		$\frac{dB}{dg}(n\delta z)\nu$	
	n^2t^2 sin.	n^2t^2 cos.	n^2t^2 sin.	n^2t^2 cos.	n^2t^3 sin.	n^2t^3 cos.	n^2t^2 sin.	n^2t^2 cos.
$\varkappa$ i' i	″	″	″	″	″	″	″	″
0 0 0		+0.010		−0.014		−0.045		+0.008
1 0 −1	+0.035	−0.010	−0.007	−0.007	−0.151	+0.024	−0.001	−0.011
−1 0 0	+0.005	+0.085	−0.047	+0.127	−0.681	+1.002	+0.019	+0.035
1 0 2	0.0	+0.1	0.0	+0.1	−0.1	+0.3		
0 2 −1					+0.3	0.0		
−1 2 −1					+3	−1		
0 3 −1			+0.5	−0.1				
0 4 −2					0	+1		
−1 5 −1	−0.08	−0.01	−0.17	−0.14	−0.40	−0.78	−0.11	−0.03
0 5 −2	+0.132	+0.033	+0.244	+0.202	+0.329	+0.761	+0.146	+0.049
1 5 −3			−0.11	−0.06	−0.14	−0.15		
−1 5 −2	+0.09	−0.20	+0.65	−0.46	+3.28	−0.43	+0.15	−0.21
0 7 −3			0.0	−0.3	−0.2	+0.3		
−1 10 −3					−0.1	+0.2		
0 10 −4	+0.004	−0.019	+0.026	−0.061	+0.114	−0.154	−0.001	−0.013

Arg= $\varkappa\gamma+i'g'+ig$	$\frac{dB}{dg'}(n'\delta z')\nu$		$\frac{dG}{dg}(n\delta z)\nu'$		$\frac{dG}{dg'}(n'\delta z')\nu'$		$\frac{1}{2}r^2\frac{d^2T}{dr^2}\nu^2$	
	n^2t^2 sin.	n^2t^2 cos.	n^2t^2 sin.	n^2t^2 cos.	n^2t^3 sin.	n^2t^2 cos.	n^2t^2 sin.	n^2t^2 cos.
$\varkappa$ i' i	″	″	″	″	″	″	″	″
0 0 0		+0.013		+0.042		−0.015		−0.002
1 0 −1	+0.073	−0.021	−0.100	+0.055	−0.160	−0.015	−0.117	+0.002
−1 0 0	+0.028	+0.252	−0.036	+0.050	−0.269	+1.045	−0.003	−0.005
1 0 −2	0.0	+0.1			0.0	+0.1		
1 2 −1					−0.4	+0.9		
0 2 −1	+0.2	0.0	−0.2	0.0				
0 3 −1	+0.6	−0.2	+1.0	−0.2	+0.2	+0.1		
0 4 −2	+0.1	+0.1	+0.1	+0.1	0.0	+0.2		
−1 5 −1	−0.33	−0.18	−0.24	−0.15	−0.56	−0.95		
0 5 −2	+0.362	+0.223	+0.329	+0.219	+0.465	+0.817	−0.040	−0.002
1 5 −3	−0.11	−0.03	−0.014	−0.05	−0.18	−0.12		
−1 5 −2	+0.87	−0.65	+0.73	−0.52	+3.67	−0.57		
−1 6 −2					+1	−1		
0 7 −3	0.0	−0.3			−0.1	+0.3		
−1 7 −3					+2	+1		
0 10 −4	+0.001	−0.037	+0.019	−0.027	+0.037	−0.084		

Arg= $\varkappa\gamma+i'g'+ig$			$rr'\frac{d^2T}{dr\,dr'}\nu\nu'$		$\frac{1}{2}r'^2\frac{d^2T}{dr'^2}\nu'^2$	
			n^2t^3 sin.	n^2t^3 cos.	n^2t^3 sin.	n^2t^3 cos.
$\varkappa$	i'	i	"	"	"	"
0	0	0		+0.093		−0.001
1	0	−1	+0.146	−0.112	−0.142	−0.007
−1	0	0	+0.014	+0.082	−0.181	+0.081
1	0	−2	+0.6	+0.3		
0	2	−1	+0.4	−0.5	−0.3	0.0
0	3	−1	+0.6	−0.1	+0.3	0.0
0	4	−2			0	+1
−1	5	−1	−0.25	+0.02	−0.25	−0.38
0	5	−2	+0.293	+0.014	+0.211	+0.344
1	5	−3	−0.10	−0.03	−0.10	−0.05
−1	5	−2	+0.16	−1.04	+1.31	−0.13
0	10	−4			−0.001	+0.006

CHAPTER XVIII.

THIRD-ORDER PERTURBATIONS OF THE MEAN ANOMALY AND RADIUS-VECTOR OF JUPITER ARISING FROM THE MUTUAL ACTION OF THIS PLANET AND SATURN.

The summation of the fourteen parts of δ^2T, given in the preceding chapter, produces the following expression:

Arg= $\varkappa\gamma+i'g'+ig$			δ^2T					
			sin.	cos.	nt sin.	nt cos.	n^2t^2 sin.	n^2t^2 cos.
$\varkappa$	i'	i	″	″	″	″	″	″
0	0	0		−0.0002131		+0.3198		+0.117
1	0	−1		−0.0000013	+0.2204	−0.0936	+0.808	−0.058
−1	0	0	−0.0038635	+0.0080094	−6.7600	−5.7051	+0.544	−0.494
0	0	−1	+0.0037	−0.0100	+8.7	+5.7		
1	0	−2	−0.000149	+0.002518	−2.18	−1.12	+0.2	+1.7
−1	0	−1	−0.036	−0.027	+25	−26		
0	0	−2	+0.040	+0.029	−27	+26		
1	0	−3	−0.008	−0.004	+4	−6		
−1	0	−2			+22	+40		
0	0	−3			−24	−40		
1	0	−4			+6	+9		
−1	1	+3			−19	−7		
0	1	+2			+59	+19		
1	1	+1			−57	−17		
−1	1	+2	+0.002	+0.003	+7	−9		
0	1	+1	−0.035	−0.030	−19	+31		
1	1	0	+0.0340	+0.0299	+17.7	−29.3		
−1	1	+1	−0.0026	−0.0003	+0.3	+1.6		
0	1	0	+0.0079	−0.0044	−3.7	−3.9		
1	1	−1	−0.0085	+0.0038	+4.5	+4.7	+3	−2
−1	1	0	−0.0001	+0.0020	−1.2	+0.2		
0	1	−1	−0.0005	−0.0015	+1.1	+0.1		
−1	1	−1	−0.0109	−0.0030	+4.3	−10.4	−1	−1
0	1	−2	+0.014	+0.001	−4	+10		
1	1	−3	−0.002	0.000				
−1	1	−2	+0.015	−0.037	+29	+18		
0	1	−3	−0.016	+0.037	−26	−16		
1	1	−4	+0.003	−0.007	+5	+3		
−1	1	−3			−24	+21		
0	1	−4			+23	−23		

Arg= $\varkappa\gamma+i'g'+ig$			δ^2T					
			sin.	cos.	nt sin.	nt cos.	n^2t^2 sin.	n^2t^2 cos.
$\varkappa$	i'	i	″	″	″	″	″	″
1	2+	1			+ 3	− 2		
−1	2+	2	+0.020	+0.005	+ 2	−25		
0	2+	1	−0.069	−0.017	− 7	+75		
1	2	0	+0.0751	+0.0156	+ 5.2	−70.5		
−1	2+	1	−0.0082	+0.0090	+ 8.4	+ 5.2		
0	2	0	+0.0230	−0.0363	−29.5	−10.5		
1	2−	1	−0.0221	+0.0360	+29.0	+10.9	− 0.4	+ 0.9
−1	2	0	−0.0003	−0.0019	− 0.65	− 1.16	− 8	− 7
0	2−	1	+0.00155	+0.00347	+ 1.19	− 0.12	+ 7.9	+ 5.4
1	2−	2	−0.0005	−0.0032	− 3.3	+ 1.1		
−1	2−	1	+0.0112	−0.0055	+ 0.1	+ 0.3	+ 1	− 1
0	2−	2	−0.0149	+0.0065	+ 0.5	+ 2.0		
−1	2−	2	−0.002	−0.012	+ 6	+ 1		
0	2−	3	0.000	+0.008	− 6	− 1		
−1	2−	3	+0.027	+0.007	− 6	+14		
0	2−	4	−0.028	−0.007	+ 6	−14		
0	3+	1	+0.002	−0.003				
1	3	0	+0.002	+0.001	+ 1	+ 1		
−1	3+	1	0.000	+0.032	+23	− 3		
0	3	0	−0.0002	−0.1027	−71.3	+ 9.5		
1	3−	1	+0.0017	+0.0918	+63.4	− 8.5		
−1	3	0	+0.0009	−0.0037	− 3.2	− 0.7		
0	3−	1	+0.00577	+0.00643	+ 4.43	− 5.66	+ 6.0	− 1.3
1	3−	2	−0.0081	−0.0077	− 4.7	+ 7.4		
−1	3−	1	+0.0076	−0.0093	+ 2.5	+ 2.0	+14	−25
0	3−	2	−0.0087	+0.0111	− 1.8	− 1.3	−14	+24
1	3−	3	+0.0006	−0.0013				
−1	3−	2	+0.0142	+0.0203	+ 2.3	− 1.0		
0	3−	3	−0.012	−0.015				
−1	3−	3	+0.008	−0.002	+ 1	+ 6		
0	3−	4	−0.008	+0.002	− 1	− 6		
−1	3−	4	0.000	+0.016	−12	− 2		
0	3−	5	0.000	−0.016	+11	+ 2		
1	4−	1	+0.0026	+0.0029	− 0.5	− 0.4		
−1	4	0	+0.0048	−0.0009	− 1.1	− 3.4		
0	4−	1	+0.0083	−0.0027	− 1.8	− 5.0		
1	4−	2	−0.0103	+0.0035	+ 2.3	+ 6.2		
−1	4−	1	−0.0006	−0.0004	+ 3.9	0.0	0	− 8
0	4−	2	−0.0004	+0.0034	+ 0.06	− 0.11	+ 3	+13
1	4−	3	+0.0004	+0.0008	− 2.9	0.0		
−1	4−	2	+0.0185	+0.0093	− 0.6	+ 0.9	+22	+ 7
0	4−	3	−0.019	−0.007	0	+ 1		
−1	4−	3	+0.002	−0.002				
−1	4−	4	+0.003	+0.006				
−1	4−	5	−0.012	+0.002				

Arg= $\varkappa\gamma+i'g'+ig$			δ^2T					
			sin.	cos.	nt sin.	nt cos.	n^2t^2 sin.	n^2t^2 cos.
$\varkappa$	i'	i	″	″	″	″	″	″
0	5—	1	+0.0027	—0.0023				
1	5—	2	—0.004247	+0.003517	+1.110	+0.808		
—1	5—	1	+0.001566	+0.002618	+2.508	—1.251	—3.65	—4.46
0	5—	2	+0.0008236	+0.0017731	—0.2405	—0.2069	+3.466	+4.538
1	5—	3	+0.000859	+0.002827	+3.268	—1.893	—1.23	—0.75
—1	5—	2	+0.003370	+0.000234	—0.954	+0.970	+22.72	—6.70
0	5—	3	—0.0021	—0.0004	—0.1	—0.7		
1	5—	4	—0.0005	—0.0005				
—1	5—	3	—0.005	+0.023	+2	0		
0	5—	4	+0.006	—0.022				
1	5—	5	0.000	+0.003				
—1	5—	4	—0.024	—0.014				
0	5—	5	+0.026	+0.014				
1	6—	2	—0.0025	+0.0075				
—1	6—	1	—0.0005	—0.0003	—0.3	+0.8		
0	6—	2	+0.0018	+0.0017				
1	6—	3	—0.0015	—0.0010	+0.6	—1.0		
—1	6—	2	+0.0084	—0.0049	—5.3	—5.7	+1	—1
0	6—	3	—0.0063	+0.0039	+4.0	+4.4		
1	6—	4	—0.0040	+0.0029	+2.5	+2.6		
—1	6—	3	—0.0007	+0.0082	+3.7	—0.5		
0	6—	4	—0.002	—0.008	—1	0		
—1	6—	4	—0.022	—0.001				
0	6—	5	+0.020	+0.001				
—1	6—	5	+0.007	—0.023				
0	6—	6	—0.007	+0.024				
1	7—	3	—0.00392	—0.00051	+0.07	+0.41		
—1	7—	2	—0.00200	—0.00678	—6.25	+1.92		
0	7—	3	+0.00247	+0.00593	+5.69	—2.52	—0.3	0.0
1	7—	4	—0.0037	—0.0015	—0.9	+3.1		
—1	7—	3	+0.0666	+0.0655	+40.5	—56.3	+3	+2
0	7—	4	—0.0731	—0.0705	—45.0	+62.2		
1	7—	5	+0.024	+0.022	+13	—19		
—1	7—	4	—0.009	+0.002	0	+1		
0	7—	5	—0.001	—0.006	—1	—2		
—1	8—	2	—0.0013	—0.0010	—0.9	+1.1		
0	8—	3	+0.00119	+0.00070	+1.23	—1.51		
1	8—	4	—0.0013	+0.0017	+0.1	+0.7		
—1	8—	3	+0.0266	+0.0067	+3.1	—25.7		
0	8—	4	—0.0282	—0.0077	—3.4	+25.2		
1	8—	5	+0.0091	0.0000	—0.9	—6.5		
—1	8—	4	—0.0529	+0.0836	+67.4	+30.1		
0	8—	5	+0.056	—0.090	—70	—29		
1	8—	6	—0.017	+0.027	+22	+9		

Arg= $\varkappa\gamma+i'g'+ig$			δ^2T					
			sin.	cos.	nt sin.	nt cos.	n^2t^2 sin.	n^2t^2 cos.
$\varkappa$	i'	i	"	"	"	"	"	"
—1	8	— 5	+0.001	+0.003	+ 3	— 2		
0	8	— 6	+0.003	—0.005				
—1	8	— 6	+0.014	—0.006				
—1	9	— 3	+0.0059	—0.0016	— 2.1	— 4.3		
0	9	— 4	—0.00479	+0.00098	+ 2.45	+ 5.85		
1	9	— 5	+0.0007	—0.0011	— 1.1	— 0.8		
—1	9	— 4	—0.0027	+0.0362	+33.8	— 2.7		
0	9	— 5	+0.005	—0.037	—30	+ 2		
1	9	— 6	+0.001	+0.012	+ 8	— 1		
—1	9	— 5	—0.080	—0.030	—14	+62		
0	9	— 6	+0.081	+0.028	+12	—64		
1	9	— 7	—0.027	—0.009	— 3	+18		
—1	9	— 6	—0.004	+0.001				
0	9	— 7	+0.006	+0.002				
—1	10	— 3	+0.00081	—0.00079	— 1.01	— 0.66	—0.1	+0.2
0	10	— 4	—0.0006663	+0.0004584	+ 0.9933	+ 0.7516	+0.188	—0.379
1	10	— 5	+0.00012	—0.00036	— 0.35	+ 0.02		
—1	10	— 4	+0.00212	+0.00753	+ 8.63	— 5.92		
0	10	— 5	—0.0030	—0.0076	— 5.3	+ 3.6		
1	10	— 6	+0.002	+0.002	+ 1	— 1		
—1	10	— 5	—0.037	+0.003	+ 8	+28		
0	10	— 6	+0.039	—0.006	— 8	—28		
1	10	— 7	—0.012	+0.004	+ 2	+ 6		
—1	10	— 6	+0.008	—0.062	—46	0		
0	10	— 7	—0.007	+0.064	+46	— 1		
1	10	— 8	+0.001	—0.020	—10	+ 1		
—1	11	— 4	+0.0006	+0.0006	+ 0.4	— 0.9		
0	11	— 5	—0.0008	—0.0006	— 0.4	+ 0.8		
—1	11	— 5	—0.0082	+0.0055	+ 5.6	+ 5.3		
0	11	— 6	+0.008	—0.007	— 5	— 3		
1	11	— 7	—0.002	+0.002				
—1	11	— 6	—0.012	—0.032	—22	+12		
0	11	— 7	+0.013	+0.033	+22	—12		
1	11	— 8	—0.003	—0.007	— 5	+ 3		
—1	11	— 7	+0.043	—0.004	— 8	—31		
0	11	— 8	—0.044	+0.005	+ 9	+31		
0	12	— 5	+0.00057	—0.00096	+ 0.01	+ 0.18		
—1	12	— 5	+0.0025	—0.0075	+ 1.5	+ 0.3		
0	12	— 6	—0.0007	+0.0022	— 1.3	— 0.4		
—1	12	— 6	—0.009	—0.008	— 4	+ 5		
0	12	— 7	+0.006	+0.006	+ 4	— 5		
—1	12	— 7	+0.025	—0.015	—10	—12		
0	12	— 8	—0.024	+0.015	+10	+12		
—1	12	— 8	+0.009	+0.026	+13	— 7		
0	12	— 9	—0.01	—0.02	—13	+ 8		

In precisely the same manner as $\overline{W_0}$ and $\overline{\delta W_0}$ have, in preceding chapters, been derived from T and δT we now get $\overline{\delta^2 W_0}$ from δ^2T. In the case of the terms depending on the arguments $5g' - 2g$ and $10g' - 4g$ the motion of the argument has been equated. We proceed as at page 275, the only difference being that here terms multiplied by n^2t^2 are present. By adding T, δT, and δ^2T we obtain

$$
\begin{aligned}
T + \delta T + \delta^2 T = \; & [-0\overset{''}{.}0718628 - 0\overset{''}{.}000139010nt + 0\overset{''}{.}00000003466n^2t^2] \sin(5g' - 2g) \\
& + [-0.1794415 + 0.000082567nt + 0.00000004538n^2t^2] \cos(5g' - 2g) \\
& + [-0.0051553 + 0.000008603nt + 0.00000000188n^2t^2] \sin(10g' - 4g) \\
& + [0.0052333 + 0.000006509nt - 0.00000000379n^2t^2] \cos(10g' - 4g)
\end{aligned}
$$

In the case of the argument $5g' - 2g$ we get $\varkappa$ and the corrected integrating factor from

$$\log \varkappa = 6.9171965n \qquad \log \mu = 1.8995167$$

In the case of the argument $10g' - 4g$ the similar quantities are

$$\log \varkappa = 7.16317n \qquad \log \mu = 1.59510$$

The expression just written can then be transformed into

$$
\begin{aligned}
& [-0\overset{''}{.}0718628 + 0\overset{''}{.}000009283nt - 0\overset{''}{.}00000000903n^2t^2] \sin(5g' - 2g + \varkappa nt) \\
& + [-0.1794415 + 0.000023179nt - 0.00000000823n^2t^2] \cos(5g' - 2g + \varkappa nt) \\
& + [-0.0051553 + 0.000000983nt - 0.00000000214n^2t^2] \sin(10g' - 4g + \varkappa nt) \\
& + [0.0052333 - 0.000000997nt + 0.00000000319n^2t^2] \cos(10g' - 4g + \varkappa nt)
\end{aligned}
$$

Integrating this the result is

$$
\begin{aligned}
W_0 + \delta W_0 + \delta^2 W_0 = \; & [5\overset{''}{.}83882 - 0\overset{''}{.}0008402nt + 0\overset{''}{.}000000716n^2t^2] \cos(5g' - 2g + \varkappa nt) \\
& + [-14.17103 + 0.0017254nt - 0.000000653n^2t^2] \sin(5g' - 2g + \varkappa nt) \\
& + [0.20113 - 0.00002881nt + 0.0000000842n^2t^2] \cos(10g' - 4g + \varkappa nt) \\
& + [0.20714 - 0.00004586nt + 0.0000001256n^2t^2] \sin(10g' - 4g + \varkappa nt) \\
= \; & [5.83882 + 0.0108709nt - 0.000002704n^2t^2] \cos(5g' - 2g) \\
& + [-14.17103 + 0.0065507nt + 0.000003492n^2t^2] \sin(5g' - 2g) \\
& + [0.20113 - 0.00033041nt - 0.0000000622n^2t^2] \cos(10g' - 4g) \\
& + [0.20714 + 0.00024699nt - 0.0000001360n^2t^2] \sin(10g' - 4g)
\end{aligned}
$$

If from the latter expression we subtract the following (obtained at page 276)

$$
\begin{aligned}
W_0 + \delta W_0 = \; & [5\overset{''}{.}92153 + 0\overset{''}{.}0107729nt] \cos(5g' - 2g) \\
& + [-14.29576 + 0.0068379nt] \sin(5g' - 2g) \\
& + [0.16574 + 0.00004901nt] \cos(10g' - 4g) \\
& + [0.17595 - 0.00003718nt] \sin(10g' - 4g)
\end{aligned}
$$

we get

$$
\begin{aligned}
\delta^2 W_0 = & \;[-\overset{\prime\prime}{0.08271} + \overset{\prime\prime}{0.0000980}nt - \overset{\prime\prime}{0.000002704}n^2t^2\,] \cos(5g' - 2g) \\
& + [\;0.12473 - 0.0002872nt + 0.000003492n^2t^2\,] \sin(5g' - 2g) \\
& + [\;0.03539 - 0.00037942nt - 0.0000000622n^2t^2] \cos(10g' - 4g) \\
& + [\;0.03119 + 0.00028417nt - 0.0000001360n^2t^2] \sin(10g' - 4g)
\end{aligned}
$$

The value of $\overline{\delta^2 W_0}$ follows:

Arg$=i'g'+ig$	$\overline{\delta^2 W_0}$					
	cos.	sin.	nt cos.	nt sin.	n^2t^2 cos.	n^2t^2 sin.
i' i	″	″	″	″	″	″
0 0			− 21.12		+ 160.7	
0 − 1	+0.0037+k_1	+0.0088+k_2	+808.55	−391.49	−2852.5	−3380.0
0 − 2	−0.0187+[8.38]k_1	+0.0138+[8.38]k_2	+ 32.3	+ 5.7	− 68.7	− 81.5
0 − 3	−0.0009	+0.0006	+ 5.8	− 8.6	− 2.5	− 2.9
1 + 2	−0.0021	+0.0018	+ 21	− 8		
1 + 1	−0.0621	+0.0549	− 32.1	− 54.6		
1 0	−0.0321	−0.0176	+ 16.4	− 16.6	+ 5	+ 3
1 − 1	−0.0006	+0.0074	+ 4.8	+ 0.3		
1 − 2	−0.0106	+0.0046	+ 0.4	+ 1.1	− 2	+ 2
1 − 3	+0.0037	+0.0111	+ 9	− 6		
2 + 2	−0.002	0.000	− 2	− 3		
2 + 1	−0.0658	+0.0076	+ 1	− 56		
2 0	−0.1349	−0.2178	+180.9	− 66.3	− 2	− 5
2 − 1	+0.0080	−0.0171	+ 4.3	− 1.3	+ 51	− 37
2 − 2	+0.0454	+0.0229	+ 0.9	− 3.2	+ 5	+ 5
2 − 3	−0.0003	+0.0071	+ 2	− 1		
2 − 4	+0.0035	−0.0010	− 1	− 2		
3 + 1	−0.0028	+0.0104	− 8	0		
3 0	−0.0100	+0.3867	−256.2	− 34.5		
3 − 1	−0.0398	+0.0387	− 24.8	− 36.8	− 29	− 6
3 − 2	−0.0467	−0.0575	− 15.1	+ 11.7	− 85	− 150
3 − 3	+0.0103	−0.0183	+ 3	+ 1		
3 − 4	+0.0021	−0.0002	+ 1	− 1		
3 − 5	0.0000	−0.0015	− 1	0		
4 − 1	−0.0427	−0.0136	+ 9.5	− 26.2		
4 − 2	+0.0003	−0.0086	− 8.2	+ 0.3	+ 8	− 46
4 − 3	+0.0338	−0.0188	− 1.5	− 2.7	+ 57	− 18
4 − 4	+0.0025	+0.0008				
4 − 5	+0.0013	−0.0025				
4 − 6	−0.0035	−0.0006				
5 − 1	+0.3583	+0.3211	− 82.6	+ 60.2		
5 − 2	−0.08342	+0.12451	+ 10.63	− 28.04	− 268.0	+ 345.6
5 − 3	−0.0120	+0.0206	− 3.4	+324.8	−1692	− 499
5 − 4	−0.0023	−0.0127	+ 2	+ 8	− 40	− 10
5 − 5	−0.0035	+0.0017				

Arg=$i'g'+ig$	$\overline{\delta^2 W_0}$					
	cos.	sin.	nt cos.	nt sin.	n^2t^2 cos.	n^2t^2 sin.
i' i	″	″	″	″	″	″
6— 2	—0.0065	+0.0056	+ 1.2	+ 2.3		
6— 3	—0.0335	—0.0203	+ 21.2	— 22.8	— 2	— 2
6— 4	—0.0030	—0.0092	+ 5	+ 1		
6— 5	—0.0062	—0.0001				
6— 6	+0.0005	+0.0022				
7— 2	—0.0327	+0.0043	+ 0.4	— 2.3		
7— 3	+0.0126	—0.0383	+ 38.3	+ 13.8	— 2	0
7— 4	+0.3000	—0.3003	+191.6	+266.7	+17	—11
7— 5	+0.0003	—0.0073	+ 5	+ 7		
8— 3	—0.0062	+0.0004	— 4.8	— 6.7		
8— 4	—0.1562	+0.0406	— 18.9	—144.7		
8— 5	—0.0450	—0.0646	+ 54	— 29		
8— 6	+0.0001	—0.0024	+ 4	0		
9— 4	—0.0214	—0.0047	+ 9.1	— 21.9		
9— 5	—0.0034	—0.0721	+ 71.4	+ 6.2		
9— 6	—0.0322	+0.0104	— 4	— 23		
9— 7	—0.0013	—0.0005	0	— 1		
10— 4	+0.03473	+0.03076	— 37.31	+ 27.77	— 6.1	—13.4
10— 5	—0.1630	+0.4065	—326.2	—223.6		
10— 6	—0.0261	+0.0082	— 3	— 22		
10— 7	+0.0010	+0.0148	— 11	— 1		
11— 5	—0.0028	+0.0024	— 1.6	— 3.5		
11— 6	—0.0100	—0.0051	+ 7	— 7		
11— 7	—0.0037	+0.0094	— 6	— 4		
11— 8	+0.0042	+0.0006	0	+ 3		
12— 5	+0.00346	+0.00572	+ 0.06	— 1.07		
12— 6	+0.0143	+0.0434	+ 7.8	— 1.5		
12— 7	—0.0048	+0.0040	— 1	— 2		
12— 8	+0.0039	+0.0022	— 2	+ 2		
12- 9	+0.0004	—0.0034	+ 2	0		

Arg=$i'g'+ig$	$\overline{\delta^2 W_0}$	
	n^3t^3 cos.	n^3t^3 sin.
i' i	″	″
0 0	+0.00000000039	
0— 1	—0.00000000165	+0.00000000181
0— 2	—0.00000000004	+0.00000000004

Also, we have the expression of the following function, needed in the determination of $\delta^2\nu$:

Arg $=i'g'+ig$	$-\frac{1}{2}\left(\overline{\frac{d\,.\,\delta^2 W_0}{d\gamma}}\right)$					
	sin.	cos.	nt sin.	nt cos.	n^2t^2 sin.	n^2t^2 cos.
i' i	″	″	″	″	″	″
0 − 1	+0.0169	−0.0366	−401.0	−193.7	+1426.3	−1690.0
0 − 2	+0.0175	+0.0111	−31.4	+2.7	+69	−81
0 − 3	+0.0009	+0.0006	−6.4	−8.7	+4	−4
1 + 2	−0.0021	−0.0018	+16	+7		
1 + 1	−0.0430	−0.0369	−19.8	+34.8		
1 0	−0.0079	+0.0030	+3.8	+4.3	+2	−1
1 − 1	−0.0001	+0.0024	−1.5	+0.2		
1 − 2	+0.0089	+0.0026	−3	+8		
1 − 3	−0.0039	+0.0108	−9	−5		
2 + 2	−0.002	0.000	−1	+2		
2 + 1	−0.0464	−0.0045	+1	+41		
2 0	−0.0572	+0.0911	+76.8	+29.2	−1	+2
2 − 1	−0.0005	−0.0024	−1.7	−0.2	−5	−4
2 − 2	−0.0289	+0.0141	−0.2	−0.7	−2	+2
2 − 3	−0.0006	+0.0057	−2	0		
2 − 4	−0.0061	−0.0016	+1	−3		
3 + 1	−0.0010	−0.0114	−7	+1		
3 0	−0.0052	−0.2208	−147.2	+19.9		
3 − 1	−0.0047	−0.0060	−4.0	+4.3		
3 − 2	+0.0181	−0.0224	+6.5	+5.0	+33	−60
3 − 3	−0.0080	−0.0139	−1	0		
3 − 4	−0.0026	−0.0001	0	−1		
3 − 5	0.0000	−0.0028	+2	0		
4 − 1	−0.0114	+0.0041	+2.6	−8.9		
4 − 2	−0.0004	−0.0005	+2.2	0.0	0	−6
4 − 3	−0.0237	−0.0120	+0.7	−1.0	−28	−9
4 − 4	−0.0018	+0.0001				
4 − 5	−0.0006	−0.0012				
4 − 6	+0.0017	−0.0003				
5 − 1	+0.1805	−0.1617	−41.3	−30.1		
5 − 2	+0.001198	+0.002714	+2.897	−1.578	−2.42	−2.58
5 − 3	+0.0047	+0.0098	+1.6	+162.0	+846	−249
5 − 4	+0.0027	−0.0106	−1	+8	+40	−10
5 − 5	+0.0061	+0.0029				
6 − 2	−0.0015	−0.0009	+0.4	−0.5		
6 − 3	+0.0090	−0.0051	−5.5	−6.0		
6 − 4	+0.0011	−0.0072	−3	0		
6 − 5	+0.0069	0.0000				
6 − 6	−0.0011	+0.0044				

Arg=$i'g'+ig$	$-\frac{1}{2}\left(\overline{\frac{d\,.\,\delta^2W_0}{d\gamma}}\right)$					
	sin.	cos.	nt sin.	nt cos.	n^2t^2 sin.	n^2t^2 cos.
i' i	″	″	″	″	″	″
7— 2	−0.0107	−0.0014	+ 0.2	+ 1.1		
7— 3	−0.0024	−0.0046	− 4.1	+ 2.2		
7— 4	−0.1698	−0.1698	−108.9	+151.1		
7— 5	−0.0046	−0.0092	− 5	+ 7		
8— 3	−0.0012	+0.0007	− 0.3	+ 0.7		
8— 4	+0.0652	+0.0153	+ 6.8	− 59.8		
8— 5	+0.0347	−0.0476	− 39	− 21		
8— 6	+0.0013	−0.0033	− 3	− 1		
9— 4	+0.0050	−0.0014	− 2.1	− 3.7		
9— 5	+0.0039	−0.0444	− 43.3	+ 3.4		
9— 6	+0.0248	+0.0074	+ 3	− 20		
9— 7	+0.0022	+0.0003				
10— 4	+0.00048	−0.00055	− 0.66	− 0.32		
10— 5	+0.0809	+0.2004	+160.8	−110.4		
10— 6	+0.0209	+0.0087	+ 4	− 18		
10— 7	−0.0019	+0.0132	+ 10	0		
11— 5	+0.0007	+0.0007	+ 0.4	− 1.0		
11— 6	+0.0067	−0.0047	− 5	− 4		
11— 7	+0.0034	+0.0092	+ 7	− 4		
11— 8	−0.0083	+0.0007	+ 1	+ 6		
12— 6	−0.0074	+0.0226	− 4.4	− 0.9		
12— 7	+0.0038	+0.0034	+ 1	− 2		

Arg=$i'g'+ig$	$-\frac{1}{2}\left(\overline{\frac{d\,.\,\delta^2W_0}{d\gamma}}\right)$	
	n^3t^3 sin.	n^3t^3 cos.
i' i	″	″
0— 1	+0.00000000082	+0.00000000090
0— 2	+0.00000000004	+0.00000000004

The rigorous equation determining z is

$$\frac{dz}{dt} = 1 + \frac{\overline{W} + \nu^2}{1 - \nu^2}$$

where for $\overline{W}$ we ought to substitute

$$\overline{W} = \overline{W}_0 + \left(\overline{\frac{dW_0}{d\gamma}}\right)n\delta z + \frac{1}{2}\left(\overline{\frac{d^2W_0}{d\gamma^2}}\right)(n\delta z)^2 + \ldots$$

Here the several factors include terms of all dimensions with respect to the disturbing forces. Limiting ourselves to the terms of three dimensions we have

$$\frac{d(n\delta^3 z)}{ndt} = \overline{\delta^3 W}_0 + \left(\frac{\overline{dW_0}}{d\gamma}\right) n\delta^2 z + \left(\frac{\overline{d\,.\,\delta W_0}}{d\gamma}\right) n\delta z + \frac{1}{2}\left(\frac{\overline{d^2 W_0}}{d\gamma^2}\right)(n\delta z)^2 + 2\nu\delta\nu + \nu^2 \frac{d\,.\,n\delta z}{ndt}$$

All the factors involved in the right member of this equation have already been given. But, as in forming the product $\left(\frac{\overline{dW_0}}{d\gamma}\right) n\delta z$ in the computation of the terms of the second order, we have corrected the two factors for the terms multiplied by nt, belonging to the arguments g, $2g$, etc., and which result from the previous computation of δW_0, these terms must be omitted from the factors $n\delta^2 z$ and $\left(\frac{\overline{d\,.\,\delta W_0}}{d\gamma}\right)$.

The values of the five additional quantities involved in the expression of $\frac{d(n\delta^3 z)}{ndt}$ follow:

Arg=$i'g'+ig$	$\left(\frac{\overline{dW_0}}{d\gamma}\right) n\delta^2 z$					
	cos.	sin.	nt cos.	nt sin.	n^2t^2 cos.	n^2t^2 sin.
i' i	″	″	″	″	″	″
0 0			+ 0.287		− 4.02	
0 − 1	+0.0154	+0.0458	+28.9	−14.4		
0 − 2	+0.001	+0.003	+ 3	+ 1		
0 − 3	+0.002	+0.002	+ 1	− 1		
1 + 2	−0.003	−0.005	+ 3	− 1		
1 + 1	−0.0050	+0.0032	− 3.1	− 4.9		
1 0	−0.0004	−0.0002	− 0.7	+ 0.6		
1 − 1	+0.0001	+0.0004	+ 1.0	− 0.1		
1 − 2	−0.001	0.000				
2 + 1	−0.013	+0.002	0	−12		
2 0	−0.0085	−0.0139	+ 9.5	− 4.0	− 3	+10
2 − 1	−0.0011	+0.0020	+ 1.8	+ 0.2	− 9	+ 7
2 − 2	−0.0009	−0.0009			−11	− 2
2 − 3	0.000	−0.001				
3 + 1	+0.001	+0.004	− 3	0		
3 0	−0.0003	+0.0818	−59.3	− 8.5		
3 − 1	+0.0022	−0.0003	− 1.3	+ 3.7	+13	+ 3
3 − 2	−0.0006	−0.0014	+ 0.8	− 2.4		
3 − 3	−0.001	+0.002				
4 0	−0.001	0.000				
4 − 1	−0.0095	−0.0018	+ 2.4	− 6.5		
4 − 2	+0.0002	−0.0021	0.0	− 0.1		
4 − 3	−0.0007	+0.0008	+ 0.2	+ 1.5		

Arg=$i'g'+ig$	$\left(\overline{\frac{dW_0}{d\gamma}}\right)n\delta^2 z$					
	cos.	sin.	nt cos.	nt sin.	n^2t^2 cos.	n^2t^2 sin.
i' i	″	″	″	″	″	″
5− 0			+ 3	− 1		
5− 1			+55.9	−19.2	+ 82	+414
5− 2	+0.000538	−0.001328	− 2.690	− 1.682	+ 35.76	− 24.37
5− 3	−0.0022	+0.0006	+24.1	+54.0	−404	+118
5− 4			+ 4	0	+ 30	+ 30
6− 2	−0.0008	+0.0007	− 0.3	+ 0.1		
6− 3	−0.0076	−0.0053	+ 4.6	− 5.1		
6− 4	−0.001	0.000				
7− 3	+0.0016	−0.0011	− 0.2	+ 0.8		
7− 4	+0.0595	−0.0562	+33.1	+45.7		
7− 5	0.000	−0.002	+ 4	− 3		
8− 3	+0.0002	+0.0007	+ 0.9	+ 0.7		
8− 4	−0.0158	+0.0032	− 1.1	−12.3		
8− 5	−0.0007	−0.0010	+ 7	− 3		
9− 4	−0.0007	−0.0002	+ 0.2	− 0.4		
9− 5	−0.0014	−0.0069	+ 5.7	+ 0.5		
9− 6	−0.007	+0.002	− 1	− 3		
10− 4	−0.00010	−0.00009	+ 1.43	− 0.39	0.0	− 0.3
10− 5	−0.0221	+0.0420	−32.2	−19.6		
10− 6	−0.001	+0.001	− 2	+ 2		

Arg=$i'g'+ig$	$\frac{1}{2}\left(\overline{\frac{d^2W_0}{d\gamma^2}}\right)(n\delta z)^2$					
	cos.	sin.	nt cos.	nt sin.	n^2t^2 cos.	n^2t^2 sin.
i' i	″	″	″	″	″	″
0 0			+0.007		−0.05	
0− 1			−1.2	+1.3		
1− 1	+0.0001	− 0.0003				
2− 1	−0.0001	+0.0002				
2− 2	+0.0023	+0.0010				
3− 1			−0.2	−0.3		
3 − 2	−0.0003	−0.0004				
5− 2	+0.000008	+0.000002	−0.020	−0.020	−0.14	−0.15
5− 3	−0.0014	−0.0001				
7− 3			−0.1	0.0		
8− 2	+0.0010	+0.0005				
8− 3	0.0000	−0.0002				
10− 4	+0.00009	+0.00009	−0.02	+0.01	−0.1	−0.1

Arg=i′g′+ig	$\left(\frac{\overline{d\,.\,\delta W_0}}{d\gamma}\right)n\delta z$					
	cos.	sin.	*nt* cos.	*nt* sin.	n^2t^2 cos.	n^2t^2 sin.
i′ i	″	″	″	″	″	″
0 0			+ 0.540		— 1.81	
0— 1	+0.0107	+0.0131	—22.2	+3.7		
0— 2	+0.002	—0.001	— 1	—1		
1+ 2	—0.001	—0.002				
1+ 1	+0.0001	—0.0020	+ 1.4	+1.3		
1 0	—0.0002	—0.0002	— 0.6	+0.3	+ 3	+ 3
1— 1	+0.0003	—0.0011	— 0.7	+0.1		
2+ 1			+ 1	—2		
2 0			— 3.3	+2.6	— 3	+ 4
2— 1	—0.0008	+0.0020	+ 0.5	—1.4	— 7	+ 4
2— 2	+0.0221	+0.0095				
2— 3	0.000	—0.001				
3+ 1			— 1	+1		
3 0	—0.0007	—0.0010				
3— 1	+0.0017	—0.0012	— 0.2	+2.9	+ 9	+ 1
3— 2	—0.0058	—0.0106	0.0	+2.0		
3— 3	+0.001	—0.005				
4 0	—0.002	+0.001				
4— 1	+0.0004	—0.0010				
4— 2	+0.0002	—0.0003	— 1.6	0.0	— 1	+ 3
4— 3	+0.0061	—0.0031				
5 0	—0.001	+0.001				
5— 1	0.0000	+0.0011			+30	+40
5— 2	+0.000560	—0.001114	— 2.548	—1.584	+35.38	—24.65
5— 3	—0.0076	—0.0016			+50	— 6
5— 4	—0.001	—0.002	+ 2	—2	+20	+30
6— 1	+0.0013	+0.0042				
6— 2	—0.0002	0.0000				
6— 3	+0.0003	+0.0008				
6— 4			— 1	0		
7— 2	—0.0091	+0.0011				
7— 3	+0.0003	+0.0001	— 1.8	—0.1		
7— 4	—0.0030	+0.0006	+ 0.4	—2.3		
7— 5	—0.003	0.000	+ 2	—5		
8— 2	+0.0097	+0.0041				
8— 3	+0.0002	+0.0005	+ 0.5	+0.3		
8— 4	—0.0004	+0.0008	— 0.5	—4.2		
8— 5	+0.001	+0.001	— 1	+1		
9— 3	—0.0003	+0.0005				
9— 4	—0.0005	—0.0002	— 0.1	—0.1		
9— 5	—0.0015	+0.0016	+ 2.0	+0.1		
9— 6	—0.002	0.000				
10— 4	—0.00027	—0.00024	+ 1.36	—0.40		
10— 5	—0.0030	+0.0171				
10— 6	+0.002	—0.001				

Arg=$i'g'+ig$	$2\nu\delta\nu$					
	cos.	sin.	nt cos.	nt sin.	n^2t^2 cos.	n^2t^2 sin.
i' i	″	″	″	″	″	″
0 0			−0.150		+2.06	
1 0			+0.3	−0.4		
2−1	+0.0008	−0.0011	−0.8	+0.1	+4	−3
3−1	−0.0011	+0.0008	0.0	−1.6	−5	−2
4−2	−0.0001	+0.0003	+0.5	0.0		
5−2	−0.000295	+0.000669	+1.344	+0.834	−17.94	+12.41
5−4					+10	+10
6−2	+0.0001	−0.0001				
7−3	−0.0002	0.0000	+0.9	+0.1		
7−5	−0.001	−0.001				
8−3	0.0000	−0.0003	−0.4	−0.2		
9−4	+0.0002	+0.0001				
10−4	+0.00003	+0.00001	−0.68	+0.18		

The single sensible term of $\frac{d\,.\,\delta z}{dt}\nu^2$ is $+0''.004nt$.

The addition of the six terms of $\frac{d\,.\,\delta^3 z}{dt}$ gives the following expression for this quantity:

Arg=$i'g'+ig$	$\frac{d\,.\,\delta^3 z}{dt}$					
	cos.	sin.	nt cos.	nt sin.	n^2t^2 cos.	n^2t^2 sin.
i' i	″	″	″	″	″	″
0 0			−20.428		+156.93	
0−1	$+0.0298+k_1$	$+0.0677+k_2$	+814.0	−400.9	−2852	−3380
0−2	$-0.0157+[8.38]k_1$	$+0.0158+[8.38]k_2$	+34.3	+5.7	−69	−81
0−3	+0.0011	+0.0026	+6.8	−9.6	−2	−3
1+2	−0.0061	−0.0052	+24	−9		
1+1	−0.0670	+0.0561	−33.8	−58.2		
1 0	−0.0327	−0.0180	+15.4	−16.1	+8	+6
1−1	−0.0001	+0.0064	+5.1	+0.3		
1−2	−0.0116	+0.0046	+0.4	+1.1		
1−3	+0.0037	+0.0111	+9	−6		
2+2	−0.002	0.000	−2	−3		
2+1	−0.0788	+0.0096	+2	−70		
2 0	−0.1434	−0.2317	+187.8	−67.7	−8	+9
2−1	+0.0068	−0.0140	+5.8	−2.4	+39	−29
2−2	+0.0689	+0.0325	+0.9	−3.2	−6	+3
2−3	−0.0003	+0.0051	+2	−1		
2−4	+0.0035	−0.0010	−1	−2		

Arg=$i'g'+ig$	$\frac{d\,.\,\delta^2 z}{dt}$					
	cos.	sin.	nt cos.	nt sin.	n^2t^2 cos.	n^2t^2 sin.
i' i	″	″	″	″	″	″
3+ 1	−0.0018	+0.0144	− 12	+ 1		
3 0	−0.0110	+0.4675	−315.5	− 43.0		
3− 1	−0.0370	+0.0380	− 26.5	− 32.1	− 12	− 4
3− 2	−0.0434	−0.0699	− 14.3	+ 11.3	− 85	−150
3− 3	+0.0103	−0.0213	+ 3	+ 1	− 2	− 3
3− 4	+0.0021	−0.0002	+ 1	− 1		
3− 5	0.0000	−0.0015	− 1	0		
4 0	−0.003	+0.001				
4− 1	−0.0518	−0.0164	+ 11.9	− 32.7		
4− 2	+0.0006	−0.0107	− 9.3	+ 0.2	+ 7	− 43
4− 3	+0.0392	−0.0211	− 1.3	− 1.2	+ 57	− 18
4− 4	+0.0025	+0.0008				
4− 5	+0.0013	−0.0025				
4− 6	−0.0035	−0.0006				
5 0	−0.001	+0.001	+ 3	− 1		
5− 1	+0.3583	+0.3222	− 26.7	+ 41.0	+ 112	+454
5− 2	−0.082609	+0.122739	+ 6.716	− 30.492	− 214.94	+308.84
5− 3	−0.0232	+0.0195	+ 20.7	+378.8	−2046	−387
5− 4	−0.0033	−0.0147	+ 8	+ 6	+ 20	+ 60
5− 5	−0.0035	+0.0017				
6− 1	+0.0013	+0.0042				
6− 2	−0.0074	+0.0062	+ 0.9	+ 2.4		
6− 3	−0.0408	−0.0248	+ 25.8	− 27.9	− 2	− 2
6− 4	−0.0040	−0.0092	+ 4	+ 1		
6− 5	−0.0062	−0.0001				
6− 6	+0.0005	+0.0022				
7− 2	−0.0236	+0.0054	+ 0.4	− 2.3		
7− 3	+0.0143	−0.0393	+ 37.1	+ 14.6	− 2	0
7− 4	+0.3565	−0.3559	+225.1	+310.1	+ 17	− 11
7− 5	−0.0037	−0.0103	+ 11	− 1		
8− 2	+0.0107	+0.0046				
8− 3	−0.0058	+0.0011	− 3.8	− 5.9		
8− 4	−0.1724	+0.0446	− 20.5	−161.2		
8− 5	−0.0510	−0.0736	+ 60	− 31		
8− 6	+0.0001	−0.0024	+ 4	0		
9− 3	−0.0003	+0.0005				
9− 4	−0.0224	−0.0050	+ 9.2	− 22.4		
9− 5	−0.0063	−0.0774	+ 79.1	+ 6.8		
9− 6	−0.0412	+0.0124	− 5	− 26		
9− 7	−0.0013	−0.0005	0	− 1		
10− 4	+0.03448	+0.03053	− 35.22	+ 27.17	− 6.2	− 13.8
10− 5	−0.1881	+0.4656	−358.4	−243.2		
10− 6	−0.0251	+0.0082	− 5	− 20		
*10− 7	+0.0010	+0.0148	− 11	− 1		

---

* The terms corresponding to the divisions 11 and 12 are omitted, since they are the same as the similar terms of $\overline{\delta^2 W_0}$, given on page 395.

Arg=$i'g'+ig$	$\frac{d\,.\,\delta^3 z}{dt}$	
	n^3t^3 cos.	n^3t^3 sin.
i' i	$''$	$''$
0— 1	—0.00000000165	+0.00000000181
0— 2	—0.00000000004	+0.00000000004

On integrating this expression we arrive at $n\delta^3 z$. In order to make the terms depending on the argument g disappear it is necessary to put $k_1 = -0''.0339$ and $k_2 = -0''.0759$. In the case of the terms involving the arguments $5g'-2g$ and $10g'-4g$ we equate the motions of the latter. By adding the values which have been obtained for $\frac{d\,.\,n\delta z}{ndt}$, $\frac{d\,.\,n\delta^2 z}{ndt}$, and $\frac{d\,.\,n\delta^3 z}{ndt}$ we get, taking the liberty of calling the sum $\frac{d\,.\,n\delta z}{ndt}$,

$$\begin{aligned}\frac{d\,.\,n\delta z}{ndt} = {} & [\quad \overset{''}{5.80177} + \overset{''}{0.0100762}nt - \overset{''}{0.000002149}n^2t^2] \cos(\ 5g'-2g) \\ & + [-14.01188 + 0.0056567nt + 0.000003088n^2t^2] \sin(\ 5g'-2g) \\ & + [\quad 0.19539 - 0.0003068nt - 0.000000062n^2t^2] \cos(10g'-4g) \\ & + [\quad 0.20289 + 0.0002363nt - 0.000000138n^2t^2] \sin(10g'-4g)\end{aligned}$$

For this expression may be substituted

$$\begin{aligned}\frac{d\,.\,n\delta z}{dt} = {} & [\quad \overset{''}{5.80177} - \overset{''}{0.0005247}nt + \overset{''}{0.000000471}n^2t^2] \cos(\ 5g'-2g+\varkappa nt) \\ & + [-14.01188 + 0.0012673nt - 0.000000525n^2t^2] \sin(\ 5g'-2g+\varkappa nt) \\ & + [\quad 0.19539 - 0.0000296nt + 0.000000079n^2t^2] \cos(10g'-4g+\varkappa nt) \\ & + [\quad 0.20289 - 0.0000307nt + 0.000000092n^2t^2] \sin(10g'-4g+\varkappa nt)\end{aligned}$$

where, in the case of $5g'-2g$, $\varkappa$ and the corrected integrating factor are determined by

$$\log \varkappa = 6.8788474n \qquad \log \mu = 1.8971166$$

and, in the case of $10g'-4g$, by

$$\log \varkappa = 7.13560n \qquad \log \mu = 1.59358$$

Integrating the last expression we obtain

$$\begin{aligned}n\delta z = {} & [\quad \overset{''}{465.230} - \overset{''}{0.047944}nt + \overset{''}{0.00003716}n^2t^2] \sin(\ 5g'-2g+\varkappa nt) \\ & + [\quad 1101.855 - 0.094137nt + 0.00004143n^2t^2] \cos(\ 5g'-2g+\varkappa nt) \\ & + [\quad 7.6077 - 0.000879nt + 0.00000310n^2t^2] \sin(10g'-4g+\varkappa nt) \\ & + [-\ 7.9932 + 0.001447nt - 0.00000361n^2t^2] \cos(10g'-4g+\varkappa nt)\end{aligned}$$

By developing this expression and subtracting therefrom the value of $n\delta z + n\delta^2 z$ we should have $n\delta^3 z$; but as this quantity has no particular interest we omit its derivation, and in the following value of $n\delta^3 z$ the terms corresponding to the arguments $5g'-2g$ and $10g'-4g$ are not given.

In writing a final form for the great inequality of Jupiter, we prefer to still further equate the motions of the arguments, so that the sum of the squares of the multipliers of nt in the coefficients may be a minimum. This gives, severally, in the cases of $5g'-2g$ and $10g'-4g$

$$\log \varkappa = 6.8752079n \qquad\qquad \log \varkappa = 7.12506n$$

and the final form of $n\delta z$ will be

$$\begin{array}{rllllll} & \quad '' & \quad '' & & {}^{\circ} & ' & '' \\ n\delta z = & [1196.045 & -0.105371nt] & \sin(5g'-2g+\varkappa nt+ & 67 & 6 & 33.71) \\ + & & 0.00005546n^2t^2 & \sin(5g'-2g+\varkappa nt+ & 48 & 46 &) \\ +[& 11.0349 & -0.001654nt] & \sin(10g'-4g+\varkappa nt+ & 313 & 35.1 &) \\ + & & 0.00000477n^2t^2 & \sin(10g'-4g+\varkappa nt+ & 311 & 21 &) \end{array}$$

The great inequality excepted, the expression of $n\delta^3 z$ follows. The proper number of decimals is restored to the factors of nt and n^2t^2:

Arg$=i'g'+ig$		$n\delta^3 z$	
		sin.	cos.
i'	i	″ ″ ″	″ ″ ″
0	0		$-0.00010214n^2t^2$ $+0.000005231n^3t^3$
0	-1	$-0.008207nt+0.00002853n^2t^2$ $+0.00000000165n^3t^3$	$-0.004066nt-0.00003380n^2t^2$ $+0.0000000181n^3t^3$
0	-2	$+0.0082-0.000172nt+0.00000034n^2t^2$ $+0.00000000002n^3t^3$	$+0.0071+0.000028nt-0.00000040n^2t^2$ $+0.00000000002n^3t^3$
0	-3	$-0.0004-0.000023nt+0.00000001n^2t^2$	$+0.0008-0.000032nt-0.00000001n^2t^2$
1	$+2$	$-0.0025+0.000100nt$	$+0.0022+0.000037nt$
1	$+1$	$-0.0481-0.000241nt$	$-0.0401+0.000415nt$
1	0	$-0.0822+0.000382nt+0.00000019n^2t^2$	$+0.0457+0.000400nt-0.00000015n^2t^2$
1	-1	$+0.0002-0.000085nt$	$+0.0109+0.000005nt$
1	-2	$+0.0073-0.000025nt$	$+0.0029+0.000069nt$
1	-3	$-0.0014-0.000035nt$	$+0.0043-0.000023nt$
2	$+2$	$-0.0007-0.000007nt$	$0.0000+0.000011nt$
2	$+1$	$-0.0439+0.000011nt$	$-0.0053+0.000388nt$
2	0	$-0.1790+0.002332nt-0.00000010n^2t^2$	$+0.2905+0.000841nt-0.00000011n^2t^2$
2	-1	$-0.0354-0.000313nt-0.00000200n^2t^2$	$-0.0704-0.000103nt-0.00000149n^2t^2$
2	-2	$-0.0577-0.000007nt+0.00000005n^2t^2$	$+0.0272-0.000027nt+0.00000003n^2t^2$
2	-3	$+0.0001-0.000009nt$	$+0.0023-0.000005nt$
2	-4	$-0.0011+0.000003nt$	$-0.0003-0.000006nt$
3	$+1$	$-0.0008-0.000054nt$	$-0.0066-0.000005nt$
3	0	$-0.0094-0.002612nt$	$-0.3892+0.000356nt$
3	-1	$-0.1850-0.001274nt-0.00000058n^2t^2$	$-0.1889+0.001538nt+0.00000019n^2t^2$
3	-2	$+0.0549+0.000176nt+0.00000107n^2t^2$	$-0.0885+0.000140nt-0.00000189n^2t^2$
3	-3	$-0.0057-0.000017nt+0.00000001n^2t^2$	$-0.0119+0.000006nt-0.00000002n^2t^2$
3	-4	$-0.0008-0.000004nt$	$-0.0001-0.000004nt$
3	-5	$0.0000+0.000003nt$	$-0.0004+0.000000nt$

Arg=$i'g'+ig$	$n\delta^3 z$	
	sin.	cos.
i' i	″ ″ ″	″ ″ ″
4 0	-0.0019	-0.0006
4 −1	$-0.0856+0.000195nt$	$+0.0272+0.000535nt$
4 −2	$-0.0015+0.000234nt-0.00000018n^2t^2$	$-0.0280+0.000005nt-0.00000110n^2t^2$
4 −3	$-0.0282+0.000009nt-0.00000041n^2t^2$	$-0.0152-0.000008nt-0.00000013n^2t^2$
4 −4	-0.0010	$+0.0003$
4 −5	-0.0004	-0.0007
4 −6	$+0.0008$	-0.0001
5 0	$-0.0005+0.000015nt$	$-0.0005+0.000005nt$
5 −1	$+0.3539-0.000255nt+0.00000111n^2t^2$	$-0.3182-0.000403nt-0.00000448n^2t^2$
5 −3	$-0.0274-0.000218nt+0.00002074n^2t^2$	$+0.0200+0.003798nt-0.00000392n^2t^2$
5 −4	$+0.0017-0.000040nt-0.00000010n^2t^2$	$-0.0074+0.000030nt+0.00000030n^2t^2$
5 −5	$+0.0012$	$+0.0006$
6 −1	$+0.0009$	-0.0030
6 −2	$-0.0175+0.000021nt$	$-0.0149-0.000058nt$
6 −3	$+0.0690-0.000442nt+0.00000003n^2t^2$	$-0.0418-0.000478nt-0.00000003n^2t^2$
6 −4	$+0.0025-0.000025nt$	$-0.0058+0.000006nt$
6 −5	$+0.0024$	0.0000
6 −6	-0.0001	$+0.0006$
7 −2	$-0.0288+0.000005nt$	$-0.0066+0.000028nt$
7 −3	$-0.0745-0.002047nt+0.00000011n^2t^2$	$-0.2058+0.000806nt+0.00000000n^2t^2$
7 −4	$-0.2996-0.001906nt-0.00000014n^2t^2$	$-0.2997+0.002625nt-0.00000009n^2t^2$
7 −5	$+0.0017-0.000050nt$	$-0.0047-0.000005nt$
8 −2	$+0.0088$	-0.0038
8 −3	$-0.0275-0.000172nt$	$-0.0059+0.000266nt$
8 −4	$+0.2187+0.000263nt$	$+0.0569-0.002070nt$
8 −5	$+0.0286-0.000337nt$	$-0.0412-0.000174nt$
8 −6	$0.0000-0.000014nt$	$-0.0009+0.000000nt$
9 −3	-0.0005	-0.0008
9 −4	$+0.0580-0.000245nt$	$-0.0128-0.000596nt$
9 −5	$+0.0046-0.000575nt$	$-0.0558+0.000049nt$
9 −6	$+0.0173+0.000021nt$	$+0.0052-0.000109nt$
9 −7	$+0.0004\quad 0.000000nt$	$-0.0001-0.000003nt$
10 −5	$+0.1907+0.003683nt$	$+0.4746-0.002499nt$
10 −6	$+0.0127+0.000025nt$	$+0.0042-0.000101nt$
10 −7	$-0.0003+0.000037nt$	$+0.0050-0.000003nt$
11 −5	$+0.0047+0.000028nt$	$+0.0042-0.000061nt$
11 −6	$+0.0064-0.000045nt$	$-0.0032-0.000045nt$
11 −7	$+0.0014+0.000023nt$	$+0.0037-0.000015nt$
11 −8	$-0.0012\quad 0.000000nt$	$+0.0002+0.000009nt$
12 −5	$-0.0210-0.000004nt$	$+0.0341-0.000064nt$
12 −6	$-0.0122-0.000067nt$	$+0.0372-0.000013nt$
12 −7	$+0.0022+0.000005nt$	$+0.0018-0.000009nt$
12 −8	$-0.0012+0.000006nt$	$+0.0007+0.000006nt$
12 −9	$-0.0001-0.000005nt$	$-0.0008+0.000000nt$

As it is not necessary for practical purposes that the radius-vector should be known to the same degree of accuracy as the longitude and latitude, we might neglect all the terms of three dimensions in it; but as it is extremely easy to derive from $\delta^2 W_0$ the portion of $\delta^2\nu$ which depends on it, and the remaining portion is probably of considerably less importance, this quantity has been derived from the equation

$$\delta^2\nu = -\frac{1}{2}\int\left(\overline{\frac{d\,.\,\delta^2 W_0}{d\gamma}}\right)ndt$$

It is thought unnecessary to go to the labor of deriving the constant term, probably very small. Also the two terms factored severally by nt and n^2t^2 and independent of g and g' have been obtained by multiplying the similar terms having the argument $-g$ by a factor whose logarithm is 8.3828:

Arg=$i'g'+ig$		$\delta^2\nu$	
		cos.	sin.
i'	i	″ ″ ″	″ ″ ″
0	0	$-0.000098nt+0.00000034n^2t^2$	
0	-1	$+0.0149-0.004044nt+0.00001427n^2t^2$ $+0.0000000082n^3t^3$	$+0.0326+0.001966nt+0.00001690n^2t^2$ $-0.0000000090n^3t^3$
0	-2	$+0.0087-0.000157nt+0.00000034n^2t^2$ $+0.0000000002n^3t^3$	$-0.0056-0.000013nt+0.00000040n^2t^2$ $-0.0000000002n^3t^3$
0	-3	$+0.0003-0.000021nt+0.00000001n^2t^2$	$-0.0002+0.000029nt+0.00000001n^2t^2$
1	$+2$	$+0.0009-0.000067nt$	$-0.0007+0.000030nt$
1	$+1$	$+0.0308+0.000141nt$	$-0.0264+0.000248nt$
1	0	$+0.0199-0.000094nt-0.00000005n^2t^2$	$+0.0077+0.000107nt-0.00000002n^2t^2$
1	-1	$-0.0002-0.000025nt$	$-0.0040-0.000003nt$
1	-2	$+0.0056-0.000019nt$	$-0.0016-0.000050nt$
1	-3	$-0.0015-0.000035nt$	$-0.0042+0.000019nt$
2	$+2$	$+0.0007+0.000004nt$	$0.0000+0.000007nt$
2	$+1$	$+0.0258-0.000005nt$	$-0.0025+0.000227nt$
2	0	$+0.0715-0.000954nt+0.00000001n^2t^2$	$+0.1144+0.000363nt+0.00000002n^2t^2$
2	-1	$-0.0026-0.000087nt-0.00000026n^2t^2$	$+0.0118+0.000005nt+0.00000020n^2t^2$
2	-2	$-0.0242-0.000002nt-0.00000002n^2t^2$	$-0.0118+0.000006nt-0.00000002n^2t^2$
2	-3	$-0.0003-0.000009nt$	$-0.0026+0.000000nt$
2	-4	$-0.0019+0.000003nt$	$+0.0005+0.000010nt$
3	$+1$	$+0.0005+0.000032nt$	$-0.0052+0.000005nt$
3	0	$+0.0045+0.001218nt$	$-0.1838+0.000165nt$
3	-1	$+0.0235+0.000192nt$	$-0.0298+0.000207nt$
3	-2	$+0.0230+0.000080nt+0.00000042n^2t^2$	$+0.0284-0.000062nt+0.00000076n^2t^2$
3	-3	$-0.0045-0.000006nt$	$+0.0078+0.000000nt$
3	-4	$-0.0009+0.000000nt$	$0.0000+0.000004nt$
3	-5	$0.0000+0.000005nt$	$+0.0007+0.000000nt$
4	-1	$+0.0185-0.000043nt$	$+0.0067-0.000146nt$
4	-2	$-0.0010+0.000057nt+0.00000000n^2t^2$	$+0.0015+0.000000nt+0.00000015n^2t^2$
4	-3	$-0.0171+0.000005nt-0.00000020n^2t^2$	$+0.0086+0.000007nt+0.00000006n^2t^2$

Arg$=i'g'+ig$	$\delta^2 v$	
	cos.	sin.
i' i	$''$ $''$ $''$	$''$ $''$ $''$
4— 4	-0.0008	0.0000
4— 5	-0.0002	$+0.0004$
4— 6	$+0.0004$	$+0.0001$
5— 1	$-0.1784+0.000408nt$	$-0.1600-0.000297nt$
5— 2	$-0.1966-0.002443nt+0.00000180n^2t^2$	$+0.3840-0.001443nt-0.00000192n^2t^2$
5— 3	$+0.0064+0.000011nt+0.00000858n^2t^2$	$-0.0099-0.001625nt+0.00000252n^2t^2$
5— 4	$+0.0014-0.000005nt+0.00000020n^2t^2$	$+0.0053-0.000040nt+0.00000005n^2t^2$
5— 5	$+0.0020$	-0.0010
6— 2	$+0.0036-0.000010nt$	$-0.0021-0.000012nt$
6— 3	$+0.0152-0.000094nt$	$+0.0086+0.000103nt$
6— 4	$+0.0007-0.000019nt$	$+0.0045+0.000000nt$
6— 5	$+0.0027$	0.0000
6— 6	-0.0003	-0.0012
7— 2	$+0.0131-0.000002nt$	$-0.0017+0.000013nt$
7— 3	$-0.0127-0.000226nt$	$+0.0243-0.000121nt$
7— 4	$-0.1427-0.000922nt$	$+0.1430-0.001279nt$
7— 5	$-0.0021-0.000023nt$	$+0.0042-0.000032nt$
8— 3	$+0.0054+0.000013nt$	$+0.0032+0.000032nt$
8— 4	$+0.0827+0.000087nt$	$-0.0195+0.000768nt$
8— 5	$+0.0195-0.000219nt$	$+0.0266+0.000118nt$
8— 6	$+0.0005-0.000011nt$	$+0.0012+0.000004nt$
9— 4	$+0.0130-0.000056nt$	$+0.0035+0.000098nt$
9— 5	$+0.0028-0.000315nt$	$+0.0320-0.000025nt$
9— 6	$+0.0104+0.000013nt$	$-0.0031+0.000084nt$
9— 7	$+0.0007$	-0.0001
10— 4	$-0.0223+0.000246nt$	$-0.0298-0.000119nt$
10— 5	$+0.0820+0.001652nt$	$-0.2042+0.001135nt$
10— 6	$+0.0105+0.000020nt$	$-0.0044+0.000091nt$
10— 7	$-0.0006+0.000034nt$	$-0.0044+0.000000nt$
11— 5	$+0.0012+0.000007nt$	$-0.0012+0.000017nt$
11— 6	$+0.0043-0.000032nt$	$+0.0030+0.000025nt$
11— 7	$+0.0013+0.000028nt$	$-0.0036+0.000015nt$
11— 8	$-0.0023+0.000003nt$	$-0.0002-0.000017nt$
12— 6	$-0.0063-0.000038nt$	$-0.0193+0.000008nt$
12— 7	$+0.0018+0.000005nt$	$-0.0016+0.000009nt$

CHAPTER XIX.

CALCULATION OF THE SEVERAL PORTIONS OF δ^2T'.

The fourteen parts of the portion of δ^2T' not factored by $n't$ or n'^2t^2 are as follows:

Arg= $\varkappa\gamma'+i'g'+ig$			$A'n'\delta^2z'$		$B'\delta\nu'$		$F'n\delta^2z$		$G'\delta\nu$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		—0.000397		+0.002185		+0.001252		—0.000696
—1	1	0		—0.000491		—0.001914		—0.001353		+0.000999
—1	2	0		+0.00113		—0.00257		—0.00207		—0.00036
0	1	0	—0.0767	+0.0411	—0.0064	+0.0064	—0.0195	+0.0080	—0.0007	+0.0012
1	0	0	+0.088669	—0.051402	+0.010129	—0.004636	+0.015432	—0.008046	+0.003443	—0.001857
—1	3	0	+0.011	—0.008	+0.003	—0.004	+0.018	—0.015	+0.004	—0.002
0	2	0	—0.161	+0.245	—0.038	+0.040	—0.046	+0.068	—0.008	+0.006
1	1	0	+0.206	—0.334	+0.042	—0.049	+0.041	—0.066	+0.007	—0.007
—1	4	0	0.000	—0.023			—0.001	—0.020		
0	3	0	+0.014	+0.431	—0.005	—0.020	+0.006	+0.187		
1	2	0	—0.004	—0.784	—0.001	+0.013	—0.002	—0.229		
0	4	0	+0.12	—0.04			+0.11	—0.03		
1	3	0	—0.064	+0.038			—0.035	+0.015		
—1—	2—	1	—0.03	—0.02			—0.01	—0.01		
—1—	1—	1	+0.652	—0.147			+0.213	—0.045	—0.009	+0.002
0—	2—	1	—0.43	+0.10			—0.18	+0.04	+0.02	0.00
1—	3—	1	+0.05	—0.01			+0.03	—0.01	—0.02	0.00
—1	0—	1	+0.263	—0.229	+0.028	—0.038	+0.065	—0.054	+0.008	—0.007
0—	1—	1	—0.193	+0.169	—0.017	+0.024	—0.072	+0.061	—0.016	+0.015
1—	2—	1	+0.016	—0.021			+0.026	—0.024	+0.014	—0.012
—1	1—	1	+0.035	—0.088	—0.001	—0.010	+0.007	—0.022	—0.001	—0.002
0	0—	1	—0.028	+0.072	+0.002	+0.008	—0.009	+0.026	—0.001	+0.003
1—	1—	1	0.000	—0.008						
—1	2—	1	—0.0014	—0.0209	—0.0047	—0.0211	—0.0017	—0.0041	+0.0007	+0.0005
0	1—	1	+0.006	+0.008	+0.006	+0.020	+0.001	+0.004	0.000	+0.001
1	0—	1	—0.001	+0.012						
—1	3—	1	—0.0069	—0.0068	+0.0046	+0.0036	+0.0001	+0.0008	—0.0011	—0.0007
0	2—	1	+0.0043	—0.0259	—0.0166	—0.0128	—0.0073	—0.0113	+0.0025	+0.0009
1	1—	1	+0.0166	+0.0443	+0.0108	+0.0120	+0.0093	+0.0132	—0.0019	—0.0007
—1	4—	1	—0.0302	—0.0050	+0.0106	+0.0002	—0.0038	+0.0031	—0.0085	—0.0023
0	3—	1	+0.1400	—0.0230	—0.0972	—0.0438	—0.0196	—0.0272	+0.0140	+0.0022
1	2—	1	+0.0773	+0.0845	+0.0826	+0.0409	+0.0477	+0.0290	—0.0111	—0.0010

Arg= $\varkappa\gamma'+i'g'+ig$			$A'n'\delta^2z'$		$B'\delta\nu'$		$F'n\delta^2z$		$G'\delta\nu$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	5	−1			+0.003	−0.001	−0.616	+0.122	+0.007	+0.002
0	4	−1	+1.145	−0.220	+0.017	0.000	+0.498	−0.098	−0.020	+0.001
1	3	−1	−0.1892	+0.0368	−0.0146	−0.0007	−0.0525	+0.0093	+0.0190	−0.0035
0	5	−1	+0.004	−0.015						
1	4	−1	+0.030	+0.014						
−1	−1	−2	+0.03	−0.05						
0	−2	−2	−0.02	+0.03						
−1	0	−2	+0.22	+0.45			+0.07	+0.16		
0	−1	−2	−0.16	−0.33			−0.07	−0.15		
1	−2	−2	+0.02	+0.06			+0.01	+0.03		
−1	1	−2	+0.25	+0.19	+0.02	+0.02	+0.06	+0.05		
0	0	−2	−0.22	−0.14	−0.02	−0.02	−0.06	−0.05		
1	−1	−2	+0.03	+0.01						
−1	2	−2	+0.094	+0.018	+0.015	0.000	+0.022	+0.004		
0	1	−2	−0.095	−0.011	−0.011	0.000	−0.021	−0.003		
−1	3	−2	+0.021	−0.007	+0.014	−0.007	+0.004	−0.002		
0	2	−2	−0.025	+0.011	−0.006	+0.003				
−1	4	−2	+0.0022	−0.0029	−0.0042	+0.0018	+0.0029	−0.0036	−0.0004	+0.0009
0	3	−2	+0.021	−0.018	+0.010	−0.005	0.000	−0.002	+0.002	−0.002
1	2	−2	−0.022	+0.020	−0.006	+0.005	−0.002	+0.005		
−1	5	−2	−0.01236	+0.04481	+0.01920	−0.05499	+0.01210	−0.03234	−0.00985	+0.01947
0	4	−2	+0.0326	−0.0799	−0.0113	+0.0148	−0.0017	+0.0112	+0.0095	−0.0219
1	3	−2	−0.0335	+0.1346	−0.0085	+0.0448	−0.0099	+0.0162	−0.0028	+0.0112
−1	6	−2	+0.14474	+0.35026	+0.00531	+0.01240	+0.00381	+0.00821	−0.00012	−0.00025
0	5	−2	+0.0042577	−0.0026910	−0.0113662	−0.0198853	−0.0060156	−0.0136926	−0.0001245	−0.0013962
1	4	−2	+0.14020	+0.34072	+0.00458	+0.00674	+0.00376	+0.01056	+0.00090	+0.00103
−1	7	−2	+0.090	+0.054	0.000	+0.003	+0.014	−0.004	−0.003	+0.003
0	6	−2	−0.0519	−0.0073	+0.0128	−0.0031	−0.0085	+0.0061	+0.0057	−0.0015
1	5	−2	−0.00406	−0.00712	−0.01093	+0.00183	−0.00374	−0.00518	−0.00365	−0.00026
−1	8	−2	+0.020	−0.001						
0	7	−2	+0.019	+0.005						
1	6	−2	−0.051	−0.006						
0	8	−2	+0.032	−0.019						
−1	0	−3	+0.05	+0.02						
−1	1	−3	−0.28	+0.21			−0.10	+0.07		
0	0	−3	+0.21	−0.17			+0.09	−0.08		
1	−1	−3	−0.04	+0.03						
−1	2	−3	−0.11	+0.25			−0.03	+0.07		
0	1	−3	+0.07	−0.19			+0.03	−0.06		
1	0	−3	−0.01	+0.03						
−1	3	−3	+0.01	+0.10			0.00	+0.02		
0	2	−3	−0.01	−0.09			0.00	−0.02		
−1	4	−3	+0.012	+0.019						
0	3	−3	−0.01	−0.02						
−1	5	−3	+0.014	+0.010						

Arg= $\kappa\gamma'+i'g'+ig$			$A'n'\delta^2 s'$		$B'\delta\nu'$		$F'n\delta^2 s$		$G'\delta\nu$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
κ	i'	i	″	″	″	″	″	″	″	″
−1	6	−3	+0.018	+0.025	+0.089	+0.006	0.000	+0.008	+0.006	+0.001
0	5	−3	−0.127	−0.009	−0.103	−0.005	−0.012	0.000		
1	4	−3	+0.003	−0.009						
−1	7	−3	+0.1584	−0.1021	−0.0002	−0.0006	+0.0496	−0.0349	−0.0179	+0.0121
0	6	−3	−0.966	+0.651	+0.009	−0.011	−0.423	+0.298	+0.022	−0.015
1	5	−3			−0.010	+0.009	+0.516	−0.365		
−1	8	−3	−0.0049	+0.1423	+0.0014	+0.0081	−0.0049	+0.0313	+0.0031	−0.0066
0	7	−3	−0.0831	−0.0234	+0.0034	−0.0036	−0.0132	−0.0132	−0.0039	+0.0084
1	6	−3	+0.0176	−0.0219	−0.0038	−0.0046	+0.0062	−0.0028	+0.0029	−0.0046
−1	9	−3	+0.0134	+0.0402	+0.0002	−0.0028	+0.0024	+0.0072	−0.0002	−0.0012
0	8	−3	−0.0247	−0.0590	+0.0016	+0.0277	−0.0038	−0.0009	+0.0006	+0.0042
1	7	−3	+0.0062	−0.0065	−0.0017	−0.0227	+0.0009	−0.0079	−0.0004	−0.0042
−1	10	−3	+0.006	+0.006			+0.025	+0.040		
0	9	−3	−0.059	−0.082	−0.004	−0.006	−0.019	−0.033	−0.002	0.000
1	8	−3	+0.0084	+0.0121	+0.0030	+0.0050	+0.0009	+0.0032	+0.0005	+0.0005
−1	2	−4	−0.18	−0.15			−0.07	−0.05		
0	1	−4	+0.15	+0.12			+0.06	+0.05		
−1	3	−4	−0.17	−0.04			−0.05	−0.01		
0	2	−4	+0.15	+0.04			+0.05	+0.01		
−1	4	−4	−0.07	+0.02						
0	3	−4	+0.06	−0.02						
−1	5	−4	−0.02	+0.02						
−1	7	−4	+0.068	−0.081	+0.009	+0.062	+0.035	−0.015	0.000	+0.015
0	6	−4	−0.105	+0.095	−0.006	−0.033	−0.095	+0.042	+0.001	−0.032
1	5	−4					+0.10	−0.04	0.00	+0.03
−1	8	−4	−0.570	−0.565	+0.014	+0.012	−0.167	−0.162		
0	7	−4	+0.308	+0.315	−0.015	−0.012	+0.132	+0.133		
1	6	−4					−0.015	−0.014		
−1	9	−4	−0.309	−0.078	−0.007	+0.007	−0.064	−0.017	−0.004	−0.001
0	8	−4	+0.226	+0.039	+0.010	−0.009	+0.063	+0.010	+0.007	−0.004
1	7	−4	−0.013	+0.002	−0.003	−0.001	−0.016	+0.007		
−1	10	−4	−0.06119	+0.01391	−0.01363	+0.00446	−0.02906	+0.00807	+0.01509	−0.00420
0	9	−4	+0.0529	−0.0154	+0.0015	−0.0004	+0.0255	−0.0080	−0.0184	+0.0057
1	8	−4	+0.021	−0.001	+0.014	−0.004	−0.002	+0.002		
−1	11	−4	+0.01289	−0.01603	+0.00086	−0.00021	−0.00177	+0.00182	−0.00004	−0.00007
0	10	−4	+0.007175	−0.005886	+0.001315	−0.001785	+0.001802	−0.001870	−0.000096	+0.000222
1	9	−4	+0.02116	−0.02153	−0.00230	+0.00256	+0.00029	+0.00019	−0.00048	+0.00005
0	11	−4	+0.0004	−0.0002						
−1	3	−5	+0.07	−0.14						
0	2	−5	−0.05	+0.12						
−1	4	−5	0.00	−0.13						
0	3	−5	0.00	+0.12						
−1	5	−5	−0.03	−0.06						
0	4	−5	+0.03	+0.05						
−1	8	−5	+0.01	−0.03	−0.04	+0.01	−0.01	+0.01		
0	7	−5			+0.02	−0.01				

Arg= $\varkappa\gamma'+i'g'+ig$			$A'n'\delta^2 z'$		$B'\delta\nu'$		$F'n\delta^2 z$		$G'\delta\nu$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	9	−5	+0.364	−0.582	−0.010	+0.014	+0.113	−0.187	−0.006	+0.010
0	8	−5	−0.24	+0.40	+0.01	−0.01	−0.09	+0.15	+0.02	−0.02
1	7	−5					+0.01	−0.02	−0.01	+0.01
−1	10	−5	+0.035	−0.311	−0.007	0.000	+0.004	−0.082	0.000	−0.004
0	9	−5	−0.026	+0.230	+0.001	−0.006	−0.004	+0.083		
1	8	−5	−0.004	−0.026			−0.001	−0.021	0.000	−0.008
−1	11	−5	−0.032	−0.081	−0.012	−0.022	−0.009	−0.018		
0	10	−5	+0.042	+0.092	+0.014	+0.026	+0.008	+0.017		
1	9	−5	−0.006	−0.006						
−1	12	−5	−0.0223	−0.0187	+0.0007	+0.0008	−0.0058	−0.0036	−0.0006	+0.0003
0	11	−5	+0.093	+0.061	−0.003	−0.001	+0.037	+0.023	−0.001	−0.001
1	10	−5					−0.042	−0.026		
−1	13	−5	+0.0094	−0.0055	+0.0002	0.0000	+0.0010	−0.0006		
0	12	−5	+0.0022	+0.0074	−0.0003	−0.0001	+0.0006	+0.0014	−0.0001	0.0000
1	11	−5	−0.0013	−0.0008						
−1	4	−6	+0.10	+0.02						
0	3	−6	−0.08	−0.01						
−1	5	−6	+0.09	−0.02						
0	4	−6	−0.08	+0.01						
−1	9	−6	+0.01	−0.01						
0	8	−6	0.00	+0.03						
−1	10	−6	+0.51	+0.19						
0	9	−6	−0.36	−0.12						
1	8	−6	+0.06	+0.02						
−1	11	−6	+0.29	−0.04						
0	10	−6	−0.22	+0.03						
1	9	−6	+0.03	−0.01						
−1	12	−6	+0.054	−0.032						
0	11	−6	−0.041	+0.041						
−1	13	−6	−0.018	+0.050						
0	12	−6	+0.008	−0.022						
−1	10	−7	+0.05	−0.02						
−1	11	−7	−0.04	+0.35						
0	10	−7	+0.03	−0.27						
−1	12	−7	+0.08	+0.22						
0	11	−7	−0.06	−0.17						
−1	13	−7	+0.05	+0.04						
0	12	−7	−0.05	−0.05						

Arg= $\varkappa\gamma'+i'g'+ig$			$\frac{1}{2}\frac{dA'}{dg'}(n'\delta z')^2$		$\frac{dA'}{dg}(n\delta z)(n'\delta z')$		$\frac{1}{2}\frac{dF'}{dg}(n\delta z)^2$		$\frac{dB'}{dg'}(n'\delta z')\nu'$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	"	"	"	"	"	"	"	"
0	0	0		+0.001323		−0.000177		−0.000105		−0.000118
−1	1	0		−0.003627		−0.000367		+0.000106		+0.000541
−1	2	0		−0.01414		+0.00014		+0.00022		−0.00290
0	1	0	−0.0063	+0.0032	+0.0044	−0.0045	+0.0015	−0.0007	+0.0030	−0.0012
1	0	0	−0.007885	−0.002139	−0.003072	−0.000028	−0.002474	+0.001186	−0.000383	+0.000970
−1	3	0	+0.001	−0.004						
0	2	0	−0.010	+0.008	−0.006	−0.001			−0.002	−0.003
1	1	0	−0.002	−0.016	−0.004	0.000			−0.003	+0.010
0	3	0	+0.007	+0.013					+0.003	−0.014
1	2	0	−0.017	−0.033					−0.007	+0.019
0	4	0	+0.02	+0.04						
1	3	0	−0.053	−0.050						
−1	0	−1	+0.006	+0.024					−0.009	+0.002
0	−1	−1	0.000	−0.008						
−1	1	−1	+0.013	+0.014	+0.004	+0.009			+0.005	−0.020
0	0	−1	−0.008	+0.012	−0.003	+0.009			−0.004	+0.015
−1	2	−1	+0.0053	−0.0187	+0.0001	−0.0063	−0.0003	−0.0012	−0.0021	−0.0047
0	1	−1	+0.012	+0.084	+0.014	+0.067	+0.003	+0.014	+0.001	+0.004
1	0	−1					−0.004	−0.018		
−1	3	−1	+0.0299	+0.0111	+0.0108	+0.0075	+0.0013	+0.0017	+0.0154	+0.0193
0	2	−1	+0.0014	+0.0231	+0.0027	+0.0076	−0.0001	−0.0001	−0.0067	−0.0098
1	1	−1	+0.0024	−0.0042	−0.0025	−0.0028			−0.0048	−0.0035
−1	4	−1	+0.0227	+0.0037	+0.0049	+0.0007			+0.0052	+0.0048
0	3	−1	−0.0119	+0.0040	−0.0004	+0.0017	+0.0016	+0.0005	+0.0019	+0.0021
1	2	−1	+0.0050	−0.0067	−0.0021	−0.0028	−0.0022	−0.0007	−0.0085	−0.0052
−1	5	−1	+0.005	−0.002					−0.007	+0.002
0	4	−1	−0.016	+0.013	−0.006	+0.005	+0.001	0.000	0.000	+0.002
1	3	−1	+0.0220	−0.0190	+0.0036	−0.0065	+0.0012	−0.0006	−0.0108	−0.0047
0	5	−1	−0.019	+0.029					−0.007	+0.005
1	4	−1	+0.038	−0.056					+0.012	−0.021
0	6	−1	−0.017	+0.051						
1	5	−1	+0.032	−0.095						
−1	1	−2	−0.03	0.00						
−1	2	−2	−0.056	−0.001	−0.013	−0.012			+0.026	+0.002
0	1	−2	+0.016	+0.013	−0.001	+0.015			−0.011	−0.001
−1	3	2	−0.186	+0.086	−0.091	+0.036	−0.012	+0.005	+0.009	−0.005
0	2	−2	+0.069	−0.028	+0.047	−0.019	+0.010	−0.004	−0.005	+0.002
−1	4	−2	−0.0817	+0.1122	−0.0350	+0.0455	−0.0038	+0.0047	−0.0131	+0.0169
0	3	−2	+0.035	−0.050	+0.022	−0.027	+0.002	−0.003	+0.007	−0.009
−1	5	−2	−0.00375	+0.04529	−0.00072	+0.01604	−0.00002	+0.00178	−0.00875	+0.00902
0	4	−2	−0.0060	−0.0211	−0.0066	−0.0033	−0.0018	−0.0013	+0.0045	−0.0023
1	3	−2	+0.0015	+0.0038	+0.0008	−0.0043	+0.0014	−0.0007	+0.0027	−0.0080
−1	6	−2	+0.00285	+0.01340	−0.00273	−0.00425	−0.00131	−0.00289	−0.00867	−0.01349
0	5	−2	−0.0025163	−0.0113586	+0.0038286	+0.0074037	+0.0016933	+0.0042305	+0.0032508	+0.0137355
1	4	−2	+0.00597	+0.00996	−0.00034	−0.00347	−0.00090	−0.00265	+0.00247	−0.01047

Arg= $\varkappa y'+i'g'+ig$			$\frac{1}{2}\frac{dA'}{dg'}(n'\delta z')^2$		$\frac{dA'}{dg}(n\delta z)(n'\delta z')$		$\frac{1}{2}\frac{dF'}{dg}(n\delta z)^2$		$\frac{dB'}{dg'}(n'\delta z')\nu'$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
—1	7—	2	+0.002	+0.003	—0.002	—0.002			—0.004	—0.002
0	6—	2	—0.0053	—0.0147	+0.0019	—0.0070	+0.0011	—0.0017	—0.0096	—0.0022
1	5—	2	+0.02512	+0.02192	+0.01108	+0.01018	+0.00134	+0.00148	+0.01919	+0.00687
—1	3—	3	—0.02	—0.05					0.00	+0.03
0	2—	3	0.00	+0.03						
—1	4—	3	—0.131	—0.186	—0.068	—0.096	—0.011	—0.015	+0.009	+0.011
0	3—	3	+0.07	+0.10	+0.05	+0.06	+0.01	+0.01		
—1	5—	3	—0.156	—0.083	—0.072	—0.034	—0.007	—0.003	—0.024	—0.013
0	4—	3	+0.083	+0.040	+0.046	+0.022	+0.006	+0.003	+0.014	+0.007
—1	6—	3	—0.085	—0.015	—0.033	—0.006			—0.016	—0.006
0	5—	3	+0.047	+0.004	+0.022	+0.002			+0.009	+0.002
—1	7—	3	—0.0264	+0.0048	—0.0039	—0.0020	—0.0009	+0.0003	+0.0124	—0.0107
0	6—	3	+0.017	—0.005	+0.008	+0.002	+0.001	0.000	—0.001	+0.004
—1	8—	3	—0.0068	—0.0018	+0.0001	—0.0025	+0.0009	—0.0020	+0.0039	—0.0062
0	7—	3	+0.0072	—0.0014	+0.0009	+0.0019	—0.0005	+0.0013	+0.0015	—0.0021
1	6—	3	—0.0080	+0.0122	—0.0028	+0.0037			—0.0040	+0.0128
—1	9—	3	—0.0024	—0.0005	—0.0001	—0.0013			+0.0003	—0.0012
0	8—	3	+0.0073	+0.0185	+0.0026	+0.0060	—0.0002	—0.0011	—0.0009	—0.0137
1	7—	3	—0.0053	+0.0081	—0.0012	+0.0086	0.0000	+0.0020	+0.0015	+0.0245
0	9—	3	+0.023	+0.034	+0.019	+0.028	+0.004	+0.006	+0.002	+0.002
1	8—	3	—0.0071	—0.0106	—0.0030	—0.0046			—0.0028	—0.0035
—1	4—	4	+0.04	—0.02						
—1	5—	4	+0.15	—0.16	+0.08	—0.09	+0.01	—0.02		
0	4—	4	—0.07	+0.08	—0.06	+0.06				
—1	6—	4	+0.03	—0.16	+0.02	—0.08			+0.01	—0.03
0	5—	4	—0.02	+0.11	—0.01	+0.05				
—1	7—	4	—0.006	—0.097	—0.001	—0.042			+0.003	—0.019
0	6—	4	+0.006	+0.061	+0.004	+0.017			+0.002	+0.010
—1	8—	4	—0.016	—0.034	0.000	—0.007			+0.013	+0.014
0	7—	4	+0.012	+0.022	—0.005	—0.005			—0.009	—0.012
—1	9—	4	—0.011	—0.006	0.000	—0.002			+0.003	+0.008
0	8—	4	+0.013	—0.002	+0.003	—0.006			+0.001	—0.004
—1	10—	4	+0.00546	—0.00744	+0.00076	—0.00152	+0.00251	—0.00068	—0.00479	+0.00209
0	9—	4	+0.0040	0.0000	—0.0056	+0.0007	—0.0018	+0.0005	—0.0009	—0.0001
—1	11—	4	+0.01013	—0.01421	+0.00007	—0.00064			—0.00013	+0.00080
0	10—	4	+0.000387	+0.001219	+0.000027	+0.000300	—0.000008	+0.000040	+0.000100	—0.000083
1	9—	4	—0.00907	+0.00932	—0.00020	+0.00008			+0.00067	—0.00107
0	11—	4	—0.0002	+0.0012						
—1	6—	5	+0.14	+0.08	+0.09	+0.05				
0	5—	5	—0.09	—0.05	—0.07	—0.03				
—1	7—	5	+0.16	0.00	+0.07	0.00				
0	6—	5	—0.09	0.00	—0.06	0.00				
—1	8—	5	+0.10	—0.02	+0.02	—0.01			+0.01	—0.01
0	7—	5	—0.05	+0.03	—0.02	+0.01				

Arg= $\varkappa\gamma'+i'g'+ig$			$\frac{1}{2}\frac{dA'}{dg'}(n'\delta z')^2$		$\frac{dA'}{dg}(n\delta z)\,(n'\delta z')$		$\frac{1}{2}\frac{dF'}{dg}(n\delta z)^2$		$\frac{dB'}{dg'}(n'\delta z')\nu'$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	9	−5	+0.033	−0.024					−0.016	+0.018
0	8	−5	−0.02	+0.01						
−1	10	−5	+0.022	+0.014	+0.008	+0.014			−0.006	−0.002
0	9	−5	−0.006	−0.007						
−1	11	−5	+0.009	−0.002					−0.014	−0.023
0	10	−5	+0.001	+0.015	+0.004	+0.008			+0.007	+0.014
−1	12	−5	0.0000	−0.0078	−0.0020	−0.0029			−0.0025	−0.0010
0	11	−5	+0.032	+0.025	+0.029	+0.018			+0.001	−0.001
−1	13	−5	+0.0179	−0.0075	+0.0039	−0.0016			−0.0002	+0.0003
0	12	−5	−0.0013	+0.0070	−0.0003	+0.0026			+0.0003	−0.0002
1	11	−5	−0.0004	−0.0004						
−1	7	−6	−0.03	+0.12						
0	6	−6	+0.02	−0.08						
−1	8	−6	+0.02	+0.11						
0	7	−6	−0.01	−0.08						
−1	9	−6	+0.03	+0.04						
0	8	−6	−0.02	−0.03						
−1	10	−6	+0.03	+0.03						
−1	11	−6	−0.01	+0.02						
−1	12	−6	−0.032	+0.041						
0	11	−6	+0.013	−0.019						
−1	13	−6	−0.036	+0.100						
0	12	−6	+0.015	−0.038						
−1	8	−7	−0.09	0.00						
0	7	−7	+0.07	0.00						
−1	9	−7	−0.08	+0.03						
0	8	−7	+0.06	−0.03						
−1	13	−7	−0.03	−0.02						
0	12	−7	+0.01	+0.02						

Arg= $\varkappa\gamma'+i'g'+ig$			$\frac{dB'}{dg}(n\delta z)\nu'$		$\frac{dG'}{dg'}(n'\delta z')\nu$		$\frac{dG'}{dg}(n\delta z)\nu$		$\frac{1}{2}r'^2\frac{d^2T'}{dr'^2}\nu'^2$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i		″	″	″	″	″	″	″
0	0	0		−0.000226		+0.000254		+0.000040		−0.000297
−1	1	0		+0.000223		−0.000147		−0.000028		+0.000338
−1	2	0		−0.00010		−0.00195		−0.00042		−0.00070
0	1	0	+0.0018	−0.0001	−0.0034	+0.0014	−0.0021	+0.0011	+0.0001	−0.0011
1	0	0	−0.002155	+0.001478	−0.001393	+0.000899	+0.001197	−0.000521	−0.000554	+0.003166
0	2	0	+0.002	0.000						
1	1	0	+0.002	+0.003	−0.001	+0.001			−0.004	+0.005
1	2	0	−0.001	+0.004						
−1	1	−1	+0.001	−0.004						
0	0	−1	−0.002	+0.006						
−1	2	−1	−0.0011	−0.0029	+0.0001	+0.0015			−0.0024	−0.0082
0	1	−1							+0.001	+0.003
1	0	−1							+0.001	+0.003
−1	3	−1	+0.0025	+0.0033	+0.0032	+0.0040	+0.0008	+0.0007	+0.0003	−0.0017
0	2	−1	−0.0024	−0.0038	−0.0020	−0.0007	−0.0003	−0.0001	+0.0013	+0.0017
1	1	−1	−0.0007	−0.0005	−0.0010	−0.0018			−0.0021	+0.0004
−1	4	−1	+0.0015	+0.0006	+0.0015	+0.0005			+0.0021	+0.0006
0	3	−1	+0.0011	+0.0006	+0.0002	+0.0005	−0.0001	+0.0001	+0.0011	+0.0009
1	2	−1	−0.0027	−0.0018	−0.0025	−0.0016	−0.0003	−0.0002	−0.0042	−0.0024
0	4	−1	+0.002	0.000	+0.011	−0.003			0.000	+0.001
1	3	−1	−0.0019	−0.0007	−0.0023	0.0000	−0.0017	+0.0002	+0.0025	−0.0020
−1	3	−2							+0.008	−0.003
−1	4	−2	−0.0029	+0.0039	−0.0033	+0.0040	−0.0008	+0.0010	+0.0032	0.0000
0	3	−2	+0.002	−0.004						
−1	5	−2	−0.00190	+0.00326	−0.00190	+0.00330	−0.00035	+0.00098	−0.00042	+0.00421
0	4	−2	+0.0021	−0.0010	+0.0015	−0.0019	+0.0005	−0.0003	+0.0001	−0.0001
1	3	−2			+0.0011	−0.0022			+0.0002	−0.0041
−1	6	−2	−0.00167	−0.00324	−0.00161	−0.00298	−0.00022	−0.00041	−0.00024	+0.00023
0	5	−2	+0.0017479	+0.0062521	−0.0003651	+0.0003996	+0.0001113	+0.0004123	−0.0009770	−0.0010015
1	4	−2	+0.00004	−0.00252	+0.00021	−0.00167	+0.00011	−0.00005	+0.00192	+0.00135
0	6	−2	−0.0041	−0.0009	−0.0018	−0.0008			−0.0013	−0.0010
1	5	−2	+0.00386	+0.00135	+0.00474	+0.00230	+0.00113	+0.00051	+0.00169	+0.00168
−1	5	−3	−0.007	−0.003						
−1	6	−3	−0.003	−0.002	−0.002	0.000				
−1	7	−3	+0.0029	−0.0028	+0.0037	−0.0029	+0.0040	−0.0025	−0.0007	−0.0007
0	6	−3	−0.002	+0.002	−0.009	+0.006				
−1	8	−3	+0.0003	−0.0017	+0.0009	−0.0023	−0.0001	+0.0002	+0.0004	−0.0009
0	7	−3	−0.0002	+0.0004	+0.0015	−0.0017	+0.0003	−0.0004	+0.0007	−0.0011
1	6	−3	−0.0008	+0.0017	−0.0022	+0.0041			−0.0012	+0.0026
−1	9	−3	+0.0001	+0.0007	−0.0003	−0.0009				
0	8	−3	−0.0005	−0.0065	+0.0001	−0.0013	+0.0001	−0.0005	0.0000	+0.0003
1	7	−3	+0.0004	+0.0056	+0.0002	+0.0038	0.0000	+0.0010		

Arg= $\varkappa\gamma'+i'g'+ig$			$\frac{dB'}{dg}(n\delta z)\nu'$		$\frac{dG'}{dg'}(n'\delta z')\nu$		$\frac{dG'}{dg}(n\delta z)\nu$		$\frac{1}{2}r'^2\frac{d^2T'}{dr'^2}\nu'^2$	
			sin.	cos.	sin.	cos.	sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
—1	8	—4			+0.004	+0.004				
—1	9	—4	0.000	+0.002	+0.002	+0.002				
0	8	—4	+0.002	—0.001						
—1	10	—4	+0.00050	—0.00018	+0.00068	—0.00033	—0.00182	+0.00039	—0.00019	+0.00002
0	9	—4			+0.0050	—0.0012	+0.0024	—0.0007		
—1	11	—4	+0.00006	—0.00002	+0.00012	—0.00018			0.00000	+0.00002
0	10	—4	—0.000053	+0.000109	—0.000083	+0.000172	—0.000024	+0.000036	+0.000012	+0.000008
1	9	—4			+0.00011	—0.00003	+0.00006	0.00000	—0.00001	—0.00001
—1	11	—5	—0.003	—0.005						
0	10	—5	+0.003	+0.006						
—1	12	—5	—0.0001	0.0000						
—1	13	—5	—0.0002	0.0000						
0	12	—5	+0.0001	0.0000	—0.0001	0.0000				

Arg=$\varkappa\gamma'+i'g'+ig$			$rr'\frac{d^2T'}{drdr'}\nu\nu'$		$\frac{1}{2}r^2\frac{d^2T'}{dr^2}\nu^2$	
			sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″
0	0	0		—0.000028		+0.000031
—1	1	—0		+0.000073		—0.000050
—1	2	—0		+0.00001		+0.00038
—1	2	—1	—0.0004	—0.0019		
—1	3	—1	—0.0002	—0.0006		
0	2	—1	+0.0007	+0.0007	+0.0002	+0.0003
0	3	—1	+0.0013	+0.0004		
1	2	—1	—0.0016	—0.0008		
—1	5	—2	—0.00012	+0.00106	—0.00005	—0.00016
0	4	—2	—0.0001	+0.0008		
—1	6	—2	—0.00001	+0.00015	+0.00002	+0.00001
0	5	—2	—0.0001883	—0.0005367	—0.0000502	—0.0000056
1	4	—2	+0.00050	+0.00070	+0.00005	+0.00006
1	5	—2	+0.00052	+0.00039	+0.00003	+0.00003
—1	7	—3	0.0000	—0.0001		
0	7	—3	+0.0002	—0.0004		
—1	10	—4	—0.00007	+0.00002		
0	10	—4	—0.000014	+0.000003		

The fourteen parts of the portion of δ^2T' factored by $n't$ follow; for convenience the coefficients are multiplied by 10000:

Arg= $\varkappa y'+i'g'+ig$			$A'n'\delta^2z'$		$B'\delta\nu'$		$F'n\delta^2z$		$G'\delta\nu$	
			$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		+ 3.3579		+ 0.7738		+ 0.8543		+0.3302
—1	1	0	+ 0.7826	— 4.6520	— 0.0752	— 1.0503	+ 0.1966	— 1.0570	+0.0030	—0.4250
—1	2	0	+ 0.49	— 0.35	+ 0.75	+ 1.41	+ 1.91	+ 1.40	—0.12	—0.10
0	1	0	+ 3.85	+ 6.32	+ 3.99	+ 4.57	+ 1.37	+ 2.24	+0.26	+0.42
1	0	0	— 11.0432	— 13.4758	— 4.8603	— 6.3974	— 3.3079	— 4.2943	—0.2432	—0.4404
—1	3	0	— 1.9	— 1.8			— 1.1	— 0.8		
0	2	0	+ 43.5	+ 18.0	+ 6.0	+ 3.5	+12.7	+ 4.8	+1.6	+0.8
1	1	0	— 71.0	— 28.5	—10.4	— 4.8	—14.8	— 5.9	—1.9	—0.9
—1	4	0	— 4.3	+ 0.7			— 3.6	+ 0.6		
0	3	0	+ 78.4	— 15.9	— 3.2	+ 2.1	+32.6	— 5.6		
1	2	0	—143.4	+ 23.2	+ 5.2	— 1.1	—39.9	+ 6.2		
—1	5	0					+ 8	+ 20		
0	4	0	— 7	— 24			— 7	— 19		
1	3	0	+ 6.6	+ 14.8			+ 3.5	+ 5.8		
0—	4—	1					+ 1	— 2		
—1—	2—	1	+ 4	— 5			+ 2	— 2		
0—	3—	1	— 3	+ 1						
—1—	1—	1	+ 10.0	+121.1	— 0.2	— 3.5	+ 2.8	+ 38.5		
0—	2—	1	— 7	— 80	0	+ 3	— 3	— 33		
1—	3—	1	+ 1	+ 10			0	+ 6		
—1	0—	1	+ 40.7	+ 64.0	+ 5.2	+ 7.7	+ 9.7	+ 14.7	+1.0	+1.3
0—	1—	1	— 28.9	— 45.6	— 3.8	— 5.3	— 8.9	— 13.4	—0.8	—1.2
1—	2—	1	+ 4.3	+ 4.6			+ 1.4	+ 1.4		
—1	1—	1	+ 26.0	+ 14.5	+ 8.3	+ 4.1	+ 5.9	+ 3.0	+1.2	+0.4
0	0—	1	— 23.2	— 11.8	— 8.4	— 3.4	— 5.3	— 3.0	—0.7	—0.2
1—	1—	1	+ 2.1	+ 0.4						
—1	2—	1	+ 6.35	+ 0.63	+ 1.88	— 0.54	+ 1.60	— 0.05	+0.41	—0.10
0	1—	1	— 7.8	— 0.3	— 2.4	+ 0.4	— 1.6	+ 0.1	—0.6	+0.3
1	0—	1	+ 3.4	— 0.6	+ 0.9	— 0.2	+ 0.2	0.0		
—1	3—	1	+ 3.42	— 2.57	— 3.79	+ 3.91	— 1.59	+ 2.23	+1.26	—1.31
0	2—	1	— 12.23	+ 5.25	+ 2.60	— 1.13	— 0.01	— 0.63	—1.87	+1.95
1	1—	1	+ 11.66	— 8.29	+ 2.10	— 3.09	+ 1.45	— 1.29	+1.18	—1.26
—1	4—	1	+ 1.31	+ 1.85	— 0.02	— 2.19	+ 0.51	+ 1.93	+0.07	—0.40
0	3—	1	— 8.84	— 32.42	— 4.34	+22.45	— 3.41	+ 1.29	—0.61	+2.03
1	2—	1	+ 14.49	— 17.72	+ 3.88	—18.56	+ 3.02	— 6.84	+0.58	—2.31
—1	5—	1			+ 0.6	+ 1.5	+35.2	+105.3	+1.5	+1.1
0	4—	1	— 69.3	—200.9	— 2.8	— 9.1	—27.6	— 84.1	—1.6	—1.2
1	3—	1	+ 10.71	+ 32.19	+ 2.13	+ 7.33	+ 1.59	+ 7.15	+0.58	+0.57
—1	6—	1	— 0.5	+ 1.7			+ 0.3	— 0.7		
0	5—	1	— 2.0	— 2.3						
1	4—	1	— 3.4	— 5.5			— 1.4	— 2.5		
—1—	1—	2	+ 7	+ 6			+ 3	+ 2		
0—	2—	2	— 3	— 5						

Arg= $\varkappa\gamma'+i'g'+ig$	$A'n'\delta^2z'$		$B'\delta\nu'$		$F'n\delta^2s$		$G'\delta\nu$	
	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.
$\varkappa$ i' i	″	″	″	″	″	″	″	″
—1 0— 2	— 88	+ 27			—30	+ 9		
0— 1— 2	+ 64	— 20			+27	— 8		
1— 2— 2	— 10	+ 3			— 6	+ 2		
—1 1— 2	— 46	+ 45	— 4	+ 5	—11	+12	+ 1	+1
0 0— 2	+ 35	— 36	+ 4	— 4	+11	—11		
1— 1— 2	— 4	+ 5						
—1 2— 2	— 7.7	+ 23.8	— 1.5	+ 7.5	— 1.8	+ 5.9	— 0.1	+0.7
0 1— 2	+ 6.2	— 19.3	+ 0.7	— 4.3	+ 1.3	— 4.8		
1 0— 2	0	+ 1						
—1 3— 2	+ 1.4	+ 6.6	+ 1.1	+ 2.5	+ 0.7	+ 1.8		
0 2— 2	— 2.0	— 5.6	— 0.3	— 0.9	— 0.2	— 0.8		
—1 4— 2	— 3.52	— 1.84	— 4.24	— 2.09	— 0.03	+ 0.27	— 1.11	—0.61
0 3— 2	+ 5.7	+ 1.2	+ 3.7	+ 1.6	+ 2.7	+ 0.8	+ 0.3	+0.2
1 2— 2	+ 4.4	+ 3.5	+ 0.8	+ 0.8	— 2.5	— 0.7	+ 0.2	+0.2
—1 5— 2	+ 8.775	— 0.555	—12.425	— 0.504	—10.576	— 0.372	+ 9.712	+0.376
0 4— 2	— 4.47	— 1.28	+ 0.75	+ 0.87	+ 8.82	+ 0.09	—11.49	—0.32
1 3— 2	+ 28.34	+ 1.66	+11.39	— 0.41	— 1.07	+ 0.34	+ 5.53	+0.08
—1 6— 2	+ 60.421	— 36.624	+ 1.743	— 0.806	+ 0.146	+ 0.229	— 0.032	—0.050
0 5— 2	— 0.7910	+ 0.7173	+ 1.3382	— 0.7277	— 0.3565	— 1.8182	— 0.5769	+0.6036
1 4— 2	+ 58.658	— 35.067	— 3.395	+ 1.954	+ 0.236	+ 0.025	— 0.322	—0.052
—1 7— 2	+ 8.2	— 19.0	+ 0.9	— 1.4			0.0	+0.3
0 6— 2	— 2.13	+ 5.56	— 0.20	+ 0.33	+ 0.05	+ 0.38	+ 0.10	—0.58
1 5— 2	+ 0.567	— 0.765	— 0.703	+ 0.990	+ 0.045	— 0.649	— 0.066	+0.487
—1 8— 2	— 0.7	— 3.4						
0 7— 2	+ 0.4	+ 1.8						
—1 0— 3	— 5	+ 8			— 2	+ 4		
0— 1— 3	+ 4	— 4			+ 2	— 3		
—1 1— 3	— 32	— 56			—11	—20		
0 0— 3	+ 26	+ 44			+11	+19		
1— 1— 3	— 5	— 8			— 3	— 4		
—1 2— 3	— 44	— 28	— 4	— 3	—11	— 7		
0 1— 3	+ 35	+ 22	+ 4	+ 2	+10	+ 6		
1 0— 3	— 5	— 3						
—1 3— 3	— 21	— 3	— 3	0	— 4	0		
0 2— 3	+ 21	+ 1	+ 3	0	+ 4	+ 1		
—1 4— 3	— 6.4	+ 2.5	— 0.8	— 0.5				
0 3— 3	+ 7	— 4						
—1 5— 3	— 0.6	+ 4.5	+ 1.2	— 2.4	— 0.2	+ 1.0	+ 0.4	—1.1
0 4— 3	+ 1.0	— 3.0	— 0.2	+ 0.4	— 0.5	+ 1.1	— 0.8	+2.2
1 3— 3					+ 0.8	— 2.5	+ 0.6	—2.0
—1 6— 3	+ 3.8	+ 1.9	— 4.3	—21.2	+ 1.7	+ 2.6	— 0.3	—1.1
0 5— 3	+ 5.6	+ 27.9	+ 5.3	+25.0	+ 0.2	+ 0.8	+ 0.1	+0.5
1 4— 3	— 1.7	— 0.3	— 0.6	— 2.6	+ 0.5	— 0.9	+ 0.1	+0.5
—1 7— 3	— 20.42	— 23.77	+ 1.91	+ 2.02	— 4.93	— 5.15	+ 0.21	+0.25
0 6— 3	+142.2	+157.6	— 3.7	— 3.1	+61.0	+65.2	— 0.9	—0.7
1 5— 3	0.0	— 0.4	+ 0.8	+ 0.8	—76.4	—81.6	+ 0.9	+0.7

Arg= $\varkappa\gamma'+i'g'+ig$	$A'n'\delta^2z'$		$B'\delta\nu'$		$F'n\delta^2z$		$G'\delta\nu$	
	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.
$\varkappa$ i' i	″	″	″	″	″	″	″	″
−1 8 − 3	+27.74	− 3.60	+ 2.58	+1.13	+ 3.49	− 0.51	− 0.06	+0.03
0 7 − 3	− 1.00	+ 15.71	− 2.90	−1.36	− 0.37	+ 3.00	0.00	−0.07
1 6 − 3	− 1.46	− 2.52	+ 0.35	+0.16	− 0.69	− 1.05		
−1 9 − 3	+ 7.55	− 4.12	+ 0.64	−0.05	+ 0.69	− 0.36		
0 8 − 3	− 3.53	+ 3.65	− 0.76	−0.02	− 0.47	+ 0.45	+ 0.02	−0.02
1 7 − 3	− 0.18	− 0.44	+ 0.13	−0.02	− 0.04	− 0.09		
−1 10 − 3	+ 0.9	− 1.2						
0 9 − 3	− 0.1	+ 0.6			+ 0.1	− 0.2		
1 8 − 3	− 0.12	− 0.01						
−1 1 − 4	− 8	− 2						
0 0 − 4	+ 5	+ 3						
−1 2 − 4	+31	− 30			+12	−11		
0 1 − 4	−25	+ 24			−10	+10		
1 0 − 4	+ 6	− 5						
−1 3 − 4	+14	− 37	+ 1	−3	+ 3	− 9		
0 2 − 4	−11	+ 30			− 4	+ 9		
1 1 − 4	+ 2	− 5						
−1 4 − 4	− 3	− 17	− 1	−2	0	− 4		
0 3 − 4	+ 2	+ 16			0	+ 3		
−1 5 − 4	− 2	− 3						
0 4 − 4	+ 2	+ 3						
−1 6 − 4	− 2	− 2	+ 1	0				
0 5 − 4	+ 4	+ 3	0	+2				
−1 7 − 4	−27.2	− 4.4	+13.1	−5.7	− 2.8	− 6.1	+ 5.1	−2.0
0 6 − 4	+26.8	+ 13.7	− 9.1	+3.9	+ 5.9	+18.3	−10.7	+4.2
1 5 − 4					− 4	−20	+ 9	−4
−1 8 − 4	−90.9	+122.5	− 0.2	0.0	−23.9	+32.9	+ 0.5	−0.6
0 7 − 4	+50.7	− 66.4	− 0.1	+0.3	+20.0	−26.4	− 0.7	+0.6
1 6 − 4	− 2.7	+ 3.3			− 2.2	+ 2.8		
−1 9 − 4	− 7.3	+ 60.2	− 0.3	+1.7	− 1.4	+11.7	0.0	−0.1
0 8 − 4	+ 5.2	− 39.6	+ 0.1	−1.0	+ 1.2	−10.4		
1 7 − 4	+ 0.8	+ 2.4			+ 0.4	+ 0.8		
−1 10 − 4	+ 5.898	+ 13.631	+ 0.185	+0.525	+ 0.859	+ 1.971	− 0.026	−0.029
0 9 − 4	− 4.10	− 9.88	− 0.18	−0.43	− 0.77	− 1.86		
1 8 − 4	+ 0.6	+ 0.4			+ 0.2	+ 0.1		
−1 11 − 4	+ 2.445	+ 1.772	+ 0.110	+0.091	+ 0.274	+ 0.200	− 0.011	−0.001
0 10 − 4	− 1.7701	− 1.3203	− 0.0777	−0.0652	− 0.2580	− 0.1905	+ 0.0075	+0.0028
1 9 − 4	+ 0.308	+ 0.077	− 0.008	−0.017	+ 0.035	− 0.005		
−1 3 − 5	+22	+ 15			+ 9	+ 5		
0 2 − 5	−20	− 13			− 8	− 5		
−1 4 − 5	+24	+ 3			+ 8	+ 1		
0 3 − 5	−20	− 3			− 7	− 2		
−1 5 − 5	+10	− 3						
0 4 − 5	− 9	+ 3						
−1 6 − 5	+ 3	− 3						
0 6 − 5					+ 1	+ 2		

Arg= $\varkappa\gamma'+i'g'+ig$		$A'n'\delta^2z'$		$B'\delta\nu'$		$F'n\delta^2z$		$G'\delta\nu$	
		$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.
$\varkappa$	i' i	″	″	″	″	″	″	″	″
—1	8— 5	— 8	— 7	+6	+9	— 2	— 1		
0	7— 5	+ 7	+ 6	—5	—6	+ 1	+ 3		
—1	9— 5	—119.2	—52.9	+0.7	+0.2	—36.4	—15.9		
0	8— 5	+ 78	+35	—1	0	+31	+13		
1	7— 5	— 9	— 4			— 6	— 2		
—1	10— 5	— 62.1	+ 4.9	—1.5	0.0	—14.7	+ 1.4		
0	9— 5	+ 45.4	— 3.3	+0.2	0.0	+13.4	— 1.2		
1	8— 5	— 4.7	+ 1.5			— 2.0	+ 0.6		
—1	11— 5	— 14.6	+ 9.8	—0.7	+0.5	— 2.8	+ 2.0		
0	10— 5	+ 11.4	— 7.4			+ 2.6	— 1.8		
1	9— 5	— 0.8	+ 1.2						
—1	12— 5	— 1.65	+ 3.38	—0.05	+0.14	— 0.27	+ 0.57		
0	11— 5	+ 1.4	— 2.6			+ 0.2	— 0.6		
1	10— 5	0.0	+ 0.3						
—1	13— 5	+ 0.07	+ 0.75	0.00	+0.02	+ 0.01	+ 0.10		
0	12— 5	— 0.08	— 0.57	0.00	—0.02	0.00	— 0.09		
—1	4— 6	— 6	+16						
0	3— 6	+ 4	—15						
—1	5— 6	+ 1	+17						
0	4— 6	— 1	—14						
—1	6— 6	+ 4	+ 7						
0	5— 6	— 4	— 6						
—1	9— 6	0	— 5			— 3	+ 2		
0	8— 6	— 1	+ 6			+ 3	0		
—1	10— 6	+ 20	—98			+ 6	—31		
0	9— 6	— 13	+71			— 5	+28		
1	8— 6	+ 2	—11			+ 2	— 6		
—1	11— 6	— 16	—52			— 4	—14		
0	10— 6	+ 11	+41			+ 4	+13		
1	9— 6	— 2	— 4			— 1	— 2		
—1	12— 6	— 12.4	—12.3			— 2.9	— 2.8		
0	11— 6	+ 9.6	+ 9.7			+ 2.7	+ 2.7		
1	10— 6	— 2	— 1						
—1	13— 6	— 3.9	— 1.1			— 0.8	— 0.2		
0	12— 6	+ 3.3	+ 0.9			+ 0.8	+ 0.2		
—1	6— 7	— 11	+ 3						
—1	10— 7	+ 3	— 4			— 2	— 3		
0	9— 7	— 4	+ 3						
—1	11— 7	+ 70	— 2			+23	0		
0	10— 7	— 55	+ 2			—21	+ 1		
1	9— 7	+ 9	0			+ 5	0		
—1	12— 7	+ 40	—21			+11	— 7		
0	11— 7	— 31	+17			—10	+ 6		
1	10— 7	+ 5	— 3						
—1	13— 7	+ 9	—13			+ 2	— 4		
0	12— 7	— 7	+11			— 2	+ 3		

Arg= $\varkappa\gamma'+i'g'+ig$			$A'n'\delta^2z'$		$B'\delta\nu'$		$F'n\delta^2z$		$G'\delta\nu$	
			$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	11	−8	+4	0						
0	10	−8	−5	+3						
−1	12	−8	+12	+46			+4	+15		
0	11	−8	−9	−34			−4	−14		
1	10	−8	+2	+7						
−1	13	−8	+22	+25			+7	+8		
0	12	−8	−18	−21			−6	−7		
1	11	−8	+4	+3						
−1	13	−9	−24	+13						
0	12	−9	+20	−11						

Arg= $\varkappa\gamma'+i'g'+ig$			$\frac{1}{2}\frac{dA'}{dg'}(n'\delta z')^2$		$\frac{dA'}{dg}(n\delta z)(n'\delta z')$		$\frac{1}{2}\frac{dF'}{dg}(n\delta z)^2$		$\frac{dB'}{dg'}(n'\delta z')\nu'$	
			$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		−0.8003		−0.6466		−0.1410		−1.2080
−1	1	0	−0.0706	+2.0525	+0.0008	+1.1755	+0.0434	+0.1754	−0.6817	+1.8936
−1	2	0	+0.15	+0.68	−0.04	+0.17	+0.04	+0.06	−0.47	−0.18
0	1	0	−1.47	−2.73	−0.73	−1.22	+0.04	−0.35	−0.60	−1.71
1	0	0	+2.9783	+5.2970	+1.1542	+2.0917	−0.0403	+0.3853	+1.4470	+3.3846
0	2	0	−2.8	−1.8	−2.2	−1.1	−0.4	−0.5	−1.5	−1.1
1	1	0	+7.6	+4.6	+3.9	+1.9	+0.4	+0.6	+3.5	+2.5
0	3	0	−2.5	+0.1					+1.1	0.0
1	2	0	+5.3	+0.3					−2.1	−0.1
0	4	0	−2	+1						
1	3	0	+4.6	−2.7						
−1	−2	−1	+2	−8					−1	+4
0	−3	−1	−2	+4						
−1	−1	−1	−1.2	−6.2					+0.5	+2.5
0	−2	−1	0	+3					0	−1
−1	0	−1	−7.5	−8.9	−2.9	−3.7	−0.7	−0.3	−3.3	−3.2
0	−1	−1	+3.4	+3.7	+2.0	+2.5			+2.2	+2.1
−1	1	−1	−7.4	−2.7	−2.6	−0.7	−0.3	0.0	−4.2	−0.8
0	0	−1	+3.6	+1.1	+2.3	+0.7			+3.0	+0.7
−1	2	−1	−3.80	−0.21	−1.67	+0.21	−0.07	+0.05	−2.20	+0.08
0	1	−1	+2.1	−0.6	+0.9	−0.3	+0.1	0.0	+2.1	−0.3
1	0	−1	−0.8	−0.2					−1.2	+0.3
−1	3	−1	−1.33	+0.29	−0.31	+0.01	+0.03	+0.08	+0.17	−0.60
0	2	−1	+0.55	+0.39	+1.00	−0.69	+0.18	−0.13	+1.82	−1.33
1	1	−1	−1.35	+0.36	−1.16	+0.82	−0.23	+0.11	−2.92	+2.31
−1	4	−1	−0.31	−0.03	0.00	−0.09			−0.07	−0.33
0	3	−1	−0.76	+3.13	−0.59	+0.32	+0.22	−0.27	+1.24	−2.63
1	2	−1	−0.11	−0.39	−0.13	+1.60	−0.26	+0.40	−1.95	+4.56

Arg= $\varkappa\gamma'+i'g'+ig$	$\frac{1}{2}\frac{dA'}{dg'}(n'\delta z')^2$		$\frac{dA'}{dg}(n\delta z)(n'\delta z')$		$\frac{1}{2}\frac{dF'}{dg}(n\delta z)^2$		$\frac{dB'}{dg'}(n'\delta z')\nu'$	
	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.
$\varkappa$ i' i	″	″	″	″	″	″	″	″
—1 5— 1					+0.2	+0.1		
0 4— 1	+0.7	+0.9	—0.3	+0.1	—0.2	—0.1	+0.2	+0.7
1 3— 1	—0.83	+0.50	+0.05	+0.33	+0.06	+0.03	—0.46	—1.33
0 5— 1	+3.2	+3.2						
1 4— 1	—2.5	—1.8	—0.6	—0.3				
—1— 1— 2	+8	+1					—5	—1
0— 2— 2	—5	0						
—1 0— 2	+4	—2					—2	+1
0— 1— 2	—3	+1						
—1 1— 2	+6	—8	+3	—4			+2	—4
0 0— 2	—2	+3	—1	+1			—2	+2
—1 2— 2	+0.7	—6.8	+0.6	—2.7			+0.2	—4.9
0 1— 2	—0.3	+3.3	0.0	+2.0			—0.2	+3.3
—1 3— 2	—0.1	—4.1	—0.3	—2.0			—1.6	—3.2
0 2— 2	+0.8	+1.9					+1.3	+2.2
-1 4— 2	+0.22	—1.44	—0.02	—0.54	+0.11	+0.01	+0.03	—0.32
0 3— 2	+0.7	+1.0	—0.4	—0.6			—0.3	+0.1
1 2— 2	—0.6	—0.7					—0.9	—0.8
—1 5— 2	+1.032	—0.269	+0.166	—0.400	+0.588	+0.315	—0.918	—0.446
0 4— 2	+0.67	+0.06	—1.20	+0.24	—0.44	—0.28	—0.37	+0.13
1 3 2	—1.41	—0.25	+0.20	—0.01			—1.41	—0.24
—1 6— 2	+0.809	—0.695	—0.139	—0.049	—0.054	—0.006	—0.576	+0.152
0 5— 2	—0.0504	+0.1314	+0.1409	—0.1522	+0.0777	—0.0295	+0.1317	+0.0589
1 4— 2	—0.557	+0.109	—0.012	—0.026	—0.064	+0.016	+0.359	—0.203
—1 7— 2	+0.8	—1.2						
0 6— 2	—0.36	+0.27	+0.16	—0.37	+0.08	—0.54	—0.08	+0.25
1 5— 2	—0.082	—0.022	+0.263	—0.403	—0.054	+0.614	+1.023	—1.775
—1 0 3	0	+7						
—1 1— 3	+3	+4						
0 0— 3	—2	—3						
—1 2— 3	+5	+2	+2	+1			+3	+1
0 1— 3	—4	—2					—2	—1
—1 3— 3	+5	—1	+2	0			+5	—1
0 2— 3	—3	+1					—2	0
—1 4— 3	+2.9	—0.1					+1.8	—2.6
0 3— 3	—1	+1						
—1 5— 3	+1.4	—0.5					+1.7	—2.4
0 4— 3	—0.4	+0.7					—0.7	+1.3
—1 6— 3	+1.1	+0.1	+0.3	+0.1			0.0	—5.4
0 5— 3	0.0	+3.2	+0.1	+1.5			0.0	+3.1
—1 7— 3	+1.81	+0.84	+0.71	+0.33	—0.08	+0.03	—0.72	—1.51
0 6— 3	—0.1	+0.6	—0.2	0.0	+0.1	—0.2	+0.4	+0.7
—1 8— 3	—0.09	—0.32	+0.68	+0.22	+0.47	—0.04	+4.35	+1.67
0 7— 3	+2.80	+1.41	+0.78	+0.37	—0.38	+0.02	—2.78	—1.04
1 6— 3	+0.43	+0.25	+0.11	+0.05			+0.38	+0.07

Arg= $\varkappa y'+i'g'+ig$			$\frac{1}{2}\frac{dA'}{dg'}(n'\delta z')^2$		$\frac{dA'}{dg}(n\delta z)(n'\delta z')$		$\frac{1}{2}\frac{dF'}{dg}(n\delta z)^2$		$\frac{dB'}{dg'}(n'\delta z')\nu'$	
			$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	9	−3	+1.38	−0.78	+0.45	−0.11			+1.48	−0.20
0	8	−3	+0.56	+0.23	−0.04	+0.01	−0.10	+0.06	−1.29	+0.10
1	7	−3	+0.34	+0.10	+0.30	+0.01			+0.76	−0.13
−1	10	−3	+0.3	−0.4					+0.2	−0.3
0	9	−3	+1.1	−0.8	+0.9	−0.7			−0.1	+0.1
1	8	−3	−0.4	+0.3	−0.16	+0.13			−0.05	−0.06
−1	3	−4	0	+5					0	+3
0	2	−4	0	−3						
−1	4	−4	0	+4					+1	+2
0	3	−4	−1	−3						
−1	5	−4	+1	+2						
−1	6	−4	−1	−1					+3	0
−1	7	−4	−7.8	+2.6	−2.0	+0.5			+5.9	−1.5
0	6	−4	+2.4	−0.5	+0.1	+0.3			−2.2	+0.5
−1	8	−4	−4.4	+4.1	−0.8	+1.1			+1.6	−1.7
0	7	−4	+1.7	−1.6					−0.8	+0.9
−1	9	−4	−2.0	+9.1	−0.5	+3.2	+0.2	+0.6	−0.7	+4.6
0	8	−4	+0.7	−3.7	+0.5	−1.6	−0.1	−0.6	+0.3	−1.7
−1	10	−4	+1.122	+4.028	+0.371	+1.322	+0.161	+0.099	+0.894	+2.097
0	9	−4	−0.61	−2.09	−0.21	−0.73	−0.17	−0.17	−0.42	−1.04
1	8	−4							+0.1	+0.2
−1	11	−4	+0.410	+0.559	+0.228	+0.248	+0.059	+0.013	+0.512	+0.353
0	10	−4	−0.5187	−0.5291	−0.1803	−0.1930	−0.0547	−0.0115	−0.3023	−0.2061
1	9	−4	+0.438	+0.297	+0.021	+0.006	+0.008	0.000	−0.010	−0.010
−1	8	−5	−4	−7					+2	+5
0	7	−5	+1	+3					−2	−2
−1	9	−5	−6.2	−3.8					+2.3	+1.5
0	8	−5	+2	+1					−2	−1
−1	10	−5	−10.4	−0.7					−4.3	+0.2
0	9	−5	+4.9	+0.2					+2.8	−0.1
−1	11	−5	−4.9	+2.4					−2.1	+1.5
0	10	−5	+3.0	−1.7					+1.3	−1.0
−1	12	−5	−1.26	+1.67					−0.44	+0.87
0	11	−5	+1.2	−2.3					+0.2	−0.5
−1	13	−5	−0.36	−0.14					0.00	+0.15
0	12	−5	+0.27	−0.34					−0.01	−0.12

Arg= $\varkappa\gamma'+i'g'+ig$			$\frac{dB'}{dg}(n\delta z)\nu'$		$\frac{dG'}{dg'}(n'\delta z')\nu$		$\frac{dG'}{dg}(n\delta z)\nu$		$\frac{1}{2}r'^2\frac{d^2T'}{dr'^2}\nu'^2$	
			n't sin.	*n't* cos.	*n't* sin.	*n't* cos.	*n't* sin.	*n't* cos.	*n't* sin.	*n't* cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		−0.4964		−0.3864		−0.1168		−0.3884
−1	1	0	−0.0364	+0.6393	+0.0729	+0.5673	+0.0447	+0.1467	+0.0866	+0.5888
−1	2	0	−0.21	−0.06	−0.07	+0.01	0.00	+0.03	−0.44	−0.75
0	1	0	−0.12	−0.67	−0.06	−0.72	0.00	−0.16	−2.6	−4.9
1	0	0	+0.3637	+1.1388	−0.0351	+1.0488	+0.0397	+0.3078	+0.8808	+1.7647
0	2	0	−0.7	−0.4	−0.4	−0.7	−0.2	−0.3		
1	1	0	+1.0	+0.7	+0.7	+1.1	+0.2	+0.3	−0.3	+0.3
−1	0	−1	−1.1	−1.1	−1.2	−0.4				
0	−1	−1	+0.8	+0.8						
−1	1	−1	−1.3	−0.2	−1.2	+0.2	−0.2	+0.1	−2.0	−0.5
0	0	−1	+0.4	+0.1	+0.5	−0.1			+0.6	+0.2
−1	2	−1	−0.71	+0.10	−0.54	+0.31	−0.15	+0.12	−0.67	+0.21
0	1	−1	+0.5	−0.1	+0.8	−0.4			+0.3	−0.1
1	0	−1			−0.3	+0.1				
−1	3	−1	+0.07	−0.37	+0.02	0.00	+0.09	−0.02	+0.94	−0.85
0	2	−1	+0.47	−0.04	+0.24	+0.10	+0.03	−0.01	+0.21	−0.10
1	1	−1	−0.73	+0.44	−0.63	−0.05	−0.14	+0.01	−1.23	+1.00
−1	4	−1	+0.03	−0.01	−0.19	−0.27	+0.04	−0.06	+0.26	−0.37
0	3	−1	+0.41	−0.92	+0.75	−0.23	+0.19	−0.15	−0.03	+0.26
1	2	−1	−0.39	+0.86	−0.89	+0.82	−0.26	+0.19	−0.26	−0.03
−1	5	−1							0.0	+0.3
0	4	−1			+0.1	+0.1	+0.1	+0.1	+0.1	+0.3
1	3	−1	−0.06	−0.32	−0.11	+0.09	−0.03	−0.12	−0.27	−1.11
−1	1	−2	+1	−1						
−1	2	−2	0.0	−0.7	−0.3	−0.7				
0	1	−2	0.0	+0.6						
−1	3	−2							−0.3	−0.3
−1	4	−2	+0.45	+0.07	−0.07	−0.04			+1.36	+1.06
0	3	−2	−0.1	+0.3	+0.3	+0.5			−0.3	−0.2
1	2	−2			−0.3	−0.6			−0.6	−0.5
−1	5	−2	+0.282	−0.036	+0.110	−0.096	−0.471	−0.416	+0.730	+0.312
0	4	−2	−0.26	+0.09	+1.27	+1.08	+0.62	+0.44	−0.34	−0.10
1	3	−2			−1.34	−1.01	−0.29	−0.21	+0.03	−0.16
−1	6	−2	−0.242	+0.125	+0.173	−0.326	+0.057	−0.010	−0.473	+0.230
0	5	−2	+0.1452	−0.1137	−0.0244	+0.2988	−0.0362	+0.0488	−0.1419	+0.0641
1	4	−2	+0.067	−0.043	−0.200	+0.096	+0.015	−0.010	+0.668	−0.354
−1	7	−2			+0.3	−1.7				
0	6	−2			−0.24	+1.78	−0.10	+0.72		
1	5	−2	+0.024	−0.134	+0.064	−0.134	+0.106	−0.623	+0.198	−0.139
−1	5	−3							−0.5	+1.0
−1	6	−3	0.0	−1.0					−0.1	+0.4
0	5	−3	0.0	+1.1						
−1	7	−3	−0.36	−0.31	+0.27	−0.22	+0.08	0.00	−0.60	−0.64
0	6	−3	+0.1	+0.1	−0.1	+0.1			+0.3	+0.3
−1	8	−3	+0.79	+0.35	+1.20	−0.15	+0.31	−0.02	−0.35	−0.23
0	7	−3	−1.07	−0.43	−0.74	+0.05	−0.24	+0.02	+0.14	+0.09
1	6	−3	+0.13	+0.06	+0.11	+0.05			+0.15	+0.11

Arg= $\varkappa\gamma'+i'g'+ig$			$\frac{dB'}{dg}(n\delta z)\nu'$		$\frac{dG'}{dg'}(n'\delta z')\nu$		$\frac{dG'}{dg}(n\delta z)\nu$		$\frac{1}{2}r'^2\frac{d^2T'}{dr'^2}\nu'^2$	
			$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	9	−3	+0.20	−0.02	+0.36	−0.33	+0.07	−0.06	−0.09	0.00
0	8	−3	−0.41	+0.03	−0.31	+0.22	−0.07	+0.06	+0.05	+0.01
1	7	−3	+0.17	−0.04	+0.13	−0.03				
−1	7	−4	+1.3	−0.3	+1.6	+0.4				
0	6	−4			−1.8	−0.4				
−1	8	−4	+0.3	−0.3					+0.7	−0.8
0	7	−4							−0.4	+0.4
−1	9	−4	−0.4	+1.2	+0.4	+1.4	+0.1	+0.4	+0.3	−0.5
0	8	−4	+0.1	−0.9	−0.3	−0.8	−0.1	−0.4	0.0	+0.2
−1	10	−4	+0.134	+0.360	+0.537	+0.434	+0.106	+0.163	−0.016	−0.135
0	9	−4	−0.12	−0.30	−0.37	−0.30	−0.11	−0.10	+0.02	+0.09
−1	11	−4	+0.070	+0.052	+0.203	+0.025	+0.038	+0.007	−0.020	−0.021
0	10	−4	−0.0499	−0.0340	−0.1606	−0.0239	−0.0372	−0.0075	+0.0122	+0.0157
1	9	−4	+0.001	−0.007	+0.015	−0.007	+0.004	−0.002		

Arg= $\varkappa\gamma'+i'g'+ig$			$rr'\frac{d^2T'}{dr\,dr'}\nu\nu'$		$\frac{1}{2}r^2\frac{d^2T'}{dr^2}\nu^2$	
			$n't$ sin.	$n't$ cos.	$n't$ sin.	$n't$ cos.
$\varkappa$	i'	i	″	″	″	″
0	0	0		−0.1840		−0.0152
−1	1	0	−0.0066	+0.2775	+0.0157	+0.0240
−1	2	0	−0.16	−0.17	−0.19	−0.12
0	1	0	0.00	−0.22	+0.15	+0.09
1	0	0	+0.2023	+0.5023	−0.0317	+0.0007
−1	2	−1	−0.28	+0.09		
−1	3	−1	+0.25	−0.33	+0.02	−0.06
0	2	−1	+0.09	0.00	0.00	+0.07
1	1	−1	−0.44	+0.29		
−1	4	−1	−0.02	−0.09		
0	3	−1	+0.06	+0.08	+0.04	+0.03
1	2	−1	−0.03	−0.10	−0.03	−0.01
1	3	−1	−0.09	−0.20		
−1	4	−2	+0.57	+0.36		
−1	5	−2	+0.128	+0.051	−0.004	−0.005
0	4	−2	−0.10	+0.03		
−1	6	−2	−0.126	+0.121	−0.011	+0.008
0	5	−2	−0.0431	+0.0853	−0.0061	+0.0082
1	4	−2	+0.167	−0.156	+0.014	−0.018
1	5	−2	+0.024	−0.032	+0.004	+0.019
−1	7	−3	−0.22	−0.15		
−1	8	−3	−0.09	−0.01		
0	7	−3	+0.07	+0.01		
−1	10	−4	−0.025	−0.026		
−1	11	−4	−0.009	−0.002		
0	10	−4	+0.0047	+0.0007	+0.0002	+0.0001

The fourteen parts of δ^2T' which are multiplied by n'^2t^2 follow; for convenience the coefficients are multiplied by 1000000:

Arg= $\varkappa\gamma'+i'g'+ig$			$A'n'\delta^2z'$		$B'\delta v'$		$F'n\delta^2 s$		$G'\delta v$	
			n'^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n'^2t^2 cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		− 0.0008		− 0.0091				
−1	1	0	−1.1555	+ 0.3559	+1.9440	− 0.0531	+0.1035	−0.1048	+0.0120	+0.1442
−1	2	0	+3.0	− 2.3	+4.4	− 3.4	−0.3	+0.3	+2.0	−1.8
0	1	0	+0.09	− 0.06	−0.12	+ 0.10	+3.05	−2.79	−4.18	+3.84
1	0	0	+3.287	− 2.497	−4.442	+ 3.342	−3.856	+3.510	+3.600	−3.307
−1	3	0	+0.5	− 1.4	+0.4	− 1.3				
0	2	0	−0.1	+ 0.4	−0.1	+ 0.4			−0.2	+0.4
1	1	0			−0.3	+ 0.6				
−1	0−	1								
0−	1−	1								
−1	1−	1	−0.8	+ 1.9	−4.0	+ 8.4			−0.2	+0.4
0	0−	1	+5.4	−11.4	+4.7	−10.0				
1−	1−	1			−0.5	+ 1.1				
−1	2−	1	−0.36	− 1.18	+0.21	+ 3.01	−0.27	+0.16	+0.14	0.00
0	1−	1	−0.2	+ 0.1	+0.1	− 2.6	+0.4	−0.6	−0.3	+0.3
1	0−	1			−0.1	− 0.3	−0.7	+0.7	+0.3	−0.3
−1	3−	1	−2.01	− 1.46	+7.32	+ 6.51	+1.48	+1.14	+0.98	+0.69
0	2−	1	+9.76	+ 8.29	−8.56	− 7.58	−1.21	−0.93	−0.76	−0.52
1	1−	1			+0.91	+ 0.79	+0.13	+0.10		
−1	4−	1	+1.31	− 0.42	+2.48	+ 0.55	+0.51	+0.14	+0.39	−0.02
0	3−	1	+1.01	+ 1.19	−2.30	− 0.77	−0.45	−0.13	−0.31	−0.01
1	2−	1	−0.11	− 0.14	0.00	− 0.01	+0.05	−0.01	+0.02	−0.02
−1	5−	1	+0.4	− 0.3	+0.2	− 0.2				
0	4−	1	−0.2	+ 0.2	−0.2	+ 0.1				
−1	2−	2	+8.3	+ 2.2	−5.8	− 1.5			−2.6	−0.8
0	1−	2	−4.5	− 1.2	+3.9	+ 0.9	+3.9	+1.4	+5.4	+1.8
1	0−	2					−5	−2		
−1	3−	2	+2.3	− 2.4	−2.6	+ 1.0			−3.3	+0.3
0	2−	2	−1.0	+ 1.5	+1.2	− 1.2				
−1	4−	2	−4.04	+ 6.60	−3.34	+ 5.81	−0.73	+1.63	−0.42	+0.94
0	3−	2	+1.9	− 3.5	+2.1	− 3.8	+0.6	−1.4	+0.3	−0.8
−1	5−	2	−0.145	+ 3.931	+0.084	+ 2.529	−0.016	+0.650	+0.050	+0.409
0	4−	2	+0.11	− 2.47	−0.05	− 1.76	−0.04	−0.62	−0.07	−0.36
1	3−	2	+0.06	+ 0.13			+0.03	+0.09		
−1	6−	2	+0.491	+ 0.972	+0.329	+ 0.501	+0.057	+0.136	+0.060	+0.076
0	5−	2	−0.3414	− 0.6896	−0.2546	− 0.3946	−0.0520	−0.1265	−0.0550	−0.0689
1	4−	2	+0.052	+ 0.027	+0.031	− 0.010	+0.013	+0.013	+0.010	+0.004
−1	7−	2	+0.1	+ 0.1						
0	6−	2	−0.11	− 0.06						
1	5−	2	+0.013	− 0.003	+0.003	− 0.004	+0.004	0.000	+0.002	−0.001
−1	3−	3	−1	+ 8	0	− 5	0	+1		
0	2−	3	0	− 5	0	+ 3				
−1	4−	3	+1.6	+ 3.1	−1.0	− 1.6				
0	3−	3	−1	− 2	+1	+ 1				

Arg= $\varkappa\gamma'+i'g'+ig$			$A'n'\delta^2 z'$		$B'\delta\nu'$		$F'n\delta^2 z$		$G'\delta\nu$	
			n'^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n'^2t^2 cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	5	−3	−6.1	−1.5	−4.5	−1.5	−1.4	−0.3	−0.8	−0.2
0	4	−3	+3.9	+0.9	+3.3	+1.1	+1.2	+0.2		
−1	6	−3	−4.0	+0.7	−2.2	+0.6	−0.7	+0.2	−0.3	+0.1
0	5	−3	+2.5	−0.5	+1.4	−0.4	+0.6	−0.2	+0.3	−0.1
−1	7	−3	−0.96	+0.80	−0.44	+0.45	−0.13	+0.13	−0.06	+0.09
0	6	−3	+0.8	−0.7	+0.3	−0.3	+0.1	−0.1		
−1	8	−3	−0.08	+0.26	−0.02	+0.12	−0.01	+0.03	0.00	+0.03
0	7	−3	+0.09	−0.22	+0.02	−0.10	+0.01	−0.03	0.00	−0.03
0	8	−3	−0.01	−0.03						
−1	5	−4	−3	+2						
0	4	−4	+1	−2						
−1	6	−4	+1	−5	0	−3				
0	5	−4	0	+4	0	+3				
−1	7	−4	−1.2	−2.9	−0.8	−1.5				
0	6	−4	+0.9	+2.3	+0.6	+1.3				
−1	8	−4	−0.8	−0.6	−0.4	−0.3				
0	7	−4	+0.7	+0.5						
−1	9	−4	−0.2	0.0						
0	8	−4	+0.2	0.0						
−1	10	−4	−0.065	+0.024	−0.024	+0.015	−0.008	+0.004	−0.004	+0.004
−1	11	−4	−0.006	+0.007						
0	10	−4	+0.0081	−0.0086	+0.0026	−0.0044	+0.0011	−0.0009	+0.0005	−0.0007

Arg= $\varkappa\gamma'+i'g'+ig$			$\frac{1}{2}\frac{dA'}{dg'}(n'\delta z')^2$		$\frac{dA'}{dg}(n\delta z)(n'\delta z')$		$\frac{1}{2}\frac{dF'}{dg}(n\delta z)^2$		$\frac{dB'}{dg'}(n'\delta z')\nu'$	
			n'^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n'^2t^2 cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
0	0	0		+0.0350		−0.1436		+0.0004		+0.0031
−1	1	0	+0.3767	−0.0569	−0.1006	+0.0212	−0.0122	+0.0006	−0.2158	+0.0016
−1	2	0	+0.3	−0.2						
0	1	0	−0.23	+0.18					−0.22	+0.16
1	0	0	+4.946	−4.641	+0.0722	−0.0313	+0.0890	−0.0310	+0.6146	−0.3815
−1	3	0							+0.3	−0.6
0	2	0	−0.2	+0.4					−0.1	+0.3
1	1	0	+0.4	−1.0					+0.5	−1.2
−1	0	−1							+2.3	−1.9
0	−1	−1							−1.0	+0.8
−1	1	−1	+0.3	−0.4					+0.3	−1.0
0	0	−1	−0.2	+0.4						
−1	2	−1	−0.10	−0.83	+0.25	+0.29			−0.08	−0.35
0	1	−1	+0.4	+2.0	−0.2	−0.1	+0.1	+0.3		
−1	3	−1	+0.61	+0.21	+0.16	+0.11			−0.01	−0.03
0	2	−1	−0.02	+0.25	−0.11	−0.12	−0.01	−0.02	−0.22	−0.12
1	1	−1	+0.9	+1.9					+0.33	+0.29

Arg= $\varkappa\gamma'+i'g'+ig$			$\frac{1}{2}\frac{dA'}{dg'}(n'\delta z')^2$		$\frac{dA'}{dg}(n\delta z)(n'\delta z')$		$\frac{1}{2}\frac{dF'}{dg}(n\delta z)^2$		$\frac{dB'}{dg'}(n'\delta z')\nu'$	
			n'^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n'^2t^2 cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
—1	4—	1	+0.02	—0.05	+0.35	—0.07	+0.10	—0.09	+2.14	+0.56
0	3—	1	+1.09	+0.29	—0.20	—0.03	—0.14	+0.04	—1.70	—0.41
1	2—	1	—0.04	+0.04	+0.09	+0.09	+0.06	+0.05	+0.70	+0.13
—1	5—	1							+0.6	—0.2
0	4—	1							—0.1	0.0
1	3—	1	+0.28	+0.02					—0.30	+0.10
—1	1—	2	—2	—1					+2	+2
—1	2—	2	+0.8	+0.2					+0.9	+0.1
—1	3—	2	—3.1	+1.3						
0	2—	2	+1.2	—0.5						
—1	4—	2	—1.30	+1.49	—0.03	+0.23				
0	3—	2	+0.6	—0.8						
—1	5—	2	—0.183	+1.960	+0.182	+0.492	+0.171	+0.004	—0.109	+2.402
0	4—	2	+0.09	—0.89	—0.19	—0.24	—0.16	0.00	+0.03	—0.95
1	3—	2	0.00	+0.18					0.00	+0.23
—1	6—	2	+0.319	+0.780	+0.171	+0.147	+0.066	+0.012	+0.511	+0.922
0	5—	2	—0.2026	—0.4598	—0.1186	—0.0885	—0.0627	—0.0158	—0.2497	—0.4606
1	4—	2	+0.081	+0.147	+0.009	+0.004	+0.011	+0.003	—0.039	—0.120
—1	7—	2	0.0	0.0						
0	6—	2	—0.12	—0.08					—0.13	—0.06
1	5—	2	+0.145	+0.085	—0.024	—0.004			—0.321	—0.168
—1	4—	3	—2.4	—3.4						
0	3—	3	+1	+2						
—1	5—	3	—2.2	—1.1						
0	4—	3	+1.3	+0.6						
—1	6—	3	—2.7	+0.3	—0.4	+0.2			—3.1	+0.6
0	5—	3	+1.8	—0.2					+1.7	—0.3
—1	7—	3	—0.97	+0.59	—0.11	+0.23			—1.09	+0.85
0	6—	3	+0.5	—0.5	0.0	—0.2			+0.5	—0.5
—1	8—	3	—0.23	+0.48	—0.03	—0.09	—0.01	—0.08	+0.17	—0.67
0	7—	3	+0.35	—0.90	+0.06	—0.15	0.00	+0.07	—0.09	+0.35
—1	9—	3	—0.17	—0.14	—0.05	—0.07			—0.07	—0.30
0	8—	3	+0.08	—0.10	+0.03	+0.04	+0.02	+0.03	+0.02	+0.18
—1	5—	4	+3	—3						
—1	6—	4	+1	—3						
0	5—	4	0	+2						
—1	7—	4	—0.5	—1.5					—1.0	—2.6
0	6—	4							+0.5	+1.2
—1	8—	4	—0.4	—0.4					—0.9	—0.7
0	7—	4	+0.5	+0.5					+0.6	+0.5
—1	9—	4	+1.3	0.0	+0.5	0.0			+0.7	0.0
0	8—	4	—0.3	0.0					—0.3	0.0
—1	10—	4	+0.697	—0.326	+0.177	—0.122	+0.034	—0.037	+0.355	—0.206
0	9—	4	—0.27	+0.14	—0.13	+0.10	—0.03	+0.03	—0.15	+0.09
—1	11—	4	+0.125	—0.160	+0.021	—0.049	+0.002	—0.013	+0.055	—0.111
0	10—	4	—0.0795	+0.1128	—0.0181	+0.0389	—0.0025	+0.0104	—0.0311	+0.0642
1	9—	4	+0.020	—0.048					—0.006	+0.002

Arg= $xy'+i'g'+ig$			$\frac{dB'}{dg}(n\delta z)\nu'$		$\frac{dG'}{dg'}(n'\delta z')\nu$		$\frac{dG'}{dg}(n\delta z)\nu$		$\frac{1}{2}r'^2\frac{d^2T'}{dr'^2}\nu'^2$	
			n'^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n'^2t^2 cos.	n'^2t^3 sin.	n'^2t^3 cos.	n'^2t^2 sin.	n'^2t^2 cos.
x	i'	i	″	″	″	″	″	″	″	″
0	0	0		−0.2450		+0.3354		−0.0047		+0.0016
−1	1	0	−0.3185	+0.2294	+0.6051	−0.3019	−0.0010	+0.0063	−1.3714	−0.0046
−1	2	0			+0.1	−0.1				
0	1	0			−0.16	+0.09			+0.11	−0.07
1	0	0	+0.0138	−0.0189	+0.1686	−0.0186	−0.0271	+0.0056	+0.0575	−0.0617
−1	3	0	0.0	−0.4	0.0	−0.7				
0	2	−0	0.0	+0.5	+0.1	+0.7				
−1	0	−1							−0.8	+0.6
−1	1	−1							0.0	+0.3
−1	2	−1	−0.35	−0.42	+0.29	+0.29			−0.37	−1.94
0	1	−1	+0.2	+0.3	−0.2	−0.2			+0.1	+0.7
1	0	−1							+0.1	+0.7
−1	3	−1	−0.22	−0.11	+0.21	−0.03			−0.42	−0.38
0	2	−1	+0.11	+0.13	−0.16	−0.01	−0.01	−0.01	+0.18	+0.19
1	1	−1			+0.02	+0.08			+0.12	+0.09
−1	4	−1	+0.51	−0.13	+0.52	−0.14	+0.14	−0.08	+0.88	+0.25
0	3	−1	−0.35	+0.11	−0.33	0.00	−0.15	+0.04	−0.27	−0.09
1	2	−1			+0.13	+0.12			−0.40	−0.11
1	3	−1							−0.21	+0.06
−1	3	−2	−0.4	0.0	+0.4	+0.5			+1.9	−0.8
0	2	−2	+0.5	0.0	−0.7	0.0			−1.0	+0.4
−1	4	−2	+0.02	−0.25	+0.12	+0.22			+0.42	−0.61
0	3	−2			0.0	−0.2			−0.2	+0.4
−1	5	−2	+0.217	+0.512	+0.274	+0.554	+0.060	+0.218	−0.052	+0.899
0	4	−2	−0.17	−0.37	−0.04	−0.64	−0.05	−0.22	+0.05	−0.43
1	3	−2			−0.14	+0.24			+0.01	−0.12
−1	6	−2	+0.218	+0.154	+0.225	+0.138	+0.080	+0.023	+0.244	+0.413
0	5	−2	−0.1600	−0.0868	−0.1569	−0.1197	−0.0695	−0.0231	−0.1153	−0.1990
1	4	−2	+0.004	−0.032	+0.038	+0.033	+0.015	−0.002	−0.053	−0.116
−1	7	−2			−0.3	0.0				
0	6	−2			+0.31	+0.03	+0.16	+0.01		
1	5	−2	+0.005	−0.003	0.000	−0.008	−0.133	−0.005	+0.008	−0.004
−1	4	−3							+1.1	+1.5
−1	5	−3							+0.8	+0.2
−1	6	−3	−0.5	+0.3	−0.5	+0.3			−0.7	+0.1
0	5	−3	+0.4	−0.2					+0.5	−0.1
−1	7	−3	−0.10	+0.28	−0.11	+0.31	−0.01	+0.09	−0.42	+0.37
0	6	−3	0.0	−0.2	0.0	−0.2			+0.2	−0.2
−1	8	−3	+0.13	−0.26	−0.01	−0.11	+0.01	−0.05	−0.03	+0.13
0	7	−3	−0.14	+0.35	0.00	+0.02	−0.01	+0.03	+0.02	−0.10
1	6	−3	+0.01	−0.04						
−1	9	−3	−0.02	−0.06	−0.08	−0.07				
0	8	−3	−0.03	+0.03	+0.02	+0.03	+0.02	+0.01		
−1	8	−4	−0.3	−0.1	−0.3	−0.1			−0.3	−0.2
−1	9	−4	+0.5	0.0	+0.1	+0.1			−0.1	+0.1
0	8	−4	−0.4	0.0						
−1	10	−4	+0.091	−0.007	+0.062	−0.083	+0.021	−0.022	−0.016	+0.012
0	9	−4	−0.09	+0.06	−0.05	+0.08				
−1	11	−4	+0.016	−0.006	−0.001	−0.034	0.000	−0.009	+0.002	+0.001
0	10	−4	−0.0118	+0.0035	−0.0009	+0.0229	+0.0002	+0.0079	+0.0008	−0.0023

Arg= $\varkappa\gamma'+i'g'+ig$			$rr'\frac{d^2T'}{dr\,dr'}\nu\nu'$		$\frac{1}{2}r^2\frac{d^2T'}{dr^2}\nu^2$	
			n'^2t^2 sin.	n'^2t^2 cos.	n'^2t^2 sin.	n'^2t^2 cos.
$\varkappa$	i'	i	$''$	$''$	$''$	$''$
0	0	0		+0.0692		+0.0007
—1	1	0	—0.2521	+0.0253	+0.0448	—0.0016
—1	2	0	0.0	—0.5		
1	0	0	+0.0604	+0.4914	—0.0099	+0.0170
0	2	0	0.0	—0.2		
—1	1	—1	—0.6	+0.1		
0	0	—1	—0.6	+0.1		
—1	2	—1	—0.25	—0.19	—0.02	—0.08
0	1	—1	+0.2	+0.1	+0.1	+0.4
1	0	—1			—0.1	—0.4
—1	3	—1	+0.44	—0.10	+0.01	0.00
0	2	—1	+0.71	—0.17	0.00	+0.02
1	1	—1	—0.70	+0.15		
—1	4	—1	+0.53	—0.32		
0	3	—1	—0.24	+0.15		
1	2	—1	—0.12	+0.06		
—1	2	—2	—0.4	—1.0		
—1	3	—2	—0.2	—0.5		
—1	4	—2	+0.41	+0.83		
0	3	—2	—0.3	—0.6		
—1	5	—2	+0.517	+0.496	—0.023	—0.024
0	4	—2	—0.37	—0.37		
—1	6	—2	+0.272	+0.073	—0.021	—0.003
0	5	—2	—0.1729	—0.0427	+0.0168	+0.0035
1	4	—2	—0.024	—0.031		
1	5	—2	+0.004	—0.003		
—1	5	—3	—1.0	+0.6		
0	4	—3	+0.6	—0.4		
—1	6	—3	—0.3	+0.8		
0	5	—3	+0.1	—0.4		
—1	7	—3	0.00	+0.32		
0	6	—3	0.0	—0.1		
—1	8	—3	+0.05	+0.07		
0	7	—3	—0.04	—0.05		
—1	10	—4	—0.003	+0.019		
0	10	—4	—0.0004	—0.0037		

CHAPTER XX.

THIRD-ORDER PERTURBATIONS OF THE MEAN ANOMALY AND RADIUS-VECTOR OF SATURN ARISING FROM THE MUTUAL ACTION OF THIS PLANET AND JUPITER.

The summation of the fourteen parts of δ^2T', given in the preceding chapter, produces the following expression:

Arg= $\varkappa\gamma'+i'g'+ig$			δ^2T'					
			sin.	cos.	$n't$ sin.	$n't$ cos.	n'^2t^2 sin.	n'^2t^2 cos.
$\varkappa$	i'	i	"	"	"	"	"	"
0	0	0		+0.003041		+ 0.9331		+ 0.0422
−1	1	0		−0.005697	+ 0.3758	+ 0.3563	−0.3410	+ 0.2616
−1	2	0		−0.02333	+ 1.64	+ 2.03	+9.5	− 8.0
0	1	0	−0.1043	+0.0548	+ 6.54	+ 5.37	−1.66	+ 1.45
1	0	0	+0.100156	−0.059616	− 12.4957	− 8.6862	+0.124	+ 0.555
−1	3	0	+0.037	−0.033	− 3.0	− 2.6	+1.2	− 4.4
0	2	0	−0.269	+0.363	+ 55.6	+ 21.2	−0.6	+ 2.9
1	1	0	+0.284	−0.453	− 81.1	− 28.1	+0.6	− 1.6
−1	4	0	−0.001	−0.043	− 7.9	+ 1.3		
0	3	0	+0.025	+0.597	+106.4	+ 19.3		
1	2	0	−0.032	−1.010	−174.9	+ 28.5		
−1	5	0			+ 8	+ 20		
0	4	0	+0.25	−0.03	− 16	− 42		
1	3	0	−0.152	+0.003	+ 14.7	+ 17.9		
0	−4	−1			+ 1	− 2		
−1	−2	−1	−0.04	−0.03	+ 7	− 11		
0	−3	−1			− 5	+ 5		
−1	−1	−1	+0.856	−0.190	+ 11.9	+152.4		
0	−2	−1	−0.59	+0.14	− 10	−108		
1	−3	−1	+0.06	−0.02	+ 1	+ 16		
−1	0	−1	+0.361	−0.302	+ 39.9	+ 70.1	+1.5	− 1.3
0	−1	−1	−0.298	+0.261	− 34.0	− 56.4	−1.0	+ 0.8
1	−2	−1	+0.056	−0.057	+ 5.7	+ 6.0		
−1	1	−1	+0.063	−0.123	+ 22.2	+ 17.4	−5.0	+ 9.7
0	0	−1	−0.053	+0.151	− 27.2	− 15.7	+9.3	−20.9
1	−1	−1	0.000	−0.008	+ 2.1	+ 0.4	−0.5	+ 1.1
−1	2	−1	−0.0079	−0.0880	+ 0.15	+ 0.90	−0.91	− 1.24
0	1	−1	+0.044	+0.205	− 5.6	− 1.3	+0.7	+ 0.7
1	0	−1	−0.004	−0.003	+ 2.2	− 0.6	−0.5	+ 0.4

Arg= $\varkappa\gamma'+i'g'+ig$			δ^2T'					
			sin.	cos.	$n't$ sin.	$n't$ cos.	n'^2t^2 sin.	n'^2t^2 cos.
$\varkappa$	i'	i	″	″	″	″	″	″
—1	3	—1	+0.0607	+0.0422	— 0.75	+ 0.41	+8.55	+ 6.55
0	2	—1	—0.0223	—0.0302	— 6.92	+ 3.70	—0.30	— 0.60
1	1	—1	+0.0261	+0.0564	+ 7.56	— 8.64	+0.90	+ 1.69
—1	4	—1	+0.0060	+0.0069	+ 1.61	— 0.06	+9.88	+ 0.18
0	3	—1	+0.0320	—0.0810	— 15.67	— 7.03	—4.34	+ 0.38
1	2	—1	+0.1774	+0.1312	+ 17.66	— 37.53	+0.38	+ 0.20
—1	5	—1	—0.608	+0.123	+ 37.5	+108.3	+1.2	— 0.7
0	4	—1	+1.632	—0.299	—100.6	—293.2	—0.5	+ 0.3
1	3	—1	—0.2247	+0.0086	+ 13.27	+ 45.11	—0.23	+ 0.18
—1	6	—1			— 0.2	+ 1.0		
0	5	—1	—0.022	+0.019	+ 1.2	+ 0.9		
1	4	—1	+0.080	—0.063	— 7.9	— 10.1		
0	6	—1	—0.017	+0.051				
1	5	—1	+0.032	—0.095				
—1	—1	—2	+0.03	—0.05	+ 13	+ 8		
0	—2	—2	—0.02	+0.03	— 8	— 5		
—1	0	—2	+0.29	+0.61	—116	+ 35		
0	—1	—2	—0.23	—0.48	+ 88	— 27		
1	—2	—2	+0.03	+0.09	— 16	+ 5		
—1	1	—2	+0.33	+0.26	— 48	+ 46	0	+ 1
0	0	—2	—0.30	—0.24	+ 45	— 45		
1	—1	—2	+0.03	+0.01	— 4	+ 5		
—1	2	—2	+0.088	+0.011	— 9.9	+ 22.1	+1.2	— 0.8
0	1	—2	—0.123	+0.013	+ 7.7	— 21.0	+8.7	+ 2.9
1	0	—2			0	+ 1	—5	— 2
—1	3	—2	—0.233	+0.103	+ 0.9	+ 1.3	—5.0	— 0.6
0	2	—2	+0.090	—0.035	— 0.4	— 3.2	+0.2	+ 0.2
—1	4	—2	—0.1369	+0.1844	— 6.25	— 5.11	—8.89	+16.89
0	3	—2	+0.101	—0.120	+ 12.3	+ 4.9	+5.0	—10.7
1	2	—2	—0.030	+0.030	+ 0.5	+ 1.2		
—1	5	—2	—0.00889	+0.06173	— 2.871	— 2.045	+1.027	+15.032
0	4	—2	+0.0232	—0.1062	— 6.54	+ 1.05	—0.86	— 9.32
1	3	—2	—0.0470	+0.1913	+ 39.97	— 0.21	—0.04	+ 0.75
—1	6	—2	+0.14015	+0.35715	+ 61.696	— 37.701	+3.022	+ 4.344
0	5	—2	—0.0067136	—0.0181338	+ 0.6473	+ 0.2683	—1.9944	— 2.7721
1	4	—2	+0.15947	+0.35029	+ 55.634	— 33.729	+0.148	— 0.080
—1	7	—2	+0.097	+0.055	+ 10.2	— 23.0	—0.2	+ 0.1
0	6	—2	—0.0610	—0.0341	— 2.72	+ 7.80	+0.11	— 0.16
1	5	—2	+0.04632	+0.03598	+ 1.413	— 2.566	—0.294	— 0.118
—1	8	—2	+0.020	—0.001	— 0.7	— 3.4		
0	7	—2	+0.019	+0.005	+ 0.4	+ 1.8		
1	6	—2	—0.051	—0.006				
0	8	—2	+0.032	—0.019				
—1	0	—3	+0.05	+0.02	— 7	+ 19		
0	—1	—3			+ 6	— 7		
—1	1	—3	—0.38	+0.28	— 40	— 72		

Arg= $\varkappa\gamma'+i'g'+ig$			δ^2T'					
$\varkappa$	i'	i	sin.	cos.	$n't$ sin.	$n't$ cos.	n'^2t^2 sin.	n'^2t^2 cos.
			″	″	″	″	″	″
0	0	−3	+0.30	−0.25	+35	+60		
1	−1	−3	−0.04	+0.03	−8	−12		
−1	2	−3	−0.14	+0.32	−49	−34		
0	1	−3	+0.10	−0.25	+43	+27		
1	0	−3	−0.01	+0.03	−5	−3		
−1	3	−3	−0.01	+0.10	−16	−5	−1	+4
0	2	−3	−0.01	−0.08	+23	+3	0	−2
−1	4	−3	−0.189	−0.267	−2.5	+0.3	−0.7	−0.4
0	3	−3	+0.12	+0.15	+6	−3	+1	+1
−1	5	−3	−0.252	−0.126	+3.4	+0.1	−15.2	−3.8
0	4	−3	+0.149	+0.072	−1.6	+2.7	+10.3	+2.4
1	3	−3			+1.4	−4.5		
−1	6	−3	−0.026	+0.011	+2.2	−23.6	−15.4	+4.2
0	5	−3	−0.164	−0.006	+11.3	+63.1	+9.3	−2.4
1	4	−3	+0.003	−0.009	−1.7	−3.3		
−1	7	−3	+0.1813	−0.1425	−22.34	−28.28	−4.39	+4.59
0	6	−3	−1.344	+0.932	+199.1	+220.6	+2.4	−3.0
1	5	−3	+0.506	−0.356	−74.7	−80.5		
−1	8	−3	−0.0057	+0.1579	+41.02	−1.48	−0.06	−0.14
0	7	−3	−0.0852	−0.0353	−5.69	+17.78	+0.27	−0.76
1	6	−3	+0.0039	+0.0032	−0.49	−2.78	+0.01	−0.04
−1	9	−3	+0.0134	+0.0402	+12.73	−6.03	−0.39	−0.64
0	8	−3	−0.0178	−0.0263	−6.35	+4.78	+0.15	+0.19
1	7	−3	+0.0006	+0.0123	+1.61	−0.64		
−1	10	−3	+0.031	+0.046	+1.4	−1.9		
0	9	−3	−0.036	−0.051	+1.9	−1.0		
1	8	−3	−0.0006	+0.0016	−0.37	+0.09		
−1	1	−4			−8	−2		
0	0	−4			+5	+3		
−1	2	−4	−0.25	−0.20	+43	−41		
0	1	−4	+0.21	+0.17	−35	+34		
1	0	−4			+6	−5		
−1	3	−4	−0.22	−0.05	+18	−41		
0	2	−4	+0.20	+0.05	−15	+36		
1	1	−4			+2	−5		
−1	4	−4	−0.03	0.00	−3	−17		
0	3	−4	+0.06	−0.02	+1	+16		
−1	5	−4	+0.22	−0.25	−1	−1	0	−1
0	4	−4	−0.13	+0.14	+2	+3	+1	−2
−1	6	−4	+0.06	−0.27	+1	−3	+2	−11
0	5	−4	−0.03	+0.16	+4	+5	0	+9
−1	7	−4	+0.108	−0.177	−12.8	−16.5	−3.5	−8.5
0	6	−4	−0.193	+0.160	+11.4	+40.0	+2.3	+4.8
1	5	−4	+0.10	−0.01	+5	−24		
−1	8	−4	−0.722	−0.738	−117.1	+157.2	−3.4	−2.4
0	7	−4	+0.423	+0.443	+70.4	−92.2	+1.8	+1.5
1	6	−4	−0.015	−0.014	−4.9	+6.1		

Arg= $\kappa\gamma'+i'g'+ig$			δ^2T'					
			sin.	cos.	$n't$ sin.	$n't$ cos.	n'^2t^2 sin.	n'^2t^2 cos.
κ	i'	i	″	″	″	″	″	″
−1	9	−4	−0.390	−0.085	−11.6	+93.5	+2.8	+0.2
0	8	−4	+0.325	+0.023	+7.6	−60.5	−0.8	0.0
1	7	−4	−0.032	+0.008	+1.2	+3.2		
−1	10	−4	−0.08575	+0.01461	+10.200	+24.440	+1.317	−0.725
0	9	−4	+0.0646	−0.0189	−7.04	−16.81	−0.72	+0.50
1	8	−4	+0.033	−0.003	+0.9	+0.7		
−1	11	−4	+0.02219	−0.02872	+4.309	+3.296	+0.214	−0.374
0	10	−4	+0.010540	−0.007515	−3.3849	−2.5618	−0.1310	+0.2400
1	9	−4	+0.01023	−0.01044	+0.812	+0.332	+0.014	−0.046
0	11	−4	+0.0002	+0.0010				
−1	3	−5	+0.07	−0.14	+31	+20		
0	2	−5	−0.05	+0.12	−28	−18		
−1	4	−5	0.00	−0.13	+32	+4		
0	3	−5	0.00	+0.12	−27	−5		
−1	5	−5	−0.03	−0.06	+10	−3		
0	4	−5	+0.03	+0.05	−9	+3		
−1	6	−5	+0.23	+0.13	+3	−3		
0	5	−5	−0.16	−0.08				
−1	7	−5	+0.23	0.00				
0	6	−5	−0.15	0.00	+1	+2		
−1	8	−5	+0.09	−0.05	−6	−1		
0	7	−5	−0.05	+0.03	+2	+4		
−1	9	−5	+0.478	−0.751	−158.8	−70.9		
0	8	−5	−0.32	+0.53	+108	+48		
1	7	−5	0.00	−0.01	−15	−6		
−1	10	−5	+0.056	−0.371	−93.0	+5.8		
0	9	−5	−0.035	+0.309	+66.7	−4.4		
1	8	−5	−0.005	−0.055	−6.7	+2.1		
−1	11	−5	−0.061	−0.151	−25.1	+16.2		
0	10	−5	+0.079	+0.178	+18.3	−11.9		
1	9	−5	−0.006	−0.006	−0.8	+1.2		
−1	12	−5	−0.0326	−0.0329	−3.67	+6.63		
0	11	−5	+0.188	+0.124	+3.0	−6.0		
1	10	−5	−0.042	−0.026	0.0	+0.3		
−1	13	−5	+0.0320	−0.0149	−0.28	+0.88		
0	12	−5	+0.0011	+0.0181	+0.18	−1.14		
1	11	−5	−0.0017	−0.0012				
−1	4	−6	+0.10	+0.02	−6	+16		
0	3	−6	−0.08	−0.01	+4	−15		
−1	5	−6	+0.09	−0.02	+1	+17		
0	4	−6	−0.08	+0.01	−1	−14		
−1	6	−6			+4	+7		
0	5	−6			−4	−6		
−1	7	−6	−0.03	+0.12				
0	6	−6	+0.02	−0.08				

Arg= $\kappa\gamma'+i'g'+ig$			δ^2T'					
			sin.	cos.	$n't$ sin.	$n't$ cos.	n'^2t^2 sin.	n'^2t^2 cos.
κ	i'	i	″	″	″	″	″	″
−1	8	−6	+0.02	+0.11				
0	7	−6	−0.01	−0.08				
−1	9	−6	+0.04	+0.03	−3	−3		
0	8	−6	−0.02	0.00	+2	+6		
−1	10	−6	+0.54	+0.22	+26	−129		
0	9	−6	−0.36	−0.12	−18	+99		
1	8	−6	+0.06	+0.02	+4	−17		
−1	11	−6	+0.28	−0.02	−20	−66		
0	10	−6	−0.22	+0.03	+15	+54		
1	9	−6	+0.03	−0.01	−3	−6		
−1	12	−6	+0.22	+0.09	−15.3	−15.1		
0	11	−6	−0.28	+0.22	+12.3	+12.4		
1	10	−6			−2	−1		
−1	13	−6	−0.054	+0.150	−4.7	−1.3		
0	12	−6	+0.023	−0.060	+4.1	+1.1		
−1	6	−7			−11	+3		
−1	8	−7	−0.09	0.00				
0	7	−7	+0.07	0.00				
−1	9	−7	−0.08	+0.03				
0	8	−7	+0.06	−0.03				
−1	10	−7	+0.05	−0.02	+1	−7		
0	9	−7			−4	+3		
−1	11	−7	−0.04	+0.35	+93	−2		
0	10	−7	+0.03	−0.27	−76	+3		
1	9	−7			+14	0		
−1	12	−7	+0.08	+0.22	+51	−28		
0	11	−7	−0.06	−0.17	−41	+23		
1	10	−7			+5	−3		
−1	13	−7	+0.02	+0.02	+11	−17		
0	12	−7	−0.04	−0.03	−9	+14		
−1	11	−8			+4	0		
0	10	−8			−5	+3		
−1	12	−8			+16	+61		
0	11	−8			−13	−48		
1	10	−8			+2	+7		
−1	13	−8			+29	+33		
0	12	−8			−24	−28		
1	11	−8			+4	+3		
−1	13	−9			−24	+13		
0	12	−9			+20	−11		

In precisely the same manner as $\overline{\mathrm{W}_0'}$ and $\overline{\delta\mathrm{W}_0'}$ have, in preceding chapters, been derived from T' and $\delta\mathrm{T}'$ we now get $\overline{\delta^2\mathrm{W}_0'}$ from $\delta^2\mathrm{T}'$. In the case of the terms depending on the arguments $5g'-2g$ and $10g'-4g$ the motion of the argument has been equated. By adding T', $\delta\mathrm{T}'$, and $\delta^2\mathrm{T}'$ we obtain

$$\begin{array}{rl}
 & \quad\;\;{}^{\prime\prime}\qquad\qquad\quad\;\;{}^{\prime\prime}\qquad\qquad\qquad\quad{}^{\prime\prime} \\
\mathrm{T}'+\delta\mathrm{T}'+\delta^2\mathrm{T}' = & [\quad 1.1238805 + 0.004092190n't - 0.0000019944n'^2t^2] \sin(\;5g'-2g) \\
 & +[\quad 2.7810935 - 0.002383674n't - 0.0000027721n'^2t^2] \cos(\;5g'-2g) \\
 & +[\quad 0.0796974 - 0.00029408n't\; - 0.0000001310n'^2t^2] \sin(10g'-4g) \\
 & +[-0.0812486 - 0.00022188n't\; + 0.0000002400n'^2t^2] \cos(10g'-4g)
\end{array}$$

In the case of the argument $5g'-2g$ we get $\varkappa$ and the corrected integrating factor from

$$\log\varkappa = 7.1938508n \qquad\qquad \log\mu = 1.4977415$$

In the case of the argument $10g'-4g$ the similar quantities are

$$\log\varkappa = 7.50649n \qquad\qquad \log\mu = 1.1972902$$

The expression just written can then be transformed into

$$\begin{array}{rl}
 & \quad\;\;{}^{\prime\prime}\qquad\qquad\quad\;\;{}^{\prime\prime}\qquad\qquad\qquad\quad{}^{\prime\prime} \\
 & [\quad 1.1238805 - 0.000253577n't + 0.0000003582n'^2t^2] \sin(\;5g'-2g+\varkappa n't) \\
 & +[\quad 2.7810935 - 0.000627487n't + 0.0000002271n'^2t^2] \cos(\;5g'-2g+\varkappa n't) \\
 & +[\quad 0.0796974 - 0.00003328n't\; + 0.0000001706n'^2t^2] \sin(10g'-4g+\varkappa n't) \\
 & +[-0.0812486 + 0.00003394n't\; - 0.0000002854n'^2t^2] \cos(10g'-4g+\varkappa n't)
\end{array}$$

Integrating this the result is

$$\begin{array}{rl}
 & \quad\;\;{}^{\prime\prime}\qquad\qquad\quad\;\;{}^{\prime\prime}\qquad\qquad\qquad\quad{}^{\prime\prime} \\
\mathrm{W}_0'+\delta\mathrm{W}_0'+\delta^2\mathrm{W}_0' = & [-35.95457 + 0.00842664n't - 0.000011267n'^2t^2] \cos(\;5g'-2g+\varkappa n't) \\
 & +[\quad 87.22464 - 0.01903107n't + 0.000007143n'^2t^2] \sin(\;5g'-2g+\varkappa n't) \\
 & +[-\;\;1.24551 + 0.0003825n't\; - 0.000002687n'^2t^2] \cos(10g'-4g+\varkappa n't) \\
 & +[-\;\;1.28572 + 0.0006191n't\; - 0.000004495n'^2t^2] \sin(10g'-4g+\varkappa n't) \\
= & [-35.95457 - 0.1278715n't\; + 0.000062367n'^2t^2] \cos(\;5g'-2g) \\
 & +[\quad 87.22464 - 0.0752141n't\; - 0.000086179n'^2t^2] \sin(\;5g'-2g) \\
 & +[-\;\;1.24551 + 0.0045096n't\; + 0.000001743n'^2t^2] \cos(10g'-4g) \\
 & +[-\;\;1.28572 - 0.0033789n't\; + 0.000003357n'^2t^2] \sin(10g'-4g)
\end{array}$$

If from the latter expression we subtract the following (obtained at page 319)

$$\begin{array}{rl}
 & \quad\;\;{}^{\prime\prime}\qquad\qquad\quad\;\;{}^{\prime\prime} \\
\mathrm{W}_0'+\delta\mathrm{W}_0' = & [-36.20315 - 0.1261798n't] \cos(\;5g'-2g) \\
 & +[\quad 87.71763 - 0.0767907n't] \sin(\;5g'-2g) \\
 & +[-\;\;1.02918 - 0.0006600n't] \cos(10g'-4g) \\
 & +[-\;\;1.09555 + 0.0005106n't] \sin(10g'-4g)
\end{array}$$

we get

$$
\begin{aligned}
\delta^2 W_0' = {} & [\quad \overset{''}{0.24858} - \overset{''}{0.0016917}n't + \overset{''}{0.000062367}n'^2t^2] \cos(\ 5g' - 2g) \\
& + [-0.49299 + 0.0015766n't - 0.000086179n'^2t^2] \sin(\ 5g' - 2g) \\
& + [-0.21633 + 0.0051696n't + 0.000001743n'^2t^2] \cos(10g' - 4g) \\
& + [-0.19017 - 0.0038895n't + 0.000003357n'^2t^2] \sin(10g' - 4g)
\end{aligned}
$$

The value of $\overline{\delta^2 W_0'}$ follows:

Arg=$i'g'+ig$	$\overline{\delta^2 W_0'}$					
	cos.	sin.	$n't$ cos.	$n't$ sin.	n'^2t^2 cos.	n'^2t^2 sin.
i' i	″	″	″	″	″	″
0 0			+ 30.0165		+ 46.8625	
1 0	+0.1045+k_1	+0.0438+k_2	−603.51	+1007.93	−437.40	−627.38
2 0	−0.164+[8.45]k_1	−0.289+[8.45]k_2	+ 37.6	+ 9.7	− 12.9	− 19.2
3 0	−0.001	−0.333	+ 55.5	+ 8.5		
4 0	−0.011	−0.020	− 0.1	− 0.1		
−4 − 1			− 0.2	+ 0.3		
−3 − 1	−0.002	+0.009	+ 0.8	+ 0.4		
−2 − 1	+0.130	+0.031	+ 1.9	− 23.3		
−1 − 1	+0.074	+0.062	+ 8.0	− 13.6	+ 0.2	+ 0.1
0 − 1	+0.021	+0.030	+ 4.6	− 5.6	+ 0.1	+ 1.7
1 − 1	+0.0089	+0.0475	− 2.63	− 0.78	− 2.07	+ 2.28
2 − 1	−0.1444	+0.1034	− 7.34	− 0.34	− 16.73	+ 12.78
3 − 1	+0.2896	−0.4214	+ 65.58	+ 65.16	+ 2.67	+ 0.44
4 − 1	−0.3839	−0.1381	+ 26.76	− 60.81	+ 0.32	+ 0.26
5 − 1	−0.031	−0.033	+ 4.1	− 3.6		
6 − 1	−0.009	−0.024	+ 0.1	− 0.2		
−2 − 2	+0.004	+0.001	+ 0.4	− 0.8		
−1 − 2	+0.025	−0.058	− 11.3	− 3.5		
0 − 2	+0.029	−0.020	− 3.9	− 3.5	0.0	− 0.3
1 − 2	−0.004	−0.008	− 1.4	− 2.3	+ 1.5	0.0
2 − 2	−0.092	−0.044	+ 0.6	+ 0.4	− 2.7	+ 0.4
3 − 2	−0.097	−0.117	+ 10.4	+ 1.1	− 7.4	+ 0.5
4 − 2	+0.1342	+0.8045	+368.07	− 44.88	− 31.78	+460.11
5 − 2	+0.26876	−0.49996	− 18.561	+ 13.919	+ 59.597	− 81.901
6 − 2	−1.6194	+1.1965	− 45.27	− 85.03	+ 8.80	− 3.64
7 − 2	−0.013	+0.030	− 1.2	− 2.5		
8 − 2	−0.011	−0.006				
−1 − 3	+0.005	−0.004	− 0.5	− 1.5		
0 − 3	−0.025	−0.015	− 2.7	+ 4.7		
1 − 3	−0.012	−0.025	− 3.1	+ 2.5		
2 − 3	−0.006	−0.006	+ 0.6	+ 0.6	− 0.2	− 0.5
3 − 3	−0.031	+0.044	+ 0.6	+ 0.8	− 0.2	− 0.1
4 − 3	−0.061	+0.030	+ 0.9	− 0.4	− 3.5	+ 0.8
5 − 3	−0.073	+0.007	+ 4.3	− 6.9	− 7.1	− 2.2
6 − 3	−0.3219	−0.1746	+ 55.95	− 56.94	− 8.11	− 8.14

Arg=$i'g'+ig$	$\overline{\delta^2 W_0'}$					
	cos.	sin.	$n't$ cos.	$n't$ sin.	n'^2t^2 cos.	n'^2t^2 sin.
i' i	″	″	″	″	″	″
7— 3	—0. 1628	+0. 3788	— 88. 71	— 39. 45	+ 0. 73	+ 1. 46
8— 3	+0. 0240	—0. 0495	+ 6. 87	+ 6. 24	— 0. 02	— 0. 06
9— 3	+0. 0121	—0. 0127	— 1. 01	— 1. 20		
0— 4			— 0. 2	0. 0		
1— 4	—0. 009	+0. 006	+ 2. 2	+ 2. 1		
2— 4	—0. 007	+0. 001	+ 0. 9	+ 2. 1		
3— 4	+0. 005	+0. 004	— 0. 4	+ 0. 6		
4— 4	+0. 023	+0. 029	+ 0. 1	— 0. 3	+ 0. 2	+ 0. 5
5— 4	+0. 010	+0. 039	+ 0. 8	— 0. 1	+ 0. 5	+ 1. 1
6— 4	—0. 003	—0. 017	— 2. 2	— 2. 0	— 0. 6	+ 1. 7
7— 4	—0. 238	+0. 236	— 38. 3	— 53. 8	— 1. 1	+ 0. 7
8— 4	—0. 199	+0. 088	— 12. 5	— 59. 5	+ 2. 0	— 0. 5
9— 4	+1. 9273	+0. 4733	—163. 35	+390. 03	—20. 53	—11. 42
10— 4	—0. 22537	—0. 20540	+ 48. 531	— 36. 168	+ 1. 557	+ 3. 055
2— 5	+0. 003	+0. 003	+ 0. 6	— 0. 4		
3— 5	0. 000	+0. 002	+ 0. 9	0. 0		
4— 5	+0. 001	+0. 001	+ 0. 3	0. 0		
5— 5	+0. 015	—0. 009	+ 0. 5	+ 0. 5		
6— 5	+0. 019	0. 000	+ 0. 2	— 0. 3		
7— 5	+0. 015	+0. 011	— 2. 2	+ 0. 1		
8— 5	+0. 068	+0. 104	— 26. 0	+ 10. 8		
9— 5	+0. 011	+0. 077	— 21. 0	— 1. 9		
10— 5	—0. 013	+0. 036	— 10. 5	— 7. 3		
11— 5	+0. 0394	—0. 0006	— 6. 68	— 11. 76		
12— 5	—0. 0543	—0. 0678	+ 0. 91	+ 4. 25		
3— 6	+0. 002	—0. 001	— 0. 2	— 0. 2		
4— 6	+0. 002	+0. 001	0. 0	— 0. 4		
5— 6			0. 0	— 0. 2		
6— 6	—0. 002	—0. 006				
7— 6	+0. 002	—0. 006				
8— 6	+0. 007	—0. 006	— 0. 1	+ 0. 3		
9— 6	+0. 060	—0. 028	+ 2. 7	+ 12. 5		
10— 6	+0. 032	+0. 001	— 2. 6	+ 7. 0		
11— 6	0. 000	—0. 011	— 2. 6	+ 2. 2		
12— 6	—0. 020	—0. 058	— 1. 1	+ 0. 3		
5— 7			— 1. 0	— 0. 3		
7— 7	—0. 003	0. 000				
8— 7	—0. 004	—0. 001				
9— 7	+0. 007	+0. 001	0. 0	+ 0. 6		
10— 7	—0. 002	—0. 019	+ 6. 3	0. 0		
11— 7	+0. 006	—0. 014	+ 3. 8	+ 2. 0		
12— 7	—0. 002	+0. 001	+ 0. 8	+ 1. 3		
10— 8			— 0. 1	— 0. 3		
11— 8			+ 0. 7	— 3. 1		
12— 8			+ 1. 5	— 1. 5		
12— 9			— 0. 7	— 0. 3		

Arg=$i'g'+ig$		$\overline{\delta^2 W_0'}$	
		n'^3t^3 cos.	n'^3t^3 sin.
i'	i	"	"
0	0	+0.00000001407	
1	0	+0.0000001850	+0.0000000413
2	0	+0.0000000051	+0.0000000011

Arg=$i'g'+ig$		$-\frac{1}{2}\left(\overline{\frac{d\,.\,\delta^2 W_0'}{d\gamma'}}\right)$					
		sin.	cos.	$n't$ sin.	$n't$ cos.	n'^2t^2 sin.	n'^2t^2 cos.
i'	i	"	"	"	"	"	"
1	0	+0.1646	−0.1232	−297.69	−500.26	−214.75	+310.36
2	0	−0.046	+0.160	+23.3	−14.5	−12.3	+17.6
3	0	+0.005	+0.259	+44.0	−8.0		
4	0	+0.025	+0.014	+0.8	+1.4		
−3	−1	−0.003	+0.005	−0.9	0.0		
−2	−1	−0.122	+0.028	−2.1	−21.2		
−1	−1	−0.068	+0.057	−7.8	−13.8	−0.2	0.0
0	−1	−0.021	+0.046	−7.2	−6.0	+1.8	−3.0
1	−1	+0.0104	+0.0926	+0.32	−1.05	+1.31	+1.57
2	−1	+0.0669	+0.0596	+1.66	−2.14	+8.45	+6.34
3	−1	+0.1711	+0.1366	+19.35	−37.77	+3.65	+0.25
4	−1	+0.1141	+0.0212	−4.37	−24.32	+0.48	−0.30
5	−1	−0.013	+0.020	+1.9	+1.0		
6	−1	−0.007	+0.020	+0.1	+0.2		
−2	−2	−0.004	+0.001	−0.4	−0.8		
−1	−2	−0.029	−0.057	+10.9	−3.5		
0	−2	−0.040	−0.032	+5.8	−5.6		
1	−2	−0.012	−0.003	+1.6	−3.6	−0.7	−0.1
2	−2	+0.063	−0.030	−0.6	−0.3	+1.5	+0.3
3	−2	+0.062	−0.069	−6.9	+1.1	+5.4	+3.9
4	−2	−0.0774	+0.4493	−166.35	−22.48	+15.46	+225.67
5	−2	+0.15093	+0.35737	+59.296	−35.980	+1.534	+2.073
6	−2	−0.7888	−0.5824	−18.14	+34.45	+4.35	+1.79
7	−2	−0.019	−0.033	−1.3	+1.8		
−1	−3	−0.001	−0.002	+0.7	−1.0		
0	−3	+0.028	−0.021	+3.0	+5.1		
1	−3	+0.013	−0.027	+4.1	+2.9		
2	−3	+0.003	+0.013	+1.8	−0.5		
3	−3	+0.031	+0.039	+0.3	0.0	+0.3	0.0
4	−3	+0.052	+0.025	−0.7	+0.5	+3.4	+0.7
5	−3	−0.002	+0.006	+0.4	+10.2	+5.6	−1.7
6	−3	−0.0922	+0.0893	+11.02	+14.70	+4.88	−5.11
7	−3	+0.0023	+0.1481	+36.51	−3.33	−0.06	−0.14
8	−3	+0.0053	+0.0270	+5.90	−2.73	−0.12	−0.20
9	−3	+0.0066	+0.0083	+0.70	−0.49		

Arg=$i'g'+ig$	$-\frac{1}{2}\left(\overline{\frac{d\,.\,\delta W_0'}{d\gamma'}}\right)$					
	sin.	cos.	$n't$ cos.	$n't$ sin.	n'^2t^2 sin.	n'^2t^2 cos.
i' i	″	″	″	″	″	″
1 − 4	+0.017	+0.012	− 2.8	+ 2.8		
2 − 4	+0.016	+0.003	− 1.2	+ 2.6		
3 − 4	+0.001	+0.001	+ 0.2	+ 1.4		
4 − 4	−0.022	+0.027	+ 0.1	+ 0.1	0.0	+0.1
5 − 4	−0.008	+0.036	+ 0.1	+ 0.5	−0.2	+1.5
6 − 4	+0.002	+0.064	+ 4.4	− 1.9	+0.6	+1.4
7 − 4	+0.194	+0.191	+30.0	− 42.7	+0.8	+0.6
8 − 4	+0.146	+0.059	+10.8	− 39.1	−0.9	−0.4
9 − 4	−0.9131	+0.2253	+78.43	+186.46	+9.88	−5.44
10 − 4	+0.01622	−0.01895	+ 2.470	+ 1.736	+0.107	−0.199
2 − 5	−0.004	+0.007	− 1.6	− 1.0		
3 − 5	0.000	+0.007	− 1.9	− 0.2		
4 − 5	+0.001	+0.003	− 0.6	+ 0.2		
5 − 5	−0.019	−0.010	− 0.2	+ 0.2		
6 − 5	−0.021	0.000				
7 − 5	−0.014	+0.011	+ 1.9	+ 0.7		
8 − 5	−0.070	+0.113	+22.9	+ 10.7		
9 − 5	−0.010	+0.073	+19.0	− 1.7		
10 − 5	+0.022	+0.055	+ 8.9	− 6.3		
11 − 5	+0.0300	+0.0323	+ 4.39	− 7.85		
12 − 5	+0.0261	−0.0134	− 0.24	+ 0.75		
3 − 6	−0.004	−0.001	+ 0.2	− 0.7		
4 − 6	−0.004	+0.001	0.0	− 0.9		
6 − 6	+0.002	−0.007				
7 − 6	−0.001	−0.008				
8 − 6	−0.006	−0.003	+ 0.1	+ 0.9		
9 − 6	−0.053	−0.021	− 2.2	+ 12.4		
10 − 6	−0.034	+0.001	+ 2.3	+ 7.9		
11 − 6	−0.003	−0.003	+ 2.3	+ 2.4		
12 − 6	+0.014	−0.039	+ 1.2	+ 0.3		
5 − 7			+ 0.5	− 0.1		
7 − 7	+0.005	0.000				
8 − 7	+0.005	−0.002				
9 − 7	−0.003	−0.001	− 0.4	+ 0.5		
10 − 7	+0.003	−0.028	− 6.8	+ 0.2		
11 − 7	−0.007	−0.020	− 4.4	+ 2.4		
12 − 7	−0.002	−0.002	− 1.2	+ 1.9		
10 − 8			− 0.2	0.0		
11 − 8			− 0.9	− 4.2		
12 − 8			− 1.9	− 2.3		
12 − 9			+ 1.3	− 0.7		

Arg=$i'g'+ig$	$-\frac{1}{2}\left(\frac{d\,.\,\delta^2 W_0'}{d\gamma'}\right)$	
	n'^3t^3 sin.	n'^3t^3 cos.
i' i	″	″
1 0	+0.0000000925	−0.0000000206
2 0	+0.0000000051	−0.0000000011

Similarly, as in the case of Jupiter, we have the equation

$$\frac{d\,.\delta^3 z}{dt}=\overline{\delta^2 W_0'}+\left(\frac{\overline{dW_0'}}{d\gamma'}\right)n'\delta^2 z'+\left(\frac{d\,.\,\delta W_0'}{d\gamma'}\right)n'\delta z'+\frac{1}{2}\left(\frac{d^2 W_0'}{d\gamma'^2}\right)(n'\delta z')^2+2\nu'\delta\nu'+\nu'^2\frac{d\,.\,\delta z'}{dt}$$

All the factors involved in the right member of this equation have already been given. But, as in forming the product $\left(\frac{\overline{dW_0'}}{d\gamma'}\right)n'\delta z'$, in treating the terms of the second order, we have corrected the two factors for the terms multiplied by $n't$, belonging to the arguments g', $2g'$, etc., and which result from the previous computation of $\delta W_0'$, these terms must be omitted from the factors $n'\delta^2 z'$ and $\left(\frac{d\,.\,\delta W_0'}{d\gamma'}\right)$.

The expressions of the five additional products involved in $\frac{d\,.\,\delta^3 z'}{dt}$ follow:

Arg=$i'g'+ig$	$\left(\frac{\overline{dW_0'}}{d\gamma'}\right)n'\delta^2 z'$					
	cos.	sin.	$n't$ cos.	$n't$ sin.	n'^2t^2 cos.	n'^2t^2 sin.
i' i	″	″	″	″	″	″
0 0			+ 1.3876		−4.2695	
1 0	−0.3845	−0.2190	+47.63	−62.77	−0.23	0.00
2 0	+0.013	+0.055	−12.8	+ 1.9	−1.9	− 3.5
3 0	+0.002	+0.131	−23.2	− 2.7	−0.1	− 0.2
4 0	+0.013	+0.008	− 2.0	+ 2.0		
−3 − 1			− 0.3	+ 0.1		
−2 − 1	−0.043	−0.010	− 0.8	+ 8.2		
−1 − 1	−0.019	−0.015	− 3.1	+ 4.6		
0 − 1	−0.003	−0.007	− 1.3	+ 0.9	−2.0	− 3.7
1 − 1	+0.0111	−0.0231	− 0.05	− 0.61	+2.13	− 4.89
2 − 1	+0.0080	+0.0113	+ 3.60	+ 3.51	−2.37	+ 2.78
3 − 1	+0.0139	+0.0689	−10.88	+ 3.47	+6.32	+ 0.60
4 − 1	+0.3762	+0.0809	−24.44	+69.92	+0.84	+ 1.12
5 − 1	+0.010	+0.0006	− 2.4	+ 2.2		
−1 − 2	−0.008	+0.016	+ 3.2	+ 1.0		
0 − 2	−0.009	+0.006	+ 1.7	+ 1.6		
1 − 2	−0.004	+0.001	+ 0.3	+ 0.6	−1.3	+ 0.3
2 − 2	+0.002	+0.001	− 0.6	+ 0.4	+0.3	− 1.1
3 − 2	−0.001	−0.001	−14.9	+ 9.5	−7.9	−23.9

Arg=$i'g'+ig$	$\left(\frac{\overline{dW'_0}}{d\gamma'}\right)n'\delta^2z'$					
	cos.	sin.	$n't$ cos.	$n't$ sin.	n'^2t^2 cos.	n'^2t^2 sin.
i' i	″	″	″	″	″	″
4— 2	+0.0015	+0.0063	—91.48	+15.79	— 3.27	—162.45
5— 2	+0.00594	—0.01510	— 9.967	— 3.022	+ 15.531	— 25.272
6— 2	+0.0381	—0.0296	—52.10	—69.91	+147.14	— 76.41
7— 2	—0.007	0.000	— 3.0	— 3.6	+ 10.6	— 4.4
0— 3	+0.006	+0.005	+ 0.7	— 1.3		
1— 3	+0.002	+0.005	+ 0.8	— 0.5		
2— 3			+ 0.4	— 0.1	0.0	+ 0.5
4— 3			0.0	+ 0.7	+ 0.4	— 0.1
5— 3	+0.044	—0.001	— 2.9	+12.7	+ 0.6	0.0
6— 3	+0.3240	+0.2040	—39.72	+50.65	— 0.12	— 0.09
7— 3	+0.0454	—0.0621	+11.93	+ 9.18	+ 0.09	+ 0.30
8— 3	+0.0009	+0.0028	+ 2.23	+ 3.53	+ 0.08	— 0.38
9— 3	—0.0188	+0.0256	+ 0.27	+ 0.52		
1— 4			— 0.4	— 0.4		
2— 4			— 0.2	— 0.4		
5— 4			0.0	+ 0.4		
6— 4	+0.017	—0.008	+ 1.9	+ 3.9		
7— 4	+0.096	—0.096	+15.9	+21.1		
8— 4	+0.004	—0.017	+ 1.9	— 3.5		
9— 4	—0.4295	—0.1185	+34.62	—80.82	— 0.36	— 0.13
10— 4	—0.00229	—0.00132	+ 3.311	— 2.898	— 0.111	— 0.153
11— 4	—0.0009	+0.0022	+ 7.28	— 1.71	— 0.04	— 0.39
7— 5	—0.004	—0.001	+ 1.0	— 0.6		
8— 5	—0.023	—0.038	+ 8.1	— 3.5		
9— 5	—0.004	—0.023	+ 4.6	+ 0.3		
10— 5	—0.002	+0.002	+ 1.5	+ 1.1		
11— 5	—0.0082	+0.0175	+ 0.42	+ 0.86		
12— 5	+0.0044	+0.0063	+ 0.05	— 0.10		
9— 6	—0.018	+0.006	— 0.6	— 3.2		
10— 6	—0.011	—0.002	+ 0.6	— 1.9		
11— 6	0.000	0.000	+ 0.6	— 0.5		
12— 6	+0.005	+0.012	+ 0.2	— 0.1		

Arg=$i'g'+ig$	$\left(\frac{\overline{dW_0'}}{d\gamma'}\right)n'\delta^2z'$	
	n'^3t^3 cos.	n'^3t^3 sin.
i' i	″	″
1 0	—0.0000000031	—0.0000000068
2 0	+0.000000091	—0.000000035
3 0	+0.000000013	—0.000000002

Arg=$i'g'+ig$	$\frac{1}{2}\left(\overline{\frac{d^2W_0'}{d\gamma'^2}}\right)(n'\delta z')^2$					
	cos.	sin.	$n't$ cos.	$n't$ sin.	n'^2t^2 cos.	n'^2t^2 sin.
i' i	″	″	″	″	″	″
0 0			+0.3610		−0.4998	
1 0	−0.0181	−0.0059	+3.55	−5.86	−0.25	+0.24
2 0	−0.008	−0.001	+2.8	−1.7	−0.1	−0.1
0 — 1	+0.004	+0.012			−0.2	+0.4
1 — 1	−0.0176	+0.0627	+0.36	−0.83	−0.27	+1.19
2 — 1	−0.0162	+0.0207	+0.12	−0.95	−0.08	+0.27
3 — 1	−0.0010	+0.0110	−1.00	−2.53	−0.06	+0.07
4 — 1			−0.29	−1.76	+0.29	+0.28
5 — 1			+1.6	−1.7	+0.3	+0.3
1 — 2	−0.001	+0.003				
2 — 2	−0.010	−0.004	−0.3	+0.1	+0.9	+0.5
3 — 2	+0.002	+0.003	−0.6	+0.3	+1.8	+2.0
4 — 2	+0.0105	+0.0365	−2.54	−0.04	−0.02	+0.50
5 — 2	+0.00426	+0.00003	−0.685	−0.245	+0.041	−0.151
6 — 2	+0.0195	−0.0079	−1.05	−1.17	+0.83	−0.70
7 — 2	−0.003	−0.001	−0.1	−0.1	+2.7	−0.1
8 — 2	−0.004	−0.002				
3 — 3	−0.003	+0.003				
4 — 3	−0.002	+0.001	+0.2	+0.5		
5 — 3	−0.001	+0.001	−0.5	+2.6	−0.4	−0.1
6 — 3	+0.0012	−0.0024	−1.49	+1.00	−0.04	−0.20
7 — 3	+0.0058	−0.0094	−2.12	+1.04	−0.13	−0.47
8 — 3	+0.0019	−0.0188	+0.68	+0.80	−0.13	+0.03
9 — 3	+0.0151	−0.0231	+0.78	+0.67		
6 — 4			−0.4	−0.1		
7 — 4	−0.002	0.000	+0.2	+0.2		
8 — 4	−0.006	−0.005	+0.7	+2.4	−0.7	0.0
9 — 4	−0.0142	−0.0062	−0.66	+2.70	−1.09	−0.43
10 — 4	−0.01892	−0.01948	+0.910	−0.704	+0.113	+0.191
11 — 4	−0.0009	−0.0017	+3.24	−0.78	+0.09	+1.17
10 — 5	−0.006	+0.011	−0.4	−0.3		
11 — 5	−0.0300	+0.0145	−0.39	−1.05	+0.09	−0.02
12 — 5	−0.0055	−0.0001	+0.02	−0.23		
11 — 6	−0.002	−0.002				
12 — 6	−0.002	−0.004				

Arg=$i'g'+ig$	$\frac{1}{2}\left(\overline{\frac{d^2W_0'}{d\gamma'^2}}\right)(n'\delta z')^2$	
	n'^3t^3 cos.	n'^3t^3 sin.
i' i	″	″
1 0	+0.0000000044	−0.0000000068
2 0	+0.000000001	−0.000000001
3 0	+0.000000005	+0.000000001

Arg=$i'g'+ig$		$\left(\frac{\overline{d\,.\,\delta W_0'}}{d\gamma'}\right)n'\delta z'$					
		cos.	sin.	$n't$ cos.	$n't$ sin.	n'^2t^2 cos.	n'^2t^2 sin.
i'	i	″	″	″	″	″	″
0	0			+ 1.2216		— 0.0010	
1	0	—0.1265	—0.1058	—20.70	+48.72	+ 0.12	+ 0.09
2	0	—0.005	—0.002	+ 3.0	— 0.5		
3	0	—0.005	—0.003	+ 2.3	+ 0.2		
4	0	—0.008	+0.006	+ 1.6	+ 1.1		
5	0	—0.009	+0.004				
—4—	1	—0.003	+0.004				
—3—	1	—0.004	+0.003	— 0.2	— 0.6		
—2—	1			+ 0.1	— 0.8		
—1—	1	+0.002	+0.009	+ 1.7	— 2.1	— 0.5	— 0.8
0—	1	—0.008	—0.004	+ 1.9	— 0.7		
1—	1	+0.0132	—0.0396	+ 2.68	+ 0.74	— 1.26	— 1.04
2—	1	+0.0650	—0.0168	— 4.27	+ 1.33	+ 0.18	— 0.31
3—	1	+0.0185	—0.0118	+ 3.16	+ 6.40	+ 5.23	+ 1.67
4—	1	—0.0031	+0.0128	— 0.18	+ 3.04	+ 0.39	+ 0.59
5—	1	+0.009	+0.024	— 1.3	+ 2.6		
6—	1	+0.007	+0.019				
7—	1	—0.003	+0.014				
0—	2			— 0.6	— 1.0		
1—	2	+0.012	—0.001	0.0	— 0.9		
2—	2	+0.074	+0.033	+ 0.1	— 0.4	+ 1.5	— 0.3
3—	2	+0.047	+0.058	— 5.5	+ 5.4	— 8.5	—14.9
4—	2	—0.0119	—0.0841	+ 2.04	+ 0.18	+ 6.39	+23.02
5—	2	+0.00045	—0.01806	— 7.214	— 2.084	+15.183	—24.804
6—	2	+0.0026	—0.0035	+ 0.81	+ 1.38	+20.27	—11.76
7—	2	+0.053	—0.006	0.0	+ 0.2	+ 1.1	— 0.6
8—	2	+0.030	+0.013				
1—	3			— 0.3	+ 0.1		
2—	3	+0.001	—0.003				
3—	3	+0.023	—0.029			+ 0.5	— 0.6
4—	3	+0.032	—0.017	— 0.4	— 0.6	+ 0.6	— 0.2
5—	3	+0.025	—0.011	+ 0.5	— 1.8	+ 1.2	+ 0.3
6—	3	—0.0023	—0.0303	+ 9.93	— 2.19	+ 0.40	+ 0.39
7—	3	—0.0054	+0.0247	+ 6.80	— 1.13	+ 0.14	+ 0.45
8—	3	+0.0045	+0.0523	+ 1.41	+ 0.88	— 0.02	— 0.08
9—	3	—0.0083	+0.0168	+ 0.09	+ 0.07		
4—	4	—0.010	—0.013				
5—	4	—0.004	—0.013				
6—	4	—0.001	—0.009	+ 2.8	+ 0.9	+ 0.2	— 0.3
7—	4	—0.007	0.000	+ 2.2	+ 1.6	+ 0.2	— 0.2
8—	4	—0.047	+0.016	— 0.7	—12.6	— 0.1	0.0
9—	4	—0.1678	—0.0215	+21.32	—50.00	— 0.16	— 0.02
10—	4	+0.03340	+0.03602	+ 3.045	— 2.678	— 0.059	— 0.082
11—	4	+0.0009	+0.0038	— 0.09	— 0.13	+ 0.02	+ 0.16

Arg=$i'g'+ig$	$\left(\frac{\overline{d\,.\,\delta W_0'}}{d\gamma'}\right)n'\delta z'$					
	cos.	sin.	$n't$ cos.	$n't$ sin.	n'^2t^2 cos.	n'^2t^2 sin.
i' i	″	″	″	″	″	″
6— 5	—0.005	0.000				
7— 5	—0.004	—0.001				
8— 5			+0.8	—0.3		
9— 5	—0.006	+0.005	+2.6	+0.3		
10— 5	0.000	—0.009	+1.7	+1.1		
11— 5	+0.0252	—0.0048	+0.51	+1.21		
12— 5	+0.0188	+0.0102	+0.03	—0.02		
10— 6			+0.3	—1.1		
11— 6	+0.010	+0.009	+0.5	—0.5		
12— 6	+0.015	+0.039	+0.3	0.0		
11— 7			—0.3	—0.1		

Arg=$i'g'+ig$	$\left(\frac{\overline{d\,.\,\delta W_0'}}{d\gamma'}\right)n'\delta z'$	
	n'^3t^3 cos.	n'^3t^3 sin.
i' i	″	″
1 0	+0.0000000006	—0.0000000042
2 0	+0.000000092	—0.000000034
3 0	+0.000000005	—0.000000002

Arg=$i'g'+ig$	$2\nu'\delta\nu'$					
	cos.	sin.	$n't$ cos.	$n't$ sin.	n'^2t^2 cos.	n'^2t^2 sin.
i' i	″	″	″	″	″	″
0 0			—0.7446		+2.1365	
1 0	—0.0038	—0.0055	+0.82	—0.28	+1.11	+ 0.77
2 0	—0.002	—0.003	+1.1	—0.2	—1.0	— 1.8
3 0			+0.4	0.0		
0— 1	+0.001	+0.002	+0.6	—0.2	—0.6	— 1.3
1— 1	+0.0026	—0.0022	+0.09	—0.38	—0.05	— 1.08
2— 1	+0.0002	—0.0096	—0.17	—1.23	+0.61	— 1.03
3— 1	—0.0147	+0.0054	—1.38	—2.72	—2.14	— 0.62
4— 1	+0.0037	+0.0007	—0.37	+1.86	+0.23	+ 0.49
5— 1			0.0	—0.1		
2— 2	+0.002	+0.001	—0.2	+0.1	+0.4	— 0.2
3— 2	0.000	+0.001	—4.2	+3.7	—4.1	— 7.8
4— 2	+0.0022	+0.0074	—1.54	+0.28	—0.62	+ 2.79
5— 2	—0.00294	+0.00605	+5.905	+2.043	—7.510	+12.476
6— 2	+0.0002	+0.0007	—0.37	—0.51	+3.28	— 1.30
7— 2					+1.2	— 0.1
5— 3	+0.013	—0.001	—0.7	+3.7		
6— 3	—0.0002	—0.0074	+3.93	—1.16	—0.16	— 0.13
7— 3	—0.0017	—0.0018	—0.26	—0.96	—0.05	— 0.15
8— 3	+0.0002	—0.0009	—0.38	—0.58	—0.02	+ 0.10

Arg=$i'g'+ig$	$2\nu'\delta\nu'$					
	cos.	sin.	$n't$ cos.	$n't$ sin.	n'^2t^2 cos.	n'^2t^2 sin.
i' i	″	″	″	″	″	″
6— 4			+0.5	+0.2		
7— 4			+0.7	+0.6		
8— 4	−0.021	+0.007	−1.0	−7.1		
9— 4	+0.0038	+0.0017	−1.23	+2.43		
10— 4	+0.00038	−0.00017	−1.132	+1.132	+0.052	+0.073
11— 4			+0.39	−0.04		
10— 5	−0.004	+0.006				
11— 5	+0.0052	+0.0018				
12— 5	−0.0008	−0.0011	0.00	−0.04		

Arg=$i'g'+ig$	$2\nu'\delta\nu'$	
	n'^3t^3 cos.	n'^3t^3 sin.
i' i	″	″
1 0	+0.0000000031	−0.0000000024
2 0	+0.000000046	−0.000000017
3 0	+0.000000004	0.000000000

Arg=$i'g'+ig$	$\frac{d\,.\,\delta z'}{dt}\nu'^2$					
	cos.	sin.	$n't$ cos.	$n't$ sin.	n'^2t^2 cos.	n'^2t^2 sin.
i' i	″	″	″	″	″	″
0 0			−0.0777		−0.0073	
1 0			−0.30	+0.47		
1— 1					−0.06	+0.31
2— 1	+0.0004	−0.0001	+0.14	+0.10	+0.18	+0.02
3— 1			+0.11	+0.08	+0.15	−0.05
4— 1			+0.02	−0.08		
2— 2					+0.3	+0.1
4— 2	−0.0001	−0.0012			−0.12	−0.94
5— 2	−0.00054	+0.00053	−0.019	+0.008	−0.076	−0.115
6— 2					−0.38	+0.25
6— 3			−0.06	+0.08		
7— 3	+0.0003	+0.0004	+0.12	+0.09		
9— 4			+0.06	−0.23		
10— 4			−0.037	+0.001	−0.001	−0.005

Arg=$i'g'+ig$	$\frac{d\,.\,\delta z'}{dt}\nu'^2$	
	n'^3t^3 cos.	n'^3t^3 sin.
i' i	″	″
1 0	−0.0000000047	+0.0000000075

The addition of the six terms of $\frac{d\,.\,\delta^3 z'}{dt}$ gives the following expression for this quantity:

Arg=$i'g'+ig$	$\frac{d\,.\,\delta^3 z'}{dt}$					
	cos.	sin.	$n't$ cos.	$n't$ sin.	n'^2t^2 cos.	n'^2t^2 sin.
i' i	″	″	″	″	″	″
0 0			+32.1644		+44.2214	
1 0	$-0.4284+k_1$	$-0.2924+k_2$	−572.51	+988.21	−436.65	−626.28
2 0	$-0.166+[8.44]k_1$	$-0.240+[8.44]k_2$	+31.7	+9.2	−15.9	−24.6
3 0	−0.004	−0.205	+35.0	+6.0	−0.1	−0.2
4 0	−0.006	−0.006	−0.5	+3.0		
5 0	−0.009	+0.004				
−4 −1	−0.003	+0.004	−0.2	+0.3		
−3 −1	−0.006	+0.012	+0.3	−0.1		
−2 −1	+0.087	+0.021	+1.2	−15.9		
−1 −1	+0.057	+0.056	+6.6	−11.1	−0.3	−0.7
0 −1	+0.015	+0.033	+5.8	−5.6	−2.7	−2.9
1 −1	+0.0182	+0.0453	+0.45	−1.86	−1.58	−3.23
2 −1	−0.0870	+0.1089	−7.92	+2.42	−18.21	+14.51
3 −1	+0.3063	−0.3479	+55.59	+69.86	+12.17	+2.11
4 −1	−0.0071	−0.0437	+1.50	+12.17	+2.07	+2.74
5 −1	−0.012	−0.003	+2.0	−0.6	+0.3	+0.3
6 −1	−0.002	−0.005	+0.1	−0.2		
7 −1	−0.003	+0.014				
−2 −2	+0.004	+0.001	+0.4	−0.8		
−1 −2	+0.017	−0.042	−8.1	−2.5		
0 −2	+0.020	−0.014	−2.8	−2.9	0.0	−0.3
1 −2	+0.003	−0.005	−1.1	−2.6	+0.2	+0.3
2 −2	−0.024	−0.013	−0.4	+0.6	+0.7	−0.6
3 −2	−0.049	−0.056	−14.8	+20.0	−26.1	−44.1
4 −2	+0.1364	+0.7694	+274.55	−28.59	−29.42	+323.03
5 −2	+0.27593	−0.52651	−30.541	+10.619	+82.766	−119.767
6 −2	−1.5590	+1.1562	−97.98	−155.24	+179.94	−93.56
7 −2	+0.030	+0.023	−4.3	−6.0	+15.6	−5.2
8 −2	+0.015	+0.005				
−1 −3	+0.005	−0.004	−0.5	−1.5		
0 −3	−0.019	−0.010	−2.0	+3.4		
1 −3	−0.010	−0.020	−2.6	+2.1		
2 −3	−0.005	−0.009	+1.0	+0.5	−0.2	0.0
3 −3	−0.011	+0.018	+0.6	+0.8	+0.3	−0.7
4 −3	−0.031	+0.014	+0.7	+0.2	−2.5	+0.5
5 −3	+0.008	−0.005	+0.7	+10.3	−5.7	−2.0
6 −3	+0.0008	−0.0107	+28.54	−8.56	−8.03	−8.17
7 −3	−0.1184	+0.3306	−72.24	−31.23	+0.78	+1.59
8 −3	+0.0315	−0.0141	+10.81	+10.87	−0.11	−0.39
9 −3	+0.0001	+0.0066	+0.13	+0.06		

Arg=$i'g'+ig$	$\frac{d\,.\,\delta^2 z'}{dt}$					
	cos.	sin.	$n't$ cos.	$n't$ sin.	n'^2t^2 cos.	n'^2t^2 sin.
i' i	″	″	″	″	″	″
0 — 4			− 0.2	0.0		
1 — 4	−0.009	+0.006	+ 1.8	+ 1.7		
2 — 4	−0.007	+0.001	+ 0.7	+ 1.7		
3 — 4	+0.005	+0.004	− 0.4	+ 0.6		
4 — 4	+0.013	+0.016	+ 0.1	− 0.3	+ 0.2	+ 0.5
5 — 4	+0.006	+0.026	+ 0.8	+ 0.3	+ 0.5	+ 1.1
6 — 4	+0.013	−0.034	+ 2.6	+ 2.9	− 0.4	+ 1.4
7 — 4	−0.151	+0.140	− 19.3	− 30.3	− 0.9	+ 0.5
8 — 4	−0.269	+0.089	− 11.6	− 80.3	+ 1.2	− 0.5
9 — 4	+1.3196	+0.3288	−109.24	+264.11	−22.14	−12.00
10 — 4	−0.21280	−0.19035	+ 54.628	− 41.315	+ 1.551	+ 3.079
11 — 4	−0.0009	+0.0043	+ 10.82	− 2.66	+ 0.07	+ 0.94
2 — 5	+0.003	+0.003	+ 0.6	− 0.4		
3 — 5	0.000	+0.002	+ 0.9	0.0		
4 — 5	+0.001	+0.001	+ 0.3	0.0		
5 — 5	+0.015	−0.009	+ 0.5	+ 0.5		
6 — 5	+0.014	0.000	+ 0.2	− 0.3		
7 — 5	+0.007	+0.009	− 1.2	− 0.5		
8 — 5	+0.045	+0.066	− 17.1	+ 7.0		
9 — 5	+0.001	+0.059	− 13.8	− 1.3		
10 — 5	−0.025	+0.046	− 7.7	− 5.4		
11 — 5	+0.0316	+0.0284	− 6.14	− 10.74	+ 0.09	− 0.02
12 — 5	−0.0374	−0.0525	+ 1.01	+ 3.86		
3 — 6	+0.002	−0.001	− 0.2	− 0.2		
4 — 6	+0.002	+0.001	0.0	− 0.4		
5 — 6			0.0	− 0.2		
6 — 6	−0.002	−0.006				
7 — 6	+0.002	−0.006				
8 — 6	+0.007	−0.006	− 0.1	+ 0.3		
9 — 6	+0.042	−0.022	+ 2.1	+ 9.3		
10 — 6	+0.021	−0.001	− 1.7	+ 4.0		
11 — 6	−0.008	−0.004	− 1.5	+ 1.2		
12 — 6	−0.002	−0.011	− 0.6	+ 0.2		
5 — 7			− 1.0	− 0.3		
7 — 7	−0.003	0.000				
8 — 7	−0.004	−0.001				
9 — 7	+0.007	+0.001	0.0	+ 0.6		
10 — 7	−0.002	−0.019	+ 6.3	0.0		
11 — 7	+0.006	−0.014	+ 3.5	+ 1.9		
12 — 7	−0.002	+0.001	+ 0.8	+ 1.3		
10 — 8			− 0.1	− 0.3		
11 — 8			+ 0.7	− 3.1		
12 — 8			+ 1.5	− 1.5		
12 — 9			− 0.7	− 0.3		

Arg$=i'g'+ig$		$\frac{d\,.\,\delta^3 z'}{dt}$	
		n'^3t^3 cos.	n'^3t^3 sin.
i'	i	$''$	$''$
1	0	+0.0000001853	+0.0000000286
2	0	+0.0000002351	—0.0000000859
3	0	+0.000000027	—0.000000003

On integrating this expression we arrive at $n'\delta^3 z'$. In order to make the terms depending on the argument g' disappear it is necessary to put $k_1=+0''.3287$ and $k_2=+0''.2339$. In the case of the terms involving the arguments $5g'-2g$ and $10g'-4g$ we equate the motions of the latter. By adding the values which have been obtained for $\frac{d\,.\,\delta z'}{dt}$, $\frac{d\,.\,\delta^2 z'}{dt}$, and $\frac{d\,.\,\delta^3 z'}{dt}$ we get, taking the liberty of calling the sum $\frac{d\,.\,\delta z'}{dt}$,

$$\begin{aligned}\frac{d\,.\,\delta z'}{dt} = \; & [-\overset{''}{35.06817}-\overset{''}{0.1504365}n't+\overset{''}{0.000082766}n'^2t^2]\cos(\;5g'-2g)\\ &+[\;84.57304-0.0853558n't-0.000119767n'^2t^2]\sin(\;5g'-2g)\\ &+[-\;1.17743+0.0047807n't+0.000001551n'^2t^2]\cos(10g'-4g)\\ &+[-\;1.22444-0.0035990n't+0.000003079n'^2t^2]\sin(10g'-4g)\end{aligned}$$

For this expression may be substituted

$$\begin{aligned}\frac{d\,.\,\delta z'}{dt} = \; & [-\overset{''}{35.06817}+\overset{''}{0.0081298}n't-\overset{''}{0.000015631}n'^2t^2]\cos(\;5g'-2g+\varkappa n't)\\ &+[\;84.57304-0.0196064n't+0.000013638n'^2t^2]\sin(\;5g'-2g+\varkappa n't)\\ &+[-\;1.17743+0.0004987n't-0.000003835n'^2t^2]\cos(10g'-4g+\varkappa n't)\\ &+[-\;1.22444+0.0005186n't-0.000006152n'^2t^2]\sin(10g'-4g+\varkappa n't)\end{aligned}$$

where, in the case of $5g'-2g$, $\varkappa$ and the corrected integrating factor are determined by

$$\log\varkappa=7.2729790n \qquad \log\mu=1.5020293$$

and, in the case of $10g'-4g$, by

$$\log\varkappa=7.54371n \qquad \log\mu=1.1992594$$

Integrating the last expression we obtain

$$\begin{aligned}n'\delta z' = \; & [-\overset{''}{1132.9347}+\overset{''}{0.285824}n't-\overset{''}{0.00049661}n'^2t^2]\sin(\;5g'-2g+\varkappa n't)\\ &+[-2677.8791+0.591358n't-0.00043329n'^2t^2]\cos(\;5g'-2g+\varkappa n't)\\ &+[-\;18.4689+0.004811n't-0.00006067n'^2t^2]\sin(10g'-4g+\varkappa n't)\\ &+[\;19.4491-0.010126n't+0.00009734n'^2t^2]\cos(10g'-4g+\varkappa n't)\end{aligned}$$

By developing this expression and subtracting therefrom the value of $n'\delta z'+n'\delta^2 z'$ we should have $n'\delta^3 z'$; but as this quantity has no particular interest we omit its derivation, and in the following value of $n'\delta^3 z'$ the terms corresponding to the arguments $5g'-2g$ and $10g'-4g$ are not given.

In writing a final form for the great inequality of Saturn, we prefer to still further equate the motions of the arguments, so that the sum of the squares of the multipliers of $n't$ in the coefficients may be a minimum. This gives, severally, in the cases of $5g'-2g$ and $10g'-4g$

$$\log \varkappa = 7.2703565n \qquad\qquad \log \varkappa = 7.52728n$$

and the final form of $n'\delta z'$ will be

$$\begin{array}{rllr}
 & \overset{\prime\prime}{} \qquad\qquad \overset{\prime\prime}{} & & \overset{\circ}{} \quad \overset{\prime}{} \quad \overset{\prime\prime}{} \\
n'\delta z' = & [2907.676 - 0.655990n't] & \sin(5g'-2g+\varkappa n't + & 247 \quad 4 \quad 5.37) \\
+ & 0.00065602n'^2t^2 & \sin(5g'-2g+\varkappa n't + & 221 \quad 41 \quad 33 \quad) \\
+ & [26.8211 - 0.010659n't] & \sin(10g'-4g+\varkappa n't + & 133 \quad 31 \quad 9.3 \;) \\
+ & 0.00011466n'^2t^2 & \sin(10g'-4g+\varkappa n't + & 122 \quad 38 \qquad)
\end{array}$$

The great inequality excepted, the expression of $n'\delta^3z'$ follows. The proper number of decimals is restored to the factors of $n't$ and n'^2t^2:

Arg=$i'g'+ig$		$n'\delta^3z'$	
		sin.	cos.
i'	i	″ ″ ″	″ ″ ″
0	0		$+0.0016082\,2n'^2t^2$ $+0.0000147405n'^3t^3$
1	0	$-0.058505n't-0.00043656n'^2t^2$ $+0.0000001853n'^3t^3$	$-0.099694n't+0.00062684n'^2t^2$ $-0.0000000286n'^3t^3$
2	0	$-0.0784+0.001595n't-0.00000800n'^2t^2$ $+0.0000001175n'^3t^3$	$+0.1177-0.000470n't-0.00001250n'^2t^2$ $+0.0000000429n'^3t^3$
3	0	$-0.0012+0.001167n't-0.00000003n'^2t^2$ $+0.0000000090n'^3t^3$	$+0.0686-0.000200n't+0.00000007n'^2t^2$ $+0.0000000010n'^3t^3$
4	0	$-0.0015-0.000012n't$	$+0.0015-0.000075n't$
5	0	-0.0018	-0.0008
−4	1	$+0.0005+0.000003n't$	$+0.0006+0.000005n't$
−3	1	$+0.0011-0.000005n't$	$+0.0022-0.000002n't$
−2	1	$-0.0194-0.000027n't$	$+0.0047-0.000355n't$
−1	1	$-0.0164-0.000189n't+0.00000009n'^2t^2$	$+0.0161-0.000319n't-0.00000020n'^2t^2$
0	1	$-0.0060-0.000234n't+0.00000109n'^2t^2$	$+0.0133-0.000226n't-0.00000117n'^2t^2$
1	1	$-0.0123-0.000033n't+0.00000107n'^2t^2$	$+0.0305-0.000127n't-0.00000218n'^2t^2$
2	1	$+0.1806+0.001763n't+0.00003768n'^2t^2$	$+0.2216+0.000346n't+0.00003003n'^2t^2$
3	1	$+0.6187+0.010777n't+0.00002356n'^2t^2$	$+0.6942-0.013431n't-0.00000408n'^2t^2$
4	1	$-0.0042+0.000102n't+0.00000136n'^2t^2$	$+0.0289-0.000800n't-0.00000181n'^2t^2$
5	1	$-0.0048+0.000079n't+0.00000012n'^2t^2$	$+0.0012+0.000024n't-0.00000012n'^2t^2$
6	1	$-0.0006+0.000003n't$	$+0.0014+0.000006n't$
7	1	-0.0007	-0.0031
−2	2	$-0.0006-0.000006n't$	$+0.0001-0.000011n't$
−1	2	$-0.0028+0.000136n't$	$-0.0070-0.000042n't$
0	2	$-0.0040+0.000056n't+0.00000000n'^2t^2$	$-0.0028-0.000058n't-0.00000006n'^2t^2$
1	2	$-0.0008+0.000028n't-0.00000005n'^2t^2$	$-0.0013-0.000066n't+0.00000008n'^2t^2$
2	2	$+0.0081+0.000013n't-0.00000024n'^2t^2$	$-0.0044+0.000020n't-0.00000020n'^2t^2$

Arg=$i'g'+ig$	$n'\delta^3z'$	
	sin.	cos.
i' i	$''$ $''$ $''$	$''$ $''$ $''$
3— 2	$+0.0254+0.000732n't+0.00001327n'^2t^2$	$-0.0290+0.001002n't-0.00002242n'^2t^2$
4— 2	$-0.1442-0.027711n't+0.00003043n'^2t^2$	$+0.8245-0.003021n't+0.00033417n'^2t^2$
6— 2	$-1.5236-0.009658n't+0.00017413n'^2t^2$	$-1.1283+0.015360n't+0.00009056n'^2t^2$
7— 2	$+0.0148-0.000216n't+0.00000767n'^2t^2$	$-0.0113+0.000305n't+0.00000256n'^2t^2$
8— 2	$+0.0049$	-0.0017
—1— 3	$-0.0006+0.000006n't$	$-0.0005-0.000018n't$
0— 3	$+0.0026+0.000027n't$	$-0.0013+0.000046n't$
1— 3	$+0.0015+0.000040n't$	$-0.0031+0.000033n't$
2— 3	$+0.0009-0.000018n't+0.00000004n'^2t^2$	$-0.0016+0.000009n't+0.00000000n'^2t^2$
3— 3	$+0.0025-0.000013n't-0.00000007n'^2t^2$	$+0.0040+0.000018n't-0.00000016n'^2t^2$
4— 3	$+0.0090-0.000020n't+0.00000072n'^2t^2$	$+0.0041+0.000006n't+0.00000014n'^2t^2$
5— 3	$-0.0032-0.000029n't+0.00000233n'^2t^2$	$-0.0020+0.000420n't-0.00000082n'^2t^2$
6— 3	$-0.0010-0.001975n't+0.00000554n'^2t^2$	$-0.0060-0.000598n't-0.00000563n'^2t^2$
7— 3	$+0.2477+0.016069n't-0.00000173n'^2t^2$	$+0.6988-0.006934n't+0.00000353n'^2t^2$
8— 3	$+0.0609+0.001963n't-0.00000020n'^2t^2$	$+0.0293-0.001976n't+0.00000071n'^2t^2$
9— 3	$+0.0001+0.000008n't$	$-0.0043-0.000004n't$
0— 4	$-0.000002n't$	$0.000000n't$
1— 4	$+0.0010-0.000020n't$	$+0.0007+0.000019n't$
2— 4	$+0.0009-0.000009n't$	$+0.0001+0.000021n't$
3— 4	$-0.0007+0.000006n't$	$+0.0006+0.000009n't$
4— 4	$-0.0022-0.000002n't-0.00000003n'^2t^2$	$+0.0027-0.000005n't+0.00000009n'^2t^2$
5— 4	$-0.0012-0.000016n't-0.00000010n'^2t^2$	$+0.0053+0.000006n't+0.00000022n'^2t^2$
6— 4	$-0.0033-0.000066n't+0.00000010n'^2t^2$	$-0.0086+0.000074n't+0.00000036n'^2t^2$
7— 4	$+0.0511+0.000658n't+0.00000030n'^2t^2$	$+0.0474-0.001033n't+0.00000017n'^2t^2$
8— 4	$+0.1371+0.000600n't-0.00000062n'^2t^2$	$+0.0455-0.004154n't-0.00000026n'^2t^2$
9— 4	$-1.3837+0.011677n't+0.00002373n'^2t^2$	$+0.3398+0.028248n't-0.00001286n'^2t^2$
11— 4	$-0.0010+0.001016n't+0.00000007n'^2t^2$	$-0.0031+0.000249n't-0.00000088n'^2t^2$
2— 5	$-0.0003-0.000006n't$	$+0.0003-0.000004n't$
3— 5	$0.0000-0.000010n't$	$+0.0002+0.000000n't$
4— 5	$-0.0001-0.000004n't$	$+0.0001+0.000000n't$
5— 5	$-0.0020-0.000007n't$	$-0.0012+0.000007n't$
6— 5	$-0.0022-0.000003n't$	$0.0000-0.000005n't$
7— 5	$-0.0013+0.000022n't$	$+0.0017-0.000009n't$
8— 5	$-0.0102+0.000387n't$	$+0.0149+0.000158n't$
9— 5	$-0.0003+0.000404n't$	$+0.0173-0.000038n't$
10— 5	$+0.0103+0.000319n't$	$+0.0190-0.000223n't$
11— 5	$-0.0229+0.000433n't-0.00000006n'^2t^2$	$+0.0198-0.000758n't-0.00000001n'^2t^2$
12— 5	$+0.0919-0.000242n't$	$-0.1255+0.000927n't$
3— 6	$-0.0002+0.000002n't$	$-0.0001-0.000002n't$
4— 6	$-0.0002+0.000000n't$	$+0.0001-0.000004n't$
5— 6	$0.000000n't$	$-0.000002n't$
6— 6	$+0.0002$	-0.0007
7— 6	-0.0003	-0.0008
8— 6	$-0.0010+0.000001n't$	$-0.0009+0.000004n't$
9— 6	$-0.0071-0.000036n't$	$-0.0037+0.000158n't$

Arg$=i'g'+ig$	$n'\delta^2 z'$	
	sin.	cos.
i' i	″ ″ ″	″ ″ ″
10— 6	$-0.0043+0.000035n't$	$-0.0002+0.000082n't$
11— 6	$+0.0020+0.000038n't$	$-0.0010+0.000031n't$
12— 6	$+0.0007+0.000021n't$	$-0.0038+0.000007n't$
5— 7	$+0.000008n't$	$-0.000002n't$
7— 7	$+0.0003$	0.0000
8— 7	$+0.0004$	-0.0001
9— 7	$-0.0009+0.000000n't$	$+0.0001+0.000007n't$
10— 7	$+0.0003-0.000085n't$	$-0.0026+0.000000n't$
11— 7	$-0.0009-0.000055n't$	$-0.0022+0.000030n't$
12— 7	$+0.0004-0.000015n't$	$+0.0002+0.000024n't$
10— 8	$+0.000001n't$	$-0.000003n't$
11— 8	$-0.000008n't$	$-0.000035n't$
12— 8	$-0.000019n't$	$-0.000019n't$
12— 9	$+0.000007n't$	$-0.000003n't$

As in the case of Jupiter, for the general value of $\delta^2\nu'$, we employ the formula

$$\delta^2\nu' = -\frac{1}{2}\int^t\left(\frac{\overline{d\,.\,\delta^2 W_0'}}{d\gamma'}\right)n'dt$$

But in the terms which involve the argument $5g'-2g$ it has been discovered that the complementary portion of the right member has a sensible value. Consequently, for this argument we make use of the complete formula

$$\frac{d\,.\,\delta^2\nu'}{n'dt} = -\frac{1}{2}\left(\frac{\overline{d\,.\,\delta^2 W_0'}}{d\gamma'}\right) - \frac{1}{2}\left(\frac{\overline{d^2 W_0'}}{d\gamma'^2}\right)n'\delta^2 z' - \frac{1}{2}\left(\frac{\overline{d^2\,.\,\delta W_0'}}{d\gamma'^2}\right)n'\delta z' - \frac{1}{4}\left(\frac{\overline{d^3 W_0'}}{d\gamma'^3}\right)(n'\delta^2 z')^2$$

The expressions for the factors involved in the right member have all been given, except those for $-\frac{1}{2}\left(\frac{\overline{d^2\delta W_0'}}{d\gamma'^2}\right)$ and $-\frac{1}{4}\left(\frac{\overline{d^3 W_0'}}{d\gamma'^3}\right)$. To a sufficient number of terms for our purpose the latter are:

Arg$=i'g'+ig$	$-\frac{1}{2}\left(\frac{\overline{d^2\delta W_0'}}{d\gamma'^2}\right)$					
	cos.	sin.	$n't$ cos.	$n't$ sin.	n'^2t^2 cos.	n'^2t^2 sin.
i' i	″	″	″	″	″	″
0 0	−0.4057		− 8.52			
1 0	+0.1443	+0.2489	− 73.09	+91.21	−1268.72	−976.23
2 0	−1.6441	−2.8744	− 11.09	+ 2.00	− 142.12	−109.38
3 0	−0.1175	−2.5142	− 0.90	− 1.19	− 13.41	− 10.35
−2— 1	+1.2195	+0.3504	+ 0.97	− 0.48		
−1— 1	+1.0988	+0.9397	+ 12.29	− 4.89		
0— 1	+0.9288	+1.0075	+151.54	−54.69		
1— 1	+0.4011	+0.4819	+ 33.83	+23.51		

Arg$=i'g'+ig$	$-\frac{1}{2}\left(\overline{\frac{d^2\delta W_0'}{d\gamma'^2}}\right)$					
	cos.	sin.	$n't$ cos.	$n't$ sin.	n'^2t^2 cos.	n'^2t^2 sin.
i' i	″	″	″	″	″	″
2— 1	+0.1176	− 0.1709	+ 229.69	+306.74		
3— 1	+2.9733	− 1.5995	+ 8.70	+ 61.11		
4— 1	+3.5992	+ 0.3508	− 4.62	+ 5.87		
1— 2	+0.5144	− 0.2865	− 35.67	− 33.89		
2— 2	+0.1096	− 0.2139	+ 4.26	+ 26.58		
3— 2	−0.5676	− 1.1824	+ 121.88	+181.27		
4— 2	−3.4317	−13.7965	+3887.57	− 13.04		
5— 2	−0.1017	+ 0.0055	+ 24.30	+ 12.76		
6— 2	−0.1658	− 0.0129	− 3.57	+ 13.95		
6— 3	+2.4988	+ 1.6449	− 51.87	+ 60.34		
7— 3	+0.1794	+ 1.5819	+ 12.32	− 4.38		
8— 3	−0.0064	+ 0.1474	+ 0.59	− 0.08		
7— 4	−1.9564	+ 1.8079	+ 12.64	+ 16.79		
8— 4	−1.2025	+ 0.6377	+ 2.38	+ 6.90		
9— 4	+5.7067	+ 1.6780	+ 9.68	− 23.94		
10— 4	+0.0459	+ 0.0441	+ 0.18	− 0.16		

Arg$=i'g'+ig$	$-\frac{1}{4}\left(\overline{\frac{d^2W_0'}{d\gamma'^2}}\right)$	
	sin.	cos.
i' i	″	″
1 0	− 18.06	− 5.29
	+16531 $n't$	+26479 $n't$
2 0	− 2.85	− 0.83
	+ 3703 $n't$	+ 5933 $n't$
1— 1	− 18.77	− 128.95
2— 1	+ 21.54	− 35.05
3— 1	+ 1.32	− 8.01
1— 2	+ 8.23	− 8.32
2— 2	+ 43.78	− 19.21
3— 2	+ 12.44	+ 12.83
4— 2	− 18.70	− 152.23
5— 2	+ 0.42	+ 0.83

The multiplication being performed we get

$$-\frac{1}{2}\left(\frac{\overline{d^2W_0'}}{d\gamma'^2}\right)n'\delta^2z' = [-0^{\prime\prime}.14019 - 62^{\prime\prime}.646n't + 4^{\prime\prime}.472n'^2t^2]\sin(5g'-2g)$$
$$+[-0.33710 + 36.383n't + 6.296n'^2t^2]\cos(5g'-2g)$$

$$-\frac{1}{2}\left(\frac{\overline{d^2\delta W_0'}}{d\gamma'^2}\right)n'\delta z' = [-0.00524 + 3.983n't - 5.024n'^2t^2]\sin(5g'-2g)$$
$$+[-0.00824 - 0.989n't - 7.943n'^2t^2]\cos(5g'-2g)$$

$$-\frac{1}{4}\left(\frac{\overline{d^3W_0'}}{d\gamma'^3}\right)(n'\delta z')^2 = [-0.00277 - 0.189n't + 1.040n'^2t^2]\sin(5g'-2g)$$
$$+[-0.00569 - 0.075n't + 2.563n'^2t^2]\cos(5g'-2g)$$

If we add these three quantities to the corresponding terms of $-\frac{1}{2}\left(\frac{\overline{d\,.\,\delta^2W_0'}}{d\gamma'}\right)$ and $\frac{d\,.\,\delta v'}{n'dt}$ and $\frac{dv'}{n'dt}$ we get

$$\frac{dv'}{n'dt} = [-0^{\prime\prime}.76561 - 30^{\prime\prime}.664n't + 2^{\prime\prime}.022n'^2t^2]\sin(5g'-2g)$$
$$+[-1.51056 + 19.032n't + 2.989n'^2t^2]\cos(5g'-2g)$$

This expression may be changed to

$$\frac{dv'}{n'dt} = [-0^{\prime\prime}.76561 + 0^{\prime\prime}.0001408n't - 0^{\prime\prime}.000000293n'^2t^2]\sin(5g'-2g+\varkappa n't)$$
$$+[-1.51056 + 0.0002776n't - 0.000000117n'^2t^2]\cos(5g'-2g+\varkappa n't)$$

where the proper number of decimals has been restored to the coefficients of $n't$ and n'^2t^2. The value of $\varkappa$ and the corrected integrating factor are given by the equations

$$\log \varkappa = 7.32699 \qquad \log \mu = 1.5054687$$

On integrating the last expression we obtain

$$v' = [\ \ 24^{\prime\prime}.7830 - 0^{\prime\prime}.004750n't + 0^{\prime\prime}.00000938n'^2t^2]\cos(5g'-2g+\varkappa n't)$$
$$+[-48.2213 + 0.008288n't - 0.00000375n'^2t^2]\sin(5g'-2g+\varkappa n't)$$

In order to put this in a final form we equate the argument still further, so that the sum of the squares of the coefficients of $n't$ may be a minimum. This makes $\log \varkappa = 7.32535$, and we have the following expression

$$v' = [54^{\prime\prime}.2171 - 0^{\prime\prime}.009543n't]\quad\cos(5g'-2g+\varkappa n't + 62^\circ\ 47'\ 58^{\prime\prime}.0)$$
$$+\,0.00001005n'^2t^2\cos(5g'-2g+\varkappa n't + 22\ \ \ 9\qquad)$$

Omitting, then, the terms corresponding to the argument $5g'-2g$ the expression for $\delta^2\nu'$ follows. The proper number of decimals are restored to the coefficients of $n't$ and n'^2t^2:

Arg=$i'g'+ig$		$\delta^2\nu'$	
		cos.	sin.
i'	i	″ ″ ″	″ ″ ″
0	0	$+0.000821n't+0.00000613n'^2t^2$ $-0.0000000026n'^3t^3$	
1	0	$+0.2151+0.030390n't+0.00021469n'^2t^2$ $-0.0000000925n'^3t^3$	$-0.1536-0.050455n't+0.00031064n'^2t^2$ $-0.0000000206n'^3t^3$
2	0	$+0.0226-0.001156n't+0.00000615n'^2t^2$ $-0.0000000025n'^3t^3$	$+0.0806-0.000731n't+0.00000880n'^2t^2$ $-0.0000000005n'^3t^3$
3	0	$-0.0018-0.001467n't$	$+0.0868-0.000267n't$
4	0	$-0.0063-0.000020n't$	$+0.0035-0.000035n't$
−3	−1	$-0.0005-0.000016n't$	$-0.0009+0.000000n't$
−2	−1	$-0.0272-0.000047n't$	$-0.0062+0.000473n't$
−1	−1	$-0.0195-0.000224n't-0.00000006n'^2t^2$	$-0.0164+0.000396n't+0.00000000n'^2t^2$
0	−1	$-0.0085-0.000290n't+0.00000072n'^2t^2$	$-0.0185+0.000242n't+0.00000121n'^2t^2$
1	−1	$+0.0069+0.000023n't+0.00000088n'^2t^2$	$-0.0624+0.000072n't-0.00000106n'^2t^2$
2	−1	$+0.1374+0.000397n't+0.00001749n'^2t^2$	$-0.1225+0.000515n't-0.00001312n'^2t^2$
3	−1	$-0.3453-0.003744n't-0.00000706n'^2t^2$	$+0.2715-0.007283n't+0.00000048n'^2t^2$
4	−1	$-0.0763+0.000288n't-0.00000032n'^2t^2$	$+0.0138-0.001603n't-0.00000020n'^2t^2$
5	−1	$+0.0052-0.000076n't$	$+0.0079+0.000040n't$
6	−1	$+0.0020-0.000003n't$	$+0.0057+0.000006n't$
−2	−2	$-0.0006-0.000006n't$	$-0.0001+0.000011n't$
−1	−2	$-0.0049+0.000183n't$	$+0.0096+0.000059n't$
0	−2	$-0.0081+0.000117n't$	$+0.0064+0.000113n't$
1	−2	$-0.0030+0.000040n't-0.00000018n'^2t^2$	$+0.0008+0.000091n't+0.00000003n'^2t^2$
2	−2	$+0.0212-0.000020n't+0.00000051n'^2t^2$	$+0.0101+0.000010n't-0.00000010n'^2t^2$
3	−2	$+0.0315-0.000351n't+0.00000275n'^2t^2$	$+0.0351-0.000051n't-0.00000198n'^2t^2$
4	−2	$-0.0825-0.016726n't+0.00001599n'^2t^2$	$-0.4821+0.002358n't-0.00023345n'^2t^2$
6	−2	$+0.7667+0.001759n't-0.00000421n'^2t^2$	$-0.5655+0.003342n't+0.00000173n'^2t^2$
7	−2	$+0.0093+0.000064n't$	$-0.0162+0.000089n't$
−1	−3	$-0.0001+0.000008n't$	$+0.0002+0.000012n't$
0	−3	$+0.0038+0.000040n't$	$+0.0028-0.000068n't$
1	−3	$+0.0020+0.000064n't$	$+0.0042-0.000045n't$
2	−3	$+0.0005+0.000033n't$	$-0.0024+0.000009n't$
3	−3	$+0.0070+0.000007n't+0.00000007n'^2t^2$	$-0.0088+0.000000n't+0.00000000n'^2t^2$
4	−3	$+0.0151-0.000020n't+0.00000099n'^2t^2$	$-0.0072-0.000014n't-0.00000020n'^2t^2$
5	−3	$-0.0008+0.000016n't+0.00000229n'^2t^2$	$-0.0025-0.000416n't+0.00000069n'^2t^2$
6	−3	$-0.0629+0.000755n't+0.00000337n'^2t^2$	$-0.0610-0.001009n't+0.00000352n'^2t^2$
7	−3	$+0.0036+0.008112n't-0.00000013n'^2t^2$	$-0.3111+0.000740n't+0.00000031n'^2t^2$
8	−3	$-0.0105-0.001074n't+0.00000022n'^2t^2$	$+0.0511-0.000496n't-0.00000036n'^2t^2$
9	−3	$-0.0043-0.000045n't$	$+0.0054-0.000032n't$
1	−4	$+0.0019-0.000031n't$	$-0.0013-0.000031n't$
2	−4	$+0.0020-0.000015n't$	$-0.0004-0.000033n't$
3	−4	$+0.0001+0.000003n't$	$-0.0001-0.000020n't$

Arg $=i'g'+ig$	$\delta^2\nu'$	
	cos.	sin.
i' i	$''$ $''$ $''$	$''$ $''$ $''$
4— 4	$-0.0037+0.000002n't+0.00000000n'^2t^2$	$-0.0046-0.000002n't-0.00000002n'^2t^2$
5— 4	$-0.0016+0.000002n't-0.00000004n'^2t^2$	$-0.0073-0.000010n't-0.00000030n'^2t^2$
6— 4	$+0.0005+0.000112n't+0.00000015n'^2t^2$	$-0.0163+0.000048n't-0.00000036n'^2t^2$
7— 4	$+0.0658+0.001023n't+0.00000027n'^2t^2$	$-0.0648+0.001455n't-0.00000020n'^2t^2$
8— 4	$+0.0745+0.000559n't-0.00000046n'^2t^2$	$-0.0300+0.002022n't+0.00000020n'^2t^2$
9— 4	$-0.9570+0.008390n't+0.00001059n'^2t^2$	$-0.2324-0.019956n't+0.00000583n'^2t^2$
10— 4	$-0.2034-0.003793n't-0.00000160n'^2t^2$	$-0.2273+0.002651n't-0.00000298n'^2t^2$
2— 5	$-0.0004-0.000015n't$	$-0.0007+0.000010n't$
3— 5	$0.0000-0.000020n't$	$-0.0008+0.000002n't$
4— 5	$+0.0001-0.000007n't$	$-0.0004-0.000002n't$
5— 5	$-0.0026-0.000003n't$	$+0.0013-0.000003n't$
6— 5	-0.0033	0.0000
7— 5	$-0.0026+0.000035n't$	$-0.0020-0.000013n't$
8— 5	$-0.0158+0.000518n't$	$-0.0254-0.000242n't$
9— 5	$-0.0030+0.000556n't$	$-0.0211+0.000050n't$
10— 5	$+0.0091+0.000368n't$	$-0.0228+0.000261n't$
11— 5	$+0.0207+0.000310n't$	$-0.0226+0.000554n't$
12— 5	$+0.0631-0.000058n't$	$+0.0319-0.000180n't$
3— 6	$-0.0003+0.000002n't$	$+0.0001+0.000006n't$
4— 6	$-0.0004+0.000000n't$	$-0.0001+0.000008n't$
6— 6	$+0.0002$	$+0.0008$
7— 6	-0.0001	$+0.0010$
8— 6	$-0.0009+0.000001n't$	$+0.0004-0.000013n't$
9— 6	$-0.0090-0.000037n't$	$+0.0036-0.000210n't$
10— 6	$-0.0069+0.000047n't$	$-0.0002-0.000161n't$
11— 6	$-0.0008+0.000059n't$	$+0.0008-0.000062n't$
12— 6	$+0.0048+0.000041n't$	$+0.0134-0.000010n't$
5— 7	$+0.000004n't$	$+0.000001n't$
7— 7	$+0.0005$	0.0000
8— 7	$+0.0005$	$+0.0002$
9— 7	$-0.0004-0.000005n't$	$+0.0001-0.000006n't$
10— 7	$+0.0004-0.000092n't$	$+0.0038-0.000003n't$
11— 7	$-0.0011-0.000069n't$	$+0.0031-0.000038n't$
12— 7	$-0.0004-0.000022n't$	$+0.0004-0.000035n't$
10— 8	$-0.000002n't$	$0.000000n't$
11— 8	$-0.000010n't$	$+0.000047n't$
12— 8	$-0.000024n't$	$+0.000029n't$
12— 9	$-0.000013n't$	$+0.000007n't$

CHAPTER XXI.

PERTURBATIONS OF SATURN OF THE SECOND ORDER FROM THE ACTION OF URANUS AND FACTORED BY $n't$.

Having now pushed the approximation to the perturbations of the longitudes and radii-vectors of Jupiter and Saturn, so far as these arise from their mutual action, to a sufficient length, it remains to consider certain terms of the second order, with respect to disturbing forces, which involve the mass of Uranus as a factor.

In the first place the perturbations of Saturn, due to the action of Uranus, which have been determined in Chapter III, are of the first order with respect to the disturbing force. The elements of both planets have been regarded as constant. In this chapter, then, we determine the additional terms which arise in the perturbations from attributing to these elements their augmentations proportional to the time. Here it will be sufficiently accurate to neglect the variations of the elements which determine the position of the planes of the orbits relative to each other. Also the effect on the latitude of Saturn may be neglected.

The more important part of the terms we wish to derive arises from the variation of the function T′ (denoted as T at page 130; but here we propose to give one accent to quantities belonging to Saturn and two to those belonging to Uranus). To find the variation of T′ we therefore employ the abbreviated formula

$$\delta \mathrm{T}' = \frac{d\mathrm{T}'}{dg'} n'\delta z' + \frac{d\mathrm{T}'}{dg''} n''\delta z'' + r'\frac{d\mathrm{T}'}{dr'}\nu' + r''\frac{d\mathrm{T}'}{dr''}\nu'' + \mathrm{C}'\delta\frac{h'}{h_0'}$$

in which the latter factors of the five terms may be limited to their secular terms. By joining together the terms of one, two, and three dimensions, obtained in the preceding chapters, we have for Saturn

$$\begin{aligned} n'\delta z' = & -[0.82487]n't \sin g' - [1.02971]n't \cos g' \\ & -[8.97104]n't \sin 2g' - [9.17599]n't \cos 2g' \\ & -[7.4183\ ]n't \sin 3g' - [7.6233\ ]n't \cos 3g' \end{aligned}$$

$$\begin{aligned} \nu' = & \ \ [8.97195]n't \\ & +[0.52384]n't \cos g' - [0.72868]n't \sin g' \\ & +[8.97104]n't \cos 2g' - [9.17599]n't \sin 2g' \\ & +[7.5944\ ]n't \cos 3g' - [7.7994\ ]n't \sin 3g' \end{aligned}$$

$$\delta\frac{h'}{h_0'} = -[9.2730\ ]n't$$

The corresponding quantities for Uranus must be derived from the theory of this planet. Availing ourselves of Professor NEWCOMB's determinations,[*] the unit of t being a Julian year, we have

$$\frac{de''}{dt} = -\,0''.05420 \qquad\qquad \frac{d\pi''}{dt} = +\,2''.846$$

These data give

$$\begin{aligned}
n''\delta z'' = &-[9.70665]n't \sin\ g'' - [0.09803]n't \cos\ g'' \\
&-[7.7756\ ]n't \sin 2g'' - [8.1671\ ]n't \cos 2g'' \\
&-[6.1456\ ]n't \sin 3g'' - [6.5370\ ]n't \cos 3g''
\end{aligned}$$

$$\begin{aligned}
\nu'' = &\ [7.7764\ ]n't \\
&+[9.40562]n't \cos\ g'' - [9.79700]n't \sin\ g'' \\
&+[7.7756\ ]n't \cos 2g'' - [8.1671\ ]n't \sin 2g'' \\
&+[6.3217\ ]n't \cos 3g'' - [6.7131\ ]n't \sin 3g''
\end{aligned}$$

The expressions for the factors $\frac{dT'}{dg'}$ and $\frac{dT'}{dg''}$ are readily derived by partial differentiation of the value of T′, given in Chapter III (pages 130–133). It only remains to show how $r'\frac{dT'}{dr'}$ and $r''\frac{dT'}{dr''}$ are obtained. We have the formulæ

$$r'\frac{dT'}{dr'} = V' + X' \qquad\qquad r''\frac{dT'}{dr''} = -V' - T'$$

where

$$\begin{aligned}
V' &= A'\frac{d}{dg'}\left(a'r'\frac{d\Omega'}{dr'}\right) + B'r'\frac{d}{dr'}\left(a'r'\frac{d\Omega'}{dr'}\right) \\
X' &= M'a'\frac{d\Omega'}{dg'} + N'ar'\frac{d\Omega'}{dr'}
\end{aligned}$$

The values of the factors involved in the right members of these equations have all been given, excepting that of $\left(r'\frac{d}{dr'}\right)^2 a'\Omega'$, A′ and B′ at page 74, M′ and N′ at page 212, $a'\frac{d\Omega'}{dg'}$ and $a'r'\frac{d\Omega'}{dr'}$ at pages 128–130. For the single remaining factor we have the equation

$$\left(r'\frac{d}{dr'}\right)^2 a'\Omega' = \frac{3}{4}\mu\left(\frac{\mathrm{a}''}{\triangle}\right)^5\left[\frac{r''^2}{\mathrm{a}''^2} - \alpha^2\frac{r'^2}{\mathrm{a}'^2}\right]^2 - \mu\left(\frac{\mathrm{a}''}{\triangle}\right)^3\left(\frac{r''}{\mathrm{a}''}\right)^2 + \frac{1}{4}\mu\frac{\mathrm{a}''}{\triangle} - (\mathrm{H})$$

The expressions for $\frac{\mathrm{a}''}{\triangle}$, $\left(\frac{\mathrm{a}''}{\triangle}\right)^3$, $\left(\frac{\mathrm{a}''}{\triangle}\right)^5$, $\frac{r''^2}{\mathrm{a}''^2}$, $\alpha^2\frac{r'^2}{\mathrm{a}'^2}$, and (H) have been given in Chapter III. We also readily get

$$\begin{aligned}
\frac{3}{4}\left[\frac{r''^2}{\mathrm{a}''^2} - \alpha^2\frac{r'^2}{\mathrm{a}'^2}\right]^2 = [9.6346] &- 2[8.7253] \cos\ g'' \qquad + 2[7.0120] \cos 2g'' \\
&+ 2[8.1956] \cos\ g' \qquad + 2[6.5608] \cos 2g' \\
&- 2[6.9890] \cos (g'' - g') - 2[6.9890] \cos (g'' + g')
\end{aligned}$$

---

[*] An Investigation of the Orbit of Uranus, pp. 80, 81. The only alteration made is the putting the mass of Neptune at $\frac{1}{19700}$ instead of $\frac{1}{17000}$.

Making the single multiplication required and availing ourselves of all the data afforded in Chapter III we obtain the following expression:

Arg= $i'g''+ig'$	$\frac{1}{\mu}\left(r'\frac{d}{dr'}\right)^2 a'\Omega'$		Arg= $i'g''+ig'$	$\frac{1}{\mu}\left(r'\frac{d}{dr'}\right)^2 a'\Omega'$		Arg= $i'g''+ig'$	$\frac{1}{\mu}\left(r'\frac{d}{dr'}\right)^2 a'\Omega'$	
	cos.	sin.		cos.	sin.		cos.	sin.
i' i			i' i			i' i		
0 0	0.4714		3− 4	−0.0597	+0.0193	6− 7	+0.0095	−0.0450
0− 1	−0.0793	−0.0495	3− 5	−0.0037	+0.0005	6− 8	+0.0025	−0.0041
0− 2	0.0000	+0.0016	4− 1	−0.0001	+0.0057	7− 3	+0.0012	−0.0010
1+ 1	−0.0067	−0.0022	4− 2	+0.0374	−0.0422	7− 4	−0.0123	−0.0014
1 0	+0.0725	+0.1167	4− 3	−0.3184	+0.0194	7− 5	+0.0302	+0.0503
1− 1	+0.1355	−0.6836	4− 4	+0.4737	+0.5111	7− 6	+0.0661	−0.1414
1− 2	−0.0124	+0.0391	4− 5	+0.0112	+0.0667	7− 7	−0.1677	+0.0119
1− 3	−0.0001	+0.0007	4− 6	−0.0004	+0.0050	7− 8	−0.0301	−0.0133
2+ 1	+0.0001	−0.0007	5− 2	+0.0081	+0.0002	8− 4	−0.0013	−0.0017
2 0	−0.0099	+0.0117	5− 3	−0.0464	−0.0466	8− 5	−0.0029	+0.0119
2− 1	+0.2466	−0.0170	5− 4	−0.0233	+0.2743	8− 6	+0.0447	−0.0183
2− 2	−1.1123	−0.4730	5− 5	+0.3947	−0.2305	8− 7	−0.0866	−0.0627
2− 3	−0.0105	−0.0325	5− 6	+0.0592	+0.0007	8− 8	−0.0161	+0.0945
2− 4	−0.0006	−0.0014	5− 7	+0.0048	+0.0016	8− 9	−0.0133	+0.0182
3 0	−0.0022	−0.0004	6− 2	+0.0006	+0.0009	9− 5	−0.0018	+0.0012
3− 1	+0.0290	+0.0287	6− 3	+0.0006	−0.0106	9− 6	+0.0111	+0.0044
3− 2	−0.0549	−0.3160	6− 4	−0.0513	+0.0416	9− 7	−0.0074	−0.0387
3− 3	−0.5614	+0.8016	6− 5	+0.2083	+0.0538	9− 8	−0.0579	+0.0487
			6− 6	−0.0844	−0.2700	9− 9	+0.0523	+0.0295
						9−10	+0.0187	+0.0055

Thence we obtain the expression for V′:

Arg= $\kappa\gamma'+i'g''+ig'$	V′		Arg= $\kappa\gamma'+i'g''+ig'$	V′	
	sin.	cos.		sin.	cos.
κ i' i	″	″	κ i' i	″	″
1 0− 1	−4.568	−0.006	−1 1 0	+1.020	+5.125
−1 0 0	−0.489	+0.353	0 1− 1	−0.522	−2.706
0 0− 1	+0.346	−0.167	1 1− 2	−0.325	−1.500
1 0− 2	+0.028	−0.131	−1 1− 1	−0.014	−0.428
−1 0− 1	+0.003	−0.010	0 1− 2	−0.077	+0.306
0 0− 2	+0.003	+0.001	1 1− 3	+0.099	−0.081
1 0− 3	+0.001	+0.002	−1 1− 2	−0.003	−0.002
−1 1+ 2	−0.005	−0.005	0 1− 3	−0.004	+0.012
0 1+ 1	−0.025	+0.006	1 1− 4	+0.005	−0.007
1 1 0	+0.040	+0.001	−1 2+ 2	0.000	0.000
−1 1+ 1	+0.377	−0.452	0 2+ 1	0.000	+0.001
0 1 0	0.000	0.000	1 2 0	+0.001	−0.002
1 1− 1	−0.396	+0.350	−1 2+ 1	−0.008	−0.053

Arg=$\varkappa\gamma'+i'g''+ig'$			V′	
			sin.	cos.
$\varkappa$	i'	i	″	″
0	2	0	0.000	0.000
1	2	—1	—0.044	+0.059
—1	2	0	+1.768	+0.131
0	2	—1	—1.247	+0.085
1	2	—2	+0.370	—0.456
—1	2	—1	—13.875	+5.936
0	2	—2	+12.592	—5.402
1	2	—3	—2.968	+1.255
—1	2	—2	+0.049	+0.430
0	2	—3	+0.504	—0.686
1	2	—4	—0.251	+0.282
—1	2	—3	—0.001	+0.011
0	2	—4	+0.031	—0.048
1	2	—5	—0.016	+0.024
—1	3	+1	—0.006	—0.001
0	3	0	0.000	0.000
1	3	—1	0.000	+0.008
—1	3	0	+0.225	—0.163
0	3	—1	—0.154	+0.130
1	3	—2	+0.014	—0.120
—1	3	—1	—1.060	+3.187
0	3	—2	+0.966	—2.761
1	3	—3	+0.029	+0.877
—1	3	—2	—7.465	—10.661
0	3	—3	+7.097	+9.956
1	3	—4	—1.938	—2.719
—1	3	—3	—0.739	—0.087
0	3	—4	+1.048	+0.522
1	3	—5	—0.392	—0.239
—1	3	—4	—0.036	+0.002
0	3	—5	+0.088	+0.026
1	3	—6	—0.040	—0.012
—1	4	0	+0.009	—0.033
0	4	—1	—0.006	+0.022
1	4	—2	—0.010	—0.011
—1	4	—1	+0.238	+0.456
0	4	—2	—0.192	—0.393
1	4	—3	+0.149	+0.092
—1	4	—2	—3.721	—0.695
0	4	—3	+3.417	+0.678
1	4	—4	—1.079	+0.027
—1	4	—3	+6.514	—7.007
0	4	—4	—6.107	+6.766
1	4	—5	+1.757	—1.961
—1	4	—4	+0.039	—0.830
0	4	—5	—0.306	+1.122
1	4	—6	+0.144	—0.410
—1	4	—5	—0.011	—0.052
0	4	—6	—0.004	+0.107
1	4	—7	+0.003	—0.045
—1	5	—1	+0.063	+0.018
0	5	—2	—0.051	—0.019
1	5	—3	+0.025	—0.009
—1	5	—2	—0.589	+0.383
0	5	—3	+0.540	—0.334
1	5	—4	—0.136	+0.187
—1	5	—3	+0.061	—3.482
0	5	—4	—0.077	+3.282
1	5	—5	—0.138	—1.025
—1	5	—4	+5.517	+3.248
0	5	—5	—5.356	—3.001
1	5	—6	+1.597	+0.880
—1	5	—5	+0.747	—0.072
0	5	—6	—0.977	—0.059
1	5	—7	+0.354	+0.041
—1	5	—6	+0.053	—0.025
0	5	—7	—0.102	+0.021
1	5	—8	+0.044	—0.006
—1	6	—1	+0.007	—0.003
0	6	—2	—0.006	+0.002
1	6	—3	+0.002	—0.002
—1	6	—2	—0.024	+0.100
0	6	—3	+0.025	—0.089
1	6	—4	+0.008	+0.041
—1	6	—3	—0.501	—0.581
0	6	—4	+0.456	+0.548
1	6	—5	—0.207	—0.135
—1	6	—4	+2.785	—0.467
0	6	—5	—2.658	+0.444
1	6	—6	+0.822	—0.243
—1	6	—5	—1.220	+3.831
0	6	—6	+1.070	—3.719
1	6	—7	—0.311	+1.126
—1	6	—6	+0.160	+0.576
0	6	—7	—0.113	—0.735
1	6	—8	+0.029	+0.264
—1	6	—7	+0.033	+0.046
0	6	—8	—0.038	—0.081
1	6	—9	+0.013	+0.034
—1	7	—2	+0.006	+0.012
0	7	—3	—0.005	—0.011
1	7	—4	+0.005	+0.003

Arg=$\varkappa\gamma'+i'g''+ig'$	V'		Arg=$\varkappa\gamma'+i'g''+ig'$	V'	
	sin.	cos.		sin.	cos.
$\varkappa$ i' i	″	″	$\varkappa$ i' i	″	″
−1 7−3	−0.132	−0.017	1 8−8	−0.357	+0.280
0 7−4	+0.119	+0.019	−1 8−7	−0.221	−1.375
1 7−5	−0.046	+0.011	0 8−8	+0.273	+1.318
−1 7−4	+0.465	−0.554	1 8−9	−0.091	−0.403
0 7−5	−0.447	+0.518	−1 8−8	−0.186	−0.236
1 7−6	+0.108	−0.207	0 8−9	+0.198	+0.293
−1 7−5	+0.743	+1.964	1 8−10	−0.068	−0.106
0 7−6	−0.721	−1.886			
1 7−7	+0.291	+0.567	−1 9−4	−0.015	−0.018
−1 7−6	−2.405	−0.188	0 9−5	+0.014	+0.017
0 7−7	+2.323	+0.098	1 9−6	−0.008	−0.003
1 7−8	−0.706	−0.019	−1 9−5	+0.138	−0.024
−1 7−7	−0.389	+0.186	0 9−6	−0.127	+0.019
0 7−8	+0.491	−0.191	1 9−7	+0.039	−0.018
1 7−9	−0.180	+0.063	−1 9−6	−0.158	+0.471
−1 8−3	−0.016	+0.011	0 9−7	+0.157	−0.441
0 8−4	+0.015	−0.009	1 9−8	−0.031	+0.144
1 8−5	−0.004	+0.006	−1 9−7	−0.711	−0.707
−1 8−4	−0.002	−0.140	0 9−8	+0.669	+0.668
0 8−5	−0.002	+0.132	1 9−9	−0.203	−0.191
1 8−6	−0.015	−0.045	−1 9−8	+0.736	−0.348
−1 8−5	+0.535	+0.307	0 9−9	−0.679	+0.336
0 8−6	−0.507	−0.299	1 9−10	+0.190	−0.070
1 8−7	+0.188	+0.066	−1 9−9	+0.169	−0.125
−1 8−6	−1.241	+0.783	0 9−10	−0.155	+0.165
0 8−7	+1.191	−0.767	1 9−11	+0.011	−0.082

In the next place we have the expression for X':

Arg=$\varkappa\gamma'+i'g''+ig'$	X'		Arg=$\varkappa\gamma'+i'g''+ig'$	X'	
	sin.	cos.		sin.	cos.
$\varkappa$ i' i	″	″	$\varkappa$ i' i	″	″
0 0 0		+0.006	−1 1+1	−0.127	+0.084
1 0−1	+1.664	−0.002	0 1 0	+0.020	+0.098
−1 0 0	+0.158	−0.071	1 1−1	+0.120	−0.120
0 0−1	−0.141	0.000	−1 1 0	−0.225	−1.164
1 0−2	+0.022	+0.041	0 1−1	−0.008	+0.001
−1 0−1	−0.002	+0.001	1 1−2	+0.133	+0.625
0 0−2	−0.002	−0.004	−1 1−1	−0.037	+0.107
1 0−3	+0.001	+0.003	0 1−2	−0.012	−0.053
−1 1+2	−0.002	+0.004	1 1−3	−0.006	+0.026
0 1+1	+0.011	−0.007	0 1−3	0.000	−0.002
1 1 0	−0.011	+0.003	1 1−4	−0.001	+0.002

$\text{Arg}=\varkappa\gamma'+i'g''+ig'$			X'	
			sin.	cos.
$\varkappa$	i'	i	″	″
—1	2+	1	—0.005	+0.010
0	2	0	+0.035	+0.001
1	2—	1	0.000	—0.014
—1	2	0	—0.417	—0.013
0	2—	1	—0.334	+0.144
1	2—	2	+0.088	+0.069
—1	2—	1	+3.965	—1.709
0	2—	2	—0.009	+0.003
1	2—	3	—0.244	+0.111
—1	2—	2	+0.005	—0.109
0	2—	3	+0.019	—0.009
1	2—	4	—0.017	+0.005
—1	2—	3	+0.001	—0.003
0	2—	4	+0.002	+0.001
1	2—	5	—0.001	—0.001
—1	3+	1	0.000	—0.001
0	3	0	+0.005	—0.002
1	3—	1	—0.001	0.000
—1	3	0	—0.056	+0.029
0	3—	1	—0.036	+0.058
1	3—	2	+0.012	—0.001
—1	3—	1	+0.416	—0.685
0	3—	2	—0.127	—0.179
1	3—	3	—0.032	+0.059
—1	3—	2	+1.512	+2.139
0	3—	3	—0.008	—0.009
1	3—	4	—0.071	—0.092
—1	3—	3	+0.131	+0.027
0	3—	4	+0.004	+0.007
1	3—	5	—0.007	—0.008
—1	3—	4	+0.006	0.000
0	3—	5	0.000	+0.001
1	3—	6	—0.001	—0.001
—1	4	0	—0.005	+0.005
0	4—	1	0.000	+0.009
—1	4—	1	—0.010	—0.101
0	4—	2	—0.054	—0.019
1	4—	3	+0.002	+0.010
—1	4—	2	+0.640	+0.233
0	4—	3	+0.083	—0.091
1	4—	4	—0.037	—0.010
—1	4—	3	—1.002	+1.094
0	4—	4	+0.005	—0.008
1	4—	5	+0.034	—0.039
—1	4—	4	—0.013	+0.117
0	4—	5	—0.003	+0.002
1	4—	6	+0.003	—0.006
—1	4—	5	+0.002	+0.008
0	4—	6	0.000	+0.001
—1	5—	1	—0.008	—0.008
0	5—	2	—0.009	+0.002
1	5—	3	+0.002	0.000
—1	5—	2	+0.108	—0.031
0	5—	3	+0.007	—0.042
1	5—	4	—0.007	+0.002
—1	5—	3	—0.071	+0.494
0	5—	4	+0.060	+0.033
1	5—	5	+0.002	—0.020
—1	5—	4	—0.700	—0.402
0	5—	5	+0.007	+0.001
1	5—	6	+0.020	+0.011
—1	5—	5	—0.087	+0.004
0	5—	6	—0.001	—0.002
1	5—	7	+0.003	0.000
—1	5—	6	—0.007	+0.003
0	5—	7	—0.001	0.000
1	5—	8	0.000	+0.001
—1	6—	2	+0.009	—0.012
0	6—	3	—0.004	—0.008
1	6—	4	—0.001	0.000
—1	6—	3	+0.046	+0.089
0	6—	4	+0.029	—0.003
1	6—	5	—0.002	—0.005
—1	6—	4	—0.335	+0.021
0	6—	5	—0.010	+0.034
1	6—	6	+0.012	—0.002
—1	6—	5	+0.125	—0.410
0	6—	6	0.000	+0.005
1	6—	7	—0.001	+0.009
—1	6—	6	—0.013	—0.058
0	6—	7	0.000	—0.001
1	6—	8	0.000	+0.001
—1	6—	7	—0.004	—0.005
—1	7—	2	0.000	—0.002
0	7—	3	—0.002	0.000
—1	7—	3	+0.014	+0.008
0	7—	4	+0.006	—0.005
1	7—	5	0.000	—0.001
—1	7—	4	—0.064	+0.050
0	7—	5	+0.005	+0.017
1	7—	6	+0.002	—0.003

$\mathrm{Arg}=\varkappa\gamma'+i'g''+ig'$			X'		$\mathrm{Arg}=\varkappa\gamma'+i'g''+ig'$			X'	
			sin.	cos.				sin.	cos.
$\varkappa$	i'	i	″	″	$\varkappa$	i'	i	″	″
−1	7	−5	−0.058	−0.205	0	8	−8	−0.002	−0.001
0	7	−6	−0.018	0.000	1	8	−9	0.000	−0.002
1	7	−7	+0.002	+0.006	−1	8	−8	+0.014	+0.019
−1	7	−6	+0.222	+0.014	0	8	−9	0.000	+0.001
0	7	−7	−0.002	+0.001	1	8	−10	0.000	−0.001
1	7	−8	−0.006	0.000					
−1	7	−7	+0.034	−0.016	−1	9	−4	+0.001	+0.002
					1	9	−6	−0.001	0.000
−1	8	−3	+0.002	−0.001	−1	9	−5	−0.013	0.000
0	8	−4	0.000	−0.002	0	9	−6	−0.002	+0.003
−1	8	−4	−0.005	+0.014	−1	9	−6	+0.019	−0.036
0	8	−5	+0.004	+0.003	0	9	−7	−0.005	−0.004
−1	8	−5	−0.045	−0.039	1	9	−8	0.000	+0.001
0	8	−6	−0.010	+0.006	−1	9	−7	+0.050	+0.057
1	8	−7	+0.001	+0.002	0	9	−8	+0.004	−0.003
−1	8	−6	+0.114	−0.062	1	9	−9	−0.001	−0.001
0	8	−7	−0.001	−0.008	−1	9	−8	−0.051	+0.023
1	8	−8	−0.003	+0.002	−1	9	−9	−0.009	+0.010
−1	8	−7	+0.019	+0.112	1	9	−11	−0.001	0.000

The foregoing developments enable us to derive the expressions for the factors $r'\frac{dT'}{dr'}$ and $r''\frac{dT'}{dr''}$:

$\mathrm{Arg}=\varkappa\gamma'+i'g''+ig'$			$r'\frac{dT'}{dr'}$		$r''\frac{dT'}{dr''}$	
			sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″
0	0	0		+0.006		
1	0	−1	−2.904	−0.008	+6.232	+0.008
−1	0	0	−0.331	+0.282	+0.644	−0.438
0	0	−1	+0.205	−0.167	−0.476	+0.211
1	0	−2	+0.050	−0.090	−0.010	+0.157
−1	0	−1	+0.001	−0.009	−0.004	+0.010
0	0	−2	+0.001	−0.003	−0.005	0.000
1	0	−3	+0.002	+0.005	0.000	−0.002
−1	1	+2	−0.007	−0.001	+0.004	+0.007
0	1	+1	−0.014	−0.001	+0.034	−0.008
1	1	0	+0.029	+0.004	−0.051	−0.002
−1	1	+1	+0.250	−0.368	−0.499	+0.561
0	1	0	+0.020	+0.098		
1	1	−1	−0.276	+0.230	+0.524	−0.429

Arg$=\varkappa y'+i'g''+ig'$			$r'\frac{dT'}{dr'}$		$r''\frac{dT'}{dr''}$	
			sin.	cos.	sin.	cos.
$\varkappa$	i'	i	$''$	$''$	$''$	$''$
−1	1	0	+0.795	+3.959	−1.297	−6.555
0	1−	1	−0.530	−2.705	+0.669	+3.504
1	1−	2	−0.192	−0.875	+0.408	+1.860
−1	1−	1	−0.051	−0.321	−0.053	+0.560
0	1−	2	−0.089	+0.253	+0.160	−0.409
1	1−	3	+0.093	−0.055	−0.138	+0.116
−1	1−	2	−0.003	−0.002	+0.003	+0.004
0	1−	3	−0.004	+0.010	+0.009	−0.018
1	1−	4	+0.004	−0.005	−0.008	+0.010
0	2+	1	0.000	+0.001	0.000	−0.001
1	2	0	+0.001	−0.002	−0.002	+0.002
−1	2+	1	−0.013	−0.043	+0.010	+0.064
0	2	0	+0.035	+0.001		
1	2−	1	−0.044	+0.045	+0.065	−0.075
−1	2	0	+1.351	+0.118	−2.415	−0.087
0	2−	1	−1.581	+0.229	+1.759	−0.179
1	2−	2	+0.458	−0.387	−0.614	+0.628
−1	2−	1	−9.910	+4.227	+19.692	−8.444
0	2−	2	+12.583	−5.399	−18.131	+7.795
1	2−	3	−3.212	+1.366	+4.555	−1.935
−1	2−	2	+0.054	+0.321	+0.017	−0.620
0	2−	3	+0.523	−0.695	−0.819	+0.986
1	2−	4	−0.268	+0.287	+0.398	−0.402
−1	2−	3	0.000	+0.008	+0.004	−0.019
0	2−	4	+0.033	−0.047	−0.051	+0.071
1	2−	5	−0.017	+0.023	+0.027	−0.036
−1	3+	1	−0.006	−0.002	+0.007	+0.001
0	3	0	+0.005	−0.002		
1	3−	1	−0.001	+0.008	+0.002	−0.011
−1	3	0	+0.169	−0.134	−0.310	+0.219
0	3−	1	−0.190	+0.188	+0.222	−0.178
1	3−	2	+0.026	−0.121	−0.039	+0.166
−1	3−	1	−0.644	+2.502	+1.644	−4.220
0	3−	2	+0.839	−2.940	−1.532	+3.704
1	3−	3	−0.003	+0.936	+0.064	−1.219
−1	3−	2	−5.953	−8.522	+9.703	+13.813
0	3−	3	+7.089	+9.947	−9.291	−12.965
1	3−	4	−2.009	−2.811	+2.596	+3.614
−1	3−	3	−0.608	−0.060	+0.958	+0.158
0	3−	4	+1.052	+0.529	−1.366	−0.727
1	3−	5	−0.399	−0.247	+0.513	+0.329
−1	3−	4	−0.030	+0.002	+0.048	0.000
0	3−	5	+0.088	+0.027	−0.116	−0.039
1	3−	6	−0.041	−0.013	+0.053	+0.019

Arg$=\varkappa\gamma'+i'g''+ig'$			$r'\frac{dT'}{dr'}$		$r''\frac{dT'}{dr''}$	
			sin.	cos.	sin.	cos.
$\varkappa$	i'	i	$''$	$''$	$''$	$''$
−1	4	0	+0.004	−0.028	−0.014	+0.043
0	4	−1	−0.006	+0.031	+0.011	−0.030
1	4	−2	−0.010	−0.011	+0.010	+0.017
−1	4	−1	+0.228	+0.355	−0.261	−0.608
0	4	−2	−0.246	−0.412	+0.206	+0.532
1	4	−3	+0.151	+0.102	−1.180	−0.137
−1	4	−2	−3.081	−0.462	+4.679	+1.020
0	4	−3	+3.500	+0.587	−4.324	−0.998
1	4	−4	−1.116	+0.017	+1.385	+0.021
−1	4	−3	+5.512	−5.913	−7.991	+8.631
0	4	−4	−6.102	+6.758	+7.506	−8.365
1	4	−5	+1.791	−2.000	−2.179	+2.451
−1	4	−4	+0.026	−0.713	−0.072	+1.021
0	4	−5	−0.309	+1.124	+0.401	−1.384
1	4	−6	+0.147	−0.416	−0.186	+0.508
−1	4	−5	−0.009	−0.044	+0.012	+0.065
0	4	−6	−0.004	+0.108	+0.008	−0.133
1	4	−7	+0.003	−0.045	−0.006	+0.056
−1	5	−1	+0.055	+0.010	−0.075	−0.029
0	5	−2	−0.060	−0.017	+0.061	+0.030
1	5	−3	+0.027	−0.009	−0.032	+0.008
−1	5	−2	−0.481	+0.352	+0.749	−0.435
0	5	−3	+0.547	−0.376	−0.691	+0.378
1	5	−4	−0.143	+0.189	+0.182	−0.220
−1	5	−3	−0.010	−2.988	−0.155	+4.219
0	5	−4	−0.017	+3.315	+0.171	−3.990
1	5	−5	−0.136	−1.045	+0.139	+1.255
−1	5	−4	+4.817	+2.846	−6.557	−3.839
0	5	−5	−5.349	−3.000	+6.380	+3.547
1	5	−6	+1.617	+0.891	−1.913	−1.044
−1	5	−5	+0.660	−0.068	−0.888	+0.073
0	5	−6	−0.978	−0.061	+1.163	+0.082
1	5	−7	+0.357	+0.041	−0.423	−0.053
−1	5	−6	+0.046	−0.022	−0.064	+0.028
0	5	−7	−0.103	+0.021	+0.122	−0.023
1	5	−8	+0.044	−0.005	−0.053	+0.006
−1	6	−1	+0.007	−0.003	−0.009	+0.003
0	6	−2	−0.006	+0.002	+0.007	−0.002
1	6	−3	+0.002	−0.002	−0.003	+0.003
−1	6	−2	−0.015	+0.088	+0.037	−0.119
0	6	−3	+0.021	−0.097	−0.038	+0.106
1	6	−4	+0.007	+0.041	−0.007	−0.049
−1	6	−3	−0.455	−0.492	+0.574	+0.713
0	6	−4	+0.485	+0.545	−0.523	−0.675
1	6	−5	−0.209	−0.140	+0.241	+0.172

Arg=$\varkappa\gamma'+i'g''+ig'$			$r'\frac{dT'}{dr'}$		$r''\frac{dT'}{dr''}$	
			sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″
−1	6	−4	+2.450	−0.446	−3.283	+0.506
0	6	−5	−2.668	+0.478	+3.139	−0.483
1	6	−6	+0.834	−0.245	−0.975	+0.273
−1	6	−5	−1.095	+3.421	+1.400	−4.439
0	6	−6	+1.070	−3.714	−1.225	+4.315
1	6	−7	−0.312	+1.135	+0.356	−1.311
−1	6	−6	+0.147	+0.518	−0.179	−0.668
0	6	−7	−0.113	−0.736	+0.126	+0.854
1	6	−8	+0.029	+0.265	−0.032	−0.307
−1	6	−7	+0.029	+0.041	−0.038	−0.054
0	6	−8	−0.038	−0.081	+0.043	+0.095
1	6	−9	+0.013	+0.034	−0.015	−0.040
−1	7	−2	+0.006	+0.010	−0.006	−0.015
0	7	−3	−0.007	−0.011	+0.005	+0.013
1	7	−4	+0.005	+0.003	−0.006	−0.004
−1	7	−3	−0.118	−0.009	+0.154	+0.027
0	7	−4	+0.125	+0.014	−0.139	−0.029
1	7	−5	−0.046	+0.010	+0.055	−0.010
−1	7	−4	+0.401	−0.504	−0.557	+0.631
0	7	−5	−0.442	+0.535	+0.536	−0.591
1	7	−6	+0.110	−0.210	−0.133	+0.238
−1	7	−5	+0.685	+1.759	−0.834	−2.267
0	7	−6	−0.739	−1.886	+0.811	+2.180
1	7	−7	+0.293	+0.573	−0.329	−0.659
−1	7	−6	−2.183	−0.174	+2.733	+0.205
0	7	−7	+2.321	+0.099	−2.643	−0.101
1	7	−8	−0.712	−0.019	+0.805	+0.018
−1	7	−7	−0.355	+0.170	+0.444	−0.210
0	7	−8	+0.491	−0.191	−0.560	+0.215
1	7	−9	−0.180	+0.063	+0.205	−0.071
−1	8	−3	−0.014	+0.010	+0.019	−0.012
0	8	−4	+0.015	−0.011	−0.018	+0.010
1	8	−5	−0.004	+0.006	+0.005	−0.007
−1	8	−4	−0.007	−0.126	−0.003	+0.161
0	8	−5	+0.002	+0.135	+0.008	−0.152
1	8	−6	−0.015	−0.045	+0.015	+0.053
−1	8	−5	+0.490	+0.268	−0.604	−0.362
0	8	−6	−0.517	−0.293	+0.573	+0.353
1	8	−7	+0.189	+0.068	−0.213	−0.080
−1	8	−6	−1.127	+0.721	+1.410	−0.876
0	8	−7	+1.190	−0.775	−1.354	+0.860
1	8	−8	−0.360	+0.282	+0.407	−0.314
−1	8	−7	−0.202	−1.263	+0.253	+1.540
0	8	−8	+0.271	+1.317	−0.312	−1.476
1	8	−9	−0.091	−0.405	+0.104	+0.452

Arg=$\varkappa\gamma'+i'g''+ig'$			$r'\frac{dT'}{dr'}$		$r''\frac{dT'}{dr''}$	
			sin.	cos.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″
−1	8	−8	−0.172	−0.217	+0.207	+0.265
0	8	−9	+0.198	+0.293	−0.221	−0.329
1	8	−10	−0.068	−0.107	+0.076	+0.119
−1	9	−4	−0.014	−0.016	+0.016	+0.021
0	9	−5	+0.014	+0.017	−0.015	−0.020
1	9	−6	−0.009	−0.003	+0.009	+0.004
−1	9	−5	+0.125	−0.024	−0.156	+0.023
0	9	−6	−0.129	+0.016	+0.145	−0.018
1	9	−7	+0.039	−0.018	−0.045	+0.019
−1	9	−6	−0.139	+0.435	+0.185	−0.526
0	9	−7	+0.152	−0.445	−0.184	+0.494
1	9	−8	−0.031	+0.145	+0.037	−0.163
−1	9	−7	−0.661	−0.650	+0.786	+0.792
0	9	−8	+0.673	+0.665	−0.744	−0.749
1	9	−9	−0.204	−0.192	+0.229	+0.215
−1	9	−8	+0.685	−0.325	−0.812	+0.384
0	9	−9	−0.679	+0.336	+0.751	−0.376
1	9	−10	+0.190	−0.070	−0.212	+0.083
−1	9	−9	+0.160	−0.115	−0.183	+0.141
0	9	−10	−0.155	+0.165	+0.172	−0.183
1	9	−11	+0.010	−0.082	−0.017	+0.088

The following is a sufficiently exact expression for C′; it is derived from the equation

$$C' = 2\,(T' + X' + T')$$

Arg=$\varkappa\gamma'+i'g''+ig'$			C′		Arg=$\varkappa\gamma'+i'g''+ig'$			C′	
			sin.	cos.				sin.	cos.
$\varkappa$	i'	i	″	″	$\varkappa$	i'	i	″	″
0	1	0	+0.024	+0.116	1	3	−1	−0.006	+0.006
1	1	−1	−0.016	−0.082	−1	3	0	+0.058	−0.054
−1	1	0	+0.104	+0.532	0	3	−1	−0.130	+0.124
0	1	−1	−0.208	−1.058	1	3	−2	+0.074	−0.094
1	1	−2	+0.100	+0.530	−1	3	−1	−0.336	+0.696
−1	1	−1	+0.060	−0.050	0	3	−2	+0.624	−1.434
−1	2	0	+0.460	−0.114	1	3	−3	−0.250	+0.802
0	2	−1	−1.022	+0.270	−1	3	−2	−1.452	−2.026
1	2	−2	+0.664	−0.206	0	3	−3	+2.928	+3.976
−1	2	−1	−3.704	+1.598	1	3	−4	−1.458	−1.974
0	2	−2	+7.358	−3.186	0	3	−4	+1.119	+0.761
1	2	−3	−3.662	+1.582	0	3	−5	+0.129	+0.067

Arg=$\varkappa\gamma'+i'g''+ig'$			C′		Arg=$\varkappa\gamma'+i'g''+ig'$			C′	
			sin.	cos.				sin.	cos.
$\varkappa$	i'	i	″	″	$\varkappa$	i'	i	″	″
−1	4−	1	+0.026	+0.102	0	5−	2	−0.022	−0.014
0	4−	2	−0.074	−0.202	−1	5−	2	−0.104	+0.042
1	4−	3	+0.066	+0.110	0	5−	4	−0.060	+0.976
−1	4−	2	−0.636	−0.184	−1	5−	4	+0.680	+0.378
0	4−	3	+1.292	+0.332	0	5−	5	−1.360	−0.712
1	4−	4	−0.686	−0.116	1	5−	6	+0.672	+0.350
−1	4−	3	+0.950	−1.060	0	6−	2	+0.002	−0.002
0	4−	4	−1.840	+2.128	−1	6−	2	−0.008	+0.014
1	4−	5	+0.912	−1.058	0	6−	5	−0.644	+0.088

In deriving the terms of $\delta T'$, all which afforded less than $0''.0000005n't$ in $n'\delta z'$ were neglected. In the following expression for $\delta T'$ the coefficients have been multiplied by 1000000 in order to escape the necessity of writing so many zeros:

Arg=$\varkappa\gamma'+i'g''+ig'$			$\delta T'$		Arg=$\varkappa\gamma'+i'g''+i'g$			$\delta T'$	
			$n't$ sin.	$n't$ cos.				$n't$ sin.	$n't$ cos.
$\varkappa$	i'	i	″	″	$\varkappa$	i'	i	″	″
−1	1+	2	+ 1	0	0	2−	2	+ 44	+ 16
0	1+	1	0	+ 1	1	2−	3	− 24	+ 3
1	1	0	− 8.4	− 5.6	−1	2−	2	− 7	− 27
−1	1+	1	− 48.8	+ 32.6	0	2−	3	− 10	+157
0	1	0	+ 48.68	− 45.15	1	2−	4	+ 11	−103
1	1−	1	+ 29.7	− 16.3	−1	2−	3	− 5	− 2
−1	1	0	+ 5.8	− 13.5	0	2−	4	+ 4	+ 26
0	1−	1	− 11.0	+ 0.1	1	2−	5	− 4	− 19
1	1−	2	+ 6.7	+ 1.7	−1	3+	1	+ 2	+ 2
−1	1−	1	+ 57.9	+ 27.7	0	3	0	− 4.1	− 2.0
0	1−	2	− 16	− 13	1	3−	1	+ 4.58	− 0.47
1	1−	3	+ 9	+ 7	−1	3	0	− 74.02	− 1.41
−1	1−	2	− 1	+ 3	0	3−	1	+111.57	+ 0.72
0	1−	3	− 2	− 6	1	3−	2	− 45.12	+ 12.19
1	1−	4	+ 1	+ 4	−1	3−	1	+208.5	−313.5
−1	2+	2	+ 1	0	0	3−	2	−250.4	+441.3
0	2+	1	0	− 1	1	3−	3	+ 84	−161
1	2	0	− 1.3	− 0.3	−1	3−	2	− 5	− 30
−1	2+	1	+ 1.7	+ 13.4	0	3−	3	0	+ 86
0	2	0	− 17.4	+ 24.1	1	3−	4	− 8	− 41
1	2−	1	+ 24.0	+ 8.9	−1	3−	3	+ 72	− 15
−1	2	0	−282.3	−213.6	0	3−	4	−150	+ 28
0	2−	1	+469.9	+334.2	1	3−	5	+ 83	− 13
1	2−	2	−178.0	−120.0	−1	3−	4	+ 10	− 7
−1	2−	1	− 3.6	− 6.1	0	3−	5	− 40	+ 10
					1	3−	6	+ 20	0

Arg$=\varkappa\gamma'+i'g''+ig'$			$\delta T'$	
			$n't$ sin.	$n't$ cos.
$\varkappa$	i'	i	$''$	$''$
−1	4	0	− 8.2	+ 5.9
0	4	−1	+ 11.8	− 8.4
1	4	−2	− 3.4	+ 6.6
−1	4	−1	− 7.0	−100.6
0	4	−2	+ 14.3	+133.1
1	4	−3	− 16.2	− 49.1
−1	4	−2	+285.6	+113.9
0	4	−3	−360	−115
1	4	−4	+129	+ 35
−1	4	−3	+ 49	− 20
0	4	−4	− 89	+ 27
1	4	−5	+ 41	− 17
−1	4	−4	+ 29	+ 69
0	4	−5	− 46	−106
1	4	−6	− 20	+ 70
−1	4	−5	0	+ 10
0	4	−6	− 10	− 20
1	4	−7	+ 10	+ 20
−1	5	−1	− 11.7	− 12.7
0	5	−2	+ 15.20	+ 16.26
1	5	−3	− 9.8	− 3.8
−1	5	−2	+102.3	− 26.7
0	5	−3	−122	+ 37
1	5	−4	+ 45	− 18
−1	5	−3	− 36	+211
0	5	−4	+ 24	−249
1	5	−5	− 7	+ 87
−1	5	−4	+ 29	+ 44
0	5	−5	− 43	− 69
1	5	−6	+ 20	+ 29
−1	5	−5	− 44	+ 36
0	5	−6	+ 70	− 40
1	5	−7	− 30	+ 40
−1	5	−6	0	+ 10
0	5	−7	+ 10	− 20
1	5	−8	− 10	+ 10
−1	6	−1	− 2.32	− 0.32
0	6	−2	+ 2.920	+ 0.467
1	6	−3	− 1.44	+ 0.39
−1	6	−2	+ 13.55	− 17.19
0	6	−3	− 16.3	+ 20.9
1	6	−4	+ 3	− 12
−1	6	−3	+ 42	+ 85
0	6	−4	− 53	− 96
1	6	−5	+ 25	+ 31
−1	6	−4	−137	+ 8
0	6	−5	+156	− 20
1	6	−6	− 53	+ 8
−1	6	−5	− 32	+ 36
0	6	−6	+ 40	− 70
1	6	−7	+ 10	+ 20
−1	6	−6	− 80	− 20
0	6	−7	+ 30	+ 30
1	6	−8	− 10	− 20
−1	6	−7	0	− 10
0	6	−8	+ 10	0
1	6	−9	− 10	− 10
−1	7	−2	− 0.1	− 3.9
0	7	−3	0.0	+ 4.1
1	7	−4	− 0.6	− 1.7
−1	7	−3	+ 19.8	+ 10.6
0	7	−4	− 22	− 8
1	7	−5	+ 10	+ 1
−1	7	−4	− 56	+ 45
0	7	−5	+ 63	− 54
1	7	−6	− 20	+ 22
−1	7	−5	− 28	− 77
0	7	−6	+ 35	+ 81
1	7	−7	− 20	− 30
−1	7	−6	− 30	− 20
0	7	−7	+ 40	+ 30
1	7	−8	− 10	− 10
−1	7	−7	+ 10	− 20
0	7	−8	− 20	+ 30
1	7	−9	0	− 10

From this expression for $\delta T'$ we derive, by the oft-repeated process, the expressions for $\overline{\delta W_0'}$ and $-\frac{1}{2}\left(\overline{\frac{d\,.\,\delta W_0'}{d\gamma'}}\right)$:

Arg $=i'g''+ig'$	$\overline{\delta W_0'}$				$-\frac{1}{2}\left(\overline{\frac{d\,.\,\delta W_0'}{d\gamma'}}\right)$			
	cos.	$n't$ cos.	sin.	$n't$ sin.	sin.	$n't$ sin.	cos.	$n't$ cos.
i' i	″	″	″	″	″	″	″	″
1+ 1		+ 24.9	−0.0001	− 14.6		+ 13.5		+ 7.3
1 0	−0.00038	− 56.9	+0.0044	− 79.6	−0.00003	+ 4.9	−0.00005	− 0.5
1− 1	−0.0001	− 28.2	0.0000	− 39.1		+ 9.3		− 17.9
1− 2	+0.0001	+ 82	+0.0001	− 39	0.0000	− 42	+0.0001	− 21
1− 3		0		− 2		− 2		− 1
2+ 1		0		− 2		+ 1		+ 1
2 0	+0.0001	+ 100.3	+0.0003	− 53.7	0.0000	+ 36.8	−0.0002	+ 16.2
2− 1	+0.00323	+1838.3	+0.00458	−1330.7	+0.00018	− 270.0	−0.00024	− 198.3
2− 2	−0.0001	+ 22.9	0.0000	− 0.8		− 10.4		+ 1.4
2− 3		− 6		− 15		+ 5		− 5
2− 4		− 2		− 2		+ 1		− 1
3 0	−0.0002	− 86.7	+0.0017	− 10.3	−0.0001	− 44.8	−0.0009	+ 5.3
3− 1	+0.000280	−2127.81	+0.041384	+ 2.04	+0.000007	− 57.73	−0.000009	+ 3.46
3− 2	−0.1161	−4240.7	+0.0772	−6428.7	+0.0583	+2030.2	+0.0387	−3064.9
3− 3	−0.0026	− 120	+0.0022	− 165	+0.0026	+ 116	+0.0022	− 158
3− 4		+ 2		− 4		− 1		− 7
3− 5		− 2		− 1		− 1		+ 1
4− 1	−0.00005	− 29.5	+0.00007	− 26.8		− 6.0		+ 6.7
4− 2	−0.00025	+ 32.5	0.00000	− 442.3	+0.00031	− 12.9	−0.00002	− 139.9
4− 3	+0.0003	+ 292	+0.0007	− 139	−0.0002	− 219	+0.0004	− 95
4− 4		+ 21		+ 3		− 22		− 1
4− 5		− 5		− 13		− 9		− 5
4− 6		0		− 4		+ 1		0
5− 2	+0.00025	+ 69.7	+0.00023	− 79.5	+0.00001	− 11.1	−0.00001	− 10.1
5− 3	−0.0004	+ 336.7	+0.0018	+ 86.0	+0.0002	− 197.6	+0.0008	+ 49.9
5− 4	+0.0002	− 8	0.0000	− 82	−0.0001	+ 1	0.0000	− 69
5− 5		+ 4		− 11		− 3		− 12
5− 6		− 3		− 10		+ 4		− 2
6− 2	+0.00004	− 27.62	+0.00027	+ 3.94		− 1.79		− 0.08
6− 3	−0.0016	− 146.9	+0.0013	− 183.0	+0.0008	+ 66.2	+0.0006	− 85.7
6− 4		+ 24		− 60		− 15		− 47
6− 5		− 31		− 2		+ 28		− 4
6− 6		− 1		+ 2		+ 9		− 4
6− 7		− 16		+ 2		+ 9		+ 1
7− 3		+ 0.7		− 15.0		+ 0.2		− 4.8
7− 4		+ 26		− 14		− 16		− 10
7− 5		− 16		− 15		+ 14		− 13
7− 6		− 8		+ 13		+ 4		+ 11
7− 7		− 2		+ 2		+ 3		+ 3
7− 8		− 2		0		− 1		0

We next must obtain the terms of $n'\delta^2 z'$ and $\delta\nu'$, which undergo but one integration. These are given by the formulæ

$$\frac{d\,.\,\delta^2 z'}{dt} = -2\frac{d\nu'}{n'dt}n'\delta z' + \nu'^2$$

$$\frac{d\,.\,\delta\nu'}{n'dt} = -\frac{1}{2}\left(\overline{\frac{d^2 W_0'}{d\gamma'^2}}\right)n'\delta z'$$

Employing the subscript $({}_{m''})$ to denote the portions of the co-ordinates of Saturn proportional to the first power of the mass of Uranus, and which have been determined in Chapter III, the formulæ just given are expanded into

$$\begin{aligned}
\frac{d\,.\,\delta^2 z'}{dt} = {} & \left(\frac{d\nu'}{n'dt}\right)_{m''} && \times \text{ secular terms of } -2n'\delta z' \\
& + (n'\delta z')_{m''} && \times \text{ secular terms of } -2\frac{d\nu'}{n'dt} \\
& + (\nu')_{m''} && \times \text{ secular terms of } \quad 2\nu'
\end{aligned}$$

$$\begin{aligned}
\frac{d\,.\,\delta\nu'}{n'dt} = {} & -\frac{1}{2}\left(\overline{\frac{d^2 W_0'}{d\gamma'^2}}\right)_{m''} && \times \text{ secular terms of } \quad n'\delta z' \\
& + (n'\delta z')_{m''} && \times \text{ secular terms of } -\frac{1}{2}\left(\overline{\frac{d^2 W_0'}{d\gamma'^2}}\right)
\end{aligned}$$

For the second factors of the terms of these equations we have the values

$$\begin{aligned}
-2n'\delta z' \times \frac{1''}{2} = {} & [5.5104]n't \sin g' + [5.7152]n't \cos g' \\
& + [3.6566]n't \sin 2g' + [3.8615]n't \cos 2g' \\
& + [2.10\ \ ]n't \sin 3g' + [2.31\ \ ]n't \cos 3g'
\end{aligned}$$

$$\begin{aligned}
-2\frac{d\nu'}{n'dt} \times \frac{1''}{2} = {} & [5.2094]n't \sin g' + [5.4142]n't \cos g' \\
& + [3.9577]n't \sin 2g' + [4.1526]n't \cos 2g' \\
& + [2.76\ \ ]n't \sin 3g' + [2.96\ \ ]n't \cos 3g'
\end{aligned}$$

$$\begin{aligned}
2\nu' \times \frac{1''}{2} = {} & [3.9585]n't \\
& + [5.2094]n't \cos g' - [5.4142]n't \sin g' \\
& + [3.6566]n't \cos 2g' - [3.8615]n't \sin 2g' \\
& + [2.28\ \ ]n't \cos 3g' - [2.48\ \ ]n't \sin 3g'
\end{aligned}$$

$$\begin{aligned}
-\frac{1}{2}\left(\overline{\frac{d^2 W_0'}{d\gamma'^2}}\right) \times \frac{1''}{2} = {} & -[4.9084]n't \cos g' + [5.1132]n't \sin g' \\
& - [3.9577]n't \cos 2g' + [4.1626]n't \sin 2g'
\end{aligned}$$

The following expression for $-\frac{1}{2}\overline{\left(\frac{d^2W_0'}{d\gamma'^2}\right)}_{m''}$ is derived from the value of $-\frac{1}{2}\frac{dW_0'}{d\gamma'}$, obtained in Chapter III:

Arg=$i'g''+ig'$	$-\frac{1}{2}\overline{\left(\frac{d^2W_0'}{d\gamma'^2}\right)}_{m''}$		Arg=$i'g''+ig'$	$-\frac{1}{2}\overline{\left(\frac{d^2W_0'}{d\gamma'^2}\right)}_{m''}$	
	cos.	sin.		cos.	sin.
i' i	$''$	$''$	i' i	$''$	$''$
1+ 1	− 0.026	−0.005	4− 4	+0.432	+0.609
1 0	− 0.147	−0.089	4− 5	+0.054	+0.108
1− 1	− 0.424	+2.144	4− 6	+0.005	+0.014
1− 2	+ 0.014	+0.336	5− 2	−0.006	+0.007
1− 3	+ 0.002	+0.032	5− 3	−0.334	−0.108
2 0	− 0.027	−0.023	5− 4	+0.006	+0.316
2− 1	− 0.406	+0.018	5− 5	+0.270	−0.118
2− 2	−10.134	−4.352	5− 6	+0.054	−0.013
2− 3	− 1.144	−0.561	6− 3	+0.060	+0.088
2− 4	− 0.108	−0.059	6− 4	−0.038	+0.090
3 0	+ 0.021	+0.029	6− 5	+0.145	+0.020
3− 1	− 0.032	−0.011	6− 6	−0.021	−0.123
3− 2	+ 5.589	+9.883	6− 7	0.000	−0.028
3− 3	− 0.663	+2.931	7− 3	0.000	+0.003
3− 4	− 0.143	+0.320	7− 4	−0.022	+0.009
3− 5	− 0.016	+0.030	7− 5	+0.031	+0.029
4− 1	− 0.001	0.000	7− 6	+0.024	−0.066
4− 2	− 0.027	+0.176	7− 7	−0.053	−0.005
4− 3	− 0.857	+0.310			

The five products being now computed, and the first three added to $\overline{\delta W_0'}$ and the last two to $-\frac{1}{2}\overline{\left(\frac{d\,.\,\delta W_0'}{d\gamma'}\right)}$, we have the following expressions for the portions of $\frac{d\,.\,\delta^2 z'}{dt}$ and $\frac{d\,.\,\delta\nu'}{n'dt}$, which have the factor $n't$, the portions independent of this factor being the same as for $\overline{\delta W_0'}$ and $-\frac{1}{2}\overline{\left(\frac{d\,.\,\delta W_0'}{d\gamma'}\right)}$:

Arg=$i'g''+ig'$	$\frac{d\,.\,\delta^2 z'}{dt}$		$\frac{d\,.\,\delta\nu'}{n'dt}$	
	$n't$ cos.	$n't$ sin.	$n't$ sin.	$n't$ cos.
i' i	$''$	$''$	$''$	$''$
1+ 1	+ 69.3	− 23.0	+ 42.4	+ 6.5
1 0	+156.9	+118.3	+ 53.3	−44.6
1− 1	− 13.5	− 65.0	+ 2.0	−23.9
1− 2	+529	−212	−229	−93
1− 3	+ 33	− 24	− 27	−13

Arg=$i'g''+ig'$	$\frac{d\,.\,\delta^2 z'}{dt}$		$\frac{d\,.\,\delta\nu'}{n'dt}$	
	$n't$ cos.	$n't$ sin.	$n't$ sin.	$n't$ cos.
i' i	″	″	″	″
2+ 1	0	+ 7	− 4	− 10
2 0	− 61.2	+ 144.0	− 50.0	− 83.6
2− 1	+1187.6	− 880.6	− 160.7	− 122.9
2− 2	+ 142.8	+ 330.8	− 43.7	+ 144.3
2− 3	+ 193	+1263	− 79	+ 544
2− 4	+ 6	+ 93	− 6	+ 70
3 0	− 941.2	+ 342.9	− 432.7	− 164.1
3− 1	−1025.44	+ 52.77	− 19.62	+ 1.60
3− 2	−4225.0	−5443.2	+2003.2	−2559.9
3− 3	+ 763	−1729	− 235	− 742
3− 4	+ 240	− 98	− 136	− 82
3− 5	+ 17	− 2	− 15	− 1
4− 1	− 4.6	− 5.4	+ 1.4	+ 0.2
4− 2	+ 19.8	− 386.2	− 8.9	− 123.4
4− 3	+ 379	− 165	− 257	− 105
4− 4	+ 94	+ 57	− 58	+ 24
4− 5	+ 12	− 60	− 21	− 29
4− 6	+ 2	− 7	− 1	− 3
5− 2	+ 49.8	− 62.6	− 8.3	− 7.6
5− 3	+ 363.6	+ 96.5	− 209.0	+ 54.7
5− 4	+ 3	− 56	− 3	− 59
5− 5	+ 24	− 21	− 14	− 19
5− 6	− 18	− 20	+ 12	− 8
5− 7	− 1	+ 2	+ 1	− 1
6− 2	− 16.56	+ 3.54	− 1.01	− 0.09
6− 3	− 141.9	− 175.9	+ 63.8	− 82.5
6− 4	+ 38	− 83	− 21	− 56
6− 5	− 29	− 4	+ 27	− 5
6− 6	− 7	− 6	+ 11	− 9
6− 7	− 21	+ 6	+ 12	+ 4
7− 3	+ 0.6	− 13.5	+ 3	− 45
7− 4	+ 28	− 14	− 18	− 11
7− 5	− 17	− 16	+ 16	− 14
7− 6	− 9	+ 13	+ 5	+ 11
7− 7	− 6	+ 3	+ 5	+ 3

By integrating the preceding expressions we obtain the values of $n'\delta^2 z'$ and $\delta\nu'$. In the following statement of these values the proper number of decimals is restored to the coefficients multiplied by $n't$:

Arg=$i'g''+ig'$	$n'\delta^2 z'$				$\delta\nu'$			
	sin.	$n't$ sin.	cos.	$n't$ cos.	cos.	$n't$ cos.	sin.	$n't$ sin.
i' i	″	″	″	″	″	″	″	″
1+ 1	0.0000	+0.000051	+0.0001	+0.000017		−0.000031		+0.000005
1 0	−0.0001	+0.000447	0.0000	−0.000337	−0.0003	−0.000152	+0.0003	−0.000127
1− 1	+0.0003	+0.000021	0.0000	−0.000100		+0.000003		+0.000037
1− 2	−0.0001	−0.000321	+0.0002	−0.000129	−0.0001	−0.000139	−0.0001	+0.000056
1− 3		−0.000012		−0.000009		−0.000010		+0.000005
2+ 1		0.000000		−0.000004		+0.000002		−0.000006
2 0	+0.0004	−0.000087	−0.0006	−0.000205	−0.0001	+0.000071	−0.0004	−0.000119
2− 1	−0.0207	−0.003975	+0.0286	−0.002948	−0.0008	−0.000538	−0.0010	+0.000411
2− 2	+0.0003	−0.000110	+0.0001	+0.000255	+0.0001	−0.000034	0.0000	−0.000111
2− 3	+0.0002	−0.000084	0.0000	+0.000549	+0.0001	−0.000034	0.0000	−0.000237
2− 4		−0.000002		+0.000028		−0.000002		−0.000021
3 0	+0.0001	−0.000895	−0.0025	−0.000326	−0.0001	+0.000403	−0.0012	−0.000156
3− 1	+0.0250	−0.019777	−1.1796	−0.001018	+0.0005	+0.000378	−0.0075	+0.000031
3− 2	+0.1164	+0.004456	+0.0767	−0.005741	+0.0586	+0.002113	−0.0386	+0.002700
3− 3	+0.0009	−0.000392	+0.0013	−0.000887	+0.0011	−0.000121	−0.0012	+0.000381
3− 4		−0.000081		−0.000033		−0.000046		+0.000028
3− 5		−0.000004		−0.000001		−0.000004		0.000000
4− 1	−0.0001	−0.000011	−0.0002	+0.000013		−0.000003		0.000000
4− 2	−0.0007	−0.000033	+0.0001	−0.000646	+0.0002	−0.000015	0.0000	+0.000206
4− 3	−0.0003	−0.000237	+0.0006	−0.000103	−0.0002	−0.000161	−0.0004	+0.000066
4− 4		−0.000036		+0.000022		−0.000022		−0.000009
4− 5		−0.000003		−0.000017		−0.000006		+0.000008
4− 6		0.000000		−0.000002		0.000000		+0.000001
5− 2	−0.0020	−0.000202	+0.0017	−0.000254	−0.0001	−0.000034	−0.0001	+0.000031
5− 3	+0.0004	−0.000292	+0.0017	+0.000077	+0.0002	−0.000168	−0.0008	−0.000044
5− 4	−0.0001	−0.000001	0.0000	−0.000025		−0.000001		+0.000026
5− 5		−0.000007		−0.000006		−0.000004		+0.000006
5− 6		+0.000004		−0.000005		+0.000003		+0.000002
6− 2	+0.0007	−0.000160	−0.0041	−0.000034		+0.000010		−0.000001
6− 3	+0.0016	+0.000158	+0.0012	−0.000196	+0.0008	+0.000071	−0.0006	+0.000092
6− 4		−0.000020		−0.000044		−0.000011		+0.000030
6− 5		+0.000010		−0.000001		+0.000009		+0.000002
6− 6		+0.000002		−0.000002		+0.000003		+0.000002
6− 7		+0.000004		+0.000001		+0.000002		−0.000001
7− 3		−0.000001		−0.000025		+0.000001		+0.000008
7− 4		−0.000018		−0.000009		−0.000012		+0.000007
7− 5		+0.000007		−0.000006		+0.000006		+0.000005
7− 6		+0.000003		+0.000004		+0.000001		−0.000003
7− 7		+0.000001		+0.000001		+0.000001		−0.000001

Thinking that possibly the second-order terms in $\delta T'$ not factored by the time might sensibly affect the long-period inequality in the longitude having the argument $3g''-g'$, I have made a rough determination of them. But as the resulting quantity was quite small, it seems unnecessary to give more details in reference to it than the following:

I found

$$
\begin{array}{rll}
\frac{dT'}{dg'}n'\delta z' = & \overset{\prime\prime}{0.000000} \sin(3g''-g') & - \overset{\prime\prime}{0.000005} \cos(3g''-g') \\
\frac{dT'}{dg''}n''\delta z'' = & +\ 0.000039 & +\ 0.000009 \\
r'\frac{dT'}{dr'}\nu' = & -\ 0.000013 & +\ 0.000006 \\
r''\frac{dT'}{dr''}\nu'' = & -\ 0.000007 & +\ 0.000035 \\
\hline
\delta T' = & +\ 0.000019 \sin(3g''-g') & +\ 0.000045 \cos(3g''-g')
\end{array}
$$

Whence, by twice integrating,

$$n'\delta^2 z' = -\ 0''.0071 \sin(3g''-g') - 0''.0167 \cos(3g''-g')$$

CHAPTER XXII.

PERTURBATIONS OF JUPITER PROPORTIONAL TO THE PRODUCT OF THE MASSES OF SATURN AND URANUS.

We now attend to the sensible inequalities of Jupiter and Saturn whose arguments involve not only the mean anomalies of these two planets but also that of Uranus. They all owe their sensible magnitude to large integrating factors. In the present chapter we investigate the inequalities of this kind which belong to Jupiter. They are only two in number, having severally the arguments $6g' - 2g - 3g''$ and $6g' - 3g - 3g''$.

If we divide the function T into the two portions, $T_{m'}$ and $T_{m''}$, severally produced by the action of Saturn and Uranus, and adopt a similar division and notation for $n\delta z$, $n'\delta z'$, $n''\delta z''$, ν, ν', and ν'', the portion of the correction δT of T, which produces the terms of the kind we are seeking, is

$$\begin{aligned}\delta T = \; & \frac{dT_{m'}}{dg}(n\delta z)_{m''} + \frac{dT_{m'}}{dg'}(n'\delta z')_{m''} + r\frac{dT_{m'}}{dr} \cdot \nu_{m''} + r'\frac{dT_{m'}}{dr'} \cdot \nu'_{m''} + C_{m'}\left(\delta\frac{h}{h_0}\right)_{m''} \\ & + \frac{dT_{m''}}{dg}(n\delta z)_{m'} + \frac{dT_{m''}}{dg''}(n''\delta z'')_{m'} + r\frac{dT_{m''}}{dr} \cdot \nu_{m'} + r''\frac{dT_{m''}}{dr''} \cdot \nu''_{m'} + C_{m''}\left(\delta\frac{h}{h_0}\right)_{m'}\end{aligned}$$

It has been assumed that all terms arising from inclination of orbits may be neglected. Of the ten terms of this formula it is discovered that two, the seventh and ninth, are quite insignificant. The first factors of the first five terms have been determined in Chapter VIII, and are there designated severally as A, F, B, G, and C. $T_{m''}$ is given in Chapter IV. The factors $r\frac{dT_{m''}}{dr}$ and $C_{m''}$ we have had no occasion to derive. The terms which here depend on them are so minute that it is accurate enough to estimate the few terms of these factors by a sort of induction. We assume that $r\frac{dT_{m''}}{dr}$ and $C_{m''}$ bear the same relation to $T_{m''}$ that $r\frac{dT_{m'}}{dr}$ and $C_{m'}$ do to $T_{m'}$. In this way we get, the coefficients in seconds of arc being expressed by their logarithms,

$$\begin{aligned}r\frac{dT_{m''}}{dr} = & -[9.36]\sin(3g''-2g) + [9.48]\cos(3g''-2g) \\ & + [9.43]\sin(3g''-3g) - [9.92]\cos(3g''-3g) \\ & + [8.52]\sin(3g''-4g) - [9.12]\cos(3g''-4g)\end{aligned}$$

$$\begin{aligned}C_{m''} = & -[8.86]\sin(3g''-2g) + [8.98]\cos(3g''-2g) \\ & + [8.93]\sin(3g''-3g) - [9.41]\cos(3g''-3g) \\ & - [8.00]\sin(3g''-4g) + [8.60]\cos(3g''-4g)\end{aligned}$$

The second factors of the formula have already been given, with the exception of $\delta\left(\frac{h}{h_0}\right)_{m''}$, which is given by the equation

$$\delta\left(\frac{h}{h_0}\right)_{m''} = -\frac{d\,.\,(n\delta z)_{m''}}{ndt} - 2\nu_{m''}$$

It is sufficient to compute in δT the coefficients of the terms having the four arguments

$$-\gamma + 6g' - 2g - 3g'' \qquad -\gamma + 6g' - g - 3g'' \qquad 6g' - 2g - 3g'' \qquad \gamma + 6g' - 3g - 3g''$$

For those having the argument $6g' - 2g - 3g''$ it is necessary to employ the complete formula, involving eight terms; but in the case of the remaining three arguments, which involve γ, it suffices to reduce the expression of δT to

$$\delta T = F(n'\delta z')_{m''} + G\nu'_{m''}$$

In computing the terms with the argument $6g' - 2g - 3g''$ the following are all the combinations of arguments (by subtraction) which give sensible results:

A, B, or C		$(n\delta z)_{m''}$, $\nu_{m''}$, or $\delta\left(\frac{h}{h_0}\right)_{m''}$
$6g' - 3g$	with	$3g'' - g$
$6g' - 4g$	with	$3g'' - 2g$
$6g' - 5g$	with	$3g'' - 3g$
$6g' - 6g$	with	$3g'' - 4g$

F or G		$(n'\delta z')_{m''}$ or $\nu'_{m''}$
$5g' - 2g$	with	$3g'' - g'$
$4g' - 2g$	with	$3g'' - 2g'$
$3g' - 2g$	with	$3g'' - 3g'$
$2g' - 2g$	with	$3g'' - 4g'$

$\frac{dT_{m''}}{dg}$, $r\frac{dT_{m''}}{dr}$, or $C_{m''}$		$(n\delta z)_{m'}$, $\nu_{m'}$, or $\delta\left(\frac{h}{h_0}\right)_{m'}$
$3g'' - 2g$	with	$6g' - 4g$
$3g'' - 3g$	with	$6g' - 5g$
$3g'' - 4g$	with	$6g' - 6g$

In the terms of δT, containing γ in their arguments, the combinations giving sensible results are

For $-\gamma + 6g' - 2g - 3g''$

F or G		$(n'\delta z')_{m''}$ or $\nu'_{m''}$
$-\gamma + 5g' - 2g$	with	$3g'' - g'$
$-\gamma + 4g' - 2g$	with	$3g'' - 2g'$
$-\gamma + 3g' - 2g$	with	$3g'' - 3g'$

For $-\gamma + 6g' - g - 3g''$

F or G		$(n'\delta z')_{m''}$ or $\nu'_{m''}$
$-\gamma + 5g' - g$	with	$3g'' - g'$
$-\gamma + 4g' - g$	with	$3g'' - 2g'$
$-\gamma + 3g' - g$	with	$3g'' - 3g'$
$-\gamma + 2g' - g$	with	$3g'' - 4g'$

For $\gamma + 6g' - 3g - 3g''$

F or G		$(n'\delta z')_{m''}$ or $\nu'_{m''}$
$\gamma + 5g' - 3g$	with	$3g'' - g'$
$\gamma + 4g' - 3g$	with	$3g'' - 2g'$
$\gamma + 3g' - 3g$	with	$3g'' - 3g'$

These details suffice to show how the terms we seek are to be obtained. But the results are limited, in point of precision, to quantities of two dimensions with respect to disturbing forces. However, in the coefficients of the terms having the arguments $6g' - 2g - 3g''$ and $-\gamma + 6g' - 2g - 3g''$, it is necessary to have their variations proportional to t, which are of three dimensions with respect to disturbing forces. As it is impossible to follow a rigorous process in getting these terms, on account of the numerous combinations, we are compelled to resort to very much abbreviated formulæ. There is no proof that the latter afford results sufficiently accurate for practical purposes, yet there is reason for thinking that they give the salient portion of these terms. It is deemed sufficient to put

$$\delta\mathrm{T} = \mathrm{F}(n'\delta z')_{m''} + \mathrm{G}\nu'_{m''}$$

Each of the four factors in the right member of this equation must be supposed to receive an augmentation proporticnal to nt. Operating then on the equation with the symbol δ we have

$$\delta^2\mathrm{T} = \mathrm{F}(n'\delta^2 z')_{m''} + \mathrm{G}\delta\nu'_{m''} + \delta\mathrm{F} \,.\, (n'\delta z')_{m''} + \delta\mathrm{G} \,.\, \nu'_{m''}$$

Of the factors involved in this $(n'\delta^2 z')_{m''}$ and $\delta\nu'_{m''}$ have been determined in the preceding chapter. Of the two new quantities $\delta\mathrm{F}$ and $\delta\mathrm{G}$ introduced into the equation, when we recall that $\mathrm{F} = \frac{d\mathrm{T}}{dg'}$, it will be seen that an approximate expression for $\delta\mathrm{F}$ will be

$$\delta\mathrm{F} = \frac{d\mathrm{F}}{dg'}n'\delta z' + \frac{d\mathrm{G}}{dg'}\nu' + \frac{d}{dg'}\left[\mathrm{A}n\delta z + \mathrm{B}\nu + \mathrm{C}\delta\frac{h}{h_0}\right]$$

Of the quantities involved in this equation $\frac{d\mathrm{F}}{dg'}$ and $\frac{d\mathrm{G}}{dg'}$ have already been used in Chapter XVII for the determination of $\delta^2\mathrm{T}$; $\mathrm{A}n\delta z + \mathrm{B}\nu$ has been computed in Chapter XI; to the factors $n'\delta z'$ and ν' we attribute only their secular terms as values. In like manner we have

$$\delta\mathrm{G} = \frac{d\mathrm{G}}{dg'}n'\delta z' + \frac{d\mathrm{G}}{dg}n\delta z + \left(r'\frac{d}{dr'}\right)^2 . \,\mathrm{T} \,.\, \nu' + \left(r\frac{d}{dr}\right)\left(r'\frac{d}{dr'}\right) . \,\mathrm{T} \,.\, \nu$$

All the first factors of the right member of this equation have been used in Chapter XVII. As before $n'\delta z'$, $n\delta z$, ν', and ν may be supposed reduced to their secular terms.

After $\delta\mathrm{T}$ has been determined, it still remains to get the portion of $n\delta^2 z$ which undergoes but one integration; this is

$$\left(\frac{\overline{d\mathrm{W}_0}}{d\gamma}\right)n\delta z + \nu^2 = \left(\frac{\overline{d\mathrm{W}_0}}{d\gamma}\right)_{m'}(n\delta z)_{m''} + \left(\frac{\overline{d\mathrm{W}_0}}{d\gamma}\right)_{m''}(n\delta z)_{m'} + 2\nu_{m'}\nu_{m''}$$

The quantities involved here are so small that there is no necessity for considering any terms of three dimensions with respect to disturbing forces.

Giving to the factors involved their previously ascertained values, performing the multiplication, and preserving only the terms which are of use, we have the following expressions for δF and δG:

Arg$=\varkappa\gamma+i'g'+ig$			δF		δG	
			nt cos.	nt sin.	nt sin.	nt cos.
$\varkappa$	i'	i	″	″	″	″
−1	3	−2	−0.000523	−0.000208	+0.000410	−0.000035
0	2	−2	−0.000041	+0.000304	−0.000067	+0.000081
−1	4	−2	−0.002623	−0.008376	+0.004163	−0.012753
0	3	−2	−0.005872	+0.002898	+0.010443	+0.005455
−1	5	−2	+0.000948	−0.004258	−0.001599	−0.005215
0	4	−2	−0.002740	−0.000160	+0.003667	−0.000542
0	5	−2	−0.000568	−0.000372	+0.000554	−0.000541

The following expression is obtained for δT:

Argument.	δT			
	sin.		cos.	
	″	″	″	″
$-\gamma+6g'-2g-3g''$	+0.00099	−0.000001013nt	−0.00381	+0.000000422nt
$-\gamma+6g'-g-3g''$	−0.00067		−0.00002	
$6g'-2g-3g''$	+0.0005921	−0.0000001410nt	+0.0000322	−0.0000002907nt
$\gamma+6g'-3g-3g''$	−0.00015		+0.00009	

The logarithms of the integrating factors are

$6g'-g-3g''$	0.0032
$6g'-2g-3g''$	2.12820n
$6g'-3g-3g''$	9.9968n

It is found that the terms from $\left(\frac{d\mathrm{W}_0}{d\gamma}\right)n\delta z+\nu^2$ are insensible.

By the application of the usual treatment to δT we get

$$\begin{aligned} n\delta z = {} & \left[\overset{''}{-0.1396}+\overset{''}{0.000135}nt\right]\sin(6g'-3g-3g'')+\left[\overset{''}{0.4897}-\overset{''}{0.000056}nt\right]\cos(6g'-3g-3g'') \\ & +\left[-9.3451+0.002545nt\right]\sin(6g'-2g-3g'')+\left[-1.2796+0.005246nt\right]\cos(6g'-2g-3g'') \end{aligned}$$

$$\begin{aligned} \nu = {} & \left[-0.0698+0.000067nt\right]\cos(6g'-3g-3g'')+\left[-0.2449+0.000028nt\right]\sin(6g'-3g-3g'') \\ & +\left[-0.0551\right]\cos(6g'-2g-3g'')+\left[-0.0047\right]\sin(6g'-2g-3g'') \end{aligned}$$

CHAPTER XXIII.

PERTURBATIONS OF SATURN PROPORTIONAL TO THE PRODUCT OF THE MASSES OF JUPITER AND URANUS.

Like those discussed in the preceding chapter these perturbations owe their sensible magnitude to large integrating factors. They are more numerous and larger than in the case of Jupiter. The formulæ for their determination are quite similar.

If we divide the function T′ into the two portions, T'_m and $T'_{m''}$, severally produced by the action of Jupiter and Uranus, the portion of the correction $\delta T'$ of T′, which produces terms of the kind we are seeking, is

$$\begin{aligned}\delta T' = \quad & A'(n'\delta z')_{m''} + B'\nu'_{m''} + C'\delta\left(\frac{h'}{h_0'}\right)_{m''} + F'(n\delta z)_{m''} + G'\nu_{m''} \\ & + \frac{dT'_{m''}}{dg'}(n'\delta z')_m + r'\frac{dT'_{m''}}{dr'}\nu'_m + C'_{m''}\delta\left(\frac{h'}{h_0'}\right)_m + \frac{dT'_{m''}}{dg''}(n''\delta z'')_m + r''\frac{dT'_{m''}}{dr''}\nu''_m\end{aligned}$$

A′, B′, C′, F′, and G′ have been determined in Chapter VIII, the remaining five first factors have been employed in Chapter XXI. The factor

$$\delta\left(\frac{h'}{h_0'}\right)_{m''} = -\left(\frac{d\,.\,n'\delta z'}{n'dt}\right)_{m''} - 2\nu'_{m''}$$

The factors $(n''\delta z'')_m$ and ν''_m, being the Jupiter-perturbations of Uranus, must be derived from the theory of the latter planet. The terms they give rise to are very small. The following are sufficiently exact expressions for these factors:*

$$\begin{aligned}(n''\delta z'')_m \times \frac{1''}{2} = & -[4.49]\sin(2g - g'') - [2.03]\cos(2g - g'') \\ & -[3.70]\sin(2g - 2g'') - [3.27]\cos(2g - 2g'') \\ & -[3.10]\sin(2g - 3g'') - [2.88]\cos(2g - 3g'')\end{aligned}$$

$$\begin{aligned}\nu''_m \times \frac{1''}{2} = & -[4.49]\cos(2g - g'') + [1.65]\cos(2g - g'') \\ & +[3.55]\cos(2g - 2g'') - [3.41]\cos(2g - 2g'') \\ & +[3.26]\cos(2g - 3g'') - [3.04]\cos(2g - 3g'')\end{aligned}$$

The remaining factors have already been given.

As in the case of Jupiter it is necessary to consider some terms of three dimensions with respect to disturbing forces and which are factored by $n't$. This, however, is only necessary for the terms of $\delta T'$ having the arguments $6g' - 2g - 3g''$ and $-\gamma' + 6g' - 2g - 3g''$. Here we can reduce the expression for $\delta T'$ to

---

*An Investigation of the Orbit of Uranus, by Prof. S. NEWCOMB, page 63.

$$\delta T' = A'(n'\delta z')_{m''} + B'\nu'_{m''} + A'_{m''}(n'\delta z')_{m} + B'_{m''}\nu'_{m}$$

Subjecting this equation to the operation δ we have

$$\begin{aligned}\delta^2 T' = {} & A'(n'\delta^2 z')_{m''} + B'\delta\nu'_{m''} + A'_{m''}(n'\delta^2 z')_{m} + B'_{m''}\delta\nu'_{m} \\ & + \delta A'(n'\delta z')_{m''} + \delta B'\nu'_{m''} + \delta A'_{m''}(n'\delta z')_{m} + \delta B'_{m''}\nu'_{m}\end{aligned}$$

In this expression four new factors, $\delta A'$, $\delta B'$, $\delta A'_{m''}$, and $\delta B'_{m''}$, appear. Remembering that

$$A' = \frac{dT'}{dg'} \qquad B' = r'\frac{dT'}{dr'} \qquad A'_{m''} = \frac{dT'_{m''}}{dg'} \qquad B'_{m''} = r'\frac{dT'_{m''}}{dr'}$$

it will be seen that, with sufficient approximation, we have

$$\begin{aligned}\delta A' &= \frac{dA'}{dg'}n'\delta z' + \frac{dB'}{dg'}\nu' + \frac{d}{dg'}[F'n\delta z + G'\nu] \\ \delta B' &= \frac{dB'}{dg'}n'\delta z' + \left(r'\frac{d}{dr'}\right)^2 T' \,.\, \nu' + \frac{dB'}{dg}n\delta z + \left(r\frac{d}{dr}\right)\left(r'\frac{d}{dr'}\right)T' \,.\, \nu \\ \delta A'_{m''} &= \frac{dA'_{m''}}{dg'}n'\delta z' + \frac{dB'_{m''}}{dg'}\nu' + \frac{dA'_{m''}}{dg''}n''\delta z'' + \frac{dG'_{m''}}{dg'}\nu'' \\ \delta B'_{m''} &= \frac{dB'_{m''}}{dg'}n'\delta z' + \left(r'\frac{d}{dr'}\right)^2 T'_{m''} \,.\, \nu' + \frac{dB'_{m''}}{dg''}n''\delta z'' + \left(r'\frac{d}{dr'}\right)\left(r''\frac{d}{dr''}\right)T'_{m''}\nu''\end{aligned}$$

Here the factors $n\delta z$, $n'\delta z'$, $n''\delta z''$, ν, ν', and ν'' are to be reduced to their secular terms. The proper expressions for $n''\delta z''$ and ν'' have already been given (page 458). $F'n\delta z + G'\nu$ has been determined in Chapter XIV. The first factors of the expressions for $\delta A'$ and $\delta B'$ have already been employed in the determination of the terms of three dimensions for Saturn. $B'_{m''}$ and $G'_{m''}$ have been derived in Chapter XXI. The terms which arise from the term having the factor $\left(r'\frac{d}{dr'}\right)^2 T'_{m''}$ are very small. It has been thought sufficiently accurate to estimate by induction the needed terms of this factor. They are

$$\begin{aligned}\left(r'\frac{d}{dr'}\right)^2 T'_{m''} = {} & [9.28\ \ ]\sin(3g'' - g') - [9.27\ \ ]\cos(3g'' - g') \\ & - [0.2248]\sin(3g'' - 2g') + [0.7693]\cos(3g'' - 2g') \\ & - [1.3276]\sin(3g'' - 3g') - [1.4748]\cos(3g'' - 3g') \\ & - [0.6241]\sin(3g'' - 4g') - [0.3256]\cos(3g'' - 4g') \\ & - [9.64\ \ ]\sin(3g'' - 5g') - [9.13\ \ ]\cos(3g'' - 5g')\end{aligned}$$

On account of the smallness of ν'' the last term of $\delta B'_{m''}$ has been neglected. Lastly, for the terms of $\delta T'$, having the argument $-\gamma' + 6g' - 2g - 3g''$, it suffices to consider $\delta A'$ and $\delta B'$ alone, omitting altogether $\delta A'_{m''}$ and $\delta B'_{m''}$.

On making the substitutions in the formulæ there has been found

Argument.	$\delta A'$		$\delta B'$	
	cos.	sin.	sin.	cos.
	″	″	″	″
$-\gamma'+3g'-2g$	$-0.0012n't$	$+0.0186n't$	$+0.0007n't$	$+0.0050n't$
$2g'-2g$	$+0.00283n't$	$-0.00572n't$	$-0.00264n't$	$+0.00107n't$
$-\gamma'+4g'-2g$	$-0.1954n't$	$+0.1102n't$	$+0.02985n't$	$+0.01707n't$
$3g'-2g$	$+0.09058n't$	$-0.04545n't$	$-0.15999n't$	$-0.08274n't$
$-\gamma'+5g'-2g$	$-0.1096n't$	$-0.0020n't$	$+0.1374n't$	$-0.0132n't$
$4g'-2g$	$+0.06126n't$	$+0.00251n't$	$-0.08552n't$	$+0.00898n't$
$5g'-2g$	$+0.01685n't$	$+0.01075n't$	$-0.01795n't$	$+0.01542n't$

Argument.	$\delta A'_{m''}$		$\delta B'_{m''}$	
	cos.	sin.	sin.	cos.
	″	″	″	″
$3g''-g'$	$-0.0002163n't$	$+0.0000054n't$	$+0.000101n't$	$-0.000031n't$
$3g''-2g'$	$+0.0007007n't$	$+0.0013700n't$	$-0.000288n't$	$+0.000444n't$
$3g''-3g'$	$+0.00001n't$	$+0.00029n't$	$+0.00023n't$	$+0.00023n't$
$3g''-4g'$	$+0.00044n't$	$+0.00010n't$	$-0.00161n't$	$+0.00021n't$

Computing the terms of two dimensions of $\delta T'$, which have the argument $6g'-2g-3g''$, we obtain severally for each of the ten terms

$$\begin{array}{rll}
A'(n'\delta z')_{m''} = & -\overset{''}{0.0044383}\sin(6g'-2g-3g'') & -\overset{''}{0.0003686}\cos(6g'-2g-3g'') \\
B'\nu'_{m''} = & -0.0026341 & +0.0003563 \\
C'\delta\left(\frac{h'}{h_0'}\right)_{m''} = & +0.0000205 & +0.0001902 \\
F'(n\delta z)_{m''} = & -0.0000102 & +0.0000402 \\
G'\nu_{m''} = & -0.0000042 & +0.0000342 \\
\frac{dT'_{m''}}{dg'}(n'\delta z')_m = & -0.0032506 & +0.0000926 \\
B'_{m''}\nu'_m = & -0.0024209 & -0.0002530 \\
C'_{m''}\delta\left(\frac{h'}{h_0'}\right)_m = & +0.0000094 & -0.0002299 \\
\frac{dT'_{m''}}{dg''}(n''\delta z'')_m = & -0.0000001 & +0.0000003 \\
r''\frac{dT'_{m''}}{dr''}\nu''_m = & +0.0000003 & +0.0000014
\end{array}$$

And their sum

$$\delta T' = -0''.0127282\sin(6g'-2g-3g'') - 0''.0001363\cos(6g'-2g-3g'')$$

In like manner the terms having the argument $-\gamma' + 6g' - 2g - 3g''$ are composed as follows:

$$\begin{array}{rll}
A'(n'\delta z')_{m''} = & +\overset{\prime\prime}{0}.02049 \sin(-\gamma' + 6g' - 2g - 3g'') & +\overset{\prime\prime}{0}.01306 \cos(-\gamma' + 6g' - 2g - 3g'') \\
B'\nu'_{m''} = & +0.01484 & +0.00461 \\
C'\delta\left(\dfrac{h'}{h_0'}\right)_{m''} = & +0.00016 & -0.00028 \\
F'(n\delta z)_{m''} = & +0.00010 & -0.00009 \\
G'\nu_{m''} = & +0.00008 & -0.00007 \\
\dfrac{dT'_{m''}}{dg'}\left(n'\delta z'\right)_{m} = & -0.00029 & +0.00034 \\
r'\dfrac{dT'_{m''}}{dr'}\nu'_{m} = & -0.00019 & +0.00022 \\
C'_{m''}\delta\left(\dfrac{h'}{h_0}\right)_{m} = & -0.00007 & +0.00020
\end{array}$$

And their sum

$$\delta T' = +0''.03512 \sin(-\gamma' + 6g' - 2g - 3g'') + 0''.01799 \cos(-\gamma' + 6g' - 2g - 3g'')$$

In computing the terms of three dimensions having these arguments, it has been found that

$$\begin{array}{rll}
A'(n'\delta^2 z')_{m''} = & +\overset{\prime\prime}{0}.0000016 91 n't \sin(6g' - 2g - 3g'') & +\overset{\prime\prime}{0}.000000669 n't \cos(6g' - 2g - 3g'') \\
B'\delta\nu'_{m''} = & +0.000000657 & -0.000000066 \\
A'_{m''}(n'\delta^2 z')_{m} = & -0.000000071 & +0.000003517 \\
B'_{m''}\delta\nu'_{m} = & +0.000000258 & +0.000002653 \\
\delta A'(n'\delta z')_{m''} = & +0.000000205 & +0.000003270 \\
\delta B'\nu'_{m''} = & +0.000001171 & +0.000002409 \\
\delta A'_{m''}(n'\delta z')_{m} = & +0.000002989 & +0.000000528 \\
\delta B'_{m''}\nu'_{m} = & +0.000000269 & -0.000000182
\end{array}$$

And their sum

$$\delta^2 T' = +0''.000007169 n't \sin(6g' - 2g - 3g'') + 0''.000012798 n't \cos(6g' - 2g - 3g'')$$

In a similar manner for the terms belonging to the argument $-\gamma' + 6g' - 2g - 3g''$

$$\begin{array}{rll}
A'(n'\delta^2 z')_{m''} = & -\overset{\prime\prime}{0}.0000071 n't \sin(-\gamma' + 6g' - 2g - 3g'') & -\overset{\prime\prime}{0}.0000074 n't \cos(-\gamma' + 6g' - 2g - 3g'') \\
B'\delta\nu'_{m''} = & -0.0000042 & -0.0000010 \\
\delta A'(n'\delta z')_{m''} = & +0.0000067 & -0.0000087 \\
\delta B'\nu'_{m''} = & -0.0000002 & -0.0000104
\end{array}$$

And their sum

$$\delta^2 T' = -\overset{\prime\prime}{0}.0000048 n't \sin(-\gamma + 6g' - 2g - 3g'') - \overset{\prime\prime}{0}.0000275 n't \cos(-\gamma' + 6g' - 2g - 3g'')$$

Adding to the previous terms those dependent on the arguments $-\gamma'+7g'-2g-3g''$ and $\gamma'+5g'-2g-3g''$, of the computation of which it is thought unnecessary to give any details, we have the following complete expression for $\delta T'$:

Argument.	sin.	cos.
	$''$ $\quad$ $''$	$''$ $\quad$ $''$
$-\gamma'+6g'-2g-3g''$	$+0.03512 \quad -0.0000048n't$	$+0.01799 \quad -0.0000275n't$
$-\gamma'+7g'-2g-3g''$	$+0.01155$	$+0.00092$
$6g'-2g-3g''$	$-0.0127282+0.00007169n't$	$-0.0001363+0.000012798n't$
$\gamma'+5g'-2g-3g''$	$+0.00419$	-0.00039

The logarithms of the integrating factors are

$7g'-2g-3g''$	0.0081
$6g'-2g-3g''$	1.73316*n*
$5g'-2g-3g''$	9.9920*n*

The very familiar process being applied to $\delta T'$ we get

$$\begin{aligned}
\overline{\delta W_0'} = {} & [\overset{''}{1.8193} - \overset{''}{0.000260}n't\,] \cos(5g'-2g-3g'') \\
& + [-0.9874 + 0.001488n't\,] \sin(5g'-2g-3g'') \\
& + [-0.65875 + 0.0003878n't] \cos(6g'-2g-3g'') \\
& + [0.02967 - 0.0006923n't] \sin(6g'-2g-3g'')
\end{aligned}$$

$$\begin{aligned}
-\frac{1}{2}\left(\frac{\overline{d\,.\,\delta W_0'}}{d\gamma'}\right) = {} & [-0.9096 + 0.000130n't\,] \sin(5g'-2g-3g'') \\
& + [-0.4937 + 0.000744n't\,] \cos(5g'-2g-3g'') \\
& + [0.00794 \qquad\qquad\quad\,] \sin(6g'-2g-3g'') \\
& + [0.00028 \qquad\qquad\quad\,] \cos(6g'-2g-3g'')
\end{aligned}$$

In order to have the value of $\frac{d\,.\,\delta^2 z'}{dt}$ it is necessary to add to the first of these expressions the terms which arise from

$$\left(\frac{\overline{dW_0'}}{d\gamma'}\right) n'\delta z' + \nu'^2$$

When this is expanded it takes the form

$$\left(\frac{\overline{dW_0'}}{d\gamma'}\right)_{m} (n'\delta z')_{m''} + \left(\frac{\overline{dW_0'}}{d\gamma'}\right)_{m''} (n'\delta z')_{m} + 2\nu'_{m}\nu'_{m''}$$

These terms are significant only in the case of the argument $6g' - 2g - 3g''$. All the factors involved have been already given. It is found that

$$\begin{array}{rll}
\left(\frac{\overline{dW_0'}}{d\gamma'}\right)_m (n'\delta z')_{m''} = & + \overset{''}{0}.03468 \cos(6g' - 2g - 3g'') & + \overset{''}{0}.01438 \sin(6g' - 2g - 3g'') \\
\left(\frac{\overline{dW_0'}}{d\gamma'}\right)_{m''} (n'\delta z')_m = & + 0.03505 & + 0.01353 \\
2\nu'_m \nu'_{m''} = & - 0.01810 & - 0.00707
\end{array}$$

And their sum $= + 0.05163 \cos(6g' - 2g - 3g'') + 0.02084 \sin(6g' - 2g - 3g'')$

It is necessary to consider the terms of three dimensions factored by $n't$ which arise from the same source, and which are given by the formula

$$\left(\frac{\overline{dW_0'}}{d\gamma'}\right)_m (n'\delta^2 z')_{m''} + \left(\frac{\overline{dW_0'}}{d\gamma'}\right)_{m''} (n'\delta^2 z')_m + 2\nu'_m \delta\nu'_{m''} + 2\nu'_{m''} \delta\nu'_m - 2\frac{d \,.\, \delta\nu'_m}{n'dt}(n'\delta z')_{m''} - 2\frac{d \,.\, \delta\nu'_{m''}}{n'dt}(n'\delta z')_m$$

All the factors of the six terms of this expression have been given in preceding chapters. It is found that

$$\begin{array}{rll}
\left(\frac{\overline{dW_0'}}{d\gamma'}\right)_m (n'\delta^2 z')_{m''} = & - \overset{''}{0}.0000094 n't \cos(6g' - 2g - 3g'') & - \overset{''}{0}.0000056 n't \sin(6g' - 2g - 3g'') \\
\left(\frac{\overline{dW_0'}}{d\gamma'}\right)_{m''} (n'\delta^2 z')_m = & - 0.0000193 & + 0.0000356 \\
2\nu'_m \delta\nu'_{m''} = & + 0.0000047 & + 0.0000028 \\
2\nu'_{m''} \delta\nu'_m = & + 0.0000100 & - 0.0000184 \\
- 2\frac{d \,.\, \delta\nu'_m}{n'dt}(n'\delta z')_{m''} = & - 0.0000211 & + 0.0000369 \\
- 2\frac{d \,.\, \delta\nu'_{m''}}{n'dt}(n'\delta z')_m = & - 0.0000093 & - 0.0000053
\end{array}$$

And their sum $= - 0.0000444 n't \cos(6g' - 2g - 3g'') + 0.0000460 n't \sin(6g' - 2g - 3g'')$

In consequence

$$\begin{aligned}
\frac{d \,.\, \delta^2 z'}{dt} = & \left[\ \ \overset{''}{1}.8193 - \overset{''}{0}.0002600 n't\right] \cos(5g' - 2g - 3g'') \\
& + \left[- 0.9874 + 0.001488 n't\right] \sin(5g' - 2g - 3g'') \\
& + \left[- 0.60712 + 0.0003434 n't\right] \cos(6g' - 2g - 3g'') \\
& + \left[\ \ 0.05051 - 0.0006463 n't\right] \sin(6g' - 2g - 3g'')
\end{aligned}$$

And, by integration,

$$\begin{aligned}
n'\delta^2 z' = & \left[- \overset{''}{1}.7848 + \overset{''}{0}.000255 n't\right] \sin(5g' - 2g - 3g'') \\
& + \left[- 0.9698 + 0.001461 n't\right] \cos(5g' - 2g - 3g'') \\
& + \left[\ \ 30.951 - 0.01858 n't\right] \sin(6g' - 2g - 3g'') \\
& + \left[\ \ 3.738 - 0.03496 n't\right] \cos(6g' - 2g - 3g'')
\end{aligned}$$

Neglecting any further consideration of terms in $\delta v'$, we get

$$\begin{aligned}\delta v' = {} & [-\overset{''}{0.8921} + \overset{''}{0.000128}n't]\cos(5g' - 2g - 3g'') + [\overset{''}{0.4848} - \overset{''}{0.000730}n't]\sin(5g' - 2g - 3g'') \\ & + [\;0.4295\;]\cos(6g' - 2g - 3g'') - [0.0151\;]\sin(6g' - 2g - 3g'')\end{aligned}$$

This completes the determination of the inequalities of Saturn having the arguments $5g' - 2g - 3g''$ and $6g' - 2g - 3g''$. But there are certain other inequalities of long period depending on all three anomalies. The following is a list of all that seemingly could be of any importance, together with the logarithms of their integrating factors:

Argument.	Log. integrating factor.	Argument.	Log. integrating factor.
$g'' + 2g' - g$	0.8771n	$6g'' - 7g' + 2g$	1.1527
$g'' - 3g' + g$	0.7796n	$7g'' - 5g' + g$	1.2051n
$2g'' - 3g' + g$	0.7340	$7g'' - 10g' + 3g$	1.0191n
$3g'' + 4g' - 2g$	1.0696	$9g'' - 8g' + 2g$	0.9133
$4g'' + g' - g$	1.0926n	$10g'' - 11g' + 3g$	1.3579n
$4g'' - 4g' + g$	0 9423n	$10g'' - 6g' + g$	1.9784n
$4g'' - 9g' + 3g$	0.8312n		

As the inequalities having these arguments are evidently quite small it will suffice to compute them by the most abbreviated formulæ. We take for $\delta T'$ only the portion independent of γ', and then have

$$n'\delta^2 z' = \iint \delta T' n'^2 dt^2$$

We may limit $\delta T'$ to the following terms

$$\delta T' = A'(n'\delta z')_{w''} + B'v'_{w''} + A'_{w''}(n'\delta z')_{w} + B'_{w''}v'_{w}$$

The following expressions have been found for $\delta T'$ and $n'\delta^2 z'$:

Argument.	$\delta T'$		$n'\delta^2 z'$	
	sin.	cos.	sin.	cos.
	″	″	″	″
$g'' + 2g' - g$	+0.000025	+0.000422	−0.001	−0.024
$g'' - 3g' + g$	+0.001579	+0.004755	−0.057	−0.172
$2g'' - 3g' + g$	+0.00137	−0.00748	−0.040	+0.220
$3g'' + 4g' - 2g$	+0.000414	+0.000272	−0.057	−0.037
$4g'' + g' - g$	+0.000024	+0.000002	−0.004	0.000
$4g'' - 4g' + g$	+0.001188	−0.000898	−0.091	+0.069
$4g'' - 9g' + 3g$	+0.000116	−0.000312	−0.005	+0.014
$6g'' - 7g' + 2g$	+0.000812	+0.000096	−0.164	−0.019
$7g'' - 5g' + g$	+0.000039	−0.000026	−0.010	+0.007
$7g'' - 10g' + 3g$	−0.000002	+0.000123	0.000	−0.013
$9g'' - 8g' + 2g$	+0.000015	+0.000009	−0.001	−0.001
$10g'' - 11g' + 3g$	−0.0000051	+0.0000072	+0.003	−0.004
$10g'' - 6g' + g$	−0.0000001	+0.0000001		

CHAPTER XXIV.

PERTURBATIONS OF THE LATITUDE OF JUPITER OF THE SECOND ORDER WITH RESPECT TO DISTURBING FORCES.

The sensible terms of these perturbations of the latitudes of our two planets arise only from their mutual action. They are quite small, and hence admit some simplifications in their computation, besides those indicated by **Hansen** in the Auseinandersetzung.

The co-ordinate u being obtained through the equations *

$$\frac{1}{n}\frac{d\mathrm{R}_0}{dt} = \frac{h}{n}r\frac{\rho}{a_0}\sin(\omega - f)\left(\frac{d\Omega}{d\mathrm{Z}}\right)\cos i = \mathrm{U}$$

$$u = \overline{\mathrm{R}}_0 + \left(\frac{\overline{d\mathrm{R}_0}}{d\gamma}\right)n\delta z$$

similarly with **Hansen**† we will put

$$\frac{d\,.\,\delta\mathrm{R}_0}{ndt} = \mathrm{A}''n\delta z + \mathrm{B}''(\nu - c) + \mathrm{C}''\delta\frac{h}{h_0} + \mathrm{D}''\frac{u}{\cos i} + \mathrm{E}''\frac{u_1}{\cos i} + \mathrm{F}''n'\delta z' + \mathrm{G}''(\nu' - c') + \mathrm{H}''\frac{u'}{\cos i'}$$

We have

$$\mathrm{A}'' = \frac{d\mathrm{U}}{dg} \qquad\qquad \mathrm{B}'' = \mathrm{Y} + \mathrm{U}$$

where

$$\mathrm{Y} = \frac{h}{n}r^2\frac{\rho}{a_0}\sin(\omega - f)\frac{d^2\Omega}{drd\mathrm{Z}}\cos i = \mathrm{C}a^2r\frac{d^2\Omega}{drd\mathrm{Z}}\cos i$$

C being the quantity so denoted (page 76); $a^2r\frac{d^2\Omega}{drd\mathrm{Z}}$ has already been derived in computing D, one of the factors involved in δT. Next we have

$$\mathrm{C}'' = \mathrm{U}$$

In the computation of D″ and E″ **Hansen** puts $\mathrm{D}'' = \mathrm{D}_1'' + \mathrm{D}_2''$ and $\mathrm{E}'' = \mathrm{E}_1'' + \mathrm{E}_2''$; but D_2'' and E_2'' are quantities of the second order with reference to sin i, sin i', and sin J, besides being of the order of the disturbing force. Hence, $\mathrm{D}_2''\frac{u}{\cos i}$ and $\mathrm{E}_2''\frac{u_1}{\cos i}$, being of the third order with reference to the inclinations and of the second with reference to disturbing forces, may be neglected. We can then write

---

*Auseinandersetzung, Abth. I, pp. 102, 103, gl. (45), (46).

†Auseinandersetzung, Abth. I, p. 133, gl. (83).

$$\begin{aligned} \mathrm{D}'' &= \mathrm{C}a^2\left[a\frac{d^2\Omega}{dZ^2} + \frac{e\sin f}{r\cos^2\varphi}\frac{d\Omega}{df} - \frac{a}{r}\frac{d\Omega}{dr}\right]\cos i \\ &= \mathrm{C}\left[a^3\frac{d^2\Omega}{dZ^2} + \frac{er\sin f}{\cos^3\varphi}\frac{d\Omega}{dg} - \frac{1 + 2e\cos f + e^2}{\cos^4\varphi}ar\frac{d\Omega}{dr}\right]\cos i \end{aligned}$$

$$\mathrm{E}'' = -\mathrm{C}\frac{a^2}{r\cos\varphi}\frac{d\Omega}{df} = -\mathrm{C}\frac{a}{r}\mathrm{T}$$

In addition we have*

$$\mathrm{F}'' = \frac{d\mathrm{U}}{dg'} \qquad \mathrm{G}'' = -(\mathrm{Y} + 2\mathrm{U}) \qquad \mathrm{H}'' = \mathrm{C}a^2a'\frac{d^2\Omega}{dZdZ'}\cos i$$

Thus, we can write

$$\begin{aligned} \delta\mathrm{U} = \frac{d\mathrm{U}}{dg}n\delta z + \frac{d\mathrm{U}}{dg'}n'dz' + \mathrm{Y}(\nu - c - \nu' + c') + \mathrm{U}\left(\nu - 2\nu' + 2c' + \delta\frac{h}{h_0}\right) \\ + \mathrm{D}''\frac{u}{\cos i} + \mathrm{E}''\frac{u_1}{\cos i} + \mathrm{H}''\frac{u'}{\cos i'} \end{aligned}$$

Here $\delta\mathrm{U}$ appears as the sum of seven products instead of eight, as with HANSEN.

It is necessary, then, to compute the developments of the four factors Y, D″, E″, and H″. By neglecting certain terms factored by $\sin^2 \mathrm{J}$ we can put†

$$a^3\frac{d^2\Omega}{dZ^2} = -\mu\alpha^3\left(\frac{\mathrm{a}'}{\triangle}\right)^3 \qquad a^2a'\frac{d^2\Omega}{dZdZ'} = \mu\alpha\left[\left(\frac{\mathrm{a}'}{\triangle}\right)^3 - \left(\frac{\mathrm{a}'}{r'}\right)^3\right]$$

Of the factors involved in D″ it is sufficient to put

$$\frac{er\sin f}{a\cos^3\varphi} = 2[8.3833]\sin g + 2[6.7655]\sin 2g$$

$$\frac{1 + 2e\cos f + e^2}{\cos^4\varphi} = [0.0010] + 2[8.6854]\cos g + 2[7.3688]\cos 2g$$

In addition

$$\left(\frac{\mathrm{a}'}{r'}\right)^3 = 1.0047 + 0.1689\cos g' + 0.0142\cos 2g' + 0.0012\cos 3g'$$

$$\mathrm{C}\frac{a}{r} = 2[9.6980]\sin(\gamma - g) - 2[8.3825]\sin\gamma + 2[8.3817]\sin(\gamma - 2g) + 2[7.12]\sin(\gamma - 3g)$$

In all these computations it is unnecessary to pay attention to the factor cos i, as we intend to take the plane of the orbit at the epoch 1850 as the plane of reference, which makes $i_0 = 0$.

---

*Auseinandersetzung, Abth. I, p. 136, gl. (97), (98), (99).

†Auseinandersetzung, Abth. I, p. 120.

The following is the result of the computation of $\frac{D''}{C}$, the only quantity independent of γ which it is worth while to give:

Arg=$i'g'+ig$	$\frac{D''}{C}$		Arg=$i'g'+ig$	$\frac{D''}{C}$		Arg=$i'g'+ig$	$\frac{D''}{C}$	
	cos.	sin.		cos.	sin.		cos.	sin.
i' i	″	″	i' i	″	″	i' i	″	″
0 0	−28.33		3 0	0.00	− 0.12	5 − 4	−1.57	−6.29
0 − 1	+ 1.41	+ 3.76	3 − 1	− 1.46	+ 0.29	5 − 5	−7.69	+4.22
0 − 2	+ 0.18	− 0.10	3 − 2	+ 6.24	+ 8.25	5 − 6	−0.99	+0.07
1 + 2	+ 0.02	+ 0.03	3 − 3	+14.58	−20.17	6 − 3	+0.28	+0.20
1 + 1	+ 0.38	− 0.30	3 − 4	+ 1.15	− 0.89	6 − 4	+0.43	−1.63
1 0	− 6.46	− 4.07	3 − 5	+ 0.11	0.00	6 − 5	−4.57	+0.16
1 − 1	− 7.99	+38.74	4 − 1	− 0.20	− 0.06	6 − 6	+1.35	+4.86
1 − 2	+ 1.31	+ 0.04	4 − 2	+ 0.23	+ 1.89	6 − 7	−0.14	+0.69
1 − 3	− 0.03	− 0.08	4 − 3	+ 7.75	− 3.73	7 − 4	+0.24	−0.26
2 + 1	+ 0.05	− 0.02	4 − 4	− 9.96	−11.21	7 − 5	−1.25	−0.62
2 0	− 0.44	− 0.88	4 − 5	− 0.42	− 1.20	7 − 6	−0.61	+3.06
2 − 1	− 6.66	+ 5.88	4 − 6	+ 0.01	− 0.08	7 − 7	+2.84	−0.10
2 − 2	+36.11	+15.71	5 − 2	− 0.12	+ 0.26	7 − 8	+0.44	+0.21
2 − 3	+ 1.26	+ 0.48	5 − 3	+ 1.90	+ 0.15			
2 − 4	+ 0.02	+ 0.07						

We have now to multiply certain expressions already obtained by the factor C (value given at page 77), and thus obtain the expressions for the four following quantities:

Arg=$\varkappa\gamma+i'g'+ig$	Y		D″		E″		H″	
	cos.	sin.	sin.	cos.	cos.	sin.	sin.	cos.
$\varkappa$ i' i	″	″	″	″	″	″	″	″
1 0 − 1	+0.032	+0.145	−28.33	+ 0.19	0.00	+0.02	+21.32	− 0.27
−1 0 0	+0.327	+0.753	− 2.75	+ 1.88	−0.02	+0.09	+ 2.68	− 2.82
1 0 − 2	−0.320	−0.749	+ 0.01	− 1.88	+0.02	−0.09	− 0.62	+ 2.83
−1 0 − 1	+0.051	−0.120	− 0.04	− 0.19	+0.01	0.00	+ 0.06	+ 0.28
1 0 − 3	−0.083	+0.048	+ 0.11	0.00	−0.01	0.00	− 0.17	− 0.01
−1 1 + 2	−0.120	+0.097	− 0.11	− 0.20	0.00	0.00	+ 0.21	+ 0.29
1 1 0	+0.181	−0.054	+ 0.42	0.00	0.00	−0.02	− 0.52	0.00
−1 1 + 1	+0.647	+0.435	+ 3.34	− 1.55	+0.02	−0.10	− 3.30	+ 2.38
1 1 − 1	−0.646	−0.435	− 2.94	+ 3.43	−0.04	+0.22	+ 2.71	− 5.10
−1 1 0	−0.150	+0.042	+ 3.74	+19.50	−0.24	+1.22	− 5.49	−28.05
1 1 − 2	+0.157	−0.100	− 4.12	−19.30	+0.22	−1.20	+ 5.92	+27.75
−1 1 − 1	+0.710	−0.153	− 0.94	− 1.38	−0.12	−0.16	+ 1.72	+ 1.91
1 1 − 3	−0.699	+0.140	+ 0.55	− 0.49	+0.14	+0.04	− 1.16	+ 0.78
−1 1 − 2	−0.068	−0.044	+ 0.07	− 0.04	0.00	−0.01	− 0.10	+ 0.07
1 1 − 4	0.000	+0.059	0.00	+ 0.04	+0.02	+0.01	− 0.02	− 0.06

Arg=$\varkappa y+i'g'+ig$			Y		D″		E″		H″	
			cos.	sin.	sin.	cos.	cos.	sin.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
—1	2+	1	+0.076	+0.179	+ 0.30	— 0.37	+0.02	—0.01	— 0.28	+ 0.56
1	2—	1	—0.022	—0.232	+ 0.02	+ 0.65	—0.12	0.00	— 0.13	— 1.00
—1	2	0	+0.561	—0.557	+ 2.88	+ 3.16	—0.84	—0.12	— 3.86	— 4.80
1	2—	2	—0.556	+0.551	— 4.64	— 2.36	+1.56	+0.44	+ 5.51	+ 4.01
—1	2—	1	+0.085	+0.164	—18.29	+ 7.64	+7.58	+3.25	+17.27	— 7.06
1	2—	3	—0.139	—0.166	+17.90	— 7.90	—7.44	—3.19	—16.82	+ 7.53
—1	2—	2	—0.016	—0.573	+ 0.68	— 0.33	—0.21	+0.18	— 0.76	+ 0.85
1	2—	4	+0.001	+0.559	+ 1.07	— 0.43	—0.51	—0.50	— 0.88	— 0.13
—1	2—	3	—0.026	+0.030	+ 0.04	+ 0.02	—0.01	0.00	— 0.04	— 0.02
1	2—	5	+0.026	+0.025	+ 0.03	— 0.04	—0.03	—0.05	0.00	— 0.01
—1	3	0	+0.233	—0.046	+ 0.65	+ 0.24	—0.14	—0.09	— 0.90	— 0.39
1	3—	2	—0.267	—0.013	— 0.96	+ 0.16	+0.23	+0.24	+ 1.22	— 0.06
—1	3—	1	—0.372	—0.565	— 3.34	+ 3.87	+0.98	+1.49	+ 3.53	— 4.26
1	3—	3	+0.364	+0.553	+ 2.57	— 4.85	—0.66	—1.91	— 2.80	+ 5.18
—1	3—	2	+0.168	—0.107	— 7.06	—10.38	+3.18	—4.59	+ 6.42	+ 9.58
1	3—	4	—0.172	+0.154	+ 7.32	+ 9.94	—3.22	+4.43	— 6.73	— 9.10
—1	3—	3	—0.408	—0.072	— 0.04	+ 0.29	+0.23	+0.12	— 0.30	— 0.27
1	3—	5	+0.390	+0.087	+ 0.74	+ 0.68	—0.53	+0.32	— 0.34	— 0.62
—1	4	0	+0.049	+0.017	+ 0.10	— 0.01	—0.01	—0.02	— 0.14	+ 0.01
1	4—	2	—0.048	—0.042	— 0.11	+ 0.10	+0.01	+0.04	+ 0.15	— 0.12
—1	4—	1	—0.006	—0.254	— 0.22	+ 0.89	—0.01	+0.27	+ 0.22	— 1.07
1	4—	3	—0.044	+0.268	— 0.18	— 1.07	+0.16	—0.34	+ 0.18	+ 1.24
—1	4—	2	—0.469	+0.193	— 3.74	— 2.06	+1.54	—0.68	+ 3.86	+ 2.06
1	4—	4	+0.456	—0.183	+ 4.23	+ 1.43	—1.76	+0.42	— 4.31	— 1.46
—1	4—	3	—0.093	—0.165	+ 5.27	— 5.48	—2.41	—2.55	— 4.82	+ 4.94
1	4—	5	+0.127	+0.173	— 4.85	+ 5.61	+2.25	+2.57	+ 4.40	— 5.08
—1	4—	4	—0.106	+0.269	— 0.15	— 0.19	+0.09	—0.22	+ 0.10	— 0.03
1	4—	6	+0.118	—0.254	— 0.33	+ 0.73	+0.13	+0.46	+ 0.34	— 0.46
—1	5—	1	+0.035	—0.053	+ 0.04	+ 0.13	—0.02	+0.04	— 0.05	— 0.17
1	5—	3	—0.058	+0.048	— 0.13	— 0.12	+0.06	—0.02	+ 0.16	+ 0.16
—1	5—	2	—0.240	—0.040	— 0.93	— 0.01	+0.33	+0.04	+ 1.04	0.00
1	5—	4	+0.241	+0.078	+ 1.01	— 0.31	—0.35	—0.18	— 1.10	+ 0.32
—1	5—	3	+0.062	+0.348	+ 0.94	— 3.10	—0.32	—1.34	— 0.90	+ 3.08
1	5—	5	—0.054	—0.335	— 0.48	+ 3.29	+0.10	+1.42	+ 0.46	— 3.26
—1	5—	4	—0.149	+0.064	+ 3.79	+ 2.34	—1.80	+1.10	— 3.41	— 2.14
1	5—	6	+0.159	—0.086	— 3.82	— 2.03	+1.80	—0.96	+ 3.45	+ 1.82
—1	5—	5	+0.160	+0.103	+ 0.21	— 0.11	—0.17	—0.09	— 0.08	+ 0.07
1	5—	7	—0.144	—0.110	— 0.58	— 0.09	+0.35	—0.01	+ 0.41	+ 0.12
—1	6—	2	—0.045	—0.045	— 0.15	+ 0.08	+0.05	+0.04	+ 0.17	— 0.10
1	6—	4	+0.037	+0.062	+ 0.12	— 0.16	—0.03	—0.06	— 0.14	+ 0.18
—1	6—	3	—0.072	+0.188	— 0.15	— 0.82	+0.10	—0.31	+ 0.19	+ 0.88
1	6—	5	+0.097	—0.182	+ 0.38	+ 0.82	—0.20	+0.31	— 0.41	— 0.87
—1	6—	4	+0.228	+0.007	+ 2.28	+ 0.20	—1.01	+0.02	— 2.22	— 0.17
1	6—	6	—0.215	—0.012	— 2.32	+ 0.12	+1.03	+0.11	+ 2.24	— 0.12

Arg=$n\gamma+i'g'+ig$	Y		D″		E″		H″	
	cos.	sin.	sin.	cos.	cos.	sin.	sin.	cos.
n i' i	″	″	″	″	″	″	″	″
−1 6−5	+0.038	+0.123	−0.84	+2.43	+0.40	+1.16	+0.77	−2.17
1 6−7	−0.051	−0.132	+0.63	−2.41	−0.30	−1.14	−0.56	+2.14
−1 6−6	+0.082	−0.087	+0.12	+0.16	−0.08	+0.11	−0.06	−0.09
1 6−8	−0.086	+0.075	−0.05	−0.40	+0.04	−0.22	+0.01	+0.30
−1 7−3			−0.11	−0.14	+0.04	−0.04	+0.12	+0.15
1 7−5			+0.16	+0.11	−0.07	+0.02	−0.19	−0.12
−1 7−4	+0.134	+0.086	+0.64	−0.26	−0.25	−0.14	−0.66	+0.29
1 7−6	−0.125	−0.102	−0.60	+0.42	+0.23	+0.20	+0.62	−0.43
−1 7−5	+0.030	−0.131	+0.23	+1.55	−0.12	+0.70	−0.21	−1.47
1 7−7	−0.033	+0.121	−0.41	−1.52	+0.20	−0.68	+0.40	+1.44
−1 7−6	+0.097	−0.009	−1.45	−0.16	+0.70	−0.07	+1.28	+0.15
1 7−8	−0.104	+0.016	+1.39	+0.02	−0.66	+0.01	−1.24	−0.02
−1 7−7			−0.12	+0.10	+0.07	+0.07	+0.08	−0.06
1 7−9			+0.25	−0.10	−0.13	−0.07	−0.20	+0.06

From the expression of R_0 given in Chapter II we derive that of $\left(\overline{\frac{dR_0}{d\gamma}}\right)$:

Arg=$i'g'+ig$	$\left(\overline{\frac{dR_0}{d\gamma}}\right)$		Arg=$i'g'+ig$	$\left(\overline{\frac{dR_0}{d\gamma}}\right)$	
	cos.	sin.		cos.	sin.
i' i	″	″	i' i	″	″
0−1	+0.12	+0.28	4−1	+0.02	+0.02
	−0.2825nt	+0.1248nt	4−2	+0.01	+0.05
0−2	+0.02	−0.02	4−3	−0.31	+0.12
	−0.0136nt	+0.0060nt	4−4	−0.03	−0.03
0−3	−0.0007nt	+0.0003nt	4−5	−0.01	+0.03
1+1	+0.14	−0.04	5−1	−0.07	−0.16
1 0	+0.18	+0.12	5−2	+0.01	0.00
1−1	+0.06	−0.03	5−3	+3.60	+0.34
1−2	+0.41	−0.10	5−4	+0.18	+0.11
1−3	+0.01	0.00	5−5	−0.01	+0.01
2 0	−0.01	+0.30	5−6	+0.01	0.00
2−1	−0.09	+0.09	6−3	+0.01	+0.01
2−2	+0.13	+0.22	6−4	−0.03	+0.07
2−3	+0.01	−0.14	6−5	+0.03	0.00
3 0	+0.04	−0.05	6−6	0.00	+0.01
3−1	+0.04	+0.01			
3−2	+0.44	+0.69	7−4	−0.04	+0.04
3−3	+0.07	0.00	7−5	+0.03	+0.01
3−4	−0.06	−0.01	7−6	0.00	−0.01

In order to obtain a higher degree of precision in the second factors of the terms of δU, we attribute to $n\delta z$, ν, $n'\delta z'$, and ν' the values which result from adding together the terms of the first, second, and third orders with respect to disturbing forces. It is thought unnecessary to give here the expressions of these second factors, as they are so easily formed from data previously given.

The terms having the argument $-\gamma$ in δU are important, as they determine the amount of the secular motion of the plane of the orbit due to the square and product of the masses of Jupiter and Saturn. The following detail of the composition of these terms is therefore given:

$$\begin{array}{rll}
 & \quad '' & \\
\frac{dU}{dg}n\delta z = & -0.0002260\cos(-\gamma) & -0.0006051\sin(-\gamma) \\
\frac{dU}{dg'}n'\delta z' = & -0.0006676 & -0.0021977 \\
U\left(\nu - 2\nu' + 2c' + \delta\frac{h}{h_0}\right) = & -0.0002280 & -0.0006215 \\
Y(\nu - c - \nu' + c') = & -0.0004473 & -0.0012610 \\
D''\frac{u}{\cos i} = & -0.0000396 & -0.0000871 \\
E''\frac{u_1}{\cos i} = & -0.0000051 & -0.0000128 \\
H''\frac{u'}{\cos i'} = & -0.0001084 & -0.0002514 \\
\hline
\text{Sum} \quad = & -0.0017220\cos(-\gamma) & -0.0050366\sin(-\gamma)
\end{array}$$

The following is the complete expression for δU:

Arg= $\varkappa\gamma+i'g'+ig$			δU			
			cos.	*nt* cos.	sin.	*nt* sin.
$\varkappa$	i'	i	″	″	″	″
−1	0	0	−0.0017220	−0.000070136	−0.0050366	+0.000033274
1	0	−2	+0.001	+0.00007	+0.002	−0.00003
−1	0	−1	−0.0025		+0.0015	
1	0	−3	+0.002		−0.001	
−1	1	+2	+0.001		−0.003	
1	1	0	−0.0025	−0.000006	+0.0028	+0.000001
−1	1	+1	+0.0005	+0.000032	+0.0008	−0.000045
1	1	−1	+0.0029	−0.000032	+0.0015	+0.000045
−1	1	0	+0.0001	−0.000021	−0.0004	0.000000
−1	1	−1	−0.0020	+0.000011	+0.0001	+0.000049
−1	1	−2	+0.001		+0.001	
−1	2	+1	−0.002		−0.003	
1	2	−1	+0.0029	−0.000005	+0.0033	−0.000005
−1	2	0	−0.0019	−0.000049	+0.0031	−0.000050
1	2	−2	+0.0017	+0.000047	−0.0020	+0.000050
−1	2	−1	−0.0008	−0.000004	+0.0003	+0.000021

Arg= $\varkappa\gamma+i'g'+ig$			δU			
			cos.	nt cos.	sin.	nt sin
$\varkappa$	i'	i	$''$	$''$	$''$	$''$
—1	3	0	—0.0055		+0.0024	
1	3—	2	+0.0039	—0.000001	—0.0017	+0.000005
—1	3—	1	+0.0024	—0.000040	+0.0024	+0.000025
—1	4+	1	—0.001		+0.002	
1	4—	1	+0.0008		—0.0012	
—1	4—	1	+0.0004	—0.000004	+0.0022	0.000000
1	4—	3	—0.0003	+0.000011	—0.0016	+0.000002
—1	4—	2	+0.0002	+0.000008	—0.0003	+0.000024
—1	5	0	+0.002		+0.002	
1	5—	2	+0.000027	+0.00000107	—0.000030	—0.00000037
—1	5—	2	+0.001389	+0.00000166	+0.000612	+0.00000084
1	5—	4	+0.002		0.000	
—1	10—	4	+0.00100		—0.00185	

By subjecting the preceding expression to the same treatment as that by which, in Chapter II, $\frac{u}{\cos i}$ was derived from $\frac{1}{n}\frac{dR_0}{dt}$, we obtain the following:

Arg= $i'g'+ig$		$\overline{\delta R_0}$	
		sin.	cos.
i'	i	$''$ $''$	$''$ $''$
0	0		+0.0001247nt +0.00000254n^2t^2
0—	1	—0.0050721nt +0.00001664n^2t^2	—0.0017371nt —0.00003507n^2t^2
0—	2	+0.0018—0.0001214nt +0.00000040n^2t^2	+0.0011—0.0000415nt —0.00000085n^2t^2
0—	3	—0.0000044nt +0.00000001n^2t^2	—0.0000015nt —0.00000003n^2t^2
1+	2	—0.0001	—0.0002
1+	1	—0.0059—0.000017nt	—0.0056—0.000005nt
1	0	—0.0039+0.000081nt	+0.0022+0.000107nt
1—	1	+0.0003—0.000055nt	+0.0005—0.000011nt
1—	2	+0.0035—0.000018nt	+0.0002+0.000082nt
1—	3	—0.0005	+0.0006
2+	1	—0.0004	+0.0004
2	0	—0.0158+0.000030nt	+0.0190—0.000030nt
2—	1	—0.0032—0.000103nt	—0.0070+0.000098nt
2—	2	+0.0047+0.000023nt	+0.0016+0.000105nt
2—	3	+0.0001	0.0000

$\mathrm{Arg}=i'g'+ig$	$\overline{\delta R_0}$	
	sin.	cos.
i' i	$''$ $''$	$''$ $''$
3 — 1	$-0.0104+0.000015nt$	$-0.0032+0.000015nt$
3 — 2	$+0.0115-0.000192nt$	$-0.0120-0.000120nt$
3 — 3	$+0.0003-0.000005nt$	$-0.0003-0.000003nt$
4 0	$+0.0009$	$+0.0012$
4 — 2	$+0.0008-0.000013nt$	$-0.0047-0.000003nt$
4 — 3	$-0.0003-0.000020nt$	$-0.0008+0.000062nt$
5 — 1	$+0.0009+0.000080nt$	$+0.0072+0.000028nt$
5 — 2	$-0.000015nt$	$+0.000002nt$
5 — 3	$+0.1071+0.000124nt$	$-0.0363-0.000063nt$
5 — 4	$+0.0026$	-0.0009
5 — 5	$+0.0001$	0.0000
10 — 5	$+0.0372$	$+0.0689$
10 — 6	$+0.0009$	$+0.0017$

In order to have the value of $\delta\left(\frac{u}{\cos i}\right)$ it is necessary to add to this expression the value of $\left(\frac{d\overline{R_0}}{d\gamma}\right)n\delta z$, which follows:

$\mathrm{Arg}=i'g'+ig$	$\left(\frac{d\overline{R_0}}{d\gamma}\right)n\delta z$	
	sin.	cos.
i' i	$''$ $''$	$''$ $''$
0 0		$+0.00000057n^2t^2$
0 — 1	$-0.00000004n^2t^2$	$+0.00000004n^2t^2$
0 — 2	$+0.00000046n^2t^2$	$+0.00000135n^2t^2$
0 — 3	$+0.00000002n^2t^2$	$+0.00000008n^2t^2$
1 + 1	$-0.0007+0.000005nt$	$-0.0007-0.000007nt$
1 0	$-0.000034nt$	$-0.000049nt$
1 — 1	$+0.000009nt$	$-0.000001nt$
1 — 2	$+0.000014nt$	$-0.000059nt$
2 0	$-0.0014-0.000090nt$	$+0.0020+0.000041nt$
2 — 1	$+0.0005-0.000100nt$	$+0.0006+0.000106nt$
2 — 2	$-0.000083nt$	$-0.000039nt$
2 — 3	$-0.000146nt$	$-0.000002nt$
2 — 4	$-0.000007nt$	$0.000000nt$
3 0	$-0.0005-0.000004nt$	$+0.0006+0.000010nt$
3 — 1	$-0.0016+0.000014nt$	$-0.0006-0.000060nt$
3 — 2	$-0.0011-0.000020nt$	$+0.0001-0.000002nt$
3 — 3	$+0.000054nt$	$-0.000030nt$
3 — 4	$-0.000002nt$	$-0.000012nt$

Arg=$i'g'+ig$	$\left(\overline{\frac{dR_0}{d\gamma}}\right)n\delta z$	
	sin.	cos.
i' i	″ ″	″ ″
4 0	+0.0008	+0.0010
4 − 1	−0.000003nt	−0.000012nt
4 − 2	0.0000−0.000011nt	+0.0007 0.000000nt
4 − 3	+0.000007nt	−0.000010nt
4 − 4	−0.000007nt	−0.000008nt
5 − 0	−0.000032nt	−0.000030nt
5 − 1	−0.0007−0.000655nt	+0.0006−0.000620nt
5 − 2	+0.000117nt	−0.000069nt
5 − 3	+0.0009+0.000012nt	0.0000−0.000904nt
5 − 4	+0.000101nt	−0.000015nt
5 − 5	+0.000005nt	+0.000002nt
6 − 4	+0.0004	+0.0019
7 − 2	+0.0008	−0.0003
7 − 4	+0.0018	−0.0001
7 − 5	+0.0015	−0.0008
8 − 5	−0.0004	+0.0006
9 − 5	0.0000	−0.0010
10 − 5	+0.0053	+0.0093
10 − 6	−0.0014	+0.0002

Thus we have the following expression for $\delta\left(\frac{u}{\cos i}\right)$:

Arg=$i'g'+ig$	$\delta\left(\frac{u}{\cos i}\right)$	
	sin.	cos.
i' i	″ ″	″ ″
0 0		+0.0001247nt
		+0.0000311n^2t^2
0 − 1	−0.0050721nt	−0.0017371nt
	+0.0001660n^2t^2	−0.0003503n^2t^2
0 − 2	+0.0018−0.0001214nt	+0.0011−0.0000415nt
	+0.00000086n^2t^2	+0.0000050n^2t^2
0 − 3	−0.0000044nt	−0.0000015nt
	+0.00000003n^2t^2	+0.00000005n^2t^2
1 + 2	−0.0001	−0.0002
1 + 1	−0.0066−0.000012nt	−0.0063−0.000012nt
1 0	−0.0039+0.000047nt	+0.0022+0.000058nt
1 − 1	+0.0003−0.000046nt	+0.0005−0.000012nt
1 − 2	+0.0035−0.000004nt	+0.0002+0.000023nt
1 − 3	−0.0005	+0.0006

Arg$=i'g'+ig$	$\delta\left(\frac{u}{\cos i}\right)$	
	sin.	cos.
i' i	$''$ $''$	$''$ $''$
$2+$ 1	-0.0004	$+0.0004$
2 0	$-0.0172-0.000060nt$	$+0.0210+0.000011nt$
$2-$ 1	$-0.0027-0.000203nt$	$-0.0064+0.000204nt$
$2-$ 2	$+0.0047-0.000060nt$	$+0.0016+0.000066nt$
$2-$ 3	$+0.0001-0.000146nt$	$0.0000-0.000002nt$
$2-$ 4	$-0.000007nt$	$0.000000nt$
3 0	$-0.0005-0.000004nt$	$+0.0006+0.000010nt$
$3-$ 1	$-0.0120+0.000029nt$	$-0.0038-0.000045nt$
$3-$ 2	$+0.0104-0.000212nt$	$-0.0119-0.000122nt$
$3-$ 3	$+0.0003+0.000049nt$	$-0.0003-0.000033nt$
$3-$ 4	$-0.000002nt$	$-0.000012nt$
4 0	$+0.0017$	$+0.0022$
$4-$ 1	$-0.000003nt$	$-0.000012nt$
$4-$ 2	$+0.0008-0.000024nt$	$-0.0040-0.000003nt$
$4-$ 3	$-0.0003-0.000013nt$	$-0.0008+0.000052nt$
$4-$ 4	$-0.000007nt$	$-0.000008nt$
5 0	$-0.000032nt$	$-0.000030nt$
$5-$ 1	$+0.0002-0.000575nt$	$+0.0078-0.000592nt$
$5-$ 2	$+0.000102nt$	$-0.000067nt$
$5-$ 3	$+0.1080+0.000136nt$	$-0.0363-0.000967nt$
$5-$ 4	$+0.0026+0.000101nt$	$-0.0009-0.000015nt$
$5-$ 5	$+0.0001+0.000005nt$	$0.0000+0.000002nt$
$6-$ 4	$+0.0004$	$+0.0019$
$7-$ 2	$+0.0008$	-0.0003
$7-$ 4	$+0.0018$	-0.0001
$7-$ 5	$+0.0015$	-0.0008
$8-$ 5	-0.0004	$+0.0006$
$9-$ 5	0.0000	-0.0010
$10-$ 5	$+0.0425$	$+0.0782$
$10-$ 6	-0.0005	$+0.0019$

The only terms of the third order with respect to disturbing forces in the latitude of Jupiter which it seems worth while to consider are the secular terms factored by the cube of the time. As these terms are scarcely appreciable after the lapse of 500 years a very rude computation of their coefficients suffices. If b, l, i, and θ denote severally the latitude, longitude, inclination of orbit, and longitude of the ascending node of Jupiter, all referred to the fixed ecliptic and equinox of 1850.0, we have

$$\sin b = \sin i \sin (l-\theta) = \sin i \cos \theta \sin l - \sin i \sin \theta \cos l$$

and

$$\frac{d(\sin i \cos \theta)}{dt} = \cos i \cos \theta \frac{di}{dt} - \sin i \sin \theta \frac{d\theta}{dt}$$

$$\frac{d(\sin i \sin \theta,}{dt} = \cos i \sin \theta \frac{di}{dt} + \sin i \cos \theta \frac{d\theta}{dt}$$

The following values of i and θ, adopted from the values given by LEVERRIER,* are sufficiently accurate for our purpose:

Date.	i	θ	$\frac{di}{dt}$	$\frac{d\theta}{dt}$
	° ′ ″	° ′ ″	″	″
1850	1 18 42.10	98 56 19.79	−0.073673	+6.29075
2350	1 18 8.30	99 50 4.24	0.061480	6.60017
2850	1 17 40.70	100 46 13.00	0.048825	6.86522
3350	1 17 19.53	101 44 21.07	0.035808	7.08005
3850	1 17 4.94	102 44 3.45	−0.022518	+7.23977

From these we derive the values of the two functions, which follow, for the same dates:

Date.	$\frac{d(\sin i \cos\theta)}{dT}$	$\frac{d(\sin i \sin\theta)}{dT}$
	″	″
1850	−13.08107	−9.51345
2350	13.73032	8.61823
2850	14.32572	7.69386
3350	14.86245	6.74492
3850	−15.33640	−5.77386

The unit of T is a century of Julian years. From these special values we obtain

$$\frac{d(\sin i \cos\theta)}{dT} = -\overset{\prime\prime}{13.08107} - \overset{\prime\prime}{0.134876}T + \overset{\prime\prime}{0.000967}T^2$$

$$\frac{d(\sin i \sin\theta)}{dT} = -\ 9.51345 + 0.175718T + 0.000714T^2$$

and by integrating these equations

$$\sin i \cos\theta = \text{const.} - \overset{\prime\prime}{13.0811}T - \overset{\prime\prime}{0.06744}T^2 + \overset{\prime\prime}{0.000322}T^3$$

$$\sin i \sin\theta = \text{const.} - \ 9.5134T + 0.08786T^2 + 0.000238T^3$$

As these values correspond to a mass of Saturn $= \frac{1}{3512}$ we augment them a little, and adopt as the terms of the sine of the latitude proportional to the cube of the time,

$$\sin b = +\ 0''.000324T^3 \sin l - 0''.000240T^3 \cos l$$

---

*Annales de l'Observatoire de Paris, Mémoires, Tome XI, pp. 7, 47.

CHAPTER XXV.

PERTURBATIONS OF THE LATITUDE OF SATURN OF THE SECOND ORDER WITH RESPECT TO DISTURBING FORCES.

The formulæ to be employed here are precisely similar to those of the preceding chapter. The co-ordinate u' is obtained through the equations

$$\frac{1}{n'}\frac{d\mathrm{R}_0'}{dt} = \frac{h'}{n'}r'\frac{\rho'}{a'}\sin(\omega' - f')\left(\frac{d\Omega'}{d\mathrm{Z}'}\right)\cos i' = \mathrm{U}'$$

$$u' = \overline{\mathrm{R}_0'} + \left(\frac{\overline{d\mathrm{R}_0'}}{d\gamma'}\right)n'\delta z'$$

For the variation of U′ we have

$$\delta\mathrm{U}' = \frac{d\mathrm{U}'}{dg'}n'\delta z' + \frac{d\mathrm{U}'}{dg}n\delta z + \mathrm{Y}'(\nu' - c' - \nu + c) + \mathrm{U}'\left(\nu' - 2\nu + 2c + \delta\frac{h'}{h_0'}\right)$$
$$+ \mathrm{D}''\frac{u'}{\cos i'} + \mathrm{E}''\frac{u_1'}{\cos i'} + \mathrm{H}''\frac{u}{\cos i}$$

Here we have

$$\mathrm{Y}' = \mathrm{C}'a'^2r'\frac{d^2\Omega'}{dr'd\mathrm{Z}'}\cos i'$$

C′ being the quantity so designated at page 78.

$$\mathrm{D}'' = \mathrm{C}'\left[a'^3\frac{d^2\Omega'}{d\mathrm{Z}'^2} + \frac{e'r'\sin f'}{\cos^3\varphi'}\frac{d\Omega'}{dg'} - \frac{1 + 2e'\cos f + e^2}{\cos^4\varphi'}a'r'\frac{d\Omega'}{dr'}\right]\cos i'$$

$$\mathrm{E}'' = -\mathrm{C}'\frac{a'}{r'}\mathrm{T}'$$

$$\mathrm{H}'' = \mathrm{C}'a'^2a\frac{d^2\Omega'}{d\mathrm{Z}d\mathrm{Z}'}\cos i'$$

By neglecting certain terms factored by $\sin^2 \mathrm{J}$ we can put

$$a'^3\frac{d^2\Omega'}{d\mathrm{Z}'^2} = -\mu'\left(\frac{\mathrm{a}'}{\Delta}\right)^3 \qquad a'^2a\frac{d^2\Omega'}{d\mathrm{Z}d\mathrm{Z}'} = \mu'\alpha\left[\left(\frac{\mathrm{a}'}{\Delta}\right)^3 - \frac{1}{\alpha^3}\left(\frac{\mathrm{a}}{r}\right)^3\right]$$

Of the factors involved in D'' it is sufficient to put

$$\frac{e'r'\sin f'}{a'\cos^3\varphi'} = 2[8.4488]\sin g' + 2[6.8961]\sin 2g'$$

$$\frac{1 + 2e'\cos f' + e'^2}{\cos^4\varphi'} = [0.0014] + 2[8.7514]\cos g' + 2[7.50]\cos 2g'$$

In addition

$$\frac{1}{a^3}\left(\frac{a}{r}\right)^3 = 6.192 + 0.895\cos g + 0.065\cos 2g + 0.005\cos 3g$$

$$C'\frac{a'}{r'} = 2[9.6976]\sin(\gamma' - g') - 2[8.4478]\sin\gamma' + 2[8.4466]\sin(\gamma' - 2g') + 2[7.25]\sin(\gamma' - 3g')$$

In all these computations it is unnecessary to attend to the factor cos i', as it is intended to take the plane of the orbit at the epoch 1850 as the plane of reference, which makes $i_0' = 0$.

The following is the expression for $\frac{D''}{C'}$, the only quantity independent of γ' it is worth while to give:

Arg=$i'g'+ig$	$\frac{D''}{C'}$ cos.	$\frac{D''}{C'}$ sin.
i' i	"	"
0 0	−176.8	
1 0	− 55.5	− 44.8
2 0	− 3.9	− 10.2
3 0	+ 0.1	− 1.4
−2 − 1	+ 0.8	+ 0.3
−1 − 1	+ 5.2	+ 4.6
0 − 1	+ 20.9	+ 46.0
1 − 1	+ 63.6	−298.4
2 − 1	− 52.6	+ 22.3
3 − 1	− 13.7	+ 0.3
4 − 1	− 2.2	− 0.7
−1 − 2	+ 0.1	− 0.3
0 − 2	+ 2.5	− 1.7
1 − 2	+ 38.1	− 64.2
2 − 2	+227.2	+ 94.9
3 − 2	+ 43.3	+ 62.7

Arg=$i'g'+ig$	$\frac{D''}{C'}$ cos.	$\frac{D''}{C'}$ sin.
i' i	"	"
4 − 2	+ 0.9	+ 16.0
5 − 2	− 1.3	+ 2.4
0 − 3	− 0.1	− 0.1
1 − 3	+ 0.8	− 6.2
2 − 3	+ 5.6	− 9.9
3 − 3	+91.1	−127.3
4 − 3	+56.1	− 24.7
5 − 3	+14.9	+ 1.8
6 − 3	+ 2.4	+ 1.7
2 − 4	− 0.3	− 0.1
3 − 4	− 0.7	− 5.7
4 − 4	−63.4	− 70.6
5 − 4	− 9.7	− 44.0
6 − 4	+ 3.8	− 12.3
7 − 4	+ 2.2	− 1.9

Arg=$i'g'+ig$	$\frac{D''}{C'}$ cos.	$\frac{D''}{C'}$ sin.
i' i	"	"
3 − 5	0.0	0.0
4 − 5	− 3.7	− 3.0
5 − 5	−48.7	+27.2
6 − 5	−31.3	+ 0.4
7 − 5	− 9.0	− 5.0
8 − 5	− 1.5	− 2.2
4 − 6	− 0.1	− 0.2
5 − 6	− 3.6	+ 1.6
6 − 6	+ 8.7	+30.9
7 − 6	− 4.5	+20.7
8 − 6	− 5.1	+ 5.9
5 − 7	− 0.1	+ 0.1
6 − 7	+ 0.1	+ 3.0
7 − 7	+18.3	− 0.9
8 − 7	+12.5	+ 5.4

We have now to multiply certain expressions already obtained by the factor C′ (value given at page 78), and thus obtain the expressions for the following quantities:

Arg= $\varkappa\gamma'+i'g'+ig$			Y		D″		E″		H″	
			cos.	sin.	sin.	cos.	cos.	sin.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	1	0	−0.99	−2.07	+175.0	+2.5	+0.2	+0.1	+426.7	−2.1
−1	2	0	+8.68	+5.75	+32.4	−22.0	−1.0	+4.8	−16.2	+18.5
1	0	0	−8.80	−6.22	−12.8	+22.5	+1.0	−4.8	+64.2	−18.9
−1	3	0	+1.32	+2.51	+2.7	−5.6	−0.5	+0.8	−2.7	+4.6
1	1	0	−0.32	−1.88	+0.4	+3.2	+0.3	−0.2	−0.5	−2.5
−1	4	0	−0.04	+0.55	+0.1	−0.8	−0.1	0.0	0.0	+0.7
1	2	0	+0.16	−0.28	+0.2	+0.3	+0.1	0.0	−0.3	−0.3
−1	−1	1	+0.18	−0.06	−0.2	0.0			+0.2	+0.1
1	−3	1	−0.38	−0.07	+0.5	−0.3			−0.3	0.0
−1	0	−1	+1.34	+0.27	−1.7	+0.4	−0.3	+1.7	−2.8	0.0
1	−2	−1	−1.87	−1.57	+2.9	−2.9	−0.3	+1.5	−3.3	+1.9
−1	1	−1	+5.23	+11.53	−7.8	+35.6	+8.9	−42.5	+57.9	−2.9
1	−1	−1	−4.85	−11.53	+11.1	−18.6	−2.7	+13.0	−53.8	+22.0
−1	2	−1	−1.95	+2.00	−34.3	−149.3	−55.2	+260.9	−31.6	−168.7
1	0	−1	+1.88	−0.06	+30.2	+150.6	+55.0	−264.7	+40.6	+169.9
−1	3	−1	+5.30	−5.43	+24.8	+6.9	+3.5	+19.6	−26.7	−32.7
1	1	−1	−5.76	+5.63	−29.2	−23.6	−9.9	+9.9	+23.4	+13.6
−1	4	−1	+2.67	−0.75	+7.4	+0.5	+1.6	+1.1	−6.6	−2.5
1	2	−1	−2.08	+0.08	−4.6	+0.7	−0.8	−0.5	+3.8	−0.6
−1	5	−1	+0.60	+0.17	+1.3	−0.4	+0.2	+0.2	−1.1	+0.2
1	3	−1	−0.32	−0.24	−0.5	+0.4	+0.0	−0.2	+0.4	−0.3
−1	0	−2	−0.16	−0.11						
1	−2	−2	+0.04	+0.20						
−1	1	−2	+0.17	−0.64	+0.3	+1.8	+0.8	−4.3	+3.6	+0.6
1	−1	−2	−1.37	+0.88	+1.8	+1.8	−0.2	+0.9	−4.5	−0.5
−1	2	−2	+10.40	−2.39	−9.5	−36.0	−2.9	+31.5	−0.6	+3.1
1	0	−2	−10.49	+2.10	+22.1	+30.6	+8.1	−29.5	−10.5	+2.0
−1	3	−2	+2.42	+2.07	−112.1	+43.9	−46.2	−16.7	+102.4	−43.0
1	1	−2	−0.90	−1.83	+112.4	−51.0	+46.2	+21.3	−103.8	+46.0
−1	4	−2	−2.84	−4.14	−24.8	+31.9	−7.6	−11.4	+23.0	−28.3
1	2	−2	+3.06	+4.59	+12.1	−27.5	+2.4	+9.4	−11.5	+24.0
−1	5	−2	−0.19	−2.30	−1.1	+8.8	+0.2	−2.5	+1.3	−7.5
1	3	−2	−0.17	+1.86	−1.4	−5.4	−0.8	+1.3	+1.0	+4.5
−1	6	−2	+0.28	−0.57	+0.6	+1.4	+0.2	−0.3	−0.5	−1.2
1	4	−2	−0.29	+0.32	−0.6	−0.5	−0.2	+0.1	+0.6	+0.4
−1	1	−3	−0.06	−0.03			0.0	−0.3		
1	−1	−3	+0.07	+0.11			0.0	+0.1		
−1	2	−3	−0.13	−0.02	−0.2	−2.7	−0.4	+2.6	−0.3	+0.2
1	0	−3	+0.12	+0.92	+0.5	+3.2	+0.6	−2.4	0.0	−0.5
−1	3	−3	−0.18	−7.96	+1.0	+0.3	+0.2	−3.5	−0.6	−2.9
1	1	−3	−0.04	+8.09	+4.1	+6.4	+2.0	+0.5	−4.0	−3.5

Arg= $\varkappa\gamma'+i'g'+ig$			Y		D″		E″		H″	
			cos.	sin.	sin.	cos.	cos.	sin.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	4	−3	+2.18	−2.23	−43.3	−62.6	−19.1	+27.4	+38.9	+55.7
1	2	−3	−1.83	+1.19	+46.1	+63.4	+20.1	−28.0	−41.3	−56.7
−1	5	−3	−2.84	+1.22	−28.7	−14.2	−11.1	+ 5.0	+25.0	+13.0
1	3	−3	+3.25	−1.38	+24.4	+ 7.0	+ 9.1	− 1.8	−21.1	− 6.6
−1	6	−3	−1.81	−0.24	− 8.1	+ 0.5	− 2.7	− 0.5	+ 7.0	− 0.2
1	4	−3	+1.51	+0.41	+ 5.1	− 1.9	+ 1.5	+ 0.9	− 4.4	+ 1.6
−1	7	−3	−0.49	−0.33	− 1.4	+ 0.9	− 0.3	− 0.3	+ 1.2	− 0.7
1	5	−3	+0.29	+0.28	+ 0.6	− 0.8	+ 0.1	+ 0.3	− 0.5	+ 0.7
−1	3	−4	+0.12	−0.11						
1	1	−4	+0.51	+0.24						
−1	4	−4	−5.48	−0.99	− 2.3	+ 0.2	− 3.1	− 0.3	+ 3.4	− 0.1
1	2	−4	+5.61	+1.17	− 1.3	+ 3.8	+ 1.5	− 1.5	− 0.3	− 3.4
−1	5	−4	−1.67	−2.11	+31.2	−33.4	+14.1	+15.2	−27.3	+29.5
1	3	−4	+1.00	+1.77	−31.7	+35.6	−14.5	−16.2	+27.9	−31.3
−1	6	−4	+0.35	+1.73	+ 5.9	−22.5	+ 2.2	+ 9.2	− 5.4	+19.6
1	4	−4	−0.47	−2.07	− 2.0	+19.2	− 0.4	− 7.6	+ 2.2	−16.6
−1	7	−4	−0.44	+1.25	− 1.7	− 6.6	− 0.8	+ 2.4	+ 1.3	+ 5.7
1	5	−4	+0.52	−1.08	+ 2.3	+ 4.2	+ 1.0	− 1.4	− 1.9	− 3.7
−1	8	−4	−0.38	+0.35	− 1.2	− 1.2	− 0.5	+ 0.3	+ 0.9	+ 1.0
1	6	−4	+0.33	−0.22	+ 0.9	+ 0.5	+ 0.3	− 0.1	− 0.8	− 0.5
−1	4	−5	−0.19	−0.25						
1	2	−5	+0.36	−0.15						
−1	5	−5	−1.43	+3.41	− 0.1	− 2.6	− 0.5	+ 2.5	+ 0.4	+ 3.1
1	3	−5	+1.59	−3.52	− 2.6	+ 1.1	− 0.7	− 1.7	+ 2.0	− 1.8
−1	6	−5	−1.89	+1.05	+23.1	+13.6	+10.6	− 6.2	−20.0	−11.6
1	4	−5	+1.60	−0.65	−24.5	−13.7	−11.4	+ 6.4	+21.4	+11.9
−1	7	−5	+0.89	+0.04	+15.9	+ 0.8	+ 6.8	− 0.7	−13.9	− 0.9
1	5	−5	−1.17	+0.02	−13.7	+ 1.0	− 5.8	− 0.3	+11.9	− 0.7
−1	8	−5	+0.79	+0.46	+ 4.8	− 2.4	+ 1.9	+ 1.0	− 4.3	+ 1.9
1	6	−5	−0.68	−0.50	− 3.2	+ 2.5	− 1.1	− 1.0	+ 2.8	− 2.0
−1	9	−5	+0.16	+0.34	+ 0.9	− 1.2	+ 0.2	+ 0.5	− 0.7	+ 1.0
1	7	−5	−0.08	−0.29	− 0.4	+ 0.9	0.0	− 0.3	+ 0.3	− 0.7
−1	5	−6	−0.31	+0.13						
1	3	−6	+0.08	−0.30						
−1	6	−6	+1.95	+1.36	+ 2.2	− 0.5	+ 1.7	+ 0.6	− 2.3	+ 0.7
1	4	−6	−2.01	−1.50	− 1.7	− 1.2	− 1.5	+ 0.2	+ 1.9	+ 0.8
−1	7	−6	+0.56	+1.58	− 4.4	+14.5	− 2.1	− 6.9	+ 3.9	−12.5
1	5	−6	−0.34	−1.36	+ 4.4	−15.6	+ 2.1	+ 7.4	− 3.9	+13.4
−1	8	−6	+0.10	−0.38	+ 1.9	+10.5	+ 0.8	− 4.5	− 1.5	− 9.1
1	6	−6	−0.10	+0.60	− 2.7	− 9.1	− 1.2	+ 3.9	+ 2.2	+ 7.9
−1	9	−6	+0.39	−0.44	+ 2.6	+ 3.2	+ 1.1	− 1.3	− 2.2	− 2.8
1	7	−6	−0.42	+0.40	− 2.3	− 2.0	− 0.9	− 0.8	+ 1.9	+ 1.9
−1	10	−6	+0.31	−0.08						
1	8	−6	−0.26	+0.03						

Arg= $\varkappa\gamma'+i'g'+ig$			Y		D″		E″		H″	
			cos.	sin.	sin.	cos.	cos.	sin.	sin.	cos.
$\varkappa$	i'	i	″	″	″	″	″	″	″	″
−1	6−	7	+0.06	+0.24						
1	4−	7	−0.19	−0.13						
−1	7−	7	+1.10	−0.98	+0.8	+1.5	+0.6	−1.1	−0.7	−1.5
1	5−	7	−1.22	+1.00	+0.3	−1.5	−0.2	+1.1	−0.1	+1.5
−1	8−	7	+1.17	−0.20	−8.6	−0.6	−4.1	+0.3	+7.3	+0.5
1	6−	7	−1.03	+0.10	+9.3	+0.4	+4.3	−0.3	−7.9	−0.3
−1	9−	7	−0.08	−0.08	−6.5	+2.7	−2.8	−1.1	+5.6	−2.3
1	7−	7	+0.24	+0.11	+5.4	−2.7	+2.4	+1.3	−4.8	+2.3
−1	10−	7	−0.23	−0.33			−0.7	−1.0		
1	8−	7	+0.22	+0.35			+0.5	+0.8		
−1	11−	7	−0.06	−0.26						
1	9−	7	+0.03	+0.22						

From the expression of R_0', given in Chapter II, we derive that of $\left(\frac{d\overline{R_0'}}{d\gamma'}\right)$:

Arg=$i'g'+ig$	$\left(\frac{d\overline{R_0'}}{d\gamma'}\right)$		Arg=$i'g'+ig$	$\left(\frac{d\overline{R_0'}}{d\gamma}\right)$	
	cos.	sin.		cos.	sin.
i' i	″	″	i' i	″	″
0 0	−0.13		2− 3	+0.01	+0.46
1 0	+1.54	+1.10	3− 3	−0.11	+0.14
	$+1.1383n't$	$-1.6004n't$	4− 3	+0.23	−0.09
2 0	+0.18	+0.43	5− 3	+0.20	+0.02
	$+0.0638n't$	$-0.0898n't$	6− 3	+0.13	+0.08
3 0	−0.01	+0.06	7− 3	−0.01	−0.01
	$+0.0040n't$	$-0.0056n't$	3− 4	+0.21	+0.04
−2− 1	−0.03	0.00	4− 4	+0.07	+0.08
−1− 1	−0.16	−0.09	5− 4	−0.02	−0.08
0− 1	+1.91	+4.04	6− 4	+0.02	−0.06
1− 1	+0.96	−0.65	7− 4	+0.03	−0.03
2− 1	+0.99	−1.02	8− 4	+0.01	0.00
3− 1	−0.37	+0.01	9− 4	−0.04	−0.01
4− 1	+0.08	+0.05	4− 5	+0.04	−0.09
−1− 2	0.00	+0.01	5− 5	+0.05	−0.03
0− 2	+0.05	+0.30	6− 5	−0.02	0.00
1− 2	−1.00	+0.22	7− 5	−0.03	−0.02
2− 2	−0.25	−0.22	8− 5	−0.01	−0.01
3− 2	+0.46	+0.29	5− 6	−0.04	−0.03
4− 2	−1.03	−8.62	6− 6	−0.01	−0.03
5− 2	−0.01	−0.02	6− 7	−0.02	+0.02
6− 2	+0.25	−0.08	7− 7	−0.02	0.00
1− 3	0.00	+0.04			

The terms having the argument γ' in $\delta U'$ are important, as they determine the amount of the secular motion of the plane of the orbit due to the square and product of the masses of Jupiter and Saturn. The following detail of the composition of these terms is therefore given:

$$\begin{array}{rlr}
 & \quad '' & '' \\
\dfrac{dU'}{dg'}n'\delta z' = & -\ 0.012147\ \cos\gamma' & -\ 0.007098\ \sin\gamma' \\
\dfrac{dU'}{dg'}n\delta z = & -\ 0.004190 & -\ 0.002847 \\
U'\left(\nu' - 2\nu + 2c + \delta\dfrac{h'}{h_0'}\right) = & +\ 0.002876 & +\ 0.002141 \\
Y'(\nu' - c' - \nu + c) = & -\ 0.013664 & -\ 0.008705 \\
D''\dfrac{u'}{\cos i'} = & +\ 0.000950 & +\ 0.000589 \\
E''\dfrac{u_1'}{\cos i'} = & -\ 0.003083 & -\ 0.002182 \\
H''\dfrac{u}{\cos i} = & -\ 0.000095 & -\ 0.000065 \\
\hline
\text{Sum} \quad = & -\ 0.029353\ \cos\gamma' & -\ 0.018167\ \sin\gamma'
\end{array}$$

The following is the complete expression for $\delta U'$:

Arg$=\kappa\gamma'+i'g'+ig$			$\delta U'$			
			cos.	$n't$ cos.	sin.	$n't$ sin.
κ	i'	i	$''$	$''$	$''$	$''$
−1	1	0	−0.0075	+0.000036	+0.0058	+0.000459
1	0	0	−0.0029353	+0.0007112	−0.018167	−0.0009439
−1	3	0	+0.013	+0.00003	+0.005	+0.00002
1	1	0	−0.0090	−0.000015	−0.0178	+0.000040
−1	1—	1	+0.0089	−0.002608	+0.0282	+0.001085
1—	1—	1	−0.006	+0.00255	−0.017	−0.00047
−1	2—	1	+0.0011	+0.000424	+0.0090	+0.000150
1	0—	1	−0.011	−0.00117	+0.008	−0.00010
−1	3—	1	+0.0181	+0.000560	−0.0189	+0.000642
1	1—	1	−0.0259	−0.000502	+0.0288	−0.000616
−1	4—	1	+0.0158	+0.000061	−0.0046	+0.000025
1	2—	1	−0.0259	−0.000015	+0.0097	−0.000020
−1	2—	2	+0.012	−0.00030	−0.003	−0.00107
1	0—	2	−0.013	+0.00022	+0.003	+0.00114
−1	4—	2	−0.0078	+0.000545	−0.0104	−0.000307
1	2—	2	+0.020	−0.00044	+0.023	+0.00025
−1	5—	2	+0.00182	+0.0000159	−0.00899	−0.0000243
1	3—	2	+0.005	0.00000	+0.027	−0.00002
−1	6—	2	−0.0025	−0.000044	+0.0001	−0.000034
1	4—	2	+0.0016	+0.000024	−0.0100	+0.000018
−1	7—	2	+0.024	−0.00001	−0.015	−0.00001
1	5—	2	−0.00003	−0.0000043	+0.00009	+0.0000035
−1	10—	4	−0.01219	−0.0000012	−0.00344	+0.0000036

By subjecting the preceding expression to the same treatment as that by which, in Chapter II, $\frac{u'}{\cos i'}$ was derived from $\frac{1}{n'}\frac{dR_0'}{dt}$, we obtain:

$\text{Arg}=i'g'+ig$	$\overline{\delta R_0'}$	
	sin.	cos.
i' i	″ ″	″ ″
0 0		$-0.0035+0.002012n't$ $-0.00002994n'^2t^2$
1 0	$+0.0525-0.018170n't$ $-0.00047195n'^2t^2$	$-0.0292-0.029324n't$ $+0.00035560n'^2t^2$
2 0	$-0.0033-0.000514n't$ $-0.00001322n'^2t^2$	$+0.0153-0.000869n't$ $+0.00000996n'^2t^2$
3 0	$-0.0005-0.000021n't$ $-0.00000056n'^2t^2$	$+0.0006-0.000036n't$ $+0.00000042n'^2t^2$
4 0	$-0.000001n't$ $-0.00000003n'^2t^2$	$-0.000002n't$ $+0.00000002n'^2t^2$
−1− 1	$-0.0003+0.000111n't$	$+0.0009+0.000031n't$
0− 1	$-0.0042+0.000961n't$	$+0.0135+0.000608n't$
1− 1	$+0.0027-0.000572n't$	$+0.0217+0.000204n't$
2− 1	$+0.0506+0.001505n't$	$+0.0547-0.001682n't$
3− 1	$+0.0614-0.000011n't$	$+0.0203+0.000036n't$
4− 1	$+0.0006-0.000002n't$	$+0.0003+0.000000n't$
0− 2	$-0.0003+0.000007n't$	$-0.0001-0.000029n't$
1− 2	$-0.0014+0.000057n't$	$-0.0004-0.000131n't$
2− 2	$+0.0011-0.000038n't$	$-0.0006+0.000020n't$
3− 2	$+0.0022-0.000403n't$	$+0.0045-0.000214n't$
4− 2	$+0.0293+0.000530n't$	$+0.2995+0.000746n't$
5− 2	$-0.0068-0.000097n't$	$-0.0331+0.000000n't$
6− 2	$+0.0141-0.000130n't$	$+0.0005-0.000103n't$
7− 2	$-0.0010-0.000004n't$	$-0.0006-0.000003n't$
7− 4	-0.0002	$+0.0001$
8− 4	$-0.0051-0.000001n't$	$+0.0014-0.000002n't$
9− 4	$-0.1820-0.000018n't$	$+0.0513-0.000054n't$
10− 4	$+0.0153$	-0.0043

In order to have $\delta\left(\frac{u'}{\cos i'}\right)$, there must be added to this the following expression:

Arg$=i'g'+ig$		$\left(\frac{dR_0'}{dy'}\right)n'\delta z'$	
		sin.	cos.
i'	i	″ ″	″ ″
0	0		$+0.0009-0.000058n't$ $-0.00000364n'^2t^2$
1	0	$-0.0525+0.000000n't$ $+0.00000249n'^2t^2$	$+0.0292+0.000006n't$ $-0.00000025n'^2t^2$
2	0	$-0.0017-0.000054n't$ $+0.00002282n'^2t^2$	$-0.0073-0.000022n't$ $-0.00005455n'^2t^2$
3	0	$-0.0005-0.000026n't$ $+0.00000159n'^2t^2$	$+0.0038+0.000003n't$ $-0.00000383n'^2t^2$
4	0	$+0.0042$ $+0.00000008n'^2t^2$	$+0.0059$ $-0.00000019n'^2t^2$
5	0	$+0.0019$	-0.0011
-3	-1	-0.0030	-0.0014
-2	-1	-0.0020	$+0.0004$
-1	-1	$+0.0002-0.000029n't$	$-0.0015-0.000079n't$
0	-1	$+0.0004-0.000059n't$	$-0.0014+0.000101n't$
1	-1	$-0.0024-0.001340n't$	$-0.0018+0.001657n't$
2	-1	$-0.0013+0.000038n't$	$+0.0094+0.000090n't$
3	-1	$-0.0119-0.001114n't$	$-0.0043-0.001708n't$
4	-1	$+0.0001-0.000215n't$	$-0.0067-0.000077n't$
5	-1	$+0.0211-0.000013n't$	$-0.0237-0.000006n't$
6	-1	-0.0002	$+0.0012$
-2	-2	$+0.0003$	-0.0015
-1	-2	$-0.0003-0.000007n't$	$-0.0007-0.000006n't$
0	-2	$+0.0004-0.000037n't$	$+0.0003+0.000027n't$
1	-2	$-0.000047n't$	$+0.000155n't$
2	-2	$-0.0014-0.000145n't$	$+0.0039-0.000010n't$
3	-2	$+0.0034-0.003065n't$	$-0.0054-0.002289n't$
4	-2	$+0.0019-0.013739n't$	$-0.0140-0.003025n't$
5	-2	$-0.0008+0.003120n't$	$-0.0010-0.001650n't$
6	-2	$-0.0122+0.007447n't$	$-0.0074-0.011948n't$
7	-2	$-0.0033+0.000417n't$	$0.0000-0.000678n't$
8	-2	$+0.000026n't$	$-0.000042n't$
-1	-3	-0.0007	-0.0001
1	-3	$-0.000012n't$	$-0.000007n't$
2	-3	$-0.000037n't$	$+0.000001n't$
3	-3	$+0.0010-0.000025n't$	$+0.0002+0.000013n't$
4	-3	$-0.0052+0.000003n't$	$-0.0032-0.000016n't$
5	-3	$-0.0304-0.000011n't$	$-0.0028-0.000012n't$
6	-3	$+0.0047+0.000010n't$	$-0.0183-0.000030n't$
7	-3	$+0.0033-0.000014n't$	$-0.0092-0.000008n't$
8	-3	$+0.0009+0.000004n't$	$+0.0024+0.000027n't$
9	-3	-0.0005	-0.0004

Arg=$i'g'+ig$		$\left(\overline{\frac{dR_0'}{d\bar{\gamma}'}}\right)n'\delta z'$	
		sin.	cos.
i'	i	$''$ $''$	$''$ $''$
5	-4	$-0.0027+0.000009n't$	$+0.0021+0.000001n't$
6	-4	$+0.0012$	$+0.0065$
7	-4	$+0.0017+0.000005n't$	$+0.0001-0.000004n't$
8	-4	$+0.0108-0.000012n't$	$+0.0013+0.000033n't$
9	-4	$+0.0591+0.000023n't$	$-0.0172+0.000104n't$
10	-4	$-0.000028n't$	$-0.000021n't$
11	-4	$-0.0002-0.000100n't$	$-0.0018-0.000014n't$
6	-5	-0.0008	-0.0001
7	-5	-0.0030	$+0.0012$
8	-5	-0.0006	$+0.0011$
9	-5	0.0000	-0.0017
10	-5	-0.0006	-0.0012
11	-5	-0.0009	-0.0006
8	-6	-0.0009	-0.0013
9	-6	-0.0007	-0.0003
10	-6	$+0.0006$	-0.0001
9	-7	$+0.0005$	-0.0005

Then we have the following expression for $\delta\left(\frac{u'}{\cos i'}\right)$:

Arg=$i'g'+ig$		$\delta\left(\frac{u'}{\cos i'}\right)$	
		sin.	cos.
i'	i	$''$ $''$	$''$ $''$
0	0		$-0.0026+0.001954n't$ $-0.00003358n'^2t^2$
1	0	$-0.018170n't$ $-0.00046946n'^2t^2$	$-0.029318n't$ $+0.00035535n'^2t^2$
2	0	$-0.0050-0.000568n't$ $+0.00000960n'^2t^2$	$+0.0080-0.000891n't$ $-0.00004459n'^2t^2$
3	0	$-0.0010-0.000047n't$ $+0.00000103n'^2t^2$	$+0.0044-0.000033n't$ $-0.00000341n'^2t^2$
4	0	$+0.0042-0.000001n't$ $+0.00000005n'^2t^2$	$+0.0059-0.000002n't$ $-0.00000017n'^2t^2$
5	0	$+0.0019$	-0.0011
-3	-1	-0.0030	-0.0014
-2	-1	-0.0020	$+0.0004$
-1	-1	$-0.0001+0.000082n't$	$-0.0006-0.000048n't$
0	-1	$-0.0038+0.000902n't$	$+0.0121+0.000709n't$
1	-1	$+0.0003-0.001912n't$	$+0.0199+0.001861n't$
2	-1	$+0.0493+0.001543n't$	$+0.0641-0.001592n't$

Arg $= i'g' + ig$		$\delta\left(\frac{u'}{\cos i'}\right)$	
		sin.	cos.
i'	i	$''$ $''$	$''$ $''$
3	1	$+0.0495-0.001125n't$	$+0.0160-0.001672n't$
4	1	$+0.0007-0.000217n't$	$-0.0064-0.000077n't$
5	1	$+0.0211-0.000013n't$	$-0.0237-0.000006n't$
6	1	-0.0002	$+0.0012$
-2	2	$+0.0003$	-0.0015
-1	2	$-0.0003-0.000007n't$	$-0.0007-0.000006n't$
0	2	$+0.0001-0.000030n't$	$+0.0002-0.000002n't$
1	2	$-0.0014+0.000010n't$	$-0.0004+0.000024n't$
2	2	$-0.0003-0.000183n't$	$+0.0033+0.000010n't$
3	2	$+0.0056-0.003468n't$	$-0.0009-0.002503n't$
4	2	$+0.0312-0.013209n't$	$+0.2855-0.002279n't$
5	2	$-0.0076+0.003023n't$	$-0.0341-0.001650n't$
6	2	$+0.0019+0.007317n't$	$-0.0069-0.012051n't$
7	2	$-0.0043+0.000413n't$	$-0.0006-0.000681n't$
8	2	$+0.000026n't$	$-0.000042n't$
-1	3	$+0.0007$	-0.0001
1	3	$-0.000012n't$	$-0.000007n't$
2	3	$-0.000037n't$	$+0.000001n't$
3	3	$+0.0010-0.000025n't$	$+0.0002+0.000013n't$
4	3	$-0.0052+0.000003n't$	$-0.0032-0.000016n't$
5	3	$-0.0304-0.000011n't$	$-0.0028-0.000012n't$
6	3	$+0.0047+0.000010n't$	$-0.0183-0.000030n't$
7	3	$+0.0033-0.000014n't$	$-0.0092-0.000008n't$
8	3	$+0.0009+0.000004n't$	$+0.0024+0.000027n't$
9	3	-0.0005	-0.0004
5	4	$-0.0027+0.000009n't$	$+0.0021+0.000001n't$
6	4	$+0.0012$	$+0.0065$
7	4	$+0.0015+0.000005n't$	$+0.0002-0.000004n't$
8	4	$+0.0057-0.000013n't$	$+0.0027+0.000031n't$
9	4	$-0.1229+0.000005n't$	$+0.0341+0.000050n't$
10	4	$+0.0153-0.000028n't$	$-0.0043-0.000021n't$
11	4	$-0.0002-0.000100n't$	$-0.0018-0.000014n't$
6	5	-0.0008	-0.0001
7	5	-0.0030	$+0.0012$
8	5	-0.0006	$+0.0011$
9	5	0.0000	-0.0017
10	5	-0.0006	-0.0012
11	5	-0.0009	-0.0006
8	6	-0.0009	-0.0013
9	6	-0.0007	-0.0003
10	6	$+0.0006$	-0.0001
9	7	$+0.0005$	-0.0005

As in the case of Jupiter the only terms in the latitude of Saturn of three dimensions, which seem large enough to be taken account of, are the secular terms proportional to the cube of the time. According to LEVERRIER* we have

$$\sin i' \sin \theta' = \quad . \quad . \quad . \quad + 0.015774 \sin (126^\circ \; 5' \; 44'' - 25''.89t)$$
$$\sin i' \cos \theta' = \quad . \quad . \quad . \quad + 0.015774 \cos (126^\circ \; 5' \; 44'' - 25''.89t)$$

But here θ' is counted from the equinox of 1800.0. Adding therefore $41' \, 53''$, the amount of precession for 50 years, the formulæ become

$$\sin i' \sin \theta' = \quad . \quad . \quad . \quad + 0.015774 \sin (126^\circ \; 47' \; 37'' - 25''.89t)$$
$$\sin i' \cos \theta' = \quad . \quad . \quad . \quad + 0.015774 \cos (126^\circ \; 47' \; 37'' - 25''.89t)$$

Whence we deduce that the sine of the latitude of Saturn, referred to the fixed ecliptic of 1850.0, contains the terms

$$\sin b' = - 0''.000858 \mathrm{T}^3 \sin l' + 0''.000641 \mathrm{T}^3 \cos l'$$

---

*Annales de l'Observatoire de Paris, Mémoires, Tome II, p. 157.

CHAPTER XXVI.

FORMULÆ FOR THE MOTION OF THE PLANE OF THE ECLIPTIC AND FOR PRECESSION.

The subject announced in the title of this chapter properly belongs to the solar theory and the general stellar theory. But in order that the preceding theory may be compared, in a satisfactory manner, with observation, it is necessary to be able to reduce the theoretical positions of our planets to the moving planes of reference unavoidably employed by observers. The formulæ given by LEVERRIER or HANSEN might be used for this purpose; but there is one imperfection attached to them: they are limited to the first and second powers of the time. On account of the lengthened series of observations we now possess it seems desirable to add to the formulæ the terms multiplied by the third power of the time. I have therefore made a partially independent investigation of this subject. The reader is, however, advised that this is only to serve a temporary purpose, and will be superceded when the solar theory, in other hands, shall have been finished.

Availing ourselves of the tables of the coefficients of the secular portion of the perturbative function given by LEVERRIER*, the rate of motion of the ecliptic has been computed for the three epochs 1600, 1850, and 2100. Thence have been inferred the two equations giving the position of the ecliptic of any date with reference to that of 1850. The following details are all it is thought necessary to give of this work.

The elements of the planets involved in the computation are, for each of the three epochs, the following:

	Date.	e	J	Π	Π'	$\pi-\Pi'$
			° ′ ″	° ′ ″	° ′ ″	° ′ ″
Mercury .	1600	0.2055529	6 59 51.58	27 38 49.01	52 29 13.15	47 4 44.97
	1850	0.2056048	7 0 7.71	28 34 4.99	53 48 31.10	46 33 8.63
	2100	0.2056561	7 0 23.14	29 29 24.41	55 8 1.40	46 1 28.61
Venus . .	1600	0.0069689	3 23 25.47	52 53 6.24	22 59 17.01	76 34 41.11
	1850	0.0068431	3 23 35.01	54 7 49.75	25 1 46.65	75 19 53.08
	2100	0.0067189	3 23 44.53	55 21 2.38	27 4 20.21	74 5 9.80
					π	
Earth . .	1600	0.0168764			99 33 58.12	
	1850	0.0167711			100 21 39.73	
	2100	0.0166642			101 9 30.01	

---

*Annales de l'Observatoire de Paris, Mémoires, Tome II, pp. 94–96.

	Date.	e	J	Π	Π'	$\pi-\Pi$
			° ′ ″	° ′ ″	° ′ ″	° ′ ″
Mars . . .	1600	0.0930287	1 51 8.91	49 37 43.23	282 14 55.75	49 56 14.89
	1850	0.0932680	1 51 2.24	51 57 45.14	284 53 57.15	48 23 54.59
	2100	0.0935058	1 50 56.75	54 18 36.97	287 33 40.79	46 50 53.04
Jupiter .	1600	0.0478384	1 19 33.49	359 39 49.79	271 28 41.54	99 54 8.33
	1850	0.0482580	1 18 42.10	1 25 19.94	272 58 11.39	98 56 19.79
	2100	0.0486717	1 17 50.89	3 10 56.45	274 28 10.06	97 58 33.56
Saturn . .	1600	0.0561499	2 30 14.84	345 54 49.18	335 3 54.64	113 39 8.94
	1850	0.0560647	2 29 40.19	348 0 50.68	337 46 8.50	112 20 49.05
	2100	0.0559786	2 29 4.84	350 7 20.96	340 29 3.36	111 2 9.05
Uranus .	1600	0.0464082	0 46 16.27	24 5 39.4	95 8 20.8	75 28 18.7
	1850	0.0463414	0 46 19.72	27 7 45.3	97 36 12.7	73 13 54.4
	2100	0.0462756	0 46 24.93	30 9 20.6	100 3 21.4	71 0 9.4
Neptune .	1600		1 48 28.84			130 52 51.4
	1850		1 47 2.13			130 6 25.1
	2100		1 45 36.00			129 19 55.1

With these values of the elements have been computed the values of the two functions

$$-\frac{d\left(\frac{a'}{\Delta}\right)}{dJ} \qquad\qquad -\frac{1}{\sin J}\left[\cos J\,\frac{d\left(\frac{a'}{\Delta}\right)}{d\Pi}+\frac{d\left(\frac{a'}{\Delta}\right)}{d\Pi'}\right]$$

(a' must be understood as belonging to the exterior of the two planets). The following are the results obtained:

	Date.	First function.	Second function.		Date.	First function.	Second function.
Mercury .	1600	0.0186901	+0.0021313	Saturn . .	1600	0.0000386728	—0.0000000065
	1850	0.0187709	+0.0021840		1850	0.0000385239	—0.0000000053
	2100	0.0188533	+0.0022337		2100	0.0000383720	—0.0000000042
Venus . .	1600	0.09254530	—0.00001803	Uranus .	1600	0.0000014416	+0.0000000006
	1850	0.09260720	—0.00001370		1850	0.0000014435	+0.0000000006
	2100	0.09266871	—0.00000945		2100	0.0000014463	+0.0000000007
Mars . .	1600	0.0229324	—0.0000471	Neptune .	1600	0.0000008752	0
	1850	0.0229104	—0.0001189		1850	0.0000008636	0
	2100	0.0228855	—0.0001901		2100	0.0000008520	0
Jupiter .	1600	0.0001327178	+0.0000000952				
	1850	0.0001313057	+0.0000000993				
	2100	0.0001298985	+0.0000001029				

In computing these quantities all the terms given by Leverrier have been employed. In the case of the action of Mars it was found that the terms of the fifth order, with respect to the eccentricities and inclination, amount to about one per cent.

of those of the first order, thus showing a lack of rapid convergence. Hence, it appears very desirable that the secular action of Mars on the Earth should be determined by the use of GAUSS' method.

If we put the first of the two quantities just obtained, equal to k cos K, and the second equal to k sin K, the rate of motion of the ecliptic of date, in reference to the ecliptic of 1850.0, is given by the formulæ

$$\frac{d(\sin i \sin \theta)}{dt} = \frac{n}{\sqrt{1-e^2}} \frac{m'}{1+m} k \cos (\pi - \Pi' - \mathrm{K})$$

$$\frac{d(\sin i \cos \theta)}{dt} = -\frac{n}{\sqrt{1-e^2}} \frac{m'}{1+m} k \sin (\pi - \Pi' - \mathrm{K})$$

in the case of the action of an interior planet. For the action of an exterior planet $\pi - \Pi$ is substituted for $\pi - \Pi'$. If the unit of t is the Julian year, and the coefficients are to be expressed in seconds of arc, we have, severally, for the three epochs:

Date.	$\log\left(\frac{n}{\sqrt{1-e^2}} \frac{1}{1+m}\right)$
1600	6.1126579
1850	6.1126572
2100	6.1126564

It is now easy to find the action of each planet in changing the position of the plane of the ecliptic when the value of its mass is known. In the cases of Mercury, Venus, and Uranus the values here adopted differ somewhat from those given in Chapter I:

	$\frac{1}{m'}$	$\frac{d(\sin i \sin \theta)}{dt}$		
		1600.	1850.	2100.
		″	″	″
Mercury	7500000	+0.0024694	+0.0025049	+0.0025401
Venus	408134	+0.0681659	+0.0744329	+0.0806656
Mars	3093500	+0.0061692	+0.0063362	+0.0065001
Jupiter	1047.879	—0.0281150	—0.0251149	—0.0221690
Saturn	3501.6	—0.0057453	—0.0054237	—0.0050999
Uranus	22640	+0.0000207	+0.0000239	+0.0000270
Neptune	19700	—0.0000377	—0.0000366	—0.0000356
Sum =		+0.0429272	+0.0527227	+0.0624283

	$\frac{d(\sin i \cos \theta)}{dt}$		
	1600.	1850.	2100.
	″	″	″
Mercury	−0.0021146	−0.0020956	−0.0020767
Venus	−0.2858925	−0.2845280	−0.2830271
Mars	−0.0073665	−0.0072112	−0.0070499
Jupiter	−0.1617378	−0.1604628	−0.1591487
Saturn	−0.0131117	−0.0131883	−0.0132566
Uranus	−0.0000799	−0.0000791	−0.0000783
Neptune	−0.0000435	−0.0000435	−0.0000434
Sum =	−0.4703465	−0.4676085	−0.4646807

From these data, adopting the century as the unit of time, it is easy to derive the following formulæ:*

$$\sin i \sin \theta = + \overset{\prime\prime}{5.2723}\mathrm{T} + \overset{\prime\prime}{0.19501}\mathrm{T}^2 - \overset{\prime\prime}{0.000240}\mathrm{T}^3$$

$$\sin i \cos \theta = - 46.7608\mathrm{T} + 0.05666\mathrm{T}^2 + 0.000506\mathrm{T}^3$$

Whence also

$$\cos i = 1 - [92.4155]\mathrm{T}^2 + [89.58]\mathrm{T}^3$$

In deriving suitable formulæ for precession we set out from the exceedingly simple differential equations first stated by POISSON

$$\frac{d\omega}{dt} = \frac{1}{Cn \sin \omega} \frac{dV}{d\psi} \qquad \frac{d\psi}{dt} = - \frac{1}{Cn \sin \omega} \frac{dV}{d\omega}$$

where

$$V = -\tfrac{3}{2}\left(C - \frac{A+B}{2}\right)\left[m' \frac{z'^2}{r'^5} + m'' \frac{z''^2}{r''^5}\right]$$

Here ω denotes the inclination of the equator to a fixed plane, ψ the amount of backward motion of its node on this plane, A, B, and C are the moments of inertia of the Earth about its principal axes, C being supposed to belong to the axis of rotation, n is the angular velocity of this rotation, supposed constant, m' and m'' are the masses severally of the Sun and Moon, r' and r'' the distances of their centers from the center of the Earth, and z' and z'' the projection of these distances on a plane perpendicular to the equator. However, denoting by δ' and δ'' the declinations of the two bodies, it is better to write V thus

$$V = -\tfrac{3}{2}\left(C - \frac{A+B}{2}\right)\left[\frac{m'}{r'^3} \sin^2 \delta' + \frac{m''}{r''^3} \sin^2 \delta''\right]$$

---

* LEVERRIER'S value of the coefficient of T^3, in the expression for $\sin i \sin \theta$, $+0''.1964$ (Annales, Tome II, p. 104), appears to agree with that found here. But HANSEN and OLUFSEN'S value, $+0''.1887$ (Tables du Soleil, p. 41), seems to be in error; at least I am unable otherwise to explain the discordance.

Were there a third body producing sensible motion in the axis of the Earth it would be only necessary to add to the last factor of V a term altogether similar to the two which are already there.

When we treat precession alone it suffices to substitute for the terms $\frac{m'}{r'^3}\sin^2\delta' + \frac{m''}{r''^3}\sin^2\delta''$ their secular portions. In the case of the first of these, neglecting all periodic perturbations, we can assume that the Sun moves about the Earth in an ellipse whose elements are slowly changing. Assuming that the ecliptic of 1850.0 is the fixed plane of reference for the measurement of ω and ψ, if β' and λ' denote the latitude and longitude of the Sun referred to the ecliptic and mean equinox of the same date, we have

$$\sin\delta' = \sin\beta'\cos\omega + \cos\beta'\sin\omega\sin(\lambda' + \psi)$$

But taking the orbit longitude l' of the Sun we have

$$\begin{aligned}\cos\beta'\cos(\lambda' - \theta) &= \cos(l' - \theta)\\ \cos\beta'\sin(\lambda' - \theta) &= \cos i\sin(l' - \theta)\\ \sin\beta' &= \sin i\sin(l' - \theta)\end{aligned}$$

Here i and θ denote the quantities which have been thus designated in the just concluded treatment of the motion of the ecliptic. By substituting these values in the equation for $\sin\delta'$ we get

$$\sin\delta' = [\sin i\cos\omega + \cos i\sin\omega\cos(\psi + \theta)]\sin(l' - \theta) + \sin\omega\sin(\psi + \theta)\cos(l' - \theta)$$

From the theory of elliptic motion we know that $\frac{a^3}{r^3}\cos 2f$ and $\frac{a^3}{r^3}\sin 2f$ have no non-periodic terms. Thus, for $\sin^2\delta'$, in the expression of V, one may substitute

$$\frac{1}{2}[\sin i\cos\omega + \cos i\sin\omega\cos(\psi + \theta)]^2 + \frac{1}{2}\sin^2\omega\sin^2(\psi + \theta)$$

and for $\frac{a'^3}{r'^3}$ may be substituted its non-periodic term $(1 - e'^2)^{-\frac{3}{2}}$. But calling the mean obliquity of date ω', and ψ' the general precession, the formulæ, which connect ω', ψ' with ω, ψ are

$$\begin{aligned}\sin\omega'\sin(\psi' + \theta) &= \sin\omega\sin(\psi + \theta)\\ \sin\omega'\cos(\psi' + \theta) &= \sin i\cos\omega + \cos i\sin\omega\cos(\psi + \theta)\\ \cos\omega' &= \cos i\cos\omega - \sin i\sin\omega\cos(\psi + \theta)\end{aligned}$$

As the addition or subtraction of a function independent of ω and ψ to V does not impair its use for our purposes, it is plain we may substitute for $\sin^2\delta'$

$$-\frac{1}{2}\cos^2\omega' = -\frac{1}{2}[\cos i\cos\omega - \sin i\sin\omega\cos(\psi + \theta)]^2$$

To ascertain the non-periodic part of $\frac{\sin^2\delta''}{r''^3}$ is more difficult, as the solar perturbations of the Moon have to be considered. However, some simplifications occur.

It is well known that the plane of the Moon's orbit follows the ecliptic in its motion. Then, if β'' and $\lambda'' + \psi'$ denote the latitude and longitude of the Moon referred to the ecliptic and mean equinox of date,

$$\sin \delta'' = \sin \beta'' \cos \omega' + \cos \beta'' \sin \omega' \sin (\lambda'' + \psi')$$

$$\begin{aligned}\frac{\sin^2 \delta''}{r''^3} = {} & \frac{\sin^2 \beta''}{r''^3} \cos^2 \omega' + \frac{\sin 2\beta''}{r''^3} \sin (\lambda'' + \psi') \sin \omega' \cos \omega' \\ & + \frac{1}{2} \frac{\cos^2 \beta''}{r''^3} \sin^2 \omega' - \frac{1}{2} \frac{\cos^2 \beta''}{r''^3} \cos 2(\lambda'' + \psi') \sin^2 \omega'\end{aligned}$$

Now it is evident that the two terms of the last equation, which have the factors $\sin (\lambda'' + \psi')$ and $\cos 2(\lambda'' + \psi')$, are wholly periodic, and thus may be rejected. Furthermore, from the expression we may subtract $\frac{\sin^2 \beta''}{r''^3}$, which does not contain ω or ψ. Thus, for our purpose, we may substitute for $\frac{\sin^2 \delta''}{r''^3}$

$$\frac{1}{2} \frac{1 - 3 \sin^2 \beta''}{r''^3} \sin^2 \omega'$$

The non-periodic term of the first factor of this, corresponding to elliptic values for the co-ordinates, is readily discovered. For the non-periodic term of $\frac{a''^3}{r''^3}$ is $(1 - e''^2)^{-\frac{3}{2}}$, and the orbit longitude and the longitude of the node of the Moon being denoted by l'' and Ω'' and its inclination by i''

$$\sin^2 \beta'' = \sin^2 i'' \sin^2 (l'' - \Omega'') = \frac{1}{2} \sin^2 i'' - \frac{1}{2} \sin^2 i'' \cos 2(l'' - \Omega'')$$

But, as before mentioned, $\frac{a''^3}{r''^3} \cos 2(l'' - \Omega'')$ does not contain any non-periodic term. Hence, the non-periodic part of $\frac{a''^3}{r''^3}(1 - 3 \sin^2 \beta'')$ is

$$(1 - e''^2)^{-\frac{3}{2}}\left(1 - \frac{3}{2} \sin^2 i''\right)$$

To obtain the terms of the constant part of this function, which arise from solar perturbation, we take from Pontécoulant* the following terms of $\frac{a''^3}{r''^3}$:

$$\begin{aligned}\frac{a''^3}{r''^3} = (1 - e''^2)^{-\frac{3}{2}} & + \frac{1}{2}m^2 - \frac{9}{32}m^4 + \frac{55}{16}m^5 + \frac{2159}{96}m^6 \\ & + \left[\frac{771}{128}m^2 + \frac{8145}{256}m^3 + \frac{681789}{4096}m^4\right]e''^2 \\ & + \left[\frac{3}{4}m^2 + \frac{153}{64}m^3 + 0m^4\right]e'^2 \\ & + \left[3m^2 + \frac{19}{2}m^3 + \frac{135}{6}me''^2\right]\cos 2\xi \\ & + 3e'' \cos \varphi + \frac{45}{8}me'' \cos (2\xi - \varphi)\end{aligned}$$

---

*Théorie Analytique du Système du Monde, Tome IV, pp. 216, 226, 303.

From the value of $\sin^2\beta''$ we have *

$$1-3\sin^2\beta''=1-\frac{3}{2}\gamma^2\Big[1+\frac{9}{64}m^2+\frac{141}{128}m^3+\frac{7103}{2048}m^4+\frac{771}{256}m^2e''^2+\frac{463}{64}m^2e'^2$$
$$-\left(\frac{3}{4}m+\frac{3}{16}m^2\right)\cos 2\xi+\frac{109}{16}m^2e''\cos\varphi+\frac{3}{2}me''\cos(2\xi-\varphi)\Big]$$

By the multiplication of these factors it is found that the constant term of $\frac{a''^3}{r''^3}(1-3\sin^2\beta'')$, which we denote by N, is

$$\mathrm{N}=(1-e''^2)^{-\frac{3}{2}}\left(1-\frac{3}{2}\sin^2 i''\right)+\frac{1}{2}m^2-\frac{9}{32}m^4+\frac{55}{16}m^5+\frac{2159}{96}m^6$$
$$+\left[\frac{771}{128}m^2+\frac{8145}{256}m^3+\frac{681789}{4096}m^4\right]e''^2$$
$$+\left[\frac{3}{4}m^2+\frac{153}{64}m^3+0m^4\right]e'^2$$
$$-\left[\frac{123}{128}m^2-\frac{9}{256}m^3+\frac{8853}{2048}m^4\right]\gamma^2$$
$$-\frac{13329}{512}m^2e''^2\gamma^2-\frac{1533}{128}m^2e'^2\gamma^2$$

This quantity may be regarded as absolutely constant, as e'^2, the only symbol which varies in it, is multiplied by a small factor. However, if it is desired to consider its variability, the resultant term unites with the similar term arising from the action of the Sun, and the final formulæ are not rendered thereby more complex. On substitution of the numerical values of the quantities involved, it is found that at the epoch 1850

$$\mathrm{N}=0.9952929$$

It will be seen that the two terms of V involve severally the factors $\frac{m'}{a'^3}$ and $\frac{m''}{a''^3}$. But it will be more convenient to replace them by the following equivalents:

$$\frac{m'}{a'^3}=\frac{m'}{m+m'}n'^2 \qquad\qquad \frac{m''}{a''^3}=\frac{m''}{m+m''}n''^2$$

where m denotes the mass of the Earth.

If, therefore, we put

$$\mathrm{H}=\frac{3}{4}\,\frac{\mathrm{C}-\frac{1}{2}(\mathrm{A}+\mathrm{B})}{\mathrm{C}}\left[\frac{m'}{m+m'}\,\frac{n'^2}{n^2}(1-e'^2)^{-\frac{3}{2}}+\frac{m''}{m+m''}\,\frac{n''^2}{n^2}\mathrm{N}\right]$$

it will be perceived that this quantity is independent of the assumed units of measurement; moreover it is nearly constant, its variability arising from that of e'^2. It is sufficiently accurate to assume that it is of the form $a+bt$. If we put

$$\Omega=\mathrm{H}\cos^2\omega'=\mathrm{H}\,[\cos i\cos\omega-\sin i\sin\omega\cos(\psi+\theta)]^2$$

---

* Théorie Analytique du Système du Monde, Tome IV, p. 338.

our differential equations become

$$\frac{1}{n}\frac{d\omega}{dt} = \frac{1}{\sin\omega}\frac{d\Omega}{d\psi} \qquad\qquad \frac{1}{n}\frac{d\psi}{dt} = -\frac{1}{\sin\omega}\frac{d\Omega}{d\omega}$$

These equations fulfill the law of dimensions as the left members are the ratios of two angular velocities, and the right members also are functions of ratios. They can be written

$$\frac{d\omega}{dt} = 2\mathrm{H}n \sin i \cos\omega' \sin(\psi + \theta)$$

$$\frac{d\psi}{dt} = 2\mathrm{H}n \cos\omega' \,[\cos i + \sin i \cot\omega \cos(\psi + \theta)]$$

It is convenient to have differential equations determining ω' and ψ' directly, and thus avoid the arriving at them through the mediation of ω and ψ. It is plain from the preceding expressions for Ω and the differential equations for ω and ψ that the complete differential of $\cos\omega'$ is equal to the partial differential of its equivalent in terms of ω and ψ taken with respect to t as far as it is implicitly involved. This gives

$$\frac{d\,.\cos\omega'}{dt} = -\Big[\sin i \cos\omega + \cos i \sin\omega \cos(\psi + \theta)\Big]\frac{di}{dt} + \sin i \sin\omega \sin(\psi + \theta)\frac{d\theta}{dt}$$

$$= -\sin\omega' \cos(\psi' + \theta)\frac{di}{dt} + \sin i \sin\omega' \sin(\psi' + \theta)\frac{d\theta}{dt}$$

Consequently

$$\frac{d\omega'}{dt} = \cos(\psi' + \theta)\frac{di}{dt} - \sin i \sin(\psi' + \theta)\frac{d\theta}{dt}$$

In order to arrive at an expression for $\frac{d\psi'}{dt}$, we differentiate the first of the equations which show the relation of ω', ψ' to ω, ψ. Thus

$$\begin{aligned}\cos\omega' \sin(\psi' + \theta)d\omega' + \sin\omega' \cos(\psi' + \theta)\,(d\psi' + d\theta) = {} & \cos\omega \sin(\psi + \theta)d\omega \\ & + \sin\omega \cos(\psi + \theta)\,(d\psi + d\theta)\end{aligned}$$

Substituting in the right member of this the values of $d\omega$ and $d\psi$ which have been just given, it reduces to

$$2\mathrm{H}n \sin\omega' \cos\omega' \cos(\psi' + \theta)dt + [\cos i \sin\omega' \cos(\psi' + \theta) - \sin i \cos\omega']d\theta$$

Also employing the valve of $d\omega'$, previously given, the equation is transformed into

$$\begin{aligned}\sin\omega' \cos(\psi' + \theta)d\psi' = {} & 2\mathrm{H}n \sin\omega' \cos\omega' \cos(\psi' + \theta)dt - \cos\omega' \sin(\psi' + \theta) \cos(\psi' + \theta)di \\ & - \Big[2 \sin^2\frac{i}{2} \sin\omega' \cos(\psi' + \theta) + \sin i \cos\omega' - \sin i \cos\omega' \sin^2(\psi' + \theta)\Big]d\theta\end{aligned}$$

Whence we derive

$$\frac{d\psi'}{dt} = 2\mathrm{H}n \cos\omega' - \cot\omega' \sin(\psi' + \theta)\frac{di}{dt} - \Big[2 \sin^2\frac{i}{2} + \sin i \cot\omega' \cos(\psi' + \theta)\Big]\frac{d\theta}{dt}$$

The expressions for $\frac{d\omega'}{dt}$ and $\frac{d\psi'}{dt}$ can be simplified by putting

$$k \cos \mathrm{K} = -\sin\theta\frac{di}{dt} - \sin i \cos\theta\frac{d\theta}{dt} = -\frac{dp}{dt} - p\tan\frac{i}{2}\frac{di}{dt}$$

$$k \sin \mathrm{K} = \cos\theta\frac{di}{dt} - \sin i \sin\theta\frac{d\theta}{dt} = \frac{dq}{dt} + q\tan\frac{i}{2}\frac{di}{dt}$$

where p and q have the usual significations

$$p = \sin i \sin\theta \qquad q = \sin i \cos\theta$$

Then the two differential equations take the form

$$\frac{d\omega'}{dt} = k\sin(\psi' + \mathrm{K})$$

$$\frac{d\psi'}{dt} = 2\mathrm{H}n\cos\omega' - 2\sin^2\frac{i}{2}\frac{d\theta}{dt} + k\cot\omega'\cos(\psi' + \mathrm{K})$$

They are readily integrated in powers of the time by the use of MACLAURIN's Theorem. Let us suppose that at the epoch 1850.0 we have $\omega' = 23^\circ 27' 31''.83$, $\frac{d\psi'}{d\mathrm{T}} = 5025''.787$. $2\mathrm{H}n$ is diminishing at the rate of $0''.003656$ in a century. The term $-2\sin^2\frac{i}{2}\frac{d\theta}{d\mathrm{T}}$, which is excessively small, is equivalent to $+0''.000023\mathrm{T}^2$. With these have been calculated, at intervals of 500 years, the following quantities:

Date.	log k	K	ω'	ψ'
		° ′ ″	° ′ ″	° ′ ″
850	1.6790634	268 26 24.02	23 35 16.72	−13 55 46.39
1350	1.6758217	266 0 11.29	23 31 25.20	− 6 58 21.06
1850	1.6726250	263 34 1.36	23 27 31.83	0 0 0.00
2350	1.6694575	261 7 42.41	23 23 38.08	+ 6 59 16.80
2850	1.6663071	258 41 0.53	23 19 45.42	+13 59 29.35

The rates of motion of ω' and ψ' can be obtained from the differential equations. They are

Date.	$\frac{d\omega'}{d\mathrm{T}}$	$\frac{d\psi'}{d\mathrm{T}}$
	″	″
850	−46.02522	+5003.78451
1350	46.53860	5014.73750
1850	46.76080	5025.78700
2350	46.68988	5036.86128
2850	−46.32694	+5047.88573

From these quantities are inferred the formulæ

$$\omega' = 23^\circ 27' 31''.83 - 46''.7608\mathrm{T} - 0''.00757\mathrm{T}^2 + 0''.001956\mathrm{T}^3$$

$$\psi' = 5025''.7870\mathrm{T} + 1''.10739\mathrm{T}^2 + 0''.000174\mathrm{T}^3 - 0''.0000488\mathrm{T}^4 - 0''.00000023\mathrm{T}^5$$

which substantially give again the values of ω' and ψ' employed in computing the rates of motion.

CHAPTER XXVII.

REFERENCE OF THE LONGITUDES AND LATITUDES OF JUPITER AND SATURN TO THE MEAN EQUINOX AND ECLIPTIC OF DATE.

In order to have the greatest simplicity in the formulæ we assume, in the case of each planet, the mean plane of its orbit at the epoch 1850.0 as the plane from which its latitude in the first instance is to be counted. In the interval, 1600–2100, that is, 250 years on each side of the epoch, the maximum latitude of Jupiter, referred thus to its plane, will be about 40″; the similar quantity for Saturn will be about 58″. Thus we can put $\cos i = 1$, $\cos b = 1$ whenever they multiply quantities of the order of the disturbing forces. Then, in HANSEN's equations (21)*, we have $i_0 = 0$, $\varkappa = 2$, and they become

$$\begin{aligned}
\cos b \sin (l - \theta_0 - \Gamma) &= \sin (f + \pi - \theta_0) - \frac{1}{2} sq \\
\cos b \cos (l - \theta_0 - \Gamma) &= \cos (f + \pi - \theta_0) + \frac{1}{2} sp \\
\sin b &= s
\end{aligned}$$

From these we derive

$$\begin{aligned}
\cos b \sin (l - f - \pi - \Gamma) &= \quad - \frac{1}{2} sq \cos (f + \pi - \theta_0) - \frac{1}{2} sp \sin (f + \pi - \theta_0) \\
\cos b \cos (l - f - \pi - \Gamma) &= 1 + \frac{1}{2} sp \cos (f + \pi - \theta_0) - \frac{1}{2} sq \sin (f + \pi - \theta_0)
\end{aligned}$$

But

$$\begin{aligned}
p \cos (v - \theta_0) - q \sin (v - \theta_0) &= - s \\
q \cos (v - \theta_0) + p \sin (v - \theta_0) &= \frac{ds}{dv}
\end{aligned}$$

Consequently

$$\begin{aligned}
\cos b \cos (l - f - \pi - \Gamma) &= 1 - \frac{1}{2} s^2 \\
\cos b \sin (l - f - \pi - \Gamma) &= - \frac{1}{2} s \frac{ds}{dv}
\end{aligned}$$

Neglecting terms of the fourth order with respect to disturbing forces these equations give

$$l = f + \pi + \Gamma - \frac{1}{2} s \frac{ds}{dv}$$

---

*Auseinandersetzung, Abth. I, s. 79.

Thus, $f + \pi$ denoting the amount of description of angle by the radius vector, the two latter terms of the equation represent the reduction required to refer this angle to the chosen fixed plane.

Since $s = \frac{a_0}{r}u$, for the computation of Γ, we have*

$$\Gamma = \frac{1}{2\cos\varphi_0}\int a_0^2\frac{d\Omega}{dZ}undt$$

The second term of the reduction can be made to undergo the following transformations:

$$-\frac{1}{2}s\frac{ds}{dv} = -\frac{1}{4}\frac{r^2(1+\nu)^2}{a^2\cos\varphi}\frac{d\,.\,s^2}{ndt} = -\frac{1}{4}\frac{r^2}{a^2\cos\varphi}\frac{d\,.\left(\frac{a^2}{r^2}u^2\right)}{ndt}$$

$$= -\frac{1}{4\cos\varphi}\left[\frac{d\,.\,u^2}{ndt} - 2\frac{d\,.\log r}{ndt}\,.\,u^2\right]$$

In the case of Jupiter it is found that the largest terms of Γ are

$$\Gamma = -0''.0001\sin(5g'-2g) - 0''.0002\cos(5g'-2g)$$

it is therefore quite insignificant. For the second term it is sufficient to take

$$u^2 = +0''.00000023n^2t^2 - 0''.000000015n^2t^2\cos(-2g) + 0''.000000017n^2t^2\sin(-2g)$$

$$\frac{d\,.\,u^2}{ndt} = +0''.00000046nt - 0''.00000030n^2t^2\sin(-2g) - 0''.00000034n^2t^2\cos(-2g)$$

Thence, with sufficient approximation,

$$-\frac{1}{2}s\frac{ds}{dv} = +0''.000000075n^2t^2\sin(-2g) + 0''.000000083n^2t^2\cos(-2g)$$

In the case of Saturn we have

$$\begin{aligned}\Gamma' = &+ 0''.0000000009n'^3t^3\\ &+ 0''.000018n't\sin(g'-g) - 0''.000003n't\cos(g'-g)\\ &- 0.000002n't\sin(3g'-g) + 0.000010n't\cos(3g'-g)\\ &+ \left\{\begin{matrix}-0.0011\\ -0.000034n't\end{matrix}\right\}\sin(5g'-2g) + \left\{\begin{matrix}-0.0021\\ +0.000021n't\end{matrix}\right\}\cos(5g'-2g)\end{aligned}$$

$$\begin{aligned}-\frac{1}{2}s'\frac{ds'}{dv'} = &- 0''.000004n't\\ &- 0.000004n't\sin 2g' - 0''.000001n't\cos 2g'\\ &- 0.000000001n'^2t^2\\ &- 0.00000016n'^2t^2\sin g' + 0.00000047n'^2t^2\cos g'\\ &+ 0.00000145n'^2t^3\sin 2g' - 0.00000419n'^2t^2\cos 2g'\\ &+ 0.00000016n'^2t^2\sin 3g' - 0.00000046n'^2t^2\cos 3g'\\ &+ 0.00000002n'^2t^2\sin 4g' - 0.00000004n'^2t^2\cos 4g'\end{aligned}$$

---

*Auseinandersetzung, Abth. I, s. 81, gl. (23).

Let i denote the inclination of the plane of the primitive orbit on the ecliptic of date, and θ the longitude of its ascending node, counted on the plane of the primitive orbit from the same point of departure as for l, and Ω the longitude of the same node, counted on the plane of the ecliptic of date from the mean equinox of date. Then, if λ and β denote the longitude and latitude of the planet referred to the mentioned equinox and ecliptic, we shall have

$$\begin{aligned}
\sin\beta &= \cos i \sin b + \sin i \cos b \sin(l - \theta) \\
\cos\beta \sin(\lambda - \Omega) &= -\sin i \sin b + \cos i \cos b \sin(l - \theta) \\
\cos\beta \cos(\lambda - \Omega) &= \cos b \cos(l - \theta)
\end{aligned}$$

In these equations i, θ, and Ω depend only on i_0, θ_0, the values of i and θ for the epoch 1850.0, and on the quantities which determine the position of the ecliptic of date with respect to the ecliptic of the epoch. If the change in the measure of orbit longitudes is denoted by $\Omega - \theta_0 + \alpha$ the foregoing equations may be written

$$\begin{aligned}
\sin\beta &= \cos i \sin b + \sin i \cos b \sin(l - \theta_0 + \alpha) \\
\cos\beta \sin(\lambda - \Omega) &= -\sin i \sin b + \cos i \cos b \sin(l - \theta_0 + \alpha) \\
\cos\beta \cos(\lambda - \Omega) &= \cos b \cos(l - \theta_0 + \alpha)
\end{aligned}$$

The three quantities, i, Ω, and α, are determined by the equations

$$\begin{aligned}
\sin i \cos(\Omega - \theta'' - \psi') &= -\sin i'' \cos i_0 + \cos i'' \sin i_0 \cos(\theta'' - \theta_0) \\
\sin i \sin(\Omega - \theta'' - \psi') &= -\sin i_0 \sin(\theta'' - \theta_0) \\
\cos i &= \cos i'' \cos i_0 + \sin i'' \sin i_0 \cos(\theta'' - \theta_0) \\
\sin i \cos\alpha &= \sin i_0 \cos i'' - \cos i_0 \sin i'' \cos(\theta'' - \theta_0) \\
\sin i \sin\alpha &= \sin i'' \sin(\theta'' - \theta_0)
\end{aligned}$$

where i'' and θ'' are the quantities denoted as i and θ in the preceding chapter, and there used to determine the position of the ecliptic of date with respect to the ecliptic of 1850.0; ψ' denotes the general precession. For the first two of these equations we may substitute

$$\begin{aligned}
\sin i \cos(\Omega - \theta_0 - \psi') &= \sin i_0 - \cos i_0 \sin i'' \cos(\theta'' - \theta_0) - 2 \sin i_0 \sin^2 \tfrac{1}{2} i'' \cos^2 (\theta'' - \theta_0) \\
\sin i \sin(\Omega - \theta_0 - \psi') &= -\cos i_0 \sin i'' \sin(\theta'' - \theta_0) - \sin i_0 \sin^2 \tfrac{1}{2} i'' \sin 2(\theta'' - \theta_0)
\end{aligned}$$

and instead of computing α it will be more accurate to derive $\Omega - \theta_0 + \alpha - \psi'$, which is a very small angle. By putting

$$\begin{aligned}
\gamma &= -\cos i_0 \sin i'' \cos(\theta'' - \theta_0) - \sin i_0 \sin^2 \tfrac{1}{2} i'' [1 + \cos^2 (\theta'' - \theta_0)] \\
\delta &= \cos^2 \tfrac{1}{2} i_0 \sin i'' \sin(\theta'' - \theta_0) + \sin i_0 \sin^2 \tfrac{1}{2} i'' \sin 2(\theta'' - \theta_0) \\
\varepsilon &= -2 \sin i_0 \sin^2 \tfrac{1}{2} i'' \sin^2 (\theta'' - \theta_0) \\
\zeta &= 2 \sin^2 \tfrac{1}{2} i_0 \sin i'' \sin(\theta'' - \theta_0) - \sin i_0 \sin^2 \tfrac{1}{2} i'' \sin 2(\theta'' - \theta_0)
\end{aligned}$$

we can write

$$\begin{aligned}
\sin i \cos(\Omega - \theta_0 - \psi') &= \sin i_0 + \gamma - \tfrac{1}{2}\varepsilon \\
\sin i \sin(\Omega - \theta_0 - \psi') &= \quad -\delta + \tfrac{1}{2}\zeta \\
\sin i \cos \alpha &= \sin i_0 + \gamma + \tfrac{1}{2}\varepsilon \\
\sin i \sin \alpha &= \quad \delta + \tfrac{1}{2}\zeta
\end{aligned}$$

From these four equations we derive

$$\sin^2 i \sin(\Omega - \theta_0 + \alpha - \psi') = \zeta \sin i_0 + \gamma\zeta - \delta\varepsilon$$

whence, since $\Omega - \theta_0 + \alpha - \psi'$ is so small an angle,

$$\Omega - \theta_0 + \alpha - \psi' = \frac{\zeta \sin i_0 + \gamma\zeta - \delta\varepsilon}{\sin^2 i}$$

In the equation which gives the value of $\sin \beta$ we can put $\cos b = 1$, since it is multiplied by the small factor $\sin i$. Thus

$$\sin \beta = \cos i \sin b + \sin i \sin(l - \theta_0 + \alpha)$$

For convenience of tabulation we shall separate β into two parts, so that $\beta = \beta_0 + \Delta\beta$, where β_0 will be obtained from the formula

$$\sin \beta_0 = \mathrm{A} \sin l + \mathrm{B} \cos l$$

A and B being expressible in powers of the time. In doing this we shall remove from the term $\cos i \sin b$ the portion which can be regarded as a function of the same form as $\sin \beta_0$, and unite it with the latter. Let what is left of $\sin b = s = \frac{a}{r}u$, after this removal, be denoted as $\Delta(\sin b)$. Then $\Delta\beta$, being purely periodic and amounting at most to a few seconds, is given with sufficient exactness by the formula

$$\Delta\beta = \frac{\cos i}{\cos \beta_0}\Delta(\sin b) = \frac{\cos i}{\cos \beta_0}\Delta\left(\frac{a}{r}u\right)$$

By putting

$$p'' = \sin i'' \sin \theta'' \qquad\qquad q'' = \sin i'' \cos \theta''$$

quantities which have been determined in the preceding chapter, we can write the formula for the latitude in either of the two forms

$$\begin{aligned}
\sin \beta = {} & \cos i \sin b + [\sqrt{1 - p''^2 - q''^2} \sin i_0 - p'' \cos i_0 \sin \theta_0 - q'' \cos i_0 \cos \theta_0] \sin(l - \theta_0) \\
& + [p'' \cos \theta_0 - q'' \sin \theta_0] \cos(l - \theta_0) \\
= {} & \cos i \sin b + \left[\sqrt{1 - p''^2 - q''^2} \sin i_0 \cos \theta_0 + p'' \sin^2 \tfrac{1}{2} i_0 \sin 2\theta_0 - q''(\cos i_0 \cos^2 \theta_0 + \sin^2 \theta_0)\right] \sin l \\
& + \left[-\sqrt{1 - p''^2 - q''^2} \sin i_0 \sin \theta_0 + p''(\cos^2 \theta_0 + \cos i_0 \sin^2 \theta_0) - q'' \sin^2 \tfrac{1}{2} i_0 \sin 2\theta_0\right] \cos l
\end{aligned}$$

The rigorous equation, which gives the value of sin b, being

$$\sin b = \frac{r_0}{r}\int \frac{a_0 n_0 dt}{\sqrt{1 - e_0^2}} r_0 \sin(\bar{f} - f)\left[\frac{d\Omega}{dZ} + \frac{r^2 + rr_0 + r_0^2}{r_0^3 r^2}\delta r \sin b\right]$$

where r_0 and f denote elliptic values and $\bar{f}$ an f which is constant in the integration, it is easy to see that the constant factor, which multiplies the force, is

$$\frac{m'}{1+m}\alpha^2\left(\frac{a_0}{\mathrm{a}}\right)^3 \sin \mathrm{J}$$

But the factor actually employed in the determination of u was

$$\frac{m'}{1+m}\alpha^2\frac{a_0}{\mathrm{a}} \sin \mathrm{J}$$

Hence, in deriving sin b from u, we ought to multiply the latter by

$$\left(\frac{a_0}{\mathrm{a}}\right)^2\frac{a}{r}$$

We have severally in the cases of Jupiter and Saturn

$$\log\left(\frac{a_0}{\mathrm{a}}\right)^2 = 0.0000085 \qquad \log\left(\frac{a_0'}{\mathrm{a}'}\right)^2 = 9.9996292$$

From the equations connecting λ and β with l and b there is obtained, with a sufficient degree of approximation,

$$\lambda - l - \Omega + \theta_0 - \alpha = -\tan^2\frac{1}{2}i \sin 2(l - \theta_0 + \alpha) + \frac{1}{2}\tan^4\frac{1}{2}i \sin 4(l - \theta_0 + \alpha)$$

$$- 2\tan\frac{1}{2}i \tan b \cos(l - \theta_0 + \alpha) + 2\tan^3\frac{1}{2}i \tan b \cos 3(l - \theta_0 + \alpha)$$

By substituting in this the value, which has been obtained for $\Omega - \theta_0 + \alpha - \psi'$, we get

$$\lambda = l + \psi' + \frac{\zeta \sin i_0 + \gamma\zeta - \delta\varepsilon}{\sin^2 i} - \tan^2\frac{1}{2}i \sin 2(l - \theta_0 + \alpha) + \frac{1}{2}\tan^4\frac{1}{2}i \sin 4(l - \theta_0 + \alpha)$$

$$- 2\tan\frac{1}{2}i \tan b \cos(l - \theta_0 + \alpha) + 2\tan^3\frac{1}{2}i \tan b \cos 3(l - \theta_0 + \alpha)$$

The part of the second line of this formula, which depends on the portion $\Delta(\sin b)$ of sin b, is, with sufficient approximation,

$$-\frac{\sin i}{\cos\beta_0}\Delta(\sin b)\cos(l - \theta_0 + \alpha) = -\tan i \,.\, \Delta\beta \,.\, \cos(l - \theta_0 + \alpha)$$

For the remainder of sin b, which can be regarded as identical with tan b, we can write

$$\tan b = \mathrm{A}\sin(l - \theta_0 + \alpha) + \mathrm{B}\cos(l - \theta_0 + \alpha)$$

A and B being expressible in powers of the time. Substituting this value in the last line of the equation for λ it becomes

$$\begin{aligned}- \mathrm{B} \tan \frac{i}{2} - \mathrm{A} \tan \frac{i}{2}\left(1 + \tan^2 \frac{i}{2}\right) \sin 2\,(l - \theta_0 + \alpha) + \mathrm{A} \tan^3 \frac{i}{2} \sin 4\,(l - \theta_0 + \alpha) \\ - \mathrm{B} \tan \frac{i}{2}\left(1 - \tan^2 \frac{i}{2}\right) \cos 2\,(l - \theta_0 + \alpha) + \mathrm{B} \tan^3 \frac{i}{2} \cos 4\,(l - \theta_0 + \alpha)\end{aligned}$$

If, therefore, we adopt two new quantities I and η, such that

$$\begin{aligned}\tan^2 \tfrac{1}{2}\mathrm{I} \cos 2\eta &= \tan^2 \frac{i}{2} + \mathrm{A} \tan \frac{i}{2}\left(1 + \tan^2 \frac{i}{2}\right) \\ \tan^2 \tfrac{1}{2}\mathrm{I} \sin 2\eta &= \qquad\qquad \mathrm{B} \tan \frac{i}{2}\left(1 - \tan^2 \frac{i}{2}\right)\end{aligned}$$

which may be replaced by the equations

$$\begin{aligned}\sin \mathrm{I} \cos \eta &= \sin i + \mathrm{A} \cos i \\ \sin \mathrm{I} \sin \eta &= \qquad\quad \mathrm{B} \cos i\end{aligned}$$

the equation for λ can be written

$$\begin{aligned}\lambda = l + \psi' + \frac{\zeta \sin i_0 + \gamma\zeta - \delta\varepsilon}{\sin^2 i} - \mathrm{B} \tan \frac{i}{2} - \tan^2 \tfrac{1}{2}\mathrm{I} \sin 2\,(l - \theta_0 + \alpha + \eta) \\ + \tfrac{1}{2} \tan^4 \tfrac{1}{2}\mathrm{I} \sin 4\,(l - \theta_0 + \alpha + \eta) \\ - \tan i \,.\, \Delta\beta \cos (l - \theta_0 + \alpha)\end{aligned}$$

As the tabulation of perturbations, both secular and periodic, for a fourth co-ordinate, the reduction to the ecliptic, appears a work of supererogation, we will give a method by which it can be avoided. Let the equation for λ be written

$$\lambda = f + \pi + ht - \tan^2 \tfrac{1}{2}i_0 \sin 2\,(f + \pi - \theta_0) + \tfrac{1}{2} \tan^4 \tfrac{1}{2}i_0 \sin 4\,(f + \pi - \theta_0) + \delta\mathrm{R}$$

In this formula ht denotes the term, proportional to the first power of the time, in the development of the expression

$$\psi' + \frac{\zeta \sin i_0 + \gamma\zeta - \delta\varepsilon}{\sin^2 i} - \mathrm{B} \tan \frac{i}{2}$$

in powers of the time; and δR denotes the remainder of this expression plus the secular and periodic perturbations of what is generally known as the reduction to the ecliptic. If we do not go beyond 300 years from the epoch, δR, for either Jupiter or Saturn, scarcely exceeds 10″. Quantities dependent on its square may then be neglected. We have

$$\frac{dz}{d\lambda} = \frac{a^2 n \cos \varphi \cos i_0}{r^2 \cos^2 \beta_0}$$

Consequently, if we equate the argument $g + n\delta z$ by applying to it the correction

$$\varDelta(n\delta z) = \frac{r^2 \cos^2\beta_0}{a^2 \cos\varphi \cos i_0}\delta \mathrm{R}$$

we shall obtain the proper value of $\lambda - \pi - ht$ by entering a table calculated for this quantity, but in which the term $\delta\mathrm{R}$ has been ignored. $\varDelta(n\delta z)$ is a quantity which can be developed as a function of g and g', and having precisely the same form as $n\delta z$, its addition in nowise complicates the latter quantity; the only change being that some of the coefficients are modified by trifling amounts.

On account of this modification of the fundamental argument the expressions for ν and β_0 must receive corrections. The correction for ν is

$$\varDelta\nu = -\frac{d\,.\log r}{dg}\varDelta(n\delta z) = \left[e \sin g - \tfrac{3}{2}e^2 \sin 2g\right]\varDelta(n\delta z)$$

The correction to sin β_0 is

$$\varDelta(\sin\beta_0) = -\frac{d(\sin\beta_0)}{dg}\varDelta(n\delta z) = -\tan i \cos^2\beta_0 \cos(f + \pi - \theta_0)\,.\,\delta\mathrm{R}$$

These corrections are quite minute.

Application of the formulæ to Jupiter.

Supposing that

$$\mathrm{A}\sin f + \mathrm{B}\cos f$$

are the terms to be removed from cos i sin b, we have, for determining $\varDelta\beta$,

$$\varDelta\beta = \frac{\cos i}{\cos\beta_0}\frac{a_0^2}{\mathrm{a}^2}\frac{a}{r}u - \mathrm{A}\frac{\sin f}{\cos\beta_0} - \mathrm{B}\frac{\cos f}{\cos\beta_0}$$

A and B can be determined so that no terms of the form $k\mathrm{T}\sin g + k'\mathrm{T}\cos g$ appear in $\varDelta\beta$. From special values computed for eight points of the circumference at two epochs, using elliptic values of the co-ordinates augmented by the secular terms, it is found that

$$\begin{aligned}
\frac{a_0^2}{\mathrm{a}^2}\frac{a}{r}\frac{\cos i}{\cos\beta_0} = {} & 0.9998907 - 0.0000077\mathrm{T} \\
& + [0.0482118 + 0.0000010\mathrm{T}]\cos g \\
& + [0.0000010 + 0.0000002\mathrm{T}]\sin g \\
& + [0.0024471 + 0.0000098\mathrm{T}]\cos 2g \\
& + [-0.0000134 + 0.0000072\mathrm{T}]\sin 2g \\
& + [0.0001444 - 0.0000019\mathrm{T}]\cos 3g \\
& + [-0.0000014 + 0.0000015\mathrm{T}]\sin 3g \\
& + [0.0000138 - 0.0000031\mathrm{T}]\cos 4g
\end{aligned}$$

In this expression g ought to be replaced by $g + n\delta z$; it is sufficient to take for $n\delta z$ the two terms having the arguments $5g' - 2g$ and $5g' - 3g$. The additional terms, which thus should be joined to the expression, are

$$\begin{aligned}
&+ 0.0000554 \cos(5g' - g) - 0.0001298 \sin(5g' - g)\\
&- 0.0000188 \cos(5g' - 2g) - 0.0000012 \sin(5g' - 2g)\\
&- 0.0000554 \cos(5g' - 3g) + 0.0001298 \sin(5g' - 3g)\\
&+ 0.0000188 \cos(5g' - 4g) + 0.0000012 \sin(5g' - 4g)
\end{aligned}$$

In a similar way have been obtained the two expressions

$$\begin{aligned}
\frac{\sin f}{\cos \beta_0} = &\qquad\qquad\qquad\qquad\qquad\qquad + 0.000001 - 0.0001796\mathrm{T}\\
&[0.998031 - 0.0000160\mathrm{T}] \sin g + [-0.000007 - 0.0000185\mathrm{T}] \cos g\\
&+ [0.048106 + 0.0001633\mathrm{T}] \sin 2g + [-0.000002 - 0.0001809\mathrm{T}] \cos 2g\\
&+ [0.002674 + 0.0000169\mathrm{T}] \sin 3g + [0.000007 - 0.0000185\mathrm{T}] \cos 3g\\
&+ [0.000159 + 0.0000015\mathrm{T}] \sin 4g + [0.000001 - 0.0000016\mathrm{T}] \cos 4g\\
&+ 0.001146 \sin(5g' - g) + 0.002687 \cos(5g' - g)\\
&- 0.000388 \sin(5g' - 2g) + 0.000024 \cos(5g' - 2g)\\
&+ 0.001146 \sin(5g' - 3g) + 0.002687 \cos(5g' - 3g)\\
&- 0.000388 \sin(5g' - 4g) + 0.000024 \cos(5g' - 4g)
\end{aligned}$$

$$\begin{aligned}
\frac{\cos f}{\cos \beta_0} = &- 0.048252 - 0.0001648\mathrm{T}\\
&+ [0.997577 - 0.0000196\mathrm{T}] \cos g + [-0.000007 + 0.0000159\mathrm{T}] \sin g\\
&+ [0.048094 + 0.0001630\mathrm{T}] \cos 2g + [0.000001 + 0.0001780\mathrm{T}] \sin 2g\\
&+ [0.002672 + 0.0000173\mathrm{T}] \cos 3g + [0.000007 + 0.0000203\mathrm{T}] \sin 3g\\
&+ [0.000159 + 0.0000014\mathrm{T}] \cos 4g + [-0.000001 + 0.0000017\mathrm{T}] \sin 4g\\
&+ 0.001145 \cos(5g' - g) - 0.002686 \sin(5g' - g)\\
&- 0.000388 \cos(5g' - 2g) - 0.000024 \sin(5g' - 2g)\\
&- 0.001145 \cos(5g' - 3g) + 0.002686 \sin(5g' - 3g)\\
&+ 0.000388 \cos(5g' - 4g) + 0.000024 \sin(5g' - 4g)
\end{aligned}$$

Adding together all the first-order terms with those of the second order, and changing the parts of the coefficients multiplied by nt and n^2t^2 into the equivalents having the factors T and T^2, we have the expression for u, which it is suitable to employ here. The equations, which then result for determining A and B, are

$$\begin{aligned}
&\mathrm{A}[0.998031 - 0.0000160\mathrm{T}] + \mathrm{B}[-0.000007 + 0.0000159\mathrm{T}] = -14''.6842\mathrm{T} - 0''.04638\mathrm{T}^2\\
&\mathrm{A}[-0.000007 - 0.0000185\mathrm{T}] + \mathrm{B}[0.997577 - 0.0000196\mathrm{T}] = 6.5083\mathrm{T} - 0.09804\mathrm{T}^2
\end{aligned}$$

Whence we derive

$$\begin{aligned}
\mathrm{A} &= -14''.7132\mathrm{T} - 0''.04681\mathrm{T}^2\\
\mathrm{B} &= +6.5240\mathrm{T} - 0.09842\mathrm{T}^2
\end{aligned}$$

Since $f = l - 11^\circ\, 56'\, 9''.33$, as far as these terms are concerned, we have, in joining to them the terms of the third order, obtained at the end of Chapter XXIV,

$$\begin{aligned} \cos i \sin b = {} & [- \overset{''}{13.0458}T - \overset{''}{0.06615}T^2 + \overset{''}{0.000324}T^3] \sin l \\ & + [9.4259T - 0.08661T^2 - 0.000240T^3] \cos l \end{aligned}$$

By employing the expressions for p'' and q'', found in Chapter XXV, we get the remaining portion of $\sin \beta_0$, which is due to the motion of the ecliptic; it is

$$\begin{aligned} \sin \beta_0 = {} & [\overset{''}{46.7603}T - \overset{''}{0.05665}T^2 - \overset{''}{0.000506}T^3] \sin l \\ & + [\ 5.2691T + 0.19508T^2 - 0.000240T^3] \cos l \end{aligned}$$

It is not worth while to add here the two portions together, as the first has been computed with the values of the masses of the planets adopted at the beginning of this investigation, and which will receive some modifications, in consequence of the comparison of the theory with observation, to be given in the following chapter. But in deriving the motion of the ecliptic, in Chapter XXV, regard has been taken of these modifications.

The following is the expression we obtain for $\Delta\beta$:

Arg=$i'g'+ig$	$\Delta\beta$	
	sin.	cos.
i' i	″ ″	″ ″
0 0		+0.0369
0 − 2	−0.0061	−0.0135
0 − 3	+0.0001	−0.0007
1 + 2	+0.0051	−0.0006
1 + 1	+0.1024−0.0005T	+0.0160+0.0002T
1 0	+0.4420+0.0041T	−0.3041+0.0057T
1 − 1	−0.1112−0.0027T	−0.0591−0.0004T
1 − 2	−0.2579−0.0010T	−0.0601+0.0042T
1 − 3	−0.0108 0.0000T	−0.0048+0.0004T
2 + 1	+0.0042+0.0004T	−0.0175+0.0001T
2 0	−0.0247+0.0014T	−0.3414−0.0015T
2 − 1	+0.4575−0.0060T	+0.4287+0.0055T
2 − 2	−0.0922+0.0008T	+0.2009+0.0058T
2 − 3	−0.0033+0.0002T	−0.0564+0.0003T
2 − 4	+0.0006+0.0002T	−0.0028 0.0000T
3 + 1	+0.0023−0.0001T	+0.0015−0.0001T
3 0	+0.0366−0.0002T	+0.0421+0.0001T
3 − 1	+0.1651+0.0005T	−0.0113+0.0004T
3 − 2	−0.5398−0.0101T	+0.8570−0.0064T
3 − 3	−0.0541−0.0005T	+0.0165−0.0004T
3 − 4	+0.0189−0.0002T	−0.0030+0.0001T
3 − 5	+0.0012	−0.0001
4 0	+0.0058	+0.0024
4 − 1	+0.0406 0.0000T	−0.0238 0.0000T
4 − 2	−0.0246−0.0007T	+0.1424−0.0001T
4 − 3	+0.2286−0.0012T	+0.0924+0.0035T
4 − 4	+0.0200 0.0000T	−0.0067+0.0002T
4 − 5	+0.0043−0.0001T	+0.0074+0.0001T
5 0	−0.0030+0.0001T	+0.0080−0.0001T
5 − 1	−0.0674+0.0036T	+0.1707+0.0005T
5 − 2	+0.1936+0.0002T	−0.0014−0.0006T
5 − 3	−3.5340+0.0065T	+0.3195−0.0042T
5 − 4	−0.1763−0.0005T	+0.0592−0.0008T
5 − 5	−0.0044−0.0001T	+0.0064+0.0004T
5 − 6	−0.0025	+0.0024
6 − 1	+0.0002	+0.0007
6 − 2	+0.0065	+0.0018
6 − 3	−0.0317	+0.0186
6 − 4	+0.0134	+0.0465
6 − 5	−0.0119	+0.0033
6 − 6	−0.0018	+0.0030
6 − 7	−0.0011	−0.0008
7 − 2	+0.0036	−0.0015
7 − 3	−0.0040	+0.0029

Arg=$i'g'+ig$	$\Delta\beta$		Arg=$i'g'+ig$	$\Delta\beta$	
	sin.	cos.		sin.	cos.
i' i	″	″	i' i	″	″
7— 4	+0.0383	+0.0373	8— 7	+0.0011	—0.0010
7— 5	—0.0075	+0.0074	8— 8	0.0000	+0.0007
7— 6	—0.0017	—0.0037	9— 5	+0.0001	+0.0038
7— 7	—0.0016	—0.0005	9— 6	—0.0025	—0.0007
7— 8	+0.0002	—0.0005	9— 7	+0.0008	—0.0015
			10— 4	+0.0012	+0.0027
8— 3	+0.0008	+0.0009	10— 5	+0.0361	+0.0631
8— 4	—0.0087	—0.0033	10— 6	—0.0008	+0.0029
8— 5	—0.0050	+0.0067	10— 7	+0.0002	—0.0010
8— 6	—0.0032	—0.0029	10— 8	+0.0007	+0.0003

At intervals of 500 years have been computed the following quantities:

Date.	$\frac{\zeta \sin i_0 + \gamma\zeta - \delta\varepsilon}{\sin^2 i}$	$-\mathrm{B} \tan \frac{i}{2}$	I			$\theta_0-\alpha-\eta$		
	″	″	°	′	″	°	′	″
850	—5.2875	—1.7083	1	22	1.106	102	42	45.06
1350	—2.6210	—0.8381	1	20	21.264	100	49	23.15
1850	0.0000	0.0000	1	18	42.100	98	56	19.79
2350	+2.5717	+0.8060	1	17	3.627	97	3	34.81
2850	+5.0899	+1.5800	1	15	25.862	95	11	8.78

Thence the reduction to the ecliptic and mean equinox is

$$\begin{aligned}\lambda - l = {} & + \overset{\prime\prime}{27.029} \sin(2l + 342^\circ\, 7'\, 20'') + 0''.002 \sin(4l + 324^\circ)\\ & + [5026.4708 + 0''.4211 \sin(2l + 104^\circ\, 37'.9)]\mathrm{T}\\ & + [\ 1.10576 + 0''.00351 \sin(2l + 223^\circ\ 9'\ )]\mathrm{T}^2\\ & + [\ 0.000169 + 0''.000020 \sin(2l + 340^\circ\ )]\mathrm{T}^3\\ & - 0.0000488\mathrm{T}^4 - 0''.00000023\mathrm{T}^5\end{aligned}$$

We have here, then, $h = 50''.264708$, and for $\delta\mathrm{R}$ the sum of the inequalities of the reduction

$$\begin{aligned}\delta\mathrm{R} = {} & \Gamma - \frac{1}{2}s\frac{ds}{dv} - \tan i \cos(l - \theta_0 + \alpha)\,.\,\Delta\beta\\ & + 0''.4211\mathrm{T} \sin(2l + 104^\circ\, 37.'9)\\ & + [1''.10576 + 0''.00351 \sin(2l + 223^\circ\, 9')]\mathrm{T}^2\\ & + [0''.000169 + 0''.000020 \sin(2l + 340^\circ\ )]\mathrm{T}^3\end{aligned}$$

From eight computed special values it is found that

$$\begin{aligned}\frac{ndz}{d\lambda} = 1.002333 &- 0.094641 \cos l - 0.019961 \sin l \\ &+ 0.002958 \cos 2l + 0.001332 \sin 2l \\ &- 0.000081 \cos 3l - 0.000060 \sin 3l\end{aligned}$$

Also, in like manner we get, when $\delta\mathrm{R}$ is limited to the three latter lines of the expression above,

$$\begin{array}{lllll} \frac{ndz}{d\lambda}\delta\mathrm{R} = & [\quad '' & & +\overset{''}{0.0019} & \\ & +0.0379 & \sin nz & -0.0477 & \cos nz \\ & -0.2606 & \sin 2nz & +0.3283 & \cos 2nz \\ & -0.0126 & \sin 3nz & +0.0158 & \cos 3nz \\ & -0.0006 & \sin 4nz & +0.0007 & \cos 4nz]\mathrm{T} \\ & & & & \\ +[& & & +1.11086 & \\ & +0.00016 & \sin nz & -0.10630 & \cos nz \\ & -0.00134 & \sin 2nz & -0.00482 & \cos 2nz \\ & +0.00004 & \sin 3nz & -0.00020 & \cos 3nz]\mathrm{T}^2 \\ & & & & \\ +[& & & +0.000169 & \\ & -0.000003 & \sin nz & -0.000016 & \cos nz \\ & +0.000020 & \sin 2nz & +0.000001 & \cos 2nz \\ & +0.000001 & \sin 3nz & 0.000000 & \cos 3nz]\mathrm{T}^3 \end{array}$$

In this, for nz, ought to be substituted $g + n\delta z$. But we may take for $n\delta z$ its eleven largest terms and neglect its square. When this is done, and we also obtain the terms which arise from the first line of the value of $\delta\mathrm{R}$, we have the following complete expression for $\varDelta(n\delta z)$:

Arg=$i'g'+ig$		$\varDelta(n\delta z)$	
		sin.	cos.
i'	i	″ ″ ″ ″	″ ″ ″ ″
0	0		$+.0019\mathrm{T}+1.11086\mathrm{T}^2+.000187\mathrm{T}^3$
0	−1	$+0.0008-.0379\mathrm{T}-.00037\mathrm{T}^2+.000004\mathrm{T}^3$	$0.0000-.0477\mathrm{T}-\ .10632\mathrm{T}^2-.000012\mathrm{T}^3$
0	−2	$+.2606\mathrm{T}+.00111\mathrm{T}^2-.000002\mathrm{T}^3$	$+.3283\mathrm{T}-\ .00461\mathrm{T}^2-.000016\mathrm{T}^3$
0	−3	$+.0126\mathrm{T}-.00007\mathrm{T}^2\quad .000000\mathrm{T}^3$	$+.0158\mathrm{T}\quad .00000\mathrm{T}^2-.000002\mathrm{T}^3$
0	−4	$+.0006\mathrm{T}\quad .00000\mathrm{T}^2$	$+.0007\mathrm{T}+\ .00001\mathrm{T}^2$
1	+2	-0.0002	$+0.0012$
1	+1	$+0.0032-.0001\mathrm{T}$	$+0.0053-.0001\mathrm{T}$
1	0	$+0.0009\qquad +.00002\mathrm{T}^2$	$-0.0025\qquad .00000\mathrm{T}^2$
1	−1	-0.0029	-0.0078
1	−2	$-0.0007\qquad -.00002\mathrm{T}^2$	$+0.0013\qquad .00000\mathrm{T}^2$
1	−3	$-0.0005+.0001\mathrm{T}$	$+0.0029-.0001\mathrm{T}$

Arg=$i'g'+ig$		$\varDelta(n\delta z)$	
		sin.	cos.
i'	i	″ ″ ″	″ ″ ″
2+	1	$+0.0039-.0002T$	$-0.0001+.0002T$
2	0	$-0.0051-.0001T \quad .00000T^2$	$+0.0049+.0004T-.00003T^2$
2−	1	$-0.0061 \quad -.00002T^2$	$-0.0007 \quad -.00005T^2$
2−	2	$+0.0051 \quad .00000T^2$	$-0.0055 \quad +.00003T^2$
2−	3	$+0.0024-.0002T+.00002T^2$	$+0.0010-.0002T+.00005T^2$
2−	4	$-0.0006-.0003T$	$0.0000-.0002T$
3	0	$-0.0001 \quad .0000T$	$+0.0019-.0002T$
3−	1	$-0.0095 \quad +.00002T^2$	$-0.0067 \quad +.00001T^2$
3−	2	-0.0003	-0.0025
3−	3	$+0.0101 \quad -.00002T^2$	$+0.0057 \quad -.00001T^2$
3−	4	$+0.0002+.0002T$	$+0.0006 \quad .0000T$
4−	1	-0.0016	-0.0004
4−	2	-0.0012	$+0.0025$
4−	3	$+0.0016$	$+0.0002$
4−	4	$+0.0010$	-0.0027
5+	1	$-.0001T$	$.0000T$
5	0	$-0.0020-.0024T+.00002T^2$	$-0.0009-.0007T+.00007T^2$
5−	1	$-0.0001+.0002T+.00027T^2$	$+0.0022-.0003T-.00013T^2$
5−	2	$+0.0004 \quad .00000T^2$	$-0.0399 \quad +.00004T^2$
5−	3	$-0.0007 \quad -.00028T^2$	$-0.0042 \quad +.00012T^2$
5−	4	$+0.0058+.0012T-.00012T^2$	$+0.0406-.0021T-.00009T^2$
5−	5	$+0.0008+.0003T+.00001T^2$	$+0.0020+.0001T-.00001T^2$

The terms of this expression not multiplied by T or its powers represent nearly the periodic perturbations of the reduction of the longitude to the ecliptic.

The correction which must be applied to the common logarithm of $\frac{r}{\bar{r}}$, on account of this change in the fundamental argument, is determined in a similar way, and is (in units of the seventh decimal):

Arg=$i'g'+ig$		$-\frac{d(\text{com. log } r)}{dz}\varDelta(\delta z)$	
		cos.	sin.
i'	i		
0	0	$-0.010T$	
0−	1	$+0.132T \quad 0.000T^2 \quad 0.0000T^3$	$-0.166T+1.098T^2-0.0002T^3$
0−	2	$+0.026T \quad 0.000T^2$	$-0.032T+0.027T^2$
0−	3	$-0.131T \quad 0.000T^2$	$+0.165T+0.070T^2$
0−	4	$-0.016T$	$+0.020T$
0−	5	$-0.001T$	$+0.002T$
5−	1	$0.00 \quad -0.003T^2$	$+0.02-0.001T$
5−	3	$0.00 \quad -0.003T^2$	$-0.04-0.001T$
5−	5	0.00	$+0.02$

The correction to be applied to sin β_0, on the same account, is

$$
\begin{aligned}
-\frac{d(\sin\beta_0)}{dz}\Delta(\delta z) = {} & [\quad \overset{\prime\prime}{0}.0044\mathrm{T} - \overset{\prime\prime}{0}.02504\mathrm{T}^2 + \overset{\prime\prime}{0}.000075\mathrm{T}^3] \sin l \\
+ {} & [\quad 0.0019\mathrm{T} + 0.00395\mathrm{T}^2 - 0.000179\mathrm{T}^3] \cos l \\
+ {} & [-0.0048\mathrm{T} + 0.00002\mathrm{T}^2] \sin 3l \\
+ {} & [-0.0005\mathrm{T} - 0.00003\mathrm{T}^2] \cos 3l
\end{aligned}
$$

Application of the Formulæ to Saturn.

Proceeding in exactly the same way as for Jupiter it is found that

$$
\begin{aligned}
\left(\frac{a_0'}{a'}\right)^2 \frac{a}{r}\frac{\cos i'}{\cos\beta_0'} = {} & \quad 0.9986723 + 0.0000228\mathrm{T} \\
+ {} & [\quad 0.0559889 + 0.0000014\mathrm{T}] \cos g' \\
+ {} & [\quad 0.0000280 - 0.0000012\mathrm{T}] \sin g' \\
+ {} & [\quad 0.0028014 - 0.0000241\mathrm{T}] \cos 2g' \\
+ {} & [-0.0003273 + 0.0000386\mathrm{T}] \sin 2g' \\
+ {} & [\quad 0.0001506 - 0.0000006\mathrm{T}] \cos 3g' \\
+ {} & [-0.0000465 + 0.0000016\mathrm{T}] \sin 3g' \\
+ {} & [\quad 0.0000081 - 0.0000002\mathrm{T}] \cos 4g' \\
+ {} & [-0.0000049 + 0.0000003\mathrm{T}] \sin 4g'
\end{aligned}
$$

In this expression g' ought to be replaced by $g' + n'\delta z'$; it is sufficient to take for $n'\delta z'$ the three terms having the arguments $2g' - g$, $4g' - 2g$, and $5g' - 2g$. The additional terms. which should be joined to the expression, are

$$
\begin{aligned}
& + 0.0000572 \cos(\ g' - \ g) \\
& - 0.0000572 \cos(3g' - \ g) \\
& - 0.0000117 \cos(3g' - 2g) - 0.0000921 \sin(3g' - 2g) \\
& + 0.0001482 \cos(4g' - 2g) - 0.0003652 \sin(4g' - 2g) \\
& + 0.0000117 \cos(5g' - 2g) + 0.0000921 \sin(5g' - 2g) \\
& - 0.0001482 \cos(6g' - 2g) + 0.0003652 \sin(6g' - 2g)
\end{aligned}
$$

In a similar way have been obtained the two expressions

$$
\begin{aligned}
\frac{r'}{a'}\sin f' = {} & \qquad 0.0000003 - 0.0005529\mathrm{T} \\
+ {} & [0.9980325 + 0.0000145\mathrm{T}] \sin\ g' + [0.0000051 - 0.0000386\mathrm{T}] \cos\ g' \\
+ {} & [0.0279551 - 0.0003436\mathrm{T}] \sin 2g' + [0.0000014 - 0.0005547\mathrm{T}] \cos 2g' \\
+ {} & [0.0011714 - 0.0000241\mathrm{T}] \sin 3g' + [0.0000051 - 0.0000387\mathrm{T}] \cos 3g' \\
+ {} & [0.0000583 - 0.0000016\mathrm{T}] \sin 4g' + [0.0000017 - 0.0000026\mathrm{T}] \cos 4g' \\
+ {} & \ 0.0000032 \qquad \sin 5g' \qquad - 0.0000001\mathrm{T} \ \cos 5g'
\end{aligned}
$$

$$
\begin{aligned}
\frac{r'}{a'}\cos f' = {} & \qquad\qquad\qquad\qquad\qquad\qquad\qquad -0.0840851 + 0.0003454\mathrm{T} \\
& + [0.0000051 + 0.0000233\mathrm{T}] \sin g' + [0.9988249 + 0.0000241\mathrm{T}] \cos g' \\
& + [-0.0000014 + 0.0005448\mathrm{T}] \sin 2g' + [0.0279697 - 0.0003438\mathrm{T}] \cos 2g' \\
& + [-0.0000051 + 0.0000388\mathrm{T}] \sin 3g' + [0.0011718 - 0.0000241\mathrm{T}] \cos 3g' \\
& + [0.0000011 + 0.0000026\mathrm{T}] \sin 4g' + [0.0000583 - 0.0000016\mathrm{T}] \cos 4g' \\
& \qquad\qquad\qquad + 0.0000001\mathrm{T} \sin 5g' + 0.0000032 \cos 5g'
\end{aligned}
$$

$\varDelta\beta'$ is obtained from its equation, put in the form

$$
\varDelta\beta' = \frac{a}{\bar{r}}\sec.\beta_0'\left[\frac{a_0'^2}{\mathrm{a}'^2}\cos i' \,.\, u' - \mathrm{A}\frac{\bar{r}'}{a'}\sin f' - \mathrm{B}\frac{\bar{r}'}{a'}\cos f'\right]
$$

It is found that

$$
\begin{aligned}
\mathrm{A} &= +\overset{''}{23.8966}\mathrm{T} - \overset{''}{0.21464}\mathrm{T}^2 \\
\mathrm{B} &= +33.4890\mathrm{T} + 0.16184\mathrm{T}^2
\end{aligned}
$$

Since $f' = l' - 90^\circ\ 6'\ 46''.22$ we have, as far as these terms are concerned, joining to them the terms of the third order, obtained in Chapter XXV,

$$
\begin{aligned}
\cos i' \sin b' = {} & [\overset{''}{33.4419}\mathrm{T} + \overset{''}{0.16226}\mathrm{T}^2 - \overset{''}{0.000858}\mathrm{T}^3] \sin l' \\
& + [-23.9625\mathrm{T} + 0.21432\mathrm{T}^2 + 0.000641\mathrm{T}^3] \cos l'
\end{aligned}
$$

The remaining portion of sin β_0', due to the motion of the ecliptic, is

$$
\begin{aligned}
\sin \beta_0' = {} & [\overset{''}{46.7526}\mathrm{T} - \overset{''}{0.05662}\mathrm{T}^2 - \overset{''}{0.000506}\mathrm{T}^3] \sin l' \\
& + [5.2524\mathrm{T} + 0.19509\mathrm{T}^2 - 0.000240\mathrm{T}^3] \cos l'
\end{aligned}
$$

For the same reason, as in the case of Jupiter, we do not add these two portions.

The following expression is obtained for $\varDelta\beta'$:

Arg=$i'g'+ig$	$\varDelta\beta'$		Arg=$i'g'+ig$	$\varDelta\beta'$	
	sin.	cos.		sin.	cos.
i' i	″ ″	″ ″	i' i	″ ″	″ ″
0 0		$-0.3294-0.0109\mathrm{T}$	3 − 1	$-0.7177+0.0002\mathrm{T}$	$-0.0636-0.0018\mathrm{T}$
2 0	$+0.0604-0.0012\mathrm{T}$	$-0.1948-0.0015\mathrm{T}$	4 − 1	$+0.0309-0.0001\mathrm{T}$	$-0.0483+0.0002\mathrm{T}$
3 0	$-0.0002-0.0003\mathrm{T}$	$-0.0186+0.0001\mathrm{T}$	5 − 1	$+0.0245+0.0002\mathrm{T}$	$-0.0288+0.0001\mathrm{T}$
4 0	$+0.0034$	$+0.0043$	6 − 1	$+0.0011$	-0.0004
5 0	$+0.0020$	-0.0011	−2 − 2	$+0.0002$	-0.0012
−3 − 1	-0.0023	-0.0013	−1 − 2	$+0.0004-0.0002\mathrm{T}$	$+0.0024-0.0001\mathrm{T}$
−2 − 1	$+0.0029$	$+0.0015$	0 − 2	$-0.0019-0.0002\mathrm{T}$	$+0.0630-0.0003\mathrm{T}$
−1 − 1	$+0.0206+0.0013\mathrm{T}$	$+0.0161-0.0015\mathrm{T}$	1 − 2	$+0.2524+0.0014\mathrm{T}$	$+0.0533-0.0025\mathrm{T}$
0 − 1	$-0.7897+0.0207\mathrm{T}$	$+1.6202+0.0131\mathrm{T}$	2 − 2	$+0.0851\ \ 0.0000\mathrm{T}$	$-0.0729+0.0008\mathrm{T}$
1 − 1	$-0.7224-0.0132\mathrm{T}$	$-0.4281+0.0040\mathrm{T}$	3 − 2	$-0.1903-0.0052\mathrm{T}$	$-0.1002-0.0016\mathrm{T}$
2 − 1	$-2.0366+0.0316\mathrm{T}$	$-2.0693-0.0364\mathrm{T}$	4 − 2	$+1.0807+0.0061\mathrm{T}$	$-8.6028+0.0143\mathrm{T}$

Arg=$i'g'+ig$	$\Delta\beta'$	
	sin.	cos.
i' i	" "	" "
5— 2	—0.1334+0.0048T	+0.3444—0.0028T
6— 2	+0.2366—0.0001T	+0.0733—0.0075T
7— 2	+0.0101—0.0003T	+0.0033—0.0008T
8— 2	+0.0004—0.0005T	+0.0001+0.0009T
—1— 3	+0.0007	—0.0001
0— 3	—0.0010	+0.0023
1— 3	+0.0008—0.0003T	+0.0072—0.0002T
2— 3	+0.0003—0.0002T	+0.0868 0.0000T
3— 3	+0.0247—0.0002T	+0.0329+0.0001T
4— 3	—0.0728+0.0001T	—0.0259 0.0000T
5— 3	—0.1169 0.0000T	+0.0080—0.0003T
6— 3	—0.0877—0.0001T	+0.0395 0.0000T
7— 3	+0.0245 0.0000T	—0.0418 0.0000T
8— 3	—0.0007 0.0000T	—0.0017+0.0001T
9— 3	—0.0005	—0.0005
2— 4	—0.0021	+0.0018
3— 4	—0.0318 0.0000T	+0.0071+0.0002T
4— 4	—0.0129+0.0001T	+0.0132+0.0001T
5— 4	—0.0010+0.0001T	—0.0142 0.0000T
6— 4	—0.0051—0.0001T	—0.0117+0.0001T
7— 4	—0.0071—0.0001T	—0.0084—0.0001T
8— 4	—0.0020—0.0001T	+0.0003—0.0002T
9— 4	—0.0825—0.0004T	+0.0272—0.0011T
10— 4	+0.0086 0.0000T	—0.0030+0.0001T
11— 4	+0.0001—0.0001T	—0.0019+0.0001T
3— 5	—0.0013	—0.0002
4— 5	—0.0057	—0.0121
5— 5	—0.0073	—0.0049
6— 5	+0.0035	—0.0007
7— 5	+0.0021	—0.0020
8— 5	+0.0013	—0.0020
9— 5	+0.0001	—0.0034
10— 5	—0.0009	—0.0021
11— 5	—0.0016	—0.0013
4— 6	—0.0004	—0.0006
5— 6	+0.0043	—0.0033
6— 6	+0.0015	—0.0038
7— 6	+0.0006	+0.0012
8— 6	+0.0008	+0.0003
9— 6	+0.0007	+0.0001
10— 6	+0.0012	—0.0002
6— 7	+0.0019	+0.0015
7— 7	+0.0019	+0.0003

Arg=$i'g''+ig'$	$\Delta\beta'$	
	sin.	cos.
i' i	" "	" "
1+ 2	—0.0001	—0.0005
1+ 1	—0.0038	—0.0202
1 0	—0.0657	—0.0557
1— 1	+0.0381	+0.0078
1— 2	+0.0183	—0.0334
1— 3	+0.0011	—0.0015
2+ 1	—0.0028	+0.0008
2 0	—0.0384	+0.0196
2— 1	+0.0615	—0.1010
2— 2	+0.0044	—0.0330
2— 3	+0.0085	+0.0003
3+ 1	+0.0008	—0.0018
3 0	+0.0114	—0.0328
3— 1	—0.0368	—0.0326
3— 2	+0.6067	+0.2214
3— 3	+0.0378	+0.0116
3— 4	+0.0014	+0.0029
4— 1	—0.0043	—0.0023
4— 2	+0.0264	—0.0051
4— 3	+0.0050	—0.0242
4— 4	+0.0007	—0.0004
5— 1	—0.0011	+0.0003
5— 2	+0.0027	—0.0014
5— 3	—0.0097	—0.0207
5— 4	+0.0046	—0.0016
6— 2	—0.0007	—0.0009
6— 3	+0.0113	+0.0071
6— 4	+0.0032	—0.0016
6— 5	+0.0005	+0.0013
7— 3	+0.0006	0.0000
7— 4	+0.0005	—0.0015
7— 5	+0.0007	+0.0007
7— 6	—0.0005	+0.0002

At intervals of 500 years have been computed the following quantities:

Date.	$\frac{\zeta \sin i_0' + \gamma\zeta - \delta\varepsilon}{\sin^2 i'}$	$-\mathrm{B} \tan \frac{i'}{2}$	I′	$\theta_0' - \alpha' - \eta'$
	″	″	° ′ ″	° ′ ″
850	−9.2472	+8.8607	2 31 47.88	117 30 7.86
1350	−4.5577	+4.3939	2 30 45.37	114 56 2.11
1850	0.0000	0.0000	2 29 40.19	112 20 49.05
2350	+4.4200	−4.3113	2 28 32.51	109 44 23.86
2850	+8.6959	−8.5306	2 27 22.41	107 6 42.41

Thence the reduction to the ecliptic and mean equinox is

$$\begin{array}{rlll}
\lambda' - l' = + & 97''.774 & \sin(2l' + 315^\circ\, 18'\, 22'') + 0''.023 & \sin(4l' + 270^\circ\, 37') \\
+ [& 5025.8141 + 1.7921 & \sin(2l' + 54\;\; 33.2\;\;) + 0.0008 & \sin(4l' + 7\;\;)]\mathrm{T} \\
+ [& 1.10629 + 0.01737 & \sin(2l' + 148\;\; 11\;\;) + 0.00002 & \sin(4l' + 90\;\;)]\mathrm{T}^2 \\
+ [& 0.000179 + 0.000115 & \sin(2l' + 239\;\; 38\;\;)]\mathrm{T}^3 & \\
+ [- & 0.0000488 + 0.0000005 & \sin(2l' + 338\;\;)]\mathrm{T}^4 & \\
- & 0.00000023\mathrm{T}^5 & &
\end{array}$$

We have, then, $h = 50''.258141$, and for $\delta\mathrm{R}'$ the sum of the inequalities of the reduction

$$\begin{aligned}
\delta\mathrm{R}' = \;& \Gamma' - \tfrac{1}{2}s'\frac{ds'}{dv'} - \tan i' \cos(l' - \theta_0' + \alpha') \,.\, \varDelta\beta' \\
&+ [1''.7921 \sin(2l' + 54^\circ\, 33'.2) + 0''.0008 \sin(4l^\circ + 7^\circ)]\mathrm{T} \\
&+ [1''.10629 + 0''.01737 \sin(2l' + 148^\circ\, 11') + 0''.00002 \sin(4l' + 90^\circ)]\mathrm{T}^2 \\
&+ [0''.000179 + 0''.000115 \sin(2l' + 239^\circ\, 38')]\mathrm{T}^3
\end{aligned}$$

By the method of computing special values, when $\delta\mathrm{R}'$ is limited to the three latter lines of the expression above, we get

$$\begin{array}{rllll}
\frac{n'dz'}{d\lambda'}\delta\mathrm{R}' = [& & & -0''.0123 & \\
& +0''.1737 & \sin n'z' & +0.2462 & \cos n'z' \\
& -1.0271 & \sin 2n'z' & -1.4547 & \cos 2n'z' \\
& -0.0575 & \sin 3n'z' & -0.0815 & \cos 3n'z' \\
& -0.0035 & \sin 4n'z' & -0.0048 & \cos 4n'z']\mathrm{T} \\
+[& & & +1.11323 & \\
& -0.00256 & \sin n'z' & -0.12278 & \cos n'z' \\
& +0.01543 & \sin 2n'z' & -0.01008 & \cos 2n'z' \\
& +0.00082 & \sin 3n'z' & -0.00052 & \cos 3n'z']\mathrm{T}^2 \\
+[& & & 0.000179 & \\
& -0.000006 & \sin n'z' & -0.000038 & \cos n'z' \\
& +0.000056 & \sin 2n'z' & +0.000102 & \cos 2n'z' \\
& 0.000000 & \sin 3n'z' & +0.000005 & \cos 3n'z']\mathrm{T}^3
\end{array}$$

In this expression for $n'z'$ ought to be substituted $g'+n'\delta z'$. For $n'\delta z'$ it will be sufficient to take its more important terms, and its square can be neglected. When this is done, and we also obtain the terms which arise from the first line of the value of $\delta R'$, we have the following complete expression for $\Delta(n'\delta z')$:

Arg=$i'g'+ig$	$\Delta(n'\delta z')$	
	sin.	cos.
i' i	″ ″ ″ ″	″ ″ ″ ″
0 0		$-0.0123T+1.11325T^2+0.000147T^3$
1 0	$+0.1739T-0.00499T^2-0.000006T^3$	$+0.2461T-.12232T^2-.000064T^3$
2 0	$-1.0272T+.01597T^2-.000011T^3$	$-1.4547T-.01217T^2+.000146T^3$
3 0	$+0.0003-.0575T-.00005T^2-.000023T^3$	$+0.0043-.0815T+.00151T^2-.000006T^3$
4 0	$+0.0002-.0035T-.00007T^2$	$+0.0004-.0048T+.00017T^2$
5 0	-0.0001	-0.0001
$-2-1$	-0.0003	-0.0005
$-1-1$	$+0.0295-.0004T$	$-0.0262-.0009T$
$0-1$	$+0.0142+.0024T-.00003T^2$	$+0.0220-.0029T-.00002T^2$
$1-1$	$+0.0235-.0008T+.00001T^2$	$+0.0175+.0001T-.00013T^2$
$2-1$	$+0.0229+.0004T$	$+0.0005-.0002T$
$3-1$	$+0.0541-.0007T\quad .00000T^2$	$+0.0255+.0012T+.00013T^2$
$4-1$	$+0.0144+.0020T-.00003T^2$	$-0.0044+.0031T+.00002T^2$
$5-1$	$-0.0001+.0002T$	$+0.0012+.0002T$
$6-1$	-0.0003	$+0.0009$
$-1-2$	$+0.0005$	-0.0013
$0-2$	-0.0047	-0.0029
$1-2$	$-0.0017+.0004T$	$-0.0003+.0003T$
$2-2$	$-0.0030+.0063T+.00004T^2$	$+0.0043+.0043T-.00005T^2$
$3-2$	$-0.0949+.0266T+.00025T^2$	$+0.1671+.0059T-.00024T^2$
$4-2$	$+0.0154-.0020T+.00081T^2$	$-0.0444-.0002T-.00032T^2$
$5-2$	$+0.0432-.0016T-.00020T^2$	$+0.1792-.0002T-.00002T^2$
$6-2$	$+0.0010-.0041T-.00082T^2$	$-0.0079+.0009T+.00031T^2$
$7-2$	$-0.0054-.0136T-.00022T^2$	$+0.0005+.0216T-.00015T^2$
$8-2$	$-0.0002-.0011T$	$0.0000+.0018T$
$0-3$	$+0.0001$	-0.0001
$1-3$	$+0.0007$	-0.0017
$2-3$	-0.0003	-0.0006
$3-3$	$+0.0007$	-0.0005
$4-3$	$+0.0014$	$+0.0002$
$5-3$	$+0.0033$	-0.0002
$6-3$	$+0.0011$	-0.0004
$7-3$	$+0.0016$	-0.0017
$8-3$	-0.0002	$+0.0010$
$2-4$	$+0.0007$	$+0.0002$
$3-4$	$+0.0003$	-0.0002
$4-4$	$+0.0003$	0.0000
$5-4$	$+0.0002$	-0.0003

Arg=$i'g'+ig$		$\Delta(n'\delta z')$	
		sin.	cos.
i'	i	″	″
6—	4	+0.0001	+0.0006
7—	4	+0.0002	+0.0002
8—	4	+0.0021	+0.0003
9—	4	—0.0006	+0.0001
10—	4	+0.0015	—0.0012
11—	4	—0.0002	+0.0002
3—	5	0.0000	+0.0002
4—	5	+0.0001	+0.0001
5—	5	+0.0002	+0.0002
6—	5	+0.0001	+0.0001

The reduction which must be applied to the common logarithm of $\frac{r'}{\bar{r}'}$, on account of this change in the fundamental argument, is determined in a similar way, and is (in units of the seventh decimal):

Arg=$i'g'+ig$		$-\frac{d(\text{com. log } r')}{dz}\Delta(\delta z')$			
		cos.		sin.	
i'	i				
0	0		$-0.05T+0.001T^2+0.0008T^3$		
1	0		$+0.60T-0.009T^2 \quad 0.0000T^3$		$-0.85T-1.311T^2-0.0002T^3$
2	0		$+0.13T-0.002T^2+0.0008T^3$		$-0.19T-0.037T^2+0.0005T^3$
3	0		$-0.59T+0.009T^2 \quad 0.0000T^3$		$+0.84T+0.004T^2-0.0001T^3$
4	0		$-0.08T$		$+0.12T$
1—	1		$0.000T^2$		$+0.001T^2$
3—	1		$0.000T^2$		$+0.001T^2$
2—	2	+0.1		+0.1	
3—	2		$+0.002T^2$		$0.000T^2$
4—	2	—0.1	$+0.008T^2$	0.0	$+0.003T^2$
5—	2		$+0.002T^2$		$0.000T^2$
6—	2	0.0	$+0.008T^2$	—0.1	$+0.003T^2$

The correction to the latitude on the same account can be divided into two portions; the first, which is periodic and dependent on the position of Jupiter, can be added to $\Delta\beta'$; the second, which is a function of l', can be applied to $\sin\beta_0'$. The first portion is

Arg=$i'g'+ig$	$\Delta(\Delta\beta')$		Arg=$i'g'+ig$	$\Delta(\Delta\beta')$	
	sin.	cos.		sin.	cos.
i' i	″	″	i' i	″	″
2 0	0.0000	−0.0001	5 − 1	−0.0003	+0.0002
4 0	0.0000	−0.0001	1 − 2	+0.0002	0.0000
−2 − 1	−0.0008	+0.0003	2 − 2	+0.0033	−0.0026
−1 − 1	−0.0001	−0.0005	3 − 2	−0.0006	+0.0007
0 − 1	−0.0008	+0.0001	4 − 2	+0.0011	−0.0082
1 − 1	−0.0010	−0.0005	5 − 2	0.0000	+0.0012
2 − 1	−0.0015	−0.0011	6 − 2	−0.0023	−0.0032
3 − 1	−0.0008	+0.0002	7 − 2	+0.0001	+0.0002
4 − 1	−0.0013	−0.0001	8 − 2	+0.0001	0.0000

The portion to be added to the expression for $\sin\beta_0'$ is

$$\begin{aligned}\Delta(\sin\beta_0') = {} & [\overset{\prime\prime}{0.0380}T - \overset{\prime\prime}{0.04441}T^2 - \overset{\prime\prime}{0.000107}T^3]\sin l' \\ & + [-0.0088T + 0.01870T^2 - 0.000428T^3]\cos l' \\ & + [-0.0208T - 0.00035T^2]\sin 3l' \\ & + [0.0330T - 0.00023T^2]\cos 3l'\end{aligned}$$

CHAPTER XXVIII.

PRELIMINARY COMPARISON OF THE PRECEDING THEORY WITH OBSERVATION AND DERIVATION OF APPROXIMATE CORRECTIONS FOR THE ELEMENTS EMPLOYED IN THE CALCULATION OF THE PERTURBATIONS.

If the elements of the orbits of Jupiter and Saturn which have been employed in the preceding investigation were sufficiently approximate the expressions arrived at would need no further modification, except for possible changes in the values of the planetary masses. But as this is almost certainly not the case, we proceed to obtain approximate corrections for the provisionally adopted elements by a comparison of the preceding theory with observation. As the adopted planes of the orbits represent quite closely the observed latitudes of the planets, we need seek only the corrections of the four elements which give the position in orbit. Consequently comparison has been made only with normals in heliocentric longitude formed about the time of opposition. The thorough investigation of the values of the attracting masses must be deferred until the whole series of the observations, properly reduced, is taken in hand. The number of normals used here is very small on account of the great labor of making comparisons without the assistance of tables. There are only as many as are absolutely necessary for our purpose.

In forming the normals, Greenwich observations, taken precisely as they stand in the published volumes, without the application of any corrections, have been exclusively employed. Before 1830 the data have been derived from the Reduction of the Greenwich Observations of the Planets from 1750 to 1830. After 1830 the tabular longitude is from the English Nautical Almanac. Equal weights have been assigned to all the observations, and afterwards, in the discussion, all the normals have received equal weight.

We take up Saturn first, as the discussion of the observations of this planet will give us some information as to the mass of Uranus, which will be of service afterwards in treating Jupiter.

The normals constructed are as follows:

Greenwich M. T.	No. of observations.	Tabular longitude.	Correction.	Observed heliocentric longitude.
		° ′ ″	″	° ′ ″
1753, June 24.0	5	272 54 10.69	−18.36	272 53 52.33
1757, Aug. 11.0	7	318 47 10.89	−17.82	318 46 53.07
1761, Oct. 2.5	7	8 7 58.71	+ 0.30	8 7 59.01
1811, June 15.0	5	263 22 22.66	− 6.31	263 22 16.35

Greenwich M. T.	No. of observations.	Tabular longitude.	Correction.	Observed heliocentric longitude.
		° ′ ″	″	° ′ ″
1822, Oct. 30.0	6	36 40 22.56	+13.86	36 40 36.42
1837, May 4.0	10	223 50 29.0	− 1.74	223 50 27.26
1844, July 26.0	11	303 57 52.1	+11.99	303 58 4.09
1851, Oct. 24.0	12	30 49 43.9	+10.48	30 49 54.38
1858, Jan. 15.0	13	114 54 24.4	− 9.29	114 54 15.11
1866, Apr. 29.0	12	219 1 5.2	− 4.81	219 1 0.39
1874, Aug. 3.0	12	310 57 53.6	+ 8.17	310 58 1.77
1882, Nov. 15.0	9	52 42 8.9	− 7.35	52 42 1.55

The values of t in Julian years, and counted from the epoch 1850.0, and of the mean anomalies of Jupiter, Saturn, Uranus, and Neptune, for the dates of these normals, are:

t	g	g'	g''	g'''
	° ′ ″	° ′ ″	° ′ ″	°
−96.51747	98 47 21.27	185 9 38.35	166 35 56	80.94
−92.38604	224 10 25.64	235 39 4.49	184 18 6	89.96
−88.24230	349 55 56.09	286 17 32.68	202 3 26	99.02
−38.54620	58 9 40.65	173 38 6.13	55 0 6	207.59
−27.17044	43 24 18.10	312 39 35.40	103 44 46	232.44
−12.65982	123 47 17.28	129 59 45.30	165 55 23	264.14
− 5.43190	343 8 55.59	218 19 45.70	196 53 39	279.93
+ 1.81246	203 0 28.66	306 51 48.83	227 56 9	295.76
8.04107	32 2 25.20	22 59 3.27	254 37 30	309.37
16.32580	283 28 26.97	124 13 59.18	290 7 28	327.47
24.58864	174 14 35.72	225 12 51.46	325 31 48	345.52
+32.87338	65 40 37.49	326 27 47.38	1 1 47	3.62

By substituting these values in the several portions of the formula obtained for $n'\delta z'$ in the preceding chapters, we get the following quantities:

Perturbations of $n'z'$ by—					
Jupiter.	Uranus.	Jupiter × Uranus.	Neptune.	$n'z'$	f'
′ ″	″	″	″	° ″	° ′ ″
−33 27.245	−56.249	+29.860	−0.730	184 35 43.99	184 6 52.44
−36 26.311	−42.067	+28.493	+1.189	235 2 25.79	229 59 9.61
−43 59.803	− 7.428	+26.439	+2.278	285 33 54.17	279 16 16.34
−34 54.392	−45.461	+25.517	−0.218	173 2 51.58	173 46 28.19
−43 10.712	−42.490	+20.603	−3.207	311 55 59.59	306 55 35.95
−52 49.437	− 2.190	+22.848	+2.479	129 7 19.00	133 53 19.05
−40 24.661	−41.397	+20.990	+0.556	217 39 1.19	213 56 8.67
−46 12.502	−10.227	+17.502	−0.288	306 5 43.32	300 41 23.28
−53 55.680	+27.624	+17.479	−3.087	22 5 49.61	24 40 47.77
−43 37.181	− 6.234	+19.561	+0.346	123 30 35.67	128 39 33.20
−22 31.183	−20.115	+17.024	+0.954	224 50 18.14	220 31 40.13
−33 15.246	+17.727	+13.680	+0.618	325 55 4.16	322 5 59.24

The $n'\delta z'$ must be understood as the $n'\delta z'$ before the modification of the preceding chapter is applied. In order to have the heliocentric longitude referred to the actual equinox of date it is necessary to add to f': first, π', precession, nutation, and the secular part of the reduction to the ecliptic, and, second, the periodic part of the last. Thus we have the following quantities:

π' + precession + nutation.	Periodic reduction.	Calculated longitude.
° ′ ″	′ ″	° ′ ″
88 46 6.19	+0 59.31	272 53 57.94
88 49 12.36	−1 19.83	318 47 2.14
88 52 38.04	−0 44.30	8 8 10.08
89 34 26.32	+1 22.38	263 22 16.89
89 44 12.09	+0 47.62	36 40 35.66
89 56 0.11	+1 6.96	223 50 26.12
90 2 30.79	−0 38.74	303 58 0.72
90 7 59.83	+0 28.48	30 49 51.59
90 13 34.10	−0 8.48	114 54 13.39
90 20 28.85	+0 53.28	219 0 55.33
90 27 14.62	−0 58.57	310 57 56.18
90 34 30.67	+1 24.68	52 41 54.59

The equations of condition under three different suppositions are

$$
\begin{array}{llllrrr}
 & & & & \text{Supp. I.} & \text{Supp. II.} & \text{Supp. III.} \\
 & & & & {}'' & {}'' & {}'' \\
0.896\varDelta L' & -0.8644(100\varDelta n') & -0.140\varDelta e' & +1.864e'\varDelta\pi' & =-\ 5.61 & -\ 7.36 & -\ 7.73 \\
0.934 & -0.8626 & -1.509 & +1.184 & =-\ 9.07 & -10.01 & -10.18 \\
1.023 & -0.9026 & -1.989 & -0.410 & =-11.07 & -\ 9.63 & -\ 9.44 \\
0.896 & -0.3453 & +0.211 & +1.857 & =-\ 0.54 & -\ 1.86 & -\ 1.89 \\
1.073 & -0.2917 & -1.631 & -1.312 & =+\ 0.76 & -\ 0.98 & -\ 0.66 \\
0.928 & -0.1175 & +1.418 & +1.281 & =+\ 1.14 & +\ 2.56 & +\ 2.98 \\
0.913 & -0.0496 & -1.094 & +1.544 & =+\ 3.37 & +\ 1.99 & +\ 2.18 \\
1.063 & +0.0193 & -1.750 & -1.125 & =+\ 2.79 & +\ 3.36 & +\ 3.96 \\
1.110 & +0.0892 & +0.859 & -1.957 & =+\ 1.72 & +\ 5.43 & +\ 6.46 \\
0.936 & +0.1528 & +1.539 & +1.149 & =+\ 5.06 & +\ 5.98 & +\ 6.47 \\
0.921 & +0.2265 & -1.276 & +1.411 & =+\ 5.59 & +\ 5.38 & +\ 5.14 \\
1.095 & +0.3602 & -1.260 & -1.705 & =+\ 6.96 & +\ 9.51 & +\ 9.48
\end{array}
$$

Supposition I is obtained by subtracting the calculated from the observed longitudes. The remaining suppositions will be explained shortly. The normal equations resulting from these equations are

$$
\begin{array}{llllrrr}
 & & & & \text{Supp. I.} & \text{Supp. II.} & \text{Supp. III.} \\
 & & & & {}'' & {}'' & {}'' \\
11.655\varDelta L' & -2.414(100\,\varDelta n') & -\ 6.836\varDelta e' & +\ 2.350e'\varDelta\pi' & =+\ 2.05 & \text{or} +\ 6.25 & \text{or} +\ 8.82 \\
-2.414 & +2.739 & +\ 3.043 & -\ 3.064 & =+27.08 & \text{or} +30.32 & \text{or} +30.59 \\
-6.836 & +3.043 & +21.554 & +\ 3.830 & =+21.55 & \text{or} +27.09 & \text{or} +27.82 \\
+2.350 & -3.064 & +\ 3.830 & +25.555 & =-16.70 & \text{or} -33.61 & \text{or} -36.63
\end{array}
$$

The solution of these equations gives

$$\begin{array}{rlll}
 & \text{I.} & \text{II.} & \text{III.} \\
 & \quad '' & \quad '' & \quad '' \\
\Delta L' = & +\,2.688 & \text{or } +\,3.688 & \text{or } +\,4.049 \\
\Delta n' = & +\,0.13188 & \text{or } +\,0.13630 & \text{or } +\,0.13662 \\
\Delta e' = & -\,0.134 & \text{or } +\,0.520 & \text{or } +\,0.694 \\
e'\Delta \pi' = & +\,0.701 & \text{or } -\,0.100 & \text{or } -\,0.272
\end{array}$$

The residuals (observation-calculation), severally, in the three suppositions are

	I.	II.	III.
	″	″	″
1753, June 24.0	+ 2.05	+ 1.36	+ 1.06
1757, Aug. 11.0	− 1.23	− 0.79	− 0.81
1761, Oct. 2.5	− 1.89	− 0.11	+ 0.02
1811, June 15.0	+ 0.33	− 0.38	− 0.44
1822, Oct. 30.0	+ 2.42	− 0.24	− 0.26
1837, May 4.0	− 0.52	+ 0.12	+ 0.20
1844, July 26.0	+ 0.33	+ 0.01	+ 0.34
1851, Oct. 24.0	+ 0.23	− 0.03	+ 0.31
1858, Jan. 15.0	− 0.95	− 0.52	− 0.38
1866, Apr. 29.0	− 0.06	− 0.23	− 0.17
1874, Aug. 3.0	− 1.04	− 0.31	− 0.41
1882, Nov. 15.0	+ 0.28	+ 1.02	+ 0.53

The residuals of Supposition I are not altogether satisfactory, and, on comparing them with the portions of the perturbations which are proportional to the mass of Uranus, it is suggested that a better agreement would be obtained by diminishing this mass. Hence, I concluded to put the value at $\frac{1}{22640}$, which is about the average of all the results which have been obtained from the observations of the satellites at the Washington Observatory. This has given rise to the numbers of the column headed Supposition II. It will be seen that the residuals of (II) are fairly satisfactory, and it does not seem worth while, in this preliminary investigation, to inquire whether we should do better with another value of the mass of Uranus.

The pertubations being now corrected for the changes in the elements shown by (II), and for the similar ones to be given hereafter for Jupiter, the resulting numbers appear under Supposition III, to which we hold as being the best which can be done at present. The residuals of (III) are to some extent better than those of (II).

We now pass to Jupiter. The normals are formed as follows:

Greenwich M. T.	No. of observations.	Tabular longitude.			Correction.	Heliocentric longitude from observation.		
		°	′	″	″	°	′	″
1757, May 3.5	7	223	44	36.85	+ 6.59	223	44	43.44
1759, July 9.5	8	287	33	42.20	+10.70	287	33	52.90
1819, Aug. 5.5	12	312	16	54.91	+ 6.78	312	17	1.69
1855, Aug. 22.0	16	327	44	57.70	− 5.46	327	44	52.24
1858, Dec. 16.0	9	77	11	8.30	+ 5.87	77	11	14.17
1861, Feb. 16.0	11	142	29	48.10	+ 8.31	142	29	56.41
1864, May 16.0	9	232	58	30.70	+17.35	232	58	48.05
1867, Aug. 23.0	6	332	18	32.80	+ 0.77	332	18	33.57
1870, Dec. 19.0	6	81	53	54.70	+ 7.63	81	54	2.33
1874, Mar. 18.0	12	176	56	16.60	+ 7.27	176	56	23.87
1877, June 19.0	11	268	41	48.00	+15.26	268	42	3.26
1878, July 20.0	7	301	49	21.10	− 0.17	301	49	20.93
1880, Oct. 7.0	12	14	30	48.20	+ 0.18	14	30	48.38

The values of t in Julian years, counted from 1850.0, and of the mean anomalies of Jupiter, Saturn, Uranus, and Neptune for the dates of these normals, are:

t	g			g'			g''			g'''
	°	′	″	°		″	°	′	″	°
−92.65847	215	54	33.83	232	19	19.25	183	8	4	89.37
−90.47637	282	7	58.96	258	59	21.65	192	29	5	94.14
−30.40520	305	14	9.63	273	7	38.17	89	53	7	225.37
+ 5.63997	319	10	21.25	353	38	24.50	244	20	11	304.12
8.95825	59	52	44.57	34	11	35.59	258	33	18	311.37
11.12936	125	46	13.19	60	43	36.17	267	51	29	316.11
14.37372	224	14	0.04	100	22	34.98	281	45	35	323.20
17.64271	323	26	39.05	140	19	37.89	295	46	2	330.34
20.96646	64	19	0.62	180	56	49.90	310	0	33	337.61
24.21082	162	46	47.47	220	35	48.71	323	54	40	344.69
27.46612	261	34	30.84	260	22	49.35	337	51	35	351.81
28.55031	294	28	45.59	273	37	49.41	342	30	20	354.17
+30.76796	1	46	59.39	300	43	57.72	352	0	29	359.02

By substituting these values in the several portions of the formula obtained for $n\delta z$ in the preceding chapters, multiplying the part due to the action of Uranus by the factor $\frac{21000}{22640}$, we get the following quantities:

Perturbation of nz by—										
Saturn.		Uranus.	Saturn × Uranus.	Neptune.	nz			f		
′	″	″	″	″	°	′	″	°	′	″
+18	36.578	−0.140	−8.244	−0.389	216	13	1.63	213	6	14.83
14	0.876	+0.205	8.205	−0.104	282	21	51.73	276	54	7.47
12	31.844	−0.164	6.581	−0.031	305	26	34.70	300	46	53.00
19	50.624	+0.518	5.380	+0.057	319	30	7.07	315	44	32.43
18	3.316	+0.047	4.846	−0.060	60	10	43.03	65	7	0.23
19	59.079	−0.164	5.161	−0.184	126	6	6.76	130	24	39.61
19	8.804	+0.052	5.609	+0.172	224	33	3.46	220	50	7.14
8	15.384	+1.595	4.957	−0.029	323	34	51.04	320	8	1.98
14	13.009	−0.098	4.496	−0.004	64	33	9.03	69	40	12.63
22	54.849	−0.133	5.074	−0.091	163	9	37.02	164	40	26.55
13	19.365	−1.185	5.089	+0.239	261	47	44.17	256	22	43.46
9	33.806	−0.584	4.830	+0.148	294	38	14.13	289	29	23.34
+ 9	27.323	+1.446	−4.256	−0.094	1	56	23.81	[illegible]	8	20.61

Applying to f the longitude of the perihelion, precession, nutation, and the reduction to the ecliptic, we have the calculated longitude as follows:

π + precession + nutation.			Periodic reduction.	Calculated longitude.		
°	′	″	″	°	′	″
10	38	20.02	+25.85	223	45	0.70
10	40	5.98	− 8.92	287	34	4.53
11	30	38.14	−25.01	312	17	6.13
12	0	43.61	−26.78	327	44	49.26
12	3	47.02	+18.66	77	11	5.91
12	5	45.97	−26.96	142	29	58.62
12	8	23.83	+26.97	232	58	57.94
12	10	52.55	−25.89	332	18	28.64
12	13	26.08	+15.28	81	53	53.99
12	16	16.18	−11.17	176	56	31.56
12	19	17.07	+ 9.68	268	42	10.21
12	20	17.64	−19.13	301	49	21.85
12	22	12.45	+ 4.92	14	30	37.98

The equations of condition under three different suppositions are

					Supp. I.	Supp. II.	Supp. III.
					″	″	″
$0.924\Delta L$	$- 0.8562(100\,\Delta n)$	$- 1.073\Delta e$	$+ 1.575e\Delta\pi$	=	− 17.26	− 22.97	− 17.26
1.015	− 0.9184	− 1.996	− 0.315	=	− 11.63	− 16.34	− 11.61
1.054	− 0.3204	− 1.744	− 1.113	=	− 4.44	− 8.81	− 4.50
1.074	+ 0.0606	− 1.422	− 1.537	=	+ 2.98	− 4.12	+ 2.96
1.045	0.0936	+ 1.837	− 0.926	=	+ 8.26	+ 1.98	+ 8.13
0.942	0.1048	+ 1.502	+ 1.209	=	− 2.21	− 8.47	− 2.39
0.932	0.1339	− 1.286	+ 1.419	=	− 9.89	− 15.82	− 9.99
1.079	0.1904	− 1.309	− 1.641	=	+ 4.93	+ 1.98	+ 4.93
1.038	0.2175	+ 1.896	− 0.776	=	+ 8.34	+ 3.44	+ 8.24
0.912	0.2208	+ 0.517	+ 1.818	=	− 7.69	− 14.65	− 7.84
0.981	0.2694	− 1.936	+ 0.397	=	− 6.95	− 11.29	− 6.98
1.026	0.2958	− 1.905	− 0.747	=	− 0.92	− 4.20	− 0.90
1.103	+ 0.3392	+ 0.077	− 2.125	=	+ 10.40	+ 0.94	+ 10.42

The normal equations resulting from these equations are

					Supp. I.		Supp. II.		Supp. III.
					″		″		″
$13.318\Delta L$	$- 0.097(100\Delta n)$	$- 7.008\Delta e$	$- 3.836e\Delta\pi$	=	− 21.25	or	− 87.59	or	− 21.94
− 0.097	+ 2.128	+ 2.599	− 1.481	=	+ 28.73	or	+ 30.40	or	+ 28.63
− 7.008	+ 2.599	+ 30.443	+ 3.462	=	+ 91.19	or	+ 116.47	or	+ 90.69
− 3.836	− 1.481	+ 3.462	+ 22.344	=	− 100.19	or	− 98.05	or	− 100.59

Their solution gives

	I.		II.		III.
	″		″		″
$\Delta L =$	− 1.540	or	− 6.923	or	− 1.615
$\Delta n =$	+ 0.07024	or	+ 0.07491	or	+ 0.06989
$\Delta e =$	+ 2.574	or	+ 2.210	or	+ 2.546
$e\Delta\pi =$	− 4.683	or	− 5.424	or	− 4.711

The residuals (observation-calculation), severally in the three suppositions, are

	I.	II.	III.
	″	″	″
1757, May 3.5	+ 0.30	+ 0.75	+ 0.35
1759, July 9.5	+ 0.05	+ 0.27	+ 0.05
1819, Aug. 5.5	− 1.29	− 1.31	− 1.36
1855, Aug. 22.0	+ 0.66	− 2.33	+ 0.65
1858, Dec. 16.0	+ 0.15	− 0.58	+ 0.12
1861, Feb. 16.0	+ 0.31	+ 0.51	+ 0.28
1864, May 16.0	+ 0.55	+ 0.16	+ 0.53
1867, Aug. 23.0	+ 0.93	+ 2.01	+ 0.94
1870, Dec. 19.0	− 0.10	+ 0.58	− 0.10
1874, Mar. 18.0	− 0.66	− 1.26	− 0.67
1877, June 19.0	− 0.49	− 0.08	− 0.48
1878, July 20.0	0.00	+ 0.92	+ 0.05
1880, Oct. 7.0	− 0.44	+ 0.33	− 0.39

Supposition I corresponds to BESSEL's value $\frac{1}{3501.6}$ of the mass of Saturn, while (II) results from using the value $\frac{1}{3482.2}$, recently derived by Prof. A. HALL from observations of Japetus. The residuals of (II) are generally larger than those of (I), and, in consequence, I shall hold to BESSEL's value, although it is possible that when the observations are more properly reduced a better showing may result for the larger mass. In fine, Supposition III results from (I) by applying to the perturbations the corrections due to the adopted changes in the elements.*

---

*It will be noticed that some of the numbers given in this chapter differ slightly from those stated in the Astr. Nachr., Nos. 2705–2706. This is because at the time of the publication of the latter the term $-\mathrm{B}\tan\frac{i}{2}$, in the reduction of orbit to ecliptic longitude (in formula on page 523), had not been noticed. HANSEN determines his $n\delta z$ in such a way that it contains no term rigorously proportional to the time. This differs from the course followed by those who employ other methods of perturbation. They make the orbit longitude, as measured along a fixed ecliptic until the node is reached and then on the plane of the orbit, to have no perturbations proportional to t. But HANSEN regards the term $2\sin^2\frac{i}{2}\frac{d\theta}{dt}$ as belonging to the reduction to the ecliptic.

CHAPTER XXIX.

RECTIFICATION OF THE FORMULÆ FOR THE PERTURBATIONS ON ACCOUNT OF THE CORRECTIONS OF THE ELEMENTS JUST DETERMINED.

Hansen has treated this matter,* but the expressions he derives are suitable to the employment of the eccentric anomaly as independent variable. As the modifications, to be made in order to render them applicable when the mean anomaly or the time is employed as independent variable, are not readily perceived, we will, as briefly as possible, develop them here.

We shall suppose that the elements which define the positions of the planes of the two orbits are known at the outset with sufficient exactitude to insure the desired degree of accuracy in the expressions of the perturbations so far as it depends on them. We can also suppose that the terms of two dimensions, with respect to disturbing forces, require no sensible correction on account of changes made in the elements; and we may assume that the same is true for u and u'. Hence, we shall limit our attention to determining the effects produced in the first-order terms of $n\delta z$, ν, $n'\delta z'$, and ν'.

Consequently, we can assume

$$\frac{1}{n}\frac{dW_0}{dt} = T = Aa_0\frac{d\Omega}{dg} + Ba_0 r\frac{d\Omega}{dr}$$

$$n\delta z = \int \left[\overline{\int T n dt}\right] n dt$$

$$\nu = \text{const.} - \frac{1}{2}\int\left(\frac{d\,.\,\left[\overline{\int T n dt}\right]}{d\gamma}\right) n dt$$

The augmentation of the elements being denoted by Δa, $\Delta a'$, Δe, $\Delta e'$, $\Delta\pi$, and $\Delta\pi'$, we have

$$\Delta T = \frac{dT}{da}\Delta a + \frac{dT}{da'}\Delta a' + \frac{dT}{de}\Delta e + \frac{dT}{de'}\Delta e' + \frac{dT}{d\pi}\Delta\pi + \frac{dT}{d\pi'}\Delta\pi'$$

$$T = \frac{a}{\cos\varphi}\left[2\frac{\rho}{r}\cos(f-\omega) - 1 + \frac{2\rho}{a\cos^2\varphi}[\cos(f-\omega) - 1]\right]\frac{d\Omega}{dv} + \frac{2a}{\cos\varphi}\frac{\rho}{r}\sin(f-\omega) r\frac{d\Omega}{dr}$$

T may be regarded, then, as a function of a, φ, ρ, r, f, ω, f', r', π, and π'; or, what is the same thing, as a function of a, φ, ρ, r, r', $f+\pi$, $\omega+\pi$, and $f'+\pi'$. And it is evident that, instead of taking the partial derivatives of T with respect to $f+\pi$, $\omega+\pi$, and $f' + \pi'$, we may take them with respect to g, γ, and g', provided that Δg, $\Delta\gamma$, and $\Delta g'$ are determined by the equations

---

*Auseinandersetzung, Abth. III, ss. 272–297.

$$\Delta g = \frac{\frac{df}{de}}{\frac{df}{dg}}\Delta e + \frac{1}{\frac{df}{dg}}\Delta\pi$$

$$\Delta\gamma = \frac{\frac{d\omega}{de}}{\frac{d\omega}{d\gamma}}\Delta e + \frac{1}{\frac{d\omega}{d\gamma}}\Delta\pi$$

$$\Delta g' = \frac{\frac{df'}{de'}}{\frac{df'}{dg'}}\Delta e' + \frac{1}{\frac{df'}{dg'}}\Delta\pi'$$

and that we conceive r, ρ, and r' to be augmented by $r\Delta\nu$, $\rho\Delta\beta$, and $r'\Delta\nu'$, such that

$$\Delta\nu = \frac{\Delta a}{a} - \frac{\cos\varepsilon + e}{\cos^2\varphi}\Delta e - \frac{e\sin\varepsilon}{\cos\varphi}\Delta\pi$$

$$\Delta\beta = \frac{\Delta a}{a} - \frac{\cos\eta + e}{\cos^2\varphi}\Delta e - \frac{e\sin\eta}{\cos\varphi}\Delta\pi$$

$$\Delta\nu' = \frac{\Delta a'}{a'} - \frac{\cos\varepsilon' + e'}{\cos^2\varphi'}\Delta e' - \frac{e'\sin\varepsilon'}{\cos\varphi'}\Delta\pi'$$

Then, we have

$$\Delta T = \frac{dT}{dg}\Delta g + \frac{dT}{dg'}\Delta g' + r\frac{dT}{dr}\Delta\nu + r'\frac{dT}{dr'}\Delta\nu' + a\frac{dT}{da}\frac{\Delta a}{a} + \frac{dT}{d\varphi}\Delta\varphi + \frac{dT}{d\gamma}\Delta\gamma + \rho\frac{dT}{d\rho}\Delta\beta$$

The coefficients of the first four terms have already been used for the computation of the second-order terms. Also

$$a\frac{dT}{da} = T - \frac{4\rho}{\cos^3\varphi}\left[\cos(f-\omega) - 1\right]\frac{d\Omega}{dv}$$

$$T = \frac{a}{\cos\varphi}\,\frac{d\Omega}{dv}$$

In computing the second-order terms we have derived the value of

$$X = -\frac{2a}{\cos\varphi}\frac{\rho}{r}\cos(f-\omega)\,.\,\frac{d\Omega}{dv} - \frac{2a}{\cos\omega}\frac{\rho}{r}\sin(f-\omega)\,.\,r\frac{d\Omega}{dr}$$

whence

$$a\frac{dT}{da} = T - 2(T + X + T) = T - C$$

where C has also been employed in treating the second-order terms. In the next place

$$\frac{dT}{d\varphi} = \frac{e}{\cos\varphi}\left[T + 2(T + X + T)\right] = \frac{e}{\cos\varphi}\left[T + C\right]$$

Consequently, as

$$\log h = \text{const.} - \frac{1}{2}\log a - \log\cos\varphi$$

and thence

$$\frac{\Delta h}{h} = -\frac{1}{2}\frac{\Delta a}{a} + \frac{e}{\cos\varphi}\Delta\varphi$$

we shall have

$$\Delta T = A\Delta g + F\Delta g' + B\Delta\nu + G\Delta\nu' + C\left(\frac{\Delta h}{h} - \frac{1}{2}\frac{\Delta a}{a}\right) + T\left(\frac{\Delta a}{a} + \frac{e\Delta e}{\cos^2\varphi}\right) + \frac{dT}{d\gamma}\Delta\gamma + (T + T)\Delta\beta$$

The computation of the first five terms of this formula is then quite similar to that of the second-order terms, and we may put

$$L = A\Delta g + F\Delta g' + B\Delta\nu + G\Delta\nu' + C\left(\frac{\Delta h}{h} - \frac{1}{2}\frac{\Delta a}{a}\right)$$

Integrating, we derive

$$\begin{aligned}\Delta W_0 &= \int Lndt + \left(\frac{\Delta a}{a} + \frac{e\Delta e}{\cos^2\varphi}\right)W_0 + \Delta\beta\left(W_0 + \int Tndt\right) + \frac{dW_0}{d\gamma}\Delta\gamma \\ &= \int Lndt + \left(\frac{\Delta a}{a} + \frac{e\Delta e}{\cos^2\varphi}\right)W_0 + \Delta\beta(W_0 + \overline{W}_0 + 2\nu) + \frac{dW_0}{d\gamma}\Delta\gamma\end{aligned}$$

$$\begin{aligned}\Delta(n\delta z) &= \int\left[\overline{\left(\int Lndt\right)} + \left(\overline{\frac{dW_0}{d\gamma}}\right)\Delta g + 2(W_0 + \nu)\Delta\nu + \left(\frac{\Delta a}{a} + \frac{e}{\cos^2\varphi}\Delta e\right)\overline{W}_0\right]ndt \\ &= \int\left[\overline{\left(\int Lndt\right)} - 2\frac{d\nu}{ndt}\Delta g + 2\left(\frac{d\,.\,\delta z}{dt} + \nu\right)\Delta\nu\right]ndt + \left(\frac{\Delta a}{a} + \frac{e}{\cos^2\varphi}\Delta e\right)n\delta z\end{aligned}$$

$$\begin{aligned}\Delta\nu = -\frac{1}{2}\int\Bigg[&\left(\overline{\frac{d\,.\int Lndt}{d\gamma}}\right) + \left(\overline{\frac{d^2W_0}{d\gamma^2}}\right)\Delta g + \left(\overline{\frac{dW_0}{d\gamma}}\right)\frac{d\,.\,\Delta g}{dg} + \left(\frac{\Delta a}{a} + \frac{e}{\cos^2\varphi}\Delta e\right)\left(\overline{\frac{dW_0}{d\gamma}}\right) \\ &+ \left(\overline{\frac{dW_0}{d\gamma}}\right)\Delta\nu + 2(W_0 + \nu)\frac{d\,.\,\Delta\nu}{dg}\Bigg]ndt\end{aligned}$$

But from formulæ to be given shortly it will be seen that

$$\frac{d\,.\,\Delta g}{dg} = 2\frac{\Delta a}{a} - 2\Delta\nu - \frac{e}{\cos^2\varphi}\Delta e$$

Hence

$$\begin{aligned}\Delta\nu &= -\frac{1}{2}\int\left[\left(\overline{\frac{d\,.\int Lndt}{d\gamma}}\right) + \left(\overline{\frac{d^2W_0}{d\gamma^2}}\right)\Delta g + \left(\overline{\frac{dW_0}{d\gamma}}\right)\left(3\frac{\Delta a}{a} - \Delta\nu\right) + 2(\overline{W}_0 + \nu)\frac{d\,.\,\Delta\nu}{dg}\right]ndt \\ &= -\frac{1}{2}\int\left[\left(\overline{\frac{d\,.\int Lndt}{d\gamma}}\right) + \left(\overline{\frac{d^2W_0}{d\gamma^2}}\right)\Delta g + 2\overline{W}_0\frac{d\,.\,\Delta\nu}{dg}\right]ndt - \nu\left(\Delta\nu - 3\frac{\Delta a}{a}\right)\end{aligned}$$

The expressions for $\varDelta g$ and $\varDelta \nu$ are

$$\varDelta g = \frac{2\varDelta e}{\cos^2 \varphi}\left[\left(J^{(0)}_{\frac{e}{2}} - J^{(2)}_{\frac{e}{2}}\right)\sin g + \frac{1}{4}(J^{(1)}_{e} - J^{(3)}_{e})\sin 2g + \frac{1}{9}\left(J^{(2)}_{\frac{3e}{2}} - J^{(4)}_{\frac{3e}{2}}\right)\sin 3g + \ldots\right]$$

$$+ \frac{\varDelta\pi}{\cos\varphi}\left[1 + \frac{3}{2}e^2 - \frac{2e}{1}\left(J^{(0)}_{\frac{e}{2}} + J^{(2)}_{\frac{e}{2}}\right)\cos g - \frac{2e}{4}(J^{(1)}_{e} + J^{(3)}_{e})\cos 2g - \frac{2e}{9}\left(J^{(2)}_{\frac{3e}{2}} + J^{(4)}_{\frac{e}{2}}\right)\cos 3g - \ldots\right]$$

$$\varDelta\nu = \frac{\varDelta a}{a} - \frac{\varDelta e}{\cos^2\varphi}\left[\frac{1}{2}e + \left(J^{(0)}_{\frac{e}{2}} - J^{(2)}_{\frac{e}{2}}\right)\cos g + \frac{1}{2}(J^{(1)}_{e} - J^{(3)}_{e})\cos 2g + \frac{1}{3}\left(J^{(2)}_{\frac{3e}{2}} - J^{(4)}_{\frac{3e}{2}}\right)\cos 3g + \ldots\right]$$

$$- \frac{e\varDelta\pi}{\cos\varphi}\left[\left(J^{(0)}_{\frac{e}{2}} + J^{(2)}_{\frac{e}{2}}\right)\sin g + \frac{1}{2}(J^{(1)}_{e} + J^{(3)}_{e})\sin 2g + \frac{1}{3}\left(J^{(2)}_{\frac{3e}{2}} + J^{(4)}_{\frac{3e}{2}}\right)\sin 3g + \ldots\right]$$

We have yet to ascertain the corrections due to the change of the integrating factors. T being a function composed of such terms as

$$T = A\sin(\varkappa\gamma + i'g' + ig) + B\cos(\varkappa\gamma + i'g' + ig)$$

we have

$$W_0 = -\frac{A}{i'\frac{n'}{n} + i}\cos(\varkappa\gamma + i'g' + ig) + \frac{B}{i'\frac{n'}{n} + i}\sin(\varkappa\gamma + i'g' + ig)$$

Whence

$$\varDelta W_0 = \left[\frac{i'A}{\left(i'\frac{n'}{n} + i\right)^2}\cos(\varkappa\gamma + i'g' + ig) - \frac{i'B}{\left(i'\frac{n'}{n} + i\right)^2}\sin(\varkappa\gamma + i'g' + ig)\right]\varDelta\frac{n'}{n}$$

But

$$\int W_0 n dt = \frac{A}{\left(i'\frac{n'}{n} + i\right)^2}\sin(\varkappa\gamma + i'g' + ig) - \frac{B}{\left(i'\frac{n'}{n} + i\right)^2}\cos(\varkappa\gamma + i'g' + ig)$$

whence

$$\varDelta W_0 = -\frac{d \,.\int W_0 n dt}{dg'}\varDelta\frac{n'}{n}$$

and

$$\varDelta \overline{W_0} = -\frac{d\,.\,(\overline{\int W_0 n dt})}{dg'}\varDelta\frac{n'}{n} = -\left(\overline{\int\frac{dW_0}{dg'}ndt}\right)\varDelta\frac{n'}{n}$$

In like manner

$$-\frac{1}{2}\left(\overline{\varDelta\frac{dW_0}{d\gamma}}\right) = \frac{1}{2}\left(\overline{\int\frac{d^2W_0}{d\gamma dg'}ndt}\right)\varDelta\frac{n'}{n}$$

Thus, we have, for getting the corrections of the perturbations due to a change in the integrating factors, the very simple formulæ

$$\varDelta(n\delta z) = -2\left[\int\frac{d\,.\,(n\delta z)}{dg'}ndt\right]\varDelta\frac{n'}{n}$$

$$\varDelta\nu = -2\left[\int\frac{d\nu}{dg'}ndt\right]\varDelta\frac{n'}{n}$$

On account of the smallness of the eccentricities of the major planets it often happens that the corrections $\Delta\pi$ and $\Delta\pi'$ are quite large, while the effects of these corrections on the positions of the planets, being more properly represented by the products $e\Delta\pi$ and $e'\Delta\pi'$, are quite small. In these cases it may be reasonably feared, since Δg and $\Delta\gamma$ contain the term

$$\frac{1+\frac{3}{2}e^2}{\cos\varphi}\Delta\pi$$

and $\Delta g'$ the term

$$\frac{1+\frac{3}{2}e'}{\cos\varphi'}\Delta\pi'$$

that it will not be sufficiently approximate to consider the corrections of the perturbations as equivalent to linear functions of Δg, $\Delta\nu$, $\Delta g'$, $\Delta\nu'$, etc., the terms involving the squares of Δg and $\Delta g'$ and their products with $\Delta\nu$, $\Delta\nu'$, etc., becoming sensible on account of the largeness of $\Delta\pi$ and $\Delta\pi'$.

A little consideration, however, shows that this difficulty may be readily surmounted. Let us suppose that putting, for the sake of brevity,

$$\mathrm{E}=\frac{1+\frac{3}{2}e^2}{\cos\varphi} \qquad \mathrm{E}'=\frac{1+\frac{3}{2}e'}{\cos\varphi'}$$

we substitute for Δg, $\Delta\gamma$, and $\Delta g'$ the expressions

$$\begin{aligned}\Delta g &= \mathrm{E}\Delta\pi + \Delta(g)\\ \Delta\gamma &= \mathrm{E}\Delta\pi + \Delta(\gamma)\\ \Delta g' &= \mathrm{E}'\Delta\pi' + \Delta(g')\end{aligned}$$

so that

$$\begin{aligned}\Delta(g) = {} & \frac{2\Delta e}{\cos^2\varphi}\left[\left(\mathrm{J}^{(0)}_{\frac{e}{2}}-\mathrm{J}^{(2)}_{\frac{e}{2}}\right)\sin g+\frac{1}{4}\left(\mathrm{J}^{(1)}_{e}-\mathrm{J}^{(3)}_{e}\right)\sin 2g+\frac{1}{9}\left(\mathrm{J}^{(2)}_{\frac{3}{2}e}-\mathrm{J}^{(4)}_{\frac{3}{2}e}\right)\sin 3g+\ldots\right]\\ & -\frac{2e\Delta\pi}{\cos\varphi}\left[\left(\mathrm{J}^{(0)}_{\frac{e}{2}}+\mathrm{J}^{(2)}_{\frac{e}{2}}\right)\cos g+\frac{1}{4}\left(\mathrm{J}^{(1)}_{e}+\mathrm{J}^{(3)}_{e}\right)\cos 2g+\frac{1}{9}\left(\mathrm{J}^{(2}_{\frac{3}{2}e}+\mathrm{J}^{(4)}_{\frac{3}{2}e}\right)\cos 3g+\ldots\right]\end{aligned}$$

with a similar expression for $\Delta(g')$. Now, when g, γ, and g' are augmented by their increments Δg, $\Delta\gamma$, and $\Delta g'$, the variation of the arguments due to the terms $\mathrm{E}\Delta\pi$ and $\mathrm{E}'\Delta\pi'$ may be retained within the functional signs sin and cos, and the corrections of the perturbations exhibited as a linear function of $\Delta(g)$, $\Delta(g')$, etc. Thus the general argument $\varkappa\gamma+i'g'+ig$ becomes

$$\varkappa\gamma+i'g'+ig+(\varkappa+i)\mathrm{E}\Delta\pi+i'\mathrm{E}'\Delta\pi'$$

The presence of the two terms ending this expression evidently has no influence on the coefficient when integrations are performed, or when we go through the operation of putting $\gamma=g$. Hence the following precept:

In the preceding formulæ, for obtaining the corrections of the perturbations arising from changes in the elements, we may everywhere substitute $\Delta(g)$ and $\Delta(g')$ for Δg and $\Delta g'$, provided that at the end, when the expressions for the perturbations have been reduced to series of terms of the form

$$\mathrm{K}\, n^k t^k \begin{matrix}\sin\\ \cos\end{matrix}\left(i'g' + ig + \varkappa\right)$$

we replace $\varkappa$ by

$$\varkappa + i\mathrm{E}\Delta\pi + i'\mathrm{E}'\Delta\pi'$$

or, which applies to any form the perturbations may be in, we everywhere replace g by $g + \mathrm{E}\Delta\pi$, and g' by $g' + \mathrm{E}'\Delta\pi'$.

In applying these formulæ to Jupiter and Saturn it was assumed that

$$\begin{aligned}
\Delta e &= +\overset{''}{2.566} & \qquad \Delta e' &= +\overset{''}{0.527}\\
e\Delta\pi &= -4.550 & e'\Delta\pi' &= -0.093\\
\Delta n &= +0.06958 & \Delta n' &= +0.12853
\end{aligned}$$

From these numbers we derive

$$\begin{aligned}
\log\frac{\Delta a}{a} &= 3.6280n & \qquad \log\frac{\Delta a'}{a'} &= 4.2895n\\
\log 2\Delta\frac{n'}{n} &= 4.2648 & \log 2\Delta\frac{n}{n'} &= 5.0548n\\
\log\left(\frac{\Delta a}{a} + \frac{e\Delta e}{\cos^2\varphi}\right) &= 3.2479 & \log\left(\frac{\Delta a'}{a'} + \frac{e'\Delta e'}{\cos^2\varphi'}\right) &= 4.2563n\\
\mathrm{E}\Delta\pi &= -1'34''.75 & \mathrm{E}'\Delta\pi' &= -1''.67
\end{aligned}$$

$$\begin{aligned}
\Delta(g) = {} & 2\,[5.0955]\sin g + 2\,[5.3440]\cos g\\
& + 2\,[3.1766]\sin 2g + 2\,[3.4251]\cos 2g\\
& + 2\,[1.5587]\sin 3g + 2\,[1.8073]\cos 3g
\end{aligned}$$

$$\begin{aligned}
\Delta(g') = {} & 2\,[4.4073]\sin g' + 2\,[3.6553]\cos g'\\
& + 2\,[2.5535]\sin 2g' + 2\,[1.8016]\cos 2g'\\
& + 2\,[1.0007]\sin 3g' + 2\,[0.2489]\cos 3g'
\end{aligned}$$

$$\begin{aligned}
\Delta v = {} & -[3.8606]\\
& - 2\,[4.7945]\cos g + 2\,[5.0430]\sin g\\
& - 2\,[3.1766]\cos 2g + 2\,[3.4251]\sin 2g\\
& - 2\,[1.7348]\cos 3g + 2\,[1.9834]\sin 3g
\end{aligned}$$

$$\begin{aligned}
\Delta v' = {} & -[4.3052]\\
& - 2\,[4.1063]\cos g' + 2\,[3.3543]\sin g'\\
& - 2\,[2.5535]\cos 2g' + 2\,[1.8016]\sin 2g'\\
& - 2\,[1.1768]\cos 3g' + 2\,[0.4250]\sin 3g'
\end{aligned}$$

Application to Jupiter.

The value of L is found to be:

Arg=$\varkappa\gamma+i'g'+ig$			L	
			sin.	cos.
$\varkappa$	i'	i	″	″
1	0	— 1		+0.0000208
1	0	— 2		—0.000013
—1	0	0	—0.0003461	+0.0006866
—1	1	+ 1	+0.0004	—0.0003
0	1	0	—0.0005	+0.0004
1	1	— 1	—0.0002	+0.0001
—1	1	0	0.0000	+0.0001
—1	2	0	+0.00201	+0.00161
0	2	— 1	—0.00317	—0.00244
1	2	— 2	+0.0011	+0.0008
—1	2	— 1	—0.0004	+0.0002
—1	3	0	+0.0007	0.0000
0	3	— 1	—0.00086	—0.00008
1	3	— 2	+0.0002	0.0000
—1	3	— 1	—0.0018	+0.0029
0	3	— 2	+0.0017	—0.0036
—1	4	— 1	+0.0001	+0.0009
0	4	— 2	—0.00014	—0.00120
—1	4	— 2	—0.0026	—0.0006
0	4	— 3	+0.002	0.000
—1	5	— 1	+0.000111	+0.000177
0	5	— 2	—0.0001230	—0.0002187
1	5	— 3	+0.000074	+0.000049
—1	5	— 2	—0.001195	+0.000201
0	5	— 3	+0.0008	—0.0002
—1	5	— 3	0.000	—0.001
—1	6	— 2	—0.0001	+0.0001
0	6	— 3	+0.0002	—0.0001
—1	6	— 3	—0.0003	—0.0005
0	7	— 3	+0.00001	—0.00006
—1	7	— 3	—0.0002	—0.0001
0	10	— 4	+0.0000010	—0.0000020

The remainder of the necessary operations being performed, we obtain:

Arg=$i'g'+ig$		$\Delta(n\delta z)$		$\Delta\nu$	
		sin.	cos.	cos.	sin.
i'	i	″	″	″	″
0	0			—0.0000052nt	
0	— 1	—0.0006872nt	—0.0003428nt	—0.0003436nt	+0.0001714nt
0	— 2	—0.0000177nt	—0.0000020nt	—0.0000177nt	+0.0000020nt
0	— 3	—0.0000006nt	0.0000000nt	—0.0000009nt	0.0000000nt
1	0	—0.0008	+0.0008	+0.0002	+0.0002
1	— 1	0.0000	+0.0003	0.0000	—0.0002
2	— 1	+0.0545	+0.0406	+0.0073	—0.0059
2	— 2	+0.0023	—0.0011	+0.0008	+0.0004
2	— 3	+0.0005	—0.0009	+0.0006	+0.0019
3	— 1	+0.0082	+0.0010	—0.0014	—0.0005
3	— 2	—0.0130	+0.0221	—0.0051	—0.0081
3	— 3	+0.0006	+0.0011		

Arg=$i'g'+ig$		$\Delta(n\delta z)$		$\Delta\nu$	
		sin.	cos.	cos.	sin.
i'	i	″	″	″	″
4—	2	+0.0015	+0.0092	0.0000	—0.0015
4—	3	+0.0038	+0.0011	+0.0024	—0.0006
5—	2	+0.1207	+0.0903	—0.0042	+0.0067
5—	3	—0.0926	+0.0155	—0.0458	—0.0076
5—	4	—0.0015	+0.0010	—0.0014	—0.0014
6—	3	—0.0009	+0.0007	—0.0002	—0.0002
6—	4	+0.0006	+0.0006		
7—	3	—0.0005	+0.0017		
7—	4	+0.0009	+0.0004	+0.0004	—0.0003
10—	4	—0.0067	+0.0082		

Application to Saturn.

The value of L′ is:

Arg=$\varkappa\gamma'+i'g'+ig$			L′	
			sin.	cos.
$\varkappa$	i'	i	″	″
—1	1	0	—0.0036	—0.000416
1	0	0	+0.002797	—0.001912
—1	2	0	—0.020	+0.01502
0	1	0	+0.0115	—0.0101
0	2	0	+0.002	—0.004
—1	1—	1	0.000	—0.007
0	0—	1	+0.004	+0.008
—1	2—	1	+0.0011	—0.0027
0	1—	1	—0.005	+0.003
1	0—	1	+0.005	+0.001
—1	3—	1	—0.0243	—0.0210
0	2—	1	+0.0195	+0.0144
1	1—	1	—0.0013	—0.0014
—1	4—	1	—0.0101	—0.0015
0	3—	1	+0.0084	+0.0011
1	2—	1	—0.0007	+0.0003
—1	2—	2	+0.0026	+0.0008
0	1—	2	—0.093	—0.030
1	0—	2	+0.09	+0.02
—1	3—	2	+0.003	—0.001
—1	4—	2	+0.017	—0.027
0	3—	2	—0.011	+0.021
—1	5—	2	+0.00055	—0.01219
0	4—	2	+0.0009	+0.0098
—1	6—	2	—0.00137	—0.00130
0	5—	2	+0.0012451	+0.0022934
1	4—	2	—0.00029	—0.00016
1	5—	2	—0.00006	+0.00001
—1	5—	3	+0.008	+0.002
0	4—	3	—0.013	—0.002
—1	6—	3	+0.010	—0.003
0	5—	3	—0.009	+0.003
—1	7—	3	+0.0021	—0.0022
0	6—	3	—0.001	+0.001
—1	8—	3	+0.0001	—0.0006
0	7—	3	—0.0001	+0.0006
—1	10—	4	+0.00012	—0.00007
0	10—	4	—0.000015	+0.000025

The remainder of the necessary operations being performed, we obtain:

$\text{Arg}=i'g'+ig$	$\Delta(n'\delta z')$		$\Delta v'$	
	sin.	cos.	cos.	sin.
i' i	″	″	″	″
0 0			+0.0006	
			+0.000027$n't$	
1 0			+0.0029	+0.0028
	−0.001886$n't$	−0.002755$n't$	+0.000942$n't$	−0.001377$n't$
2 0	−0.0005	+0.0010		
	−0.000030$n't$	−0.000046$n't$	+0.000030$n't$	−0.000046$n't$
3 0	−0.000001$n't$	−0.000002$n't$	+0.000001$n't$	−0.000002$n't$
0 −1	−0.0004	+0.0012	0.0000	−0.0004
1 −1	−0.0027	+0.0015	−0.0008	−0.0019
2 −1	−0.1877	−0.1455	−0.0523	+0.0397
3 −1	−0.0221	−0.0017	+0.0083	−0.0004
1 −2	−0.0010	+0.0003	+0.0013	−0.0003
2 −2	−0.0007	+0.0007	−0.0003	−0.0003
3 −2	−0.0061	+0.0020	−0.0046	−0.0020
4 −2	+0.0178	−0.4020	+0.0094	+0.1967
5 −2	−0.2931	−0.2678	+0.0075	+0.0108
6 −2	+0.0017	−0.0003	−0.0009	−0.0002
4 −3	+0.0003	0.0000	−0.0006	0.0000
5 −3	−0.0012	+0.0004	−0.0012	−0.0004
6 −3	−0.0028	+0.0029	−0.0016	−0.0017
7 −3	+0.0009	−0.0053	+0.0002	+0.0011
9 −4	+0.0019	−0.0012	+0.0010	+0.0005
10 −4	+0.0159	−0.0187		

In addition to the corrections here stated it must be understood that in all the arguments g is to be replaced by $g+E\Delta\pi$, and g' by $g'+E'\Delta\pi'$.

CHAPTER XXX.

ADDITION OF THE SEVERAL PORTIONS OF THE EXPRESSIONS FOR THE CO-ORDINATES OF JUPITER AND SATURN AND REDUCTION OF THEM TO THEIR FINAL FORM.

In this final chapter we shall be engaged in putting the expressions we have arrived at in a final form. In the first place, it is determined to change the values of the masses of four of the major planets so that they stand as follows:

$$\text{Mercury, } \frac{1}{7500000} \qquad \text{Earth, } \frac{1}{327000}$$

$$\text{Venus, } \frac{1}{408134} \qquad \text{Uranus, } \frac{1}{22640}$$

The remaining four still retaining the values of their masses, which were stated in Chapter I. Consequently to the terms of the perturbations of Jupiter, which have its own mean anomaly as argument, ought to be added the following corrections:

$$\begin{aligned}
\varDelta(n\delta z) = &+ \overset{\prime\prime}{0}.0001609nt \sin(-\ g) + \overset{\prime\prime}{0}.0015450nt \cos(-\ g) \\
&+ 0.0000019nt \sin(-2g) + 0.0000186nt \cos(-2g) \\
&\ 0.0000000nt \sin(-3g) + 0\ 0000004nt \cos(-3g)
\end{aligned}$$

$$\begin{aligned}
\varDelta v = &+ 0.0020 \\
&+ 0.0000019nt \\
&- 0.0007 \qquad \cos(-\ g) - 0.0001 \sin(-g) \\
&+ 0.0000804nt \cos(-\ g) - 0.0007725nt \sin(-\ g) \\
&+ 0.0000019nt \cos(-2g) - 0.0000186nt \sin(-2g) \\
&+ 0.0000001nt \cos(-3g) - 0.0000007nt \sin(-3g)
\end{aligned}$$

$$\begin{aligned}
\varDelta\left(\frac{u}{\cos i}\right) = &\qquad\qquad\qquad\qquad\qquad - 0.0000051n \\
&+ 0.0001414nt \sin(-\ g) + 0.0000702nt \cos(-\ g) \\
&+ 0.0000034nt \sin(-2g) + 0.0000017nt \cos(-2g) \\
&+ 0.0000001nt \sin(-3g) + 0.0000001nt \cos(-3g)
\end{aligned}$$

In addition the small terms dependent on the elongations of Jupiter, severally from Venus and the Earth, ought to be modified to suit the new values of the masses of the latter planets; also the remaining periodic terms due to the action of Uranus ought to be multiplied by the factor $\frac{21000}{22640}$.

For the same reason the terms of the perturbations of Saturn, having its own anomaly as argument, ought to receive the additions

$$
\begin{aligned}
\Delta(n'\delta z') = & -\overset{''}{0.006157}n't \sin g' + \overset{''}{0.011276}n't \cos g' \\
& + 0.0002 \sin 2g' + 0.0001 \cos 2g' \\
& - 0.000086n't \sin 2g' + 0.000158n't \cos 2g' \\
& - 0.000002n't \sin 3g' + 0.000004n't \cos 3g'
\end{aligned}
$$

$$
\begin{aligned}
\Delta\nu' = & + 0.0192 \\
& + 0.000086n't \\
& - 0.0058 \cos g' + 0.0024 \sin g' \\
& + 0.003078n't \cos g' + 0.005638n't \sin g' \\
& - 0.0002 \cos 2g' + 0.0001 \sin 2g' \\
& + 0.000086n't \cos 2g' + 0.000158n't \sin 2g' \\
& + 0.000004n't \cos 3g' + 0.000007n't \sin 3g'
\end{aligned}
$$

$$
\begin{aligned}
\Delta\left(\frac{u'}{\cos i'}\right) = & \qquad\qquad + 0.0008 \\
& \qquad\qquad + 0.000026n't \\
& - 0.002382n't \sin g' - 0.003110n't \cos g' \\
& + 0.0002 \sin 2g' \quad 0.0000 \cos 2g' \\
& - 0.000067n't \sin 2g' - 0.000087n't \cos 2g' \\
& - 0.000003n't \sin 3g' - 0.000004n't \cos 3g'
\end{aligned}
$$

In addition the small terms dependent on the elongations of Saturn, severally from Venus and the Earth, ought to be modified to suit the new values of the masses of the latter planets; also the remaining periodic terms due to the action of Uranus ought to be multiplied by the factor $\frac{21000}{22640}$.

Hansen's co-ordinate ν is not that which it is convenient to tabulate. We adopt $\log(1+\nu)$ in its place. Consequently, neglecting all higher powers of ν than the second, we add to ν the quantity $-\frac{1}{2}\nu^2$. The expressions for this, severally in the cases of Jupiter and Saturn, follow:

Arg $=i'g'+ig$	$-\frac{1}{2}\nu^2$	
	cos.	sin.
i' i	$''$ $''$ $''$	$''$ $''$ $''$
0 0	$-0.0249+.000001nt-.00000110n^2t^2$	
0 − 1	$-0.0071-.000007nt-.00000010n^2t^2$	$-0.0047+.000008nt+.00000005n^2t^2$
0 − 2	$+.00000009n^2t^2$	$+.00000118n^2t^2$
0 − 3		$+.00000005n^2t^2$
1 + 1	$+0.0004+.000002nt$	$+0.0005+.000002nt$
1 0	$+0.0090+.000050nt$	$+0.0037-.000029nt$
1 − 1	$+0.0021+.000005nt$	$-0.0030-.000006nt$
1 − 2	$-0.0007-.000036nt$	$+0.0003-.000048nt$
1 − 3	$-.000002nt$	$-.000001nt$

$\text{Arg} = i'g' + ig$		$-\frac{1}{2}\nu^2$	
		cos.	sin.
i'	i	″ ″ ″	″ ″ ″
2+	1	$.000000nt$	$+.000002nt$
2	0	$+0.0017+.000028nt$	$+0.0036+.000038nt$
2−	1	$-0.0079+.000090nt-.00000002n^2t^2$	$+0.0094+.000237nt+.00000001n^2t^2$
2−	2	$+0.0011+.000038nt$	$+0.0013-.000020nt$
2−	3	$+.000228nt$	$-.000103nt$
2−	4	$+.000012nt$	$-.000005nt$
3	0	$+0.0009-.000001nt$	$-0.0029-.000008nt$
3−	1	$+0.0197+.000016nt+.00000002n^2t^2$	$-0.0074-.000071nt+.00000001n^2t^2$
3−	2	$+0.0029+.000018nt$	$+0.0004-.000006nt$
3−	3	$-0.0035-.000080nt$	$+0.0060-.000008nt$
3−	4	$-.000007nt$	$-.000025nt$
4−	1	$+0.0006+.000007nt$	$+0.0004-.000006nt$
4−	2	$+0.0004+.000017nt$	$+0.0047+.000007nt$
4−	3	$-0.0023-.000006nt$	$-0.0031-.000012nt$
4−	4	$-0.0095+.000006nt$	$-0.0105-.000020nt$
4−	5	$-.000006nt$	$.000000nt$
5−	1	$-.000026nt$	$+.000001nt$
5−	2	$+0.0002-.000116nt+.00000009n^2t^2$	$+0.0002-.000142nt-.00000006n^2t^2$
5−	3	$+0.0006+.000001nt$	$+0.0019+.000015nt$
5−	4	$+0.0025-.000130nt-.00000005n^2t^2$	$+0.0095+.000120nt+.00000005n^2t^2$
5−	5	$-0.0025-.000010nt$	$+0.0020+.000004nt$
6−	2	$0.0000\quad .000000nt$	$+0.0001-.000002nt$
6−	3	$-0.0001+.000002nt$	$-0.0003-.000001nt$
6−	4	$+0.0013-.000002nt$	$-0.0051\quad .000000nt$
6−	5	$-0.0015-.000001nt$	$0.0000-.000002nt$
7−	3	$+0.0005+.000003nt$	$-0.0003\quad .000000nt$
7−	4	$+0.0056$	-0.0011
7−	5	$+0.0195$	$+0.0105$
8−	3	$0.0000+.000002nt$	$+0.0005+.000001nt$
8−	4	-0.0013	-0.0003
8−	5	-0.0035	-0.0060
8−	6	$+0.0010$	-0.0020
9−	4	-0.0001	0.0000
9−	5	0.0000	-0.0007
9−	6	$+0.0015$	-0.0010
10−	4	$0.0000+.000003nt$	$+0.0001-.000001nt$
10−	5	-0.0011	$+0.0015$
10−	6	-0.0070	-0.0010

Arg$=i'g'+ig$		$-\frac{1}{2}\nu'^2$	
		cos.	sin.
i'	i	$''\quad ''\quad ''$	$''\quad ''\quad ''$
0	0	$-0.2024+.000068n't-.00004837n'^2t^2$	
1	0	$+0.0509-.001417n't-.00000468n'^2t^2-\frac{15''}{10^{10}}n'^3t^3$	$+0.0844+.002225n't+.00000199n'^2t^2+\frac{12''}{10^{10}}n'^3t^3$
2	0	$+0.0144-.000216n't+.00002110n'^2t^2-230n'^3t^3$	$+0.0157+.000052n't+.00004341n'^2t^2+85n'^3t^3$
3	0	$+0.0023-.000036n't+.00000116n'^2t^2-\ 20n'^3t^3$	$-0.0038-.000002n't+.00000238n'^2t^2\quad 0n'^3t^3$
4	0	$-.000001n't+.00000006n'^2t^2$	$.000000n't+.00000013n'^2t^2$
—1—	1	$-0.0039-.000104n't$	$-0.0037+.000068n't$
0—	1	$-0.0074-.002537n't+.00000030n'^2t^2$	$-0.0040+.001055n't+.00000065n'^2t^2$
1—	1	$-0.0274+.000878n't+.00000002n'^2t^2$	$+0.0745+.001653n't+.00000054n'^2t^2$
2—	1	$+0.0580+.002085n't-.00000030n'^2t^2$	$+0.1051+.001722n't+.00000051n'^2t^2$
3—	1	$+0.1415+.001139n't+.00000107n'^2t^2$	$-0.0577-.001260n't+.00000031n'^2t^2$
4—	1	$-0.0207-.000115n't-.00000011n'^2t^2$	$-0.0065-.000062n't-.00000024n'^2t^2$
5—	1	$0.0000-.000007n't$	$0.0000+.000007n't$
0—	2	$-0.0014-.000029n't$	$+0.0009+.000037n't$
1—	2	$-0.0008+.000072n't$	$+0.0028+.000465n't$
2—	2	$+0.0487+.000125n't-.00000020n'^2t^2$	$+0.0195+.000239n't+.00000010n'^2t^2$
3—	2	$+0.0137+.004334n't+.00000205n'^2t^2$	$-0.0439-.003474n't+.00000390n'^2t^2$
4—	2	$-0.0373-.000415n't+.00000031n'^2t^2$	$-0.1328-.000207n't-.00000139n'^2t^2$
5—	2	$-0.0013-.004672n't+.00000375n'^2t^2$	$+0.0199-.002169n't-.00000624n'^2t^2$
6—	2	$+0.0001+.000289n't-.00000164n'^2t^2$	$-0.0009+.000643n't+.00000065n'^2t^2$
7—	2	$+.000013n't-.00000060n'^2t^2$	$+.000010n't+.00000005n'^2t^2$
1—	3	$.000000n't$	$+.000003n't$
2—	3	$+.000105n't$	$+.000007n't$
3—	3	$+0.0093+.000067n't$	$-0.0135+.000033n't$
4—	3	$-0.0018-.000017n't$	$-0.0094-.000060n't$
5—	3	$-0.1480+.000040n't$	$-0.0109-.000220n't$
6—	3	$+0.0247-.000160n't+.00000008n'^2t^2$	$+0.1088+.000006n't+.00000006n'^2t^2$
7—	3	$+0.0129+.000051n't+.00000002n'^2t^2$	$-0.0203+.000028n't+.00000007n'^2t^2$
8—	3	$-0.0014\quad .000000n't+.00000001n'^2t^2$	$+0.0005+.000025n't-.00000005n'^2t^2$
2—	4	$+.000002n't$	$.000000n't$
3—	4	$+.000010n't$	$-.000031n't$
4—	4	$-0.0034+.000015n't$	$-0.0037-.000017n't$
5—	4	$-0.0039-.000021n't$	$-0.0009+.000001n't$
6—	4	$-0.0081-.000043n't$	$+0.0221-.000016n't$
7—	4	$-0.0055-.000041n't$	$+0.0100-.000037n't$
8—	4	$+0.1315+.000044n't$	$-0.0346+.000337n't$
9—	4	$-0.0394+.000061n't$	$-0.0148-.000122n't$
10—	4	$+0.0016+.000047n't-.00000003n'^2t^2$	$+0.0032-.000047n't-.00000004n'^2t^2$
11—	4	$-.000017n't$	$+.000001n't$
4—	5	$-.000010n't$	$-.000005n't$
5—	5	$-.000005n't$	$-.000007n't$
6—	5	$-.000001n't$	$+.000005n't$
7—	5	$+0.0047-.000003n't$	$+0.0026+.000006n't$
8—	5	$+0.0020-.000003n't$	$+0.0034+.000001n't$
9—	5	$+0.0002-.000001n't$	$+0.0021\quad .000000n't$
10—	5	$+0.0008$	-0.0012
11—	5	-0.0017	-0.0014
12—	5	$+0.0002\quad .000000n't$	$+0.0005+.000002n't$
7—	6	$+.000002n't$	$+.000001n't$
8—	6	$+.000002n\ t$	$+.000002n't$
9—	6	$.000000n't$	$+.000002n't$

If we apply to the elements of Jupiter and Saturn, given in Chapter I, the corrections of Supposition III in Chapter XXVIII, we obtain the following system of values:

Epoch 1850, Jan. 0.0, Greenwich M. T.

$$
\begin{array}{rrrr@{\qquad\qquad}rrrr}
 & {}^{\circ} & {}' & {}'' & & {}^{\circ} & {}' & {}'' \\
L = & 159 & 56 & 24.98 & L' = & 14 & 49 & 38.09 \\
= & 11 & 54 & 31.67 & \pi' = & 90 & 6 & 41.37 \\
\theta = & 98 & 56 & 19.79 & \theta' = & 112 & 20 & 49.05 \\
i = & 1 & 18 & 42.10 & i' = & 2 & 29 & 40.19
\end{array}
$$

$e = 0.04825511$ $\qquad$ $e' = 0.05606025$

$n = 109256''.62552$ $\qquad$ $n' = 43996''.21506$

In order to tabulate the radius vector of a planet it is necessary to have a clear understanding of the linear unit one wishes to employ. Let us suppose that it is desired that the semi-axis major of the Earth's orbit connected with its sidereal mean motion by the well-known equation should be represented by unity. Then m_0 denoting the Earth's mass and n_0 its mean motion the semi-axis major of Jupiter is given by the equation

$$a = \left[\frac{1 + m}{1 + m_0}\,\frac{n_0^2}{n^2}\right]^{\frac{1}{3}}$$

and that of Saturn by an equation entirely similar. For m_0 we take the value given at the beginning of this chapter, and we put

$$n_0 = 1295977''.41516$$

It is here understood that n_0 is the constant of the Earth's orbit, which is exactly analogous to the constants n and n', severally belonging to the orbits of Jupiter and Saturn. This gives

$$\log a = 0.7162374088 \qquad\qquad \log a' = 0.9794956385$$

In the expressions for the co-ordinates which follow, the inequalities of the fundamental argument and of the latitude are given the form

$$k_0 \sin(\chi + K_0) + k_1 T \sin(\chi + K_1) + k_2 T^2 \sin(\chi + K_2) + k_3 T^3 \sin(\chi + K_3)$$

and that of common logarithm $\left(\frac{r}{\bar{r}} = 1 + \nu\right)$ the form

$$k_0 \cos(\chi + K_0) + k_1 T \cos(\chi + K_1) + k_2 T^2 \cos(\chi + K_2) + k_3 T^3 \cos(\chi + K_3)$$

The unit of T is a century of Julian years, and it is counted from 1850.0. K is so taken that k may be positive, except in the absolute terms, where K is supposed to vanish and k receives its proper sign. The k belonging to the common log $(1+\nu)$ are uniformily expressed in units of the seventh decimal.

The values of the constituents of the arguments occurring in the formulae are

$$\begin{array}{rrrrlr}
 & \circ & ' & '' & & '' \\
g = & 148 & 1 & 53.31 & + & 109256.62552t \\
g' = & 284 & 42 & 56.72 & + & 43996.21506t \\
g'' = & 220 & 10 & 10.35 & + & 15425.752t \\
g''' = & 291 & 48 & 8.61 & + & 7864.935t \\
♀ - ♃ = & 84 & 1 & & + & 1997384.73t \\
♁ - ♃ = & 299 & 52 & & + & 1186720.79t \\
♀ - ♄ = & 229 & 8 & & + & 2062645.15t \\
♁ - ♄ = & 84 & 59 & & + & 1251981.21t
\end{array}$$

Inequalities of the fundamental argument of Jupiter.

To form the value of $n\delta z$ it is necessary to add together the following expressions:

I. The first-order terms due to the action of Saturn (pages 103–105).

II. The first-order terms due to the action of Uranus; as given at page 160 they must be multiplied by the factor $\frac{21000}{22640}$.

III. The first-order terms due to the action of Neptune (page 191).

IV. The small terms arising from the action of the four interior planets, given at pages 193–196. Those coming from Venus and Earth must be modified to correspond to the masses adopted at the beginning of this chapter.

V. The second-order terms arising from the mutual action of Jupiter and Saturn (pages 290–292).

VI. The third-order terms arising from the mutual action of Jupiter and Saturn (pages 404, 405)

VII. The second-order terms arising from the joint action of Saturn and Uranus; as given at page 479 they must be multiplied by the factor $\frac{21000}{22640}$.

VIII. The terms of $\varDelta(n\delta z)$, given at pages 528, 529.

IX. The terms of $\varDelta(n\delta z)$, given at pages 551, 552.

In passing from terms multiplied by nt to those multiplied by T the logarithm of the factor to be employed is 1.7240226, and in deriving the K we remember that $E\varDelta\pi = -98''.082$ and $E'\varDelta\pi' = -4''.883$.

χ	$n\delta z$							
	k_0	K_0	k_1	K_1	k_2	K_2	k_3	K_3
g' g	″	° ′ ″	″	° ′ ″	″	° ′	″	° ′
0 0			+ 0.0019		+0.83320		+0.016208	
0 − 1			100.6962	227 27 58.93	0.69738	298 22.3	0.000358	45 55
0 − 2	0.236	35 8	0.7960	224 50.8	0.02672	284 46	0.000013	274
0 − 3	0.047	137	0.0108	223 54	0.00073	275 38	0.000002	90
0 − 4	0.002	103	0.0002	0	0.00001	0		
1 + 3	0.005	147						
1 + 2	0.128	123 20	0.0057	21 16				
1 + 1	1.237	215 13.9	0.0332	116 11				
1 0	11.156	150 56 7	0.1755	49 46	0.00070	322 44		
1 − 1	79.843	79 12 7	0.0045	244 58				
1 − 2	1.508	90 37.7	0.0237	131 4	0.00002	180		
1 − 3	0.108	108 27	0.0026	199 51				
1 − 4	0.018	212 27						
2 + 2	0.013	205 33	0.0007	123				
2 + 1	0.487	184 19	0.0213	86 39				
2 0	6.813	123 49.3	0.1752	13 51.3	0.00044	230 21		
2 − 1	123.012	1 24 45.6	1.2671	301 24.3	0.00704	216 56		
2 − 2	194.634	336 53 42.3	0.0222	354 34	0.00018	39		
2 − 3	2.811	331 31.6	0.0649	22 42	0.00005	68		
2 − 4	0.054	305 46	0.0024	10 29				
2 − 5	0.002	300						
3 + 1	0.062	275 52	0.0029	185 11				
3 0	3.685	270 58.7	0.1418	174 15				
3 − 1	14.038	312 11 30	0.2316	210 12.5	0.00170	161 23		
3 − 2	82.649	127 22 51	1.1498	30 1.0	0.00609	299 34		
3 − 3	16.228	57 42 44	0.0147	150 34	0.00007	279		
3 − 4	0.405	38 13	0.0078	100 26				
3 − 5	0.014	327 36	0.0004	50				
4 0	0.015	177 16						
4 − 1	0.684	191 30	0.0304	84 0				
4 − 2	16.838	98 28 1	0.4607	0 32.9	0.00313	260 45		
4 − 3	14.978	26 2 35	0.2044	288 17.2	0.00121	197 39		
4 − 4	3.611	129 27.5	0.0039	36 49				
4 − 5	0.152	104 21	0.0024	168 36				
4 − 6	0.009	33						
5 0	0.004	45	0.0048	17 56	0.00007	74		
5 − 1	0.776	1 46.6	0.2567	11 47.2	0.01314	284 55		
5 − 2 $-81''.97009t$	1196.138	67 8 55.03	5.5814	247 9.1	0.15562	48 49.7		
5 − 3	160.938	176 27 45.4	4.7607	80 53.6	0.05892	349 26.2		

χ	$n\delta z$							
	k_0	K_0	k_1	K_1	k_2	K_2	k_3	K_3
g' g	″	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″
5— 4	3.666	133 33.4	0.0293	72 9	0.00085	118 10		
5— 5	1.121	206 52.3	0.0015	134 17	0.00001	315		
5— 6	0.068	178 43	0.0009	245				
5— 7	0.004	120						
6— 1	0.004	320						
6— 2	0.150	29 31	0.0088	290 27				
6— 3	1.181	150 52.8	0.0944	289 28	0.00012	315		
6— 4	1.522	74 35.9	0.0398	336 28				
6— 5	0.803	179 13	0.0114	82 55				
6— 6	0.373	285 43	0.0003	158				
6— 7	0.032	254 31	0.0004	310				
6— 8	0.002	225						
7— 2	0.008	213	0.0015	88 4				
7— 3	1.916	214 9.8	0.0775	116 10.0	0.00031	0		
7— 4	2.897	223 47.6	0.1111	125 23.8	0.00046	212 21		
7— 5	0.294	161 34	0.0093	64 35				
7— 6	0.305	258 47	0.0041	159 35				
7— 7	0.138	2 15	0.0001	270				
7— 8	0.015	329 46	0.0002	342				
7— 9	0.001	301						
8— 2	0.010	340 29						
8— 3	0.278	198 1	0.0132	104 13				
8— 4	1.862	13 32.6	0.0878	277 18				
8— 5	0.319	304 25	0.0132	207 56				
8— 6	0.137	234 50	0.0044	139 1				
8— 7	0.124	336 33	0.0014	238 51				
8— 8	0.054	77 42						
8— 9	0.008	47						
8—10	0.001	16						
9— 3	0.009	170						
9— 4	0.528	344 38	0.0281	247 56				
9— 5	0.504	272 23	0.0251	175 17				
9— 6	0.107	14 51	0.0035	280 37				
9— 7	0.063	312 30	0.0017	218 51				
9— 8	0.054	53 34	0.0007	318				
9— 9	0.022	154 15						
9—10	0.004	124						
10— 4 —145″.72t	11.024	313 41.0	0.0876	133 41	0.01338	311 27		
10— 5	3.578	63 18.1	0.2075	325 50.1				
10— 6	0.097	16 23	0.0044	289 54				
10— 7	0.034	93 32	0.0011	352				
10— 8	0.030	28 18	0.0008	285				
10— 9	0.025	129 29						
10—10	0.009	230						
10—11	0.002	201						

χ	$n\delta z$							
	k_0	K_0	k_1	K_1	k_2	K_2	k_3	K_3
g' g	″	° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″
11—4	0.005	286						
11—5	0.097	34 14	0.0029	294 49				
11—6	0.079	321 52	0.0029	225 9				
11—7	0.040	66 2	0.0010	328				
11—8	0.012	168 13	0.0001	90				
11—9	0.015	104 11	0.0003	0				
11—10	0.012	208 35						
11—11	0.004	304						
11—12	0.001	276						
12—5	0.065	35 13	0.0028	266 49				
12—6	0.055	293 31	0.0030	190 14				
12—7	0.023	38 45	0.0004	293				
12—8	0.017	144 9	0.0004	40				
12—9	0.004	223	0.0002	198				
12—10	0.007	184						
12—11	0.005	284						
12—12	0.002	12						
g'' g								
1+1	0.010	183						
1 0	0.273	174 41						
1—1	0.910	156 57						
1—2	0.006	188						
2 0	0.010	190						
2—1	0.519	136 42						
2—2	0.464	132 49						
2—3	0.012	130 44						
3 0	0.001	235						
3—1	0.091	132 12						
3—2	0.145	126 54						
3—3	0.034	287 32						
3—4	0.002	283						
4—1	0.015	128 38						
4—2	0.034	121 9						
4—3	0.013	282 16						
4—4	0.004	83						
5—1	0.003	127						
5—2	0.008	115						
5—3	0.003	277						
5—4	0.002	78						
5—5	0.001	237						
6—1	0.001	117						
6—2	0.002	109						
6—3	0.001	270						
6—4	0.001	72						
7—1	0.015	116 6						
7—2	0.004	103						

χ	$n\delta z$							
	k_0	K_0	k_1	K_1	k_2	K_2	k_3	K_3
g' g g''		° ′ ″	″	° ′ ″	″	° ′ ″	″	° ′ ″
6— 2— 3	8.749	187 50.0	0.2864	64 10				
6— 3— 3	0.472	105 59	0.0072	337 27				
g''' g								
1 0	0.011	99 21						
1— 1	0.286	31 37						
1— 2	0.004	35						
2 0	0.002	61						
2— 1	0.178	243 29						
2— 2	0.101	242 47						
2— 3	0.002	242						
3— 1	0.002	209						
3— 2	0.002	151						
3— 3	0.006	273						
♀—♃	0.070	0						
♂—♃	0.121	0						

Inequalities of the logarithm of the radius-vector of Jupiter.

To form the expression for the common logarithm $\left(\frac{r}{\bar{r}} = 1 + \nu\right)$ it is necessary to add the nine portions correspondent to those of $n\delta z$, and, in addition, the terms of $-\frac{1}{2}\nu^2$, given at pages 555, 556. The logarithm of the factor for passing from seconds of arc in ν to units of the seventh decimal in the final form of the co-ordinate is 1.3233592.

χ	Common log $\frac{r}{\bar{r}}$							
	k_0	K_0	k_1	K_1	k_2	K_2	k_3	K_3
g' g		° ′ ″		° ′ ″		° ′		° ′
0— 0	— 40.83		— 17.308		—0.024			
0— 1	18.17	323 32	1059.214	227 27 21.2	7.291	297 58.8	0.0037	45 25
0— 2	3.89	31 43	25.498	227 13.5	0.366	285 46	0.0001	45
0— 3	0.80	133 10	1.155	229 24	0.087	272 26		
0— 4	0.07	111	0.065	228 49	0.001	270		
1+ 3	0.13	323 49						
1+ 2	2.08	308 0	0.081	208 37				
1+ 1	16.58	33 51	0.451	294 30				
1 0	46.87	341 13.9	0.857	229 1	0.003	149		
1— 1	545.14	79 11 23	0.051	236 42				
1— 2	23.70	87 58.9	0.289	130 59				
1— 3	2.09	107 4	0.055	196 40				
1— 4	0.33	206 40						

χ	Common log $\frac{r}{\bar{r}}$							
	k_0	K_0	k_1	K_1	k_2	K_2	k_3	K_3
g' g		° ′ ″		° ′ ″		° ′		° ′
2+ 2	0.31	18 52	0.009	299				
2+ 1	7.42	1 54	0.298	265 2				
2 0	61.05	305 11.4	1.601	193 19.5	0.001	297		
2− 1	383.02	356 11 17	2.917	300 58.5	0.021	217		
2− 2	2303.37	336 53 56.2	0.242	352 6	0.002	135		
2− 3	62.33	333 10.5	0.874	22 59				
2− 4	1.94	319 56	0.041	3 11				
2− 5	0.10	329						
3+ 1	1.39	94 40	0.058	355 38				
3 0	43.89	90 51	1.688	353 42.1				
3− 1	56.45	133 2.4	0.858	29 1	0.001	333		
3− 2	738.42	126 35 32	10.215	30 3.6	0.051	298 58		
3− 3	241.37	58 30 46	0.154	121 7				
3− 4	9.52	44 11	0.121	98 36				
3− 5	0.34	356 55	0.009	45				
4 0	0.23	355 51	0.006	248				
4− 1	4.61	24 58	0.083	91 34				
4− 2	85.28	94 3.3	2.283	358 30.5	0.009	270		
4− 3	193.21	27 0.6	2.652	288 26.2	0.012	197		
4− 4	59.81	127 50.9	0.051	358 51				
4− 5	3.50	109 14	0.040	168 36				
4− 6	0.20	52 55						
5 0	0.12	215	0.152	197 54				
5− 1	8.14	180 47	2.691	192 9.0	0.003	158		
5− 2	229.34	237 53.6	9.058	143 57.1	0.162	46 23		
5− 3	1679.20	176 23 44	49.701	80 52.5	0.525	343 42		
5− 4	65.06	141 13.3	0.931	73 6	0.011	326		
5− 5	20.58	204 48	0.042	243 34				
5− 6	1.56	184 1	0.017	241				
5− 7	0.11	129 51	0.003	207				
6− 1	0.05	137						
6− 2	0.92	203 41	0.040	102 57				
6− 3	8.78	145 29	0.365	46 48				
6− 4	20.79	76 42	0.565	337 5				
6− 5	13.52	180 38	0.192	80 47				
6− 6	6.92	283 56	0.008	117				
6− 7	0.71	260 4	0.006	307				
6− 8	0.06	236						
7− 2	0.18	7 25	0.019	283				
7− 3	5.50	214 14	0.216	118 29				
7− 4	34.30	223 11.6	1.313	125 13				
7− 5	5.17	167 55	0.159	68 47				
7− 6	5.43	259 28	0.074	158 59				
7− 7	2.68	0 22	0.004	147				
7− 8	0.34	335 14	0.003	27				
7− 9	0.03	312						

χ	Common log $\frac{r}{\bar{r}}$			
	k_0	K_0	k_1	K_1
g' g		° ′		° ′
8—3	1.09	13 26	0.024	259
8—4	16.42	12 48	0.775	276 18
8—5	4.89	304 1	0.193	208 46
8—6	2.42	239 46	0.073	142 35
8—7	2.31	337 34	0.029	232 53
8—8	1.08	75 50	0.003	243
8—9	0.18	50 5		
9—3	0.08	359	0.003	117
9—4	2.61	340 31	0.109	240 8
9—5	6.53	272 59	0.312	175 24
9—6	1.75	10 57	0.066	275 1
9—7	1.18	316 51	0.033	211
9—8	1.04	54 49	0.016	315
9—9	0.45	151 37		
9—10	0.09	125		
10—4	3.47	123 36	0.190	31 11
10—5	37.04	63 11.2	2.298	325 46
10—6	1.81	22 44	0.082	296 1
10—7	0.68	88 14	0.028	356
10—8	0.57	33 57	0.015	287
10—9	0.49	131 13	0.007	31
10—10	0.19	226 10		
10—11	0.04	203		
11—5	0.65	31 58	0.017	290
11—6	1.10	322 57	0.045	220
11—7	0.70	67 0	0.031	330
11—8	0.25	162 23	0.011	79
11—9	0.29	112 53	0.009	7
11—10	0.23	208 41		
11—11	0.08	299 36		
12—6	0.49	296 9	0.037	189
12—7	0.39	39 39	0.009	299
12—8	0.26	145 5	0.004	237
12—9	0.09	236 55	0.005	346
12—10	0.15	186 6	0.003	90
12—11	0.11	284		
12—12	0.04	10		
g' g				
1+1	0.12	3		
1 0	0.24	8		
1—1	8.46	156 57		

χ	Common log $\frac{r}{\bar{r}}$			
	k_0	K_0	k_1	K_1
g'' g		° ′		° ′
1—2	0.13	177		
2 0	0.06	114		
2—1	4.55	136 22		
2—2	6.70	132 49		
2—3	0.27	130		
2—4	0.01	132		
3—1	0.71	131 32		
3—2	1.96	127 7		
3—3	0.56	287		
3—4	0.04	285		
4—1	0.09	125		
4—2	0.44	122		
4—3	0.21	282		
4—4	0.08	83		
5—2	0.09	116		
5—3	0.05	277		
5—4	0.04	80		
5—5	0.01	239		
6—2	0.03	110		
6—3	0.01	270		
6—4	0.01	75		
7—2	0.04	103		
g' g g''				
6—2—3	1.08	175 11		
6—3—3	4.97	105 59	0.076	337 27
g''' g				
1 0	0.06	22		
1—1	2.83	31 37		
1—2	0.07	34		
2 0	0.04	242		
2—1	1.75	243 22		
2—2	1.52	242 44		
2—3	0.06	242		
3—1	0.02	207		
3—2	0.03	161		
3—3	0.10	274		
♀—♃	1.48	0		
♂—♃	2.55	0		

Periodic inequalities of the latitude of Jupiter.

We have only to change the expression given (pages 526, 527) to the form here adopted and to take account of the corrections to the angles K arising from $E\varDelta\pi$ and $E'\varDelta\pi'$.

χ	$\varDelta\beta$			
	k_0	K_0	k_1	K_1
g' g	$''$	$°$ $'$	$''$	$°$ $'$
0 0	+0.037			
0 − 2	0.015	66		
0 − 3	0.001	82		
1 + 2	0.005	353		
1 + 1	0.104	8 51	0.0005	158
1 0	0.536	325 28	0.0070	54 16
1 − 1	0.126	208 1	0.0027	188 27
1 − 2	0.265	193 10	0.0043	103 27
1 − 3	0.012	204	0.0004	90
2 + 1	0.018	283	0.0004	14
2 0	0.342	265 52	0.0021	313
2 − 1	0.627	43 9	0.0081	137 30
2 − 2	0.221	114 42	0.0059	82 11
2 − 3	0.056	267	0.0004	57
2 − 4	0.003	282	0.0002	0
3 + 1	0.003	33	0.0001	225
3 0	0.056	49	0.0002	153
3 − 1	0.165	356 6	0.0006	38
3 − 2	1.013	122 15	0.0120	212 25
3 − 3	0.057	163 7	0.0006	218
3 − 4	0.019	351	0.0002	153
3 − 5	0.001	355		
4 0	0.006	22		
4 − 1	0.047	329 38		
4 − 2	0.144	99 51	0.0007	188
4 − 3	0.247	22 5	0.0037	109
4 − 4	0.021	342	0.0002	90
4 − 5	0.009	60	0.0001	135
5 0	0.009	111	0.0001	315
5 − 1	0.184	111 34	0.0036	8
5 − 2	0.194	359 38	0.0006	288
5 − 3	3.548	174 54.5	0.0077	327 12
5 − 4	0.187	161 37	0.0009	238
5 − 5	0.008	125	0.0004	104
5 − 6	0.003	136		
6 − 1	0.001	74		
6 − 2	0.007	16		
6 − 3	0.037	150		
6 − 4	0.048	74		
6 − 5	0.012	165		
6 − 6	0.003	121		
6 − 7	0.001	216		
7 − 2	0.004	337		
7 − 3	0.005	144		
7 − 4	0.053	44		
7 − 5	0.011	135		
7 − 6	0.004	245		
7 − 7	0.002	198		
7 − 8	0.001	292		
8 − 3	0.001	48		
8 − 4	0.009	201		
8 − 5	0.008	127		
8 − 6	0.004	222		
8 − 7	0.001	318		
8 − 8	0.001	90		
9 − 5	0.004	89		
9 − 6	0.003	196		
9 − 7	0.002	298		
10 − 4	0.003	66		
10 − 5	0.073	60 20		
10 − 6	0.003	106		
10 − 7	0.001	281		
10 − 8	0.001	23		

We derive f from $nz = g + n\delta z$ by the formula

$$\begin{aligned} f = nz &+ 19900''.870 \sin nz + 599''.861 \sin 2nz + 25''.072 \sin 3nz \\ &+ 1.198 \sin 4nz + 0.062 \sin 5nz + 0.003 \sin 6nz \end{aligned}$$

Then

$$\text{com. log } r = 0.71522495 - \text{com. log } (1 + e \cos f)$$

In addition $l = f + \pi$, and we derive R from

$$R = +\,27''.029 \sin (2l + 342^\circ\ 7'\ 20'') + 0''.002 \sin (4l + 324^\circ)$$

The heliocentric longitude of Jupiter, referred to the mean equinox of date, is

$$\lambda = f + R + \pi + 50''.264708t$$

In order to get the equation which determines $\sin \beta_0$, we note that the correction of $\frac{u}{\cos i}$ (page 554), on account of changes in the values of the disturbing masses, requires that the A and B of page 525 should receive severally the corrections $-0''.0075T$ and $+0''.0037T$, so that the equation for $\cos i \sin b$ now becomes

$$\begin{aligned} \cos i \sin b = &\ [-13''.0524T - 0''.06615T^2 + 0''.000324T^3] \sin l \\ &+ [\quad 9.4311T - 0.08661T^2 - 0.000240T^3] \cos l \end{aligned}$$

If we add this to the expression for the portion of $\sin \beta_0$, which arises from the motion of the ecliptic (page 526), and also take account of the correction $-\frac{d(\sin \beta_0)}{dz} \varDelta(\delta z)$, given at page 530, we obtain

$$\begin{aligned} \sin \beta_0 = &\ [\quad \sin i_0 \cos \theta_0 + 33''.7123T - 0''.14784T^2 - 0''.000107T^3] \sin l \\ &+ [-\sin i_0 \sin \theta_0 + 14.7021T + 0.11242T^2 - 0.000659T^3] \cos l \\ &+ [-0''.0048T + 0''.00002T^2] \sin 3l \\ &+ [-0.0005T - 0.00003T^2] \cos 3l \end{aligned}$$

Inequalities of the fundamental argument of Saturn.

To form the value of $n'\delta z'$ it is necessary to add together the following expressions:

I. The first-order terms due to the action of Jupiter (pages 106–108).

II. The first-order terms due to the action of Uranus; as given at pages 138, 139, they must be multiplied by the factor $\frac{21000}{22640}$.

III. The first-order terms due to the action of Neptune (page 179).

IV. The small terms arising from the action of the four interior planets, given at pages 196–198. Those coming from Venus and the earth must be modified to correspond to the masses adopted at the beginning of this chapter.

V. The second-order terms arising from the mutual action of Jupiter and Saturn (pages 335–337).

VI. The third-order terms arising from the mutual action of Jupiter and Saturn (pages 450–452).

VII. The second-order terms due to the action of Uranus, and given at pages 474, 475; they must be multiplied by the factor $\frac{21000}{22640}$.

VIII. The second-order terms due to the combined action of Jupiter and Uranus. As given at pages 485, 486 they need to be multiplied by $\frac{21000}{22640}$.

IX. The terms of $\Delta(n'\delta z')$ given at pages 534, 535.

X. The terms of $\Delta(n'\delta z')$ given at page 553.

In adding these several portions we pass from terms factored by $n't$ to those factored by T by multiplying by the factor whose logarithm is 1.3289902; and, in deriving the K, we take account of the corrections dependent on the quantities $E\Delta\pi$ and $E'\Delta\pi'$.

χ	$n'\delta z'$							
	k_0	K_0	k_1	K_1	k_2	K_2	k_3	K_3
g' g	″	° ′ ″	″	° ′ ″	″	° ′	″	° ′
0 0			— 0.0123		+1.79397		—0.028256	
1 0			268.8347	237 59 29.70	1.72526	142 22.2	0.001825	349 12
2 0	2.612	121 24.3	5.4941	237 21 17	0.10462	119 22.0	0.001261	26 28
3 0	0.648	91 39	0.1945	243 35.3	0.00676	113 0	0.000064	4
4 0	0.026	4	0.0113	239 34	0.00044	115 45		
5 0	0.003	214	0.0005	239				
—4— 1	0.006	21	0.0001	59				
—3— 1	0.006	76	0.0001	202				
—2— 1	0.195	165 51	0.0078	264 34				
—1— 1	0.362	141 48	0.0177	228 48	0.00010	294		
0— 1	12.089	86 45 53	0.1460	209 10	0.00072	310 12		
1— 1	7.196	189 35 2	0.2961	303 39.0	0.00123	293 56		
2— 1	421.948	181 25 43.70	4.1702	122 26 58	0.02192	38 34		
3— 1	33.511	121 13 45.6	0.8283	31 8.0	0.01086	350 52		
4— 1	0.101	90 31	0.0323	16 39	0.00100	306 15		
5— 1	0.043	159 30	0.0034	29 24	0.00008	315		
6— 1	0.003	124	0.0001	135				
7— 1	0.003	257						
—2— 2	0.004	141	0.0003	241				
—1— 2	0.076	244 22	0.0031	342				
0— 2	0.164	114 12	0.0020	276	0.00003	270		
1— 2	2.764	250 7.6	0.0385	289 55	0.00004	122		
2— 2	32.025	156 58 9	0.0156	7 59	0.00017	235		
3— 2	26.138	135 33 5	0.9096	42 1.5	0.01219	301 7		
4— 2	683.664	277 23 44.39	16.5281	179 34 55	0.15242	84 31.9		
5— 2 $-82''.0017n'$	2907.855	247 6 43.27	13.9914	67 6 36	0.29863	221 43.0		
6— 2	1.719	255 17.3	2.0642	125 59.7	0.08871	27 56.8		
7— 2	0.034	323 7	0.0809	125 33	0.00340	15 41		
8— 2	0.006	339	0.0041	124 44				

χ	$n'\delta z'$							
	k_0	K_0	k_1	K_1	k_2	K_2	k_3	K_3
g' g	″	° ′ ″	″	° ′ ″	″	° ′	″	° ′
−1— 3	0.003	208	0.0004	289				
0— 3	0.029	335	0.0010	62				
1— 3	0.139	269 30	0.0015	348				
2— 3	0.190	142 54	0.0019	345	0.00002	0		
3— 3	6.513	234 22.9	0.0022	357	0.00008	246		
4— 3	4.600	203 15.5	0.0660	107 21	0.00033	11		
5— 3	3.250	174 37.3	0.0903	77 49	0.00112	340 41		
6— 3	3.339	157 20.7	0.1382	58 30	0.00359	314 36		
7— 3	6.247	31 24.2	0.2540	289 53.7	0.00179	116 10		
8— 3	0.654	18 10	0.0451	303 37	0.00034	106		
9— 3	0.057	110 32	0.0002	130				
10— 3	0.002	59						
0— 4	0.001	291						
1— 4	0.011	22	0.0006	135				
2— 4	0.021	25	0.0005	93				
3— 4	0.122	205 21	0.0006	356				
4— 4	1.910	312 8.4	0.0004	62	0.00004	109		
5— 4	1.290	281 50.3	0.0194	185 6	0.00011	115		
6— 4	0.692	249 33	0.0201	152 59	0.00017	75		
7— 4	0.375	41 51	0.0134	300 15	0.00016	30		
8— 4	1.486	14 35.8	0.0774	277 44	0.00031	203		
9— 4	8.824	163 42 22	0.5281	67 33.6	0.01228	331 39		
10— 4 −148″.145t	26.795	133 37 11	0.2274	313 37.1	0.05217	122 44		
11— 4	0.002	197	0.0199	13 57	0.00040	275		
1— 5	0.001	0						
2— 5	0.006	115	0.0002	219				
3— 5	0.010	106	0.0002	194				
4— 5	0.069	280 55	0.0003	353				
5— 5	0.661	29 42	0.0003	132				
6— 5	0.479	0 7	0.0073	263 43				
7— 5	0.219	332 12	0.0062	237 19				
8— 5	0.120	121 33	0.0054	22 6				
9— 5	0.145	90 5	0.0068	355 15				
10— 5	0.129	59 45	0.0070	326 14				
11— 5	0.211	39 34	0.0166	300 19				
12— 5	0.241	213 4	0.0181	108 0				
2— 6	0.001	73						
3— 6	0.003	194	0.0001	333				
4— 6	0.006	200	0.0002	286				
5— 6	0.038	356 15	0.0003	86				
6— 6	0.251	106 44	0.0003	215				
7— 6	0.200	78 29	0.0030	346				
8— 6	0.092	50 55	0.0024	312				
9— 6	0.047	199 40	0.0019	105				

χ	$n'\delta z'$			
	k_0	K_0	k_1	K_1
g' g	″	° ′	″	° ′
10—6	0.052	169 12	0.0012	65
11—6	0.026	135 9	0.0007	39
12—6	0.013	103 13	0.0004	24
5—7	0.003	298	0.0002	343
6—7	0.021	72 38	0.0003	155
7—7	0.099	183 15		
8—7	0.086	156 23	0.0013	60
9—7	0.045	130 9	0.0012	34
10—7	0.017	275 8	0.0011	177
11—7	0.023	242 24	0.0011	153
12—7	0.010	219	0.0005	114
6—8	0.002	25		
7—8	0.011	152		
8—8	0.041	260 19	0.0001	135
9—8	0.040	233 52	0.0005	138
10—8	0.023	205 38	0.0005	109
11—8	0.007	352	0.0005	256
12—8	0.011	325	0.0005	225
8—9	0.006	227		
9—9	0.017	336		
10—9	0.019	313	0.0001	217
11—9	0.011	286	0.0002	195
12—9	0.002	57	0.0001	346
9—10	0.003	302		
10—10	0.007	50		
11—10	0.009	29		
12—10	0.006	2		
10—11	0.001	20		
11—11	0.003	125		
12—11	0.004	106		
11—12	0.001	97		
12—12	0.002	195		
g'' g'				
1+1	0.021	179 15	0.0011	20
1 0	0.926	145 45	0.0111	322 51
1—1	8.036	79 2.1	0.0020	280 47
1—2	0.153	99 26	0.0068	201 39
1—3	0.004	97	0.0003	213
2+1	0.002	153	0.0001	270
2 0	0.113	139 36	0.0044	246 39
2—1	7.682	354 17.1	0.0979	216 34
2—2	12.380	336 43.3	0.0054	113 4
2—3	0.235	330 22	0.0110	98 45
2—4	0.007	330	0.0006	90

χ	$n'\delta z'$			
	k_0	K_0	k_1	K_1
g'' g'	″	° ′ ″	″	° ′
3+1	0.001	305		
3 0	0.060	306 36	0.0189	200 8
3—1	28.520	321 46 31	0.3917	182 56.8
3—2	23.356	119 19 47	0.1437	307 48
3—3	1.372	66 35	0.0192	246 2
3—4	0.044	50 6	0.0017	202 38
3—5	0.002	45		
4 0	0.001	284		
4—1	0.054	288 22	0.0003	123
4—2	0.912	83 39	0.0128	267 4
4—3	0.703	18 8	0.0052	203 20
4—4	0.257	129 39	0.0009	148
4—5	0.014	111	0.0004	256
4—6	0.001	106		
5—1	0.003	242		
5—2	0.297	48 8	0.0064	231 28
5—3	0.429	341 6	0.0060	164 40
5—4	0.140	92 57	0.0005	270
5—5	0.072	207 39	0.0002	207
5—6	0.006	187	0.0001	315
6—2	0.119	4 38	0.0032	191 48
6—3	0.244	124 25	0.0050	309 0
6—4	0.055	61 29	0.0009	245
6—5	0.043	172 12	0.0002	0
6—6	0.023	284 39		
6—7	0.002	263		
7—3	0.016	89 21	0.0005	270
7—4	0.019	22 15	0.0004	207
7—5	0.015	135 29	0.0001	315
7—6	0.016	250 29		
7—7	0.008	1		
7—8	0.001	340		
8—3	0.007	53		
8—4	0.011	347		
8—5	0.005	98		
8—6	0.006	214		
8—7	0.006	328		
8—8	0.003	77		
9—4	0.003	131		
9—5	0.001	73		
9—6	0.002	177		
9—7	0.002	290		
9—8	0.002	45		
9—9	0.001	153		

χ	$n'\delta z'$			
	k_0	K_0	k_1	K_1
g'' g'	″	° ′	″	° ′
10— 7	0.001	256		
10— 8	0.001	9		
10— 9	0.001	124		
g''' g'				
1+ 1	0.002	270		
1 0	0.101	287 29		
1— 1	1.717	312 59		
1— 2	0.027	309 1		
1— 3	0.001	303		
2 0	0.012	269 30		
2 - 1	0.904	84 44		
2— 2	1.052	86 17		
2— 3	0.026	87 22		
2— 4	0.001	90		
3— 1	0.031	166 12		
3— 2	0.103	197 18		
3— 3	0.093	39 58		
3— 4	0.004	42		
4— 1	0.001	284		
4— 2	0.009	308		
g''' g'	″	° ′	″	° ′
4— 3	0.010	151		
4— 4	0.015	353		
4— 5	0.001	354		
5— 2	0.001	67		
5— 3	0.001	262		
5— 4	0.002	102		
5— 5	0.003	308		
6— 6	0.001	261		
g' g g''				
2— 1+ 1	0.022	270		
3— 1— 1	0.168	288 21		
3— 1— 2	0.207	79 43		
4— 2+ 3	0.063	213 2		
4— 1— 4	0.106	37 11		
5— 2— 3	1.884	208 34	0.0294	80 14
6— 2— 3	28.917	6 56.0	0.7830	242 4
7— 2— 6	0.153	353 26		
♀ — ♄	0.038	0		
♂ — ♄	0.066	0		

Inequalities of the logarithm of the radius-vector of Saturn.

To form the expression for log $\frac{r'}{\bar{r}'}$ we combine together the ten portions of ν corresponding to the ten portions of $n'\delta z'$ and besides apply to the sum the value of $-\frac{1}{2}\nu'^2$ (page 557).

χ	Common log $\frac{r'}{\bar{r}'}$							
	k_0	K_0	k_1	K_1	k_2	K_2	k_3	K_3
g' g		° ′ ″		° ′ ″		° ′		° ′
0 0	+1825.0		+ 41.95		+0.674		+0.0003	
1 0	187.3	295 24.7	2832.89	57 59 18.5	18.084	322 22.6	0.0197	167 40
2 0	49.9	293 9	78.60	58 38.5	1.838	303 11	0.0049	205 31
3 0	14.2	271 43	2.12	77 46	0.127	302 49	0.0004	166
4 0	0.6	311	0.04	82	0.008	299		
—3— 1	0.2	111						
—2— 1	4.6	165	0.22	263 45				
—1— 1	10.4	140 34	0.36	235 18	0.001	180		
0— 1	82.0	110 49	1.15	219 39	0.020	299		
1— 1	3780.8	79 45 10	3.19	304 47	0.010	25		

χ	Common log $\frac{r'}{\bar{r}'}$							
	k_0	K_0	k_1	K_1	k_2	K_2	k_3	K_3
g' g		° ′ ″		° ′ ″		° ′		° ′
2— 1	2442.1	176 2 37	21.60	121 29.6	0.204	36 17		
3— 1	241.2	305 54.4	6.21	207 37	0.058	188 31		
4— 1	35.1	342 36	0.45	126 52	0.006	134		
5— 1	0.7	309	0.08	214				
6— 1	0.1	294						
—2— 2	0.1	158						
—1— 2	1.8	241 2	0.09	341				
0— 2	3.7	210 18	0.11	316				
1— 2	55.2	98 52	0.26	257 18	0.002	189		
2— 2	643.5	156 34.5	0.32	14 5	0.003	0		
3— 2	420.9	141 57.8	11.31	46 59	0.051	339 5		
4— 2	7001.9	277 15 19	170.48	179 38.4	2.252	85 53		
5— 2 —88″.928t	1141.0	62 49 32	4.36	242 49	0.077	6 8		
6— 2	18.3	77 17	19.45	306 10	0.055	209 17		
7— 2	0.6	114	1.06	306 55	0.006	185		
8— 2			0.06	307				
—1— 3	0.1	224	0.01	303				
0— 3	0.8	319	0.04	58				
1— 3	1.0	46	0.04	61				
2— 3	5.3	178 39	0.05	342				
3— 3	147.1	233 56.0	0.04	32	0.001	0		
4— 3	102.0	206 23.8	1.36	107 2	0.010	11 30		
5— 3	59.7	177 52	1.80	78 59	0.023	343 18		
6— 3	17.3	178 3	2.86	51 41	0.048	314 0		
7— 3	34.6	32 39	2.39	340 42	0.004	254		
8— 3	4.9	210 27	0.39	153 48	0.005	61		
9— 3	0.7	275	0.02	139				
0— 4	0.1	298						
1— 4	0.4	43	0.02	134				
2— 4	0.5	17	0.02	115				
3— 4	2.8	229 44	0.01	8				
4— 4	44.5	311 30	0.01	122				
5— 4	31.5	285 3	0.42	184 23	0.003	98		
6— 4	14.9	259 21	0.35	157 31	0.004	67		
7— 4	8.1	37 4	0.52	302 47	0.003	37		
8— 4	21.5	15 52	0.91	284 18	0.005	204		
9— 4	93.1	163 39	8.55	67 13	0.116	331 16		
10— 4	11.0	306 25	1.81	215 3	0.033	118 28		
11— 4	0.2	102	0.02	17				
2— 5	0.2	113	0.01	214				
3— 5	0.2	106	0.01	199				
4— 5	1.5	296 38	0.01	0				

χ	Common log $\frac{r'}{\bar{r}'}$			
	k_0	K_0	k_1	K_1
g' g		° ′		°
5— 5	15.6	28 45	0.01	184
6— 5	11.9	3 8	0.16	263
7— 5	5.5	337 30	0.14	240
8— 5	2.7	116 7	0.17	24
9— 5	3.7	118 23	0.21	354
10 — 5	2.7	63 43	0.18	325
11— 5	3.6	36 26	0.26	300
12— 5	0.7	263	0.07	113
3— 6	0.1	191		
4— 6	0.1	191		
5— 6	0.8	8	0.01	69
6— 6	5.9	105 37		
7— 6	5.0	80 27	0.07	341
8— 6	2.4	56 33	0.06	317
9— 6	1.0	191 57	0.06	96
10— 6	1.3	171 26	0.06	74
11— 6	0.7	144	0.03	47
12— 6	0.1	120	0.02	15
5— 7	0.1	279		
6— 7	0.4	83		
7— 7	2.4	182 5		
8— 7	2.2	158 51	0.03	59
9— 7	1.1	135 7	0.03	35
10— 7	0.4	265	0.02	171
11— 7	0.6	247	0.03	150
12— 7	0.3	222	0.02	122
7— 8	0.2	158		
8— 8	1.0	258		
9— 8	1.0	236	0.02	140
10— 8	0.6	214	0.02	113
11— 8	0.1	334	0.01	252
12— 8	0.2	325	0.01	228
8— 9	0.1	231		
9— 9	0.4	333		
10— 9	0.5	313	0.01	216
11— 9	0.3	293	0.01	191
9—10	0.1	304		
10—10	0.2	48		
11—10	0.2	29		
12—10	0.2	8		
11—11	0.1	122		
12—11	0.1	105		

χ	Common log $\frac{r}{\bar{r}}$			
	k_0	K_0	k_1	K_1
g'' g'		° ″		° ′
1+ 1	0.3	356	0.01	188
1 0	3.2	345 3	0.08	140
1— 1	59.0	79 3	0.01	277
1— 2	2.5	95 57	0.06	202
1— 3	0.1	95		
2 0	1.0	328 4	0.06	59
2— 1	35.5	350 35	0.28	217 25
2— 2	154.1	336 43.3	0.05	106
2— 3	5.3	332 13	0.10	98
2— 4	0.2	332	0.01	90
3 0	0.6	126	0.18	20 33
3— 1	26.4	137 55	0.16	355 1
3— 2	237.4	119 5.6	1.43	308 4
3— 3	22.1	69 58	0.17	252 12
3— 4	1.1	57 13	0.02	211
3— 5	0.1	52		
4— 1	0.4	104		
4— 2	6.7	80 4	0.09	266
4— 3	9.7	19 51	0.07	202
4— 4	4.4	128 12	0.01	158
4— 5	0.3	115		
5— 2	1.1	38 31	0.02	225
5— 3	5.2	342 27	0.07	166
5— 4	2.2	93 59	0.01	270
5— 5	1.3	206 15		
5— 6	0.1	190		
6— 2	0.2	172		
6— 3	2.4	123 27	0.05	307
6— 4	0.8	65 50	0.02	256
6— 5	0.7	173 49		
6— 6	0.4	283		
6— 7	0.1	266		
7— 3	0.1	84		
7— 4	0.3	26	0.01	327
7— 5	0.3	139		
7— 6	0.3	252		
7— 7	0.2	359		
8— 4	0.1	348		
8— 5	0.1	104		
8— 6	0.1	218		
8— 7	0.1	329		
8— 8	0.1	75		

χ	Common log $\frac{r'}{\bar{r}'}$				χ	Common log $\frac{r'}{\bar{r}'}$			
	k_0	K_0	k_1	K_1		k_0	K_0	k_1	K_1
g''' g'		° ′		° ′	g''' g'		° ′		° ′
1 0	0.2	337			4— 2	0.1	307		
1— 1	15.4	312 58			4— 3	0.1	148		
1— 2	0.5	310			4— 4	0.3	353		
2— 0	0.2	86							
2— 1	7.6	84 55			5— 5	0.1	307		
2— 2	14.8	86 17			g' g g''				
2— 3	0.6	87			5— 2— 3	19.8	208 34	0.31	80 14
3— 1	0.2	173			6— 2— 3	8.4	2 4		
3— 2	1.3	195 39							
3— 3	1.5	40 12			♀—♄	0.8	0		
3— 4	0.1	42			♅—♄	1.4	0		

Periodic inequalities of the latitude of Saturn.

We take the terms of $\Delta\beta'$ given at pages 531, 532, and multiply the co-efficients which involve g'' in their arguments by $\frac{21000}{22640}$, and afterwards add the first portion of $\Delta(\Delta\beta')$ given on page 536. To these can be joined the very small terms due to the action of Neptune (page 179). The whole is then changed to the form adopted in the chapter.

χ	$\Delta\beta'$				χ	$\Delta\beta'$			
	k_0	K_0	k_1	K_1		k_0	K_0	k_1	K_1
g' g	″	° ′	″	° ′	g' g	″	° ′	″	° ′
0 0	—0.329		—0.0109		—2— 2	0.001	279		
2 0	0.204	287 13	0.0019	231	—1— 2	0.002	81	0.0002	207
3 0	0.019	269	0.0003	162	0— 2	0.063	91 47	0.0004	237
4 0	0.005	51			1— 2	0.258	11 58	0.0029	299 18
5 0	0.002	331			2— 2	0.116	319 33	0.0008	90
—3— 1	0.003	209			3— 2	0.215	207 35	0.0054	197 9
—2— 1	0.002	41			4— 2	8.679	277 12.5	0.0155	66 57
—1— 1	0.026	37	0.0020	311	5— 2	0.370	111 9	0.0056	329 47
0— 1	1.803	116 2	0.0245	32 22	6— 2	0.245	16 42	0.0075	269 18
1— 1	0.841	210 41	0.0138	163 10	7— 2	0.011	19	0.0009	249
2— 1	2.905	225 28.5	0.0482	310 59	—1— 3	0.001	352		
3— 1	0.721	185 4	0.0018	276	0— 3	0.003	114		
4— 1	0.057	301 28	0.0002	117	1— 3	0.007	84		
5— 1	0.037	310 15	0.0002	27	2— 3	0.087	89 53		
6— 1	0.001	340			3— 3	0.041	53 11		

χ	$\Delta\beta'$			
	k_0	K_0	k_1	K_1
g' g	$''$	° ′	$''$	° ′
4— 3	0.077	199 40		
5— 3	0.117	176 9		
6— 3	0.096	155 49		
7— 3	0.048	300 27		
8— 3	0.002	247		
9— 3	0.001	225		
2— 4	0.003	139		
3— 4	0.033	167 31		
4— 4	0.018	134 26		
5— 4	0.014	266 3		
6— 4	0.013	246 33		
7— 4	0.011	230		
8— 4	0.002	171		
9— 4	0.087	161 51	0.0012	250
10— 4	0.009	341		
11— 4	0.002	273		
3— 5	0.001	189		
4— 5	0.013	245		
5— 5	0.009	214		
6— 5	0.004	349		
7— 5	0.003	317		
8— 5	0.002	303		
9— 5	0.003	272		
10— 5	0.002	247		
11— 5	0.002	219		
4— 6	0.001	237		
5— 6	0.005	323		
6— 6	0.004	292		
7— 6	0.001	63		
8— 6	0.001	21		
9— 6	0.001	8		
10— 6	0.001	351		
6— 7	0.002	38		
7— 7	0.002	9		
g'' g'				
1+ 1	0.019	259 21		
1 0	0.080	220 17		
1— 1	0.036	11 34		

χ	$\Delta\beta'$			
	k_0	K_0	k_1	K_1
g'' g'	$''$	° ′	$''$	° ′
1— 2	0.035	298 43		
1— 3	0.002	306		
2+ 1	0.003	164		
2 0	0.040	152 57		
2— 1	0.110	301 20		
2— 2	0.031	277 36		
2— 3	0.008	2		
3+ 1	0.002	294		
3 0	0.032	289 10		
3— 1	0.046	221 32		
3— 2	0.599	20 3		
3— 3	0.037	17 4		
3— 4	0.003	64		
4— 1	0.005	208		
4— 2	0.025	349 4		
4— 3	0.023	281 40		
4— 4	0.001	331		
5— 1	0.001	165		
5— 2	0.003	333		
5— 3	0.021	244 54		
5— 4	0.005	341		
6— 2	0.001	232		
6— 3	0.012	32		
6— 4	0.003	333		
6— 5	0.001	69		
7— 3	0.001	0		
7— 4	0.001	288		
7— 5	0.001	45		
g''' g'				
1+ 1	0.002	137		
1 0	0.005	146		
1— 1	0.001	4		
1— 2	0.002	120		
2 0	0.003	276		
2— 1	0.018	98 27		
2— 2	0.001	111		
3— 2	0.004	232		

We derive f' from $n'z' = g' + n'\delta z'$ by the formula

$$= n'z' + 23117''.434 \sin n'z' + 809''.366 \sin 2n'z' + 39''.292 \sin 3n'z'$$
$$+ 2.180 \sin 4n'z' + 0.130 \sin 5n'z' + 0.008 \sin 6n'z'$$

Then

$$\text{com. log } r' = 0.97812861 - \text{com. log } (1 + e' \cos f')$$

In addition, $l' = f' + \pi'$, and we derive R′ from

$$R' = +97''.774 \sin (2l' + 315^\circ\ 18'\ 22'') + 0''.023 \sin (4l' + 270^\circ\ 37')$$

The heliocentric longitude of Saturn, referred to the mean equinox of date, is

$$\lambda' = f' + R' + \pi' + 50''.258141t$$

In order to get the equation which determines $\sin \beta_0'$, we note that the correction of $\frac{u'}{\cos i'}$ (page 555), on account of changes in the values of the disturbing masses, requires that the A and B of page 531 should receive severally the corrections $-0''.0507T$ and $-0''.0662T$, so that the equation for $\cos i' \sin b'$ now becomes

$$\begin{aligned} \cos i' \sin b' = & [33\overset{''}{.}3758T + 0\overset{''}{.}16226T^2 - 0\overset{''}{.}000858T^3] \sin l' \\ & + [- 23.9117T + 0.21432T^2 + 0.000641T^3] \cos l' \end{aligned}$$

If we add to this the expression for the portion of $\sin \beta_0'$, which arises from the motion of the ecliptic (page 531), and also take account of the correction $-\frac{d(\sin \beta_0')}{dz'} \Delta(\delta z')$, given at page 536, we obtain

$$\begin{aligned} \sin \beta_0' = & [\sin i_0' \cos \theta_0' + 80\overset{'}{.}1664T + 0\overset{''}{.}06123T^2 - 0.001471T^3] \sin l' \\ & + [- \sin i_0' \sin \theta_0' - 18.6681T + 0.42811T^2 - 0.000027T^3] \cos l' \\ & + [- 0\overset{''}{.}0208T - 0\overset{''}{.}00035T^2] \sin 3l' \\ & + [0.0330T - 0.00023T^2] \cos 3l' \end{aligned}$$

ADDENDA.

Page 301. Insert in the table beginning in the middle of this page the following line:

1 0 0	−0.018886	+0.033370	+0.000304	−0.000132	+0.000034	−0.00066	+0.000147	−0.000095

Page 377. Insert in the table on this page the following line:

−1 0 0	+0.0000130	−0.0000937	−0.0001570	−0.0004719	−0.0008621	−0.0011553	+0.0000327	−0.0000799

Page 379. Insert in the table on this page the following line:

−1 0 0	+0.0000605	−0.0001759	+0.0001224	−0.0002322	+0.0002096	−0.0004344	−0.0000007	−0.0000032

Page 380. Insert in the table occupying the lower half of the page the following line:

−1 0 0	+0.0000403	−0.0000593	+0.0002053	−0.0000693

Page 384. In the columns headed $\frac{1}{2}\frac{dF}{dg'}(n'\delta z')^2$ add the terms corresponding to the argument −1 9− 4:

+3.5	0.0

Page 416. Insert in the table occupying the lower half of the page the following line:

1 0 0	−0.000544	+0.000973	−0.000254	+0.000341

PLEASE DO NOT REMOVE
CARDS OR SLIPS FROM THIS POCKET

UNIVERSITY OF TORONTO LIBRARY

QB
3
H55
v.3

Hill, George William
Collected mathematical
works

Physical &
Applied Sci.